H. NEUBER

LÖSUNGEN ZUR AUFGABENSAMMLUNG MESTSCHERSKI

VEB DEUTSCHER VERLAG DER WISSENSCHAFTEN

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HOCHSCHULBÜCHER FÜR PHYSIK

HERAUSGEGEBEN VON OTTO LUCKE UND ROBERT ROMPE

BAND 19

LÖSUNGEN ZUR AUFGABENSAMMLUNG MESTSCHERSKI

VON H. NEUBER

ordentlicher Professor für techn. Mechanik

an der Technischen Hochschule München

5., unveränderte Auflage

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Statik starrer Körper

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Vorwort

Die vorliegenden Lösungen für die Aufgabensammlung zur Mechanik von I. W. MESTSCHERSKI (Hochschulbücher für Physik, Band 13) verfolgen den Zweck, den Studierenden anzuleiten und ihm auch ohne Zuhilfenahme eines speziellen Lehrbuches die Lösung komplizierterer Aufgaben zu ermöglichen. Darüber hinaus eignet sich diese Zusammenstellung für den Techniker und den Physiker sowohl als Nachschlagewerk als auch zum Selbststudium und Vertiefen seiner Kenntnisse.

Die Herausgeber

Erster Teil

Statik starrer Körper

I. Ebenes Kräftesystem

1. Geradlinig wirkende Kräfte

Lösung 1

1. Die Kräfte werden algebraisch addiert:

$$P_1 + P_2 + P_3 + P_4 = 10 + 20 + 12 + 18 = 60\,\text{kg}.$$

2. In der einen Richtung wirkt $P_1 + P_2 = 10 + 20 = 30\,\text{kg}$, in der entgegengesetzten Richtung $P_3 + P_4 = 12 + 18 = 30\,\text{kg}$.
Die resultierende Kraft hat die Größe:

$$(P_1 + P_2) - (P_3 + P_4) = 0\,\text{kg}.$$

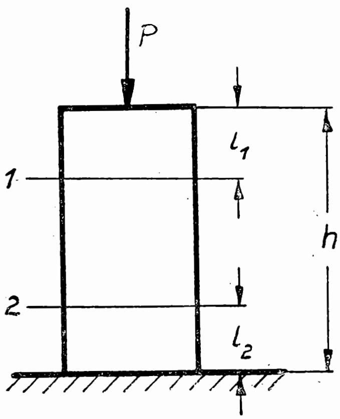
Lösung 2

Die Reaktion muß gleich der Aktion sein

1\. 10 kg; 2. 10 kg.

Lösung 3 1. $F_1 = G_1 = 10\,\text{kg}$; 2. $F_2 = G_1 + G_2 = 15\,\text{kg}$.

Lösung 4



Fundamentdruck $= P + Q = 7\,\text{t}$,

Kraft im Schnitt $1 = P + \frac{Q \cdot l_1}{h} = 4{,}3\,\text{t}$,

Kraft im Schnitt $2 = P + \frac{Q\,(h - l_2)}{h} = 6{,}7\,\text{t}$.

Lösung 5

Der erste Kahn muß mit $\frac{1800-600}{200}=6$ Seilen befestigt werden;

der zweite Kahn muß mit $\frac{400+200}{200}=3$ Seilen befestigt werden;

der dritte Kahn muß mit $\frac{200}{200}=1$ Seil befestigt werden.

Lösung 6

1. $P=30\,\mathrm{kg}$; $F_A=30\,\mathrm{kg}$; $F_B=32{,}5\,\mathrm{kg}$; $F_C=30\,\mathrm{kg}$;
2. $P=25\,\mathrm{kg}$; $F_A=30\,\mathrm{kg}$; $F_B=27{,}5\,\mathrm{kg}$; $F_C=25\,\mathrm{kg}$;
3. $P=35\,\mathrm{kg}$; $F_A=30\,\mathrm{kg}$; $F_B=32{,}5\,\mathrm{kg}$; $F_C=35\,\mathrm{kg}$.

Lösung 7

1. Der Druck des Mannes auf die Schachtsohle beträgt $64\,\mathrm{kg}-48\,\mathrm{kg}=16\,\mathrm{kg}$.
2. Der Mann kann höchstens 64 kg halten.

Lösung 8

$$\text{Zugkraft der Lokomotive} = 180\cdot 0{,}005 = 0{,}9\,\mathrm{t} \mathrel{\hat{=}} 900\,\mathrm{kg}.$$

Lösung 9

Die Lokomotivenkupplung hat zu übertragen (Zugkraft der Lokomotive):

$$(5\cdot 48+20+45)\cdot\frac{1}{200}=1{,}525\,\mathrm{t}\mathrel{\hat{=}}1525\,\mathrm{kg}.$$

Die Kupplung des letzten Wagens hat zu übertragen:

$$48\cdot 1000\cdot\frac{1}{200}=240\,\mathrm{kg}.$$

Die Kupplung des vorletzten Wagens hat zu übertragen:

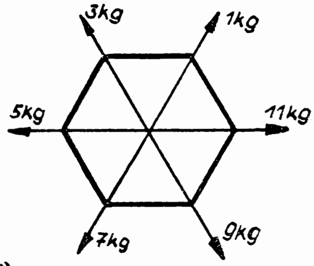
$$2\cdot 240=480\,\mathrm{kg}\ \text{usw.}$$

Lösung 10

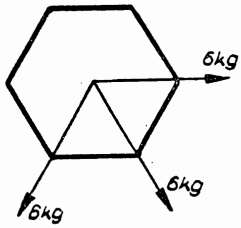
$$\begin{aligned}P&=\frac{\pi}{4}\left[p_1\left(D_1^2-d_1^2\right)-p_2\left(D_1^2-d_2^2\right)+p_2\left(D_2^2-d_2^2\right)-p_3\left(D_2^2-d_3^2\right)\right]\\&=\frac{\pi}{4}\left[p_1\left(D_1^2-d_1^2\right)+p_2\left(D_2^2-D_1^2\right)-p_3\left(D_2^2-d_3^2\right)\right]\\&=\frac{\pi}{4}\left[9{,}5\,(32^2-6^2)+2{,}5\,(60^2-32^2)-0{,}1\,(60^2-10^2)\right]\\&=12\,100\,\mathrm{kg}\mathrel{\hat{=}}12{,}1\,\mathrm{t}.\end{aligned}$$

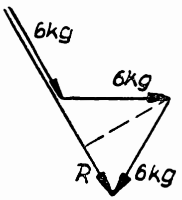
2. Kräfte, deren Wirkungslinien sich in einem Punkt schneiden

Lösung 11



Die auf gleicher Wirkungslinie liegenden Kräfte werden addiert:



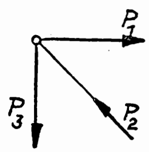


Aus der Symmetrie der Kräfte folgt nun:

$$R = 6 + 2 \cdot 6 \cdot \sin 30^\circ$$

$$\underline{\underline{R = 12\,\text{kg}}}$$

Lösung 12



Die auf der Wirkungslinie von P_2 liegende Resultierende hat die Größe:

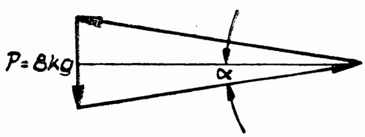
$$R = P_2 - P_1 \cdot \frac{\sqrt{2}}{2} - P_3 \frac{\sqrt{2}}{2}$$

$$P_1 = P_3 = 141\ \text{kg}$$

$$= 100 - 141\sqrt{2} = \underline{\underline{-100\,\text{kg}}}$$

R ist also entgegen P_2 gerichtet.

Lösung 13

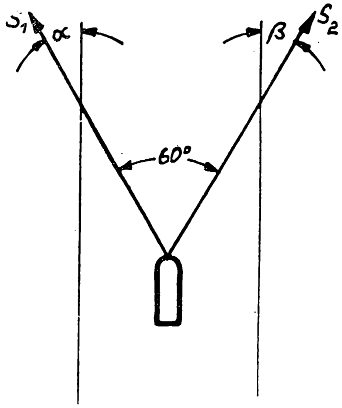


Die Größe der Teilkräfte ist nur abhängig von ihrem Richtungswinkel α

Lösung 14

$$S = N = \frac{Q\sqrt{2}}{2} = 177\ \text{kg}$$

Lösung 15



$$S_1 \sin\alpha = S_2 \sin\beta; \quad \alpha + \beta = 60°$$

$$\frac{S_1}{S_2} = \frac{\sin 60° \cos\alpha}{\sin\alpha} - \frac{\cos 60° \sin\alpha}{\sin\alpha}$$

$$\operatorname{ctg}\alpha = \left(\frac{S_1}{S_2} + \cos 60°\right) \cdot \frac{1}{\sin 60°}$$

$$\operatorname{ctg}\alpha = \frac{1{,}334}{0{,}866} = 1{,}54; \quad \underline{\underline{\alpha = 33°}}$$

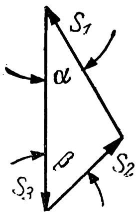
$$\underline{\underline{\beta = 27°}}$$

Wasserwiderstand: $P = S_1 \cos\alpha + S_2 \cos\beta$

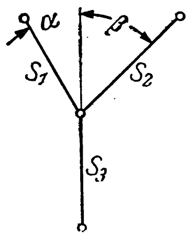
$= 80 \cdot 0{,}838 + 96 \cdot 0{,}891$

$\underline{\underline{P = 153\ \text{kg}}}$

Lösung 16



Lageplan



Kraftplan

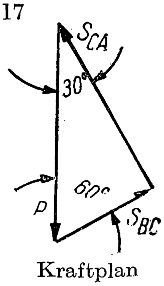
Kosinussatz: $S_1^2 = S_2^2 + S_3^2 - 2 S_2 S_3 \cos\beta; \quad \cos\beta = \dfrac{S_3^2 + S_2^2 - S_1^2}{2 S_2 S_3}$

$S_1 = 8$ kg
$S_2 = 7$ kg $\quad \cos\beta = \dfrac{154}{2 \cdot 91} = 0{,}846; \quad \underline{\underline{\beta = 32{,}2°}}$
$S_3 = 13$ kg

$$\frac{S_1}{S_2} = \frac{\sin\beta}{\sin\alpha}; \quad \sin\alpha = \frac{7}{8}\,0{,}532 = 0{,}465$$

$$\underline{\underline{\alpha = 27{,}8°}}$$

Lösung 17

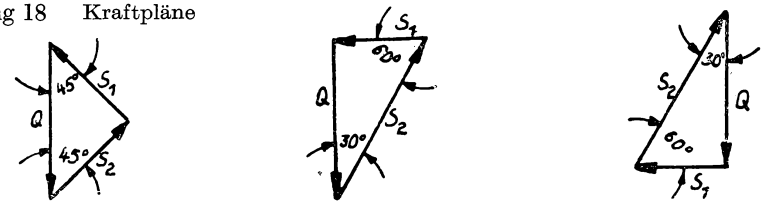


Kraftplan

$$S_{BC} = P \cdot \sin 30° = \frac{P}{2} = 500\ \text{kg}$$

$$S_{CA} = P \cos 30° = P\,\frac{\sqrt{3}}{2} = 866\ \text{kg}$$

Lösung 18 Kraftpläne

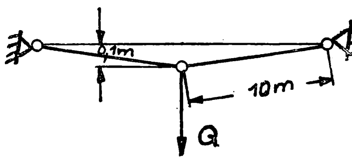


$S_1 = S_2 = \frac{Q\sqrt{2}}{2} = 707$ kg; $S_2 = -\frac{Q}{\cos 30^\circ} = -1154$ kg; $S_1 = -Q \operatorname{tg} 30^\circ = -577$ kg

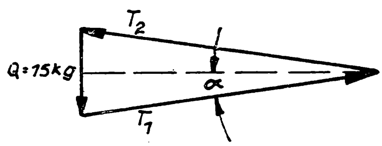
$S_1 = Q \operatorname{tg} 30^\circ = 577$ kg; $S_2 = \frac{Q}{\cos 30^\circ} = 1154$ kg

Das negative Vorzeichen entsteht, wenn der Kraftpfeil auf das betrachtete Gelenk zeigt.

Lösung 19



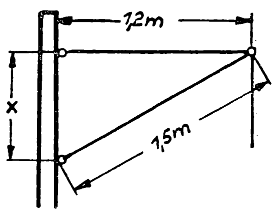
Lageplan



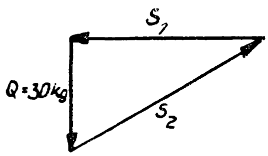
Kraftplan

$$T = T_1 = T_2 = \frac{Q}{2 \sin \alpha} = \frac{10 \cdot 15}{2 \cdot 0{,}1} = 750 \text{ kg}$$

Lösung 20



Lageplan



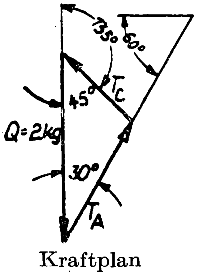
Kraftplan

$$x = \sqrt{1{,}5^2 - 1{,}2^2} = 0{,}9 \text{ m}$$

$$\frac{Q}{0{,}9} = \frac{S_1}{1{,}2}; \quad S_1 = Q \cdot \frac{1{,}2}{0{,}9} = \underline{\underline{40 \text{ kg}}}$$

$$\frac{Q}{0{,}9} = \frac{S_2}{1{,}5}; \quad S_2 = -\underline{\underline{50 \text{ kg}}}$$

Lösung 21



Kraftplan

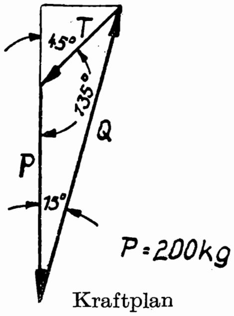
$$Q = T_A \cdot \cos 30^\circ + T_C \cdot \cos 45^\circ$$

$$T_C \cdot \sin 45^\circ = T_A \cdot \sin 30^\circ; \quad T_A = T_C \cdot \sqrt{2}$$

$$Q = T_C \left(\frac{\sqrt{2}}{2} \sqrt{3} + \frac{\sqrt{2}}{2} \right)$$

$$T_C = \frac{\sqrt{2} \cdot Q}{(\sqrt{3} + 1)} = \underline{\underline{1{,}04 \text{ kg}}}; \quad T_A = \frac{2\,Q}{(\sqrt{3} + 1)} = \underline{\underline{1{,}46 \text{ kg}}}$$

Lösung 22



Kraftplan

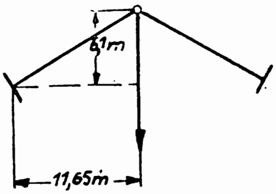
$$P + \frac{T\sqrt{2}}{2} = Q \cdot \cos 15^\circ$$

$$T\frac{\sqrt{2}}{2} = Q \cdot \sin 15^\circ; \quad P + \frac{T\sqrt{2}}{2} = \frac{T\sqrt{2}}{2\,\mathrm{tg}\,15^\circ}$$

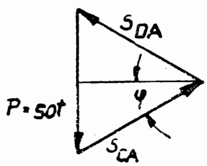
$$T = \frac{P\sqrt{2}}{\mathrm{ctg}\,15^\circ - 1} = \frac{283}{3{,}73 - 1} = \underline{\underline{104\ \mathrm{kg}}}$$

$$Q = \frac{P}{(\mathrm{ctg}\,15^\circ - 1) \cdot \sin 15^\circ} = \frac{200}{0{,}707} = \underline{\underline{283\ \mathrm{kg}}}$$

Lösung 23



Lageplan



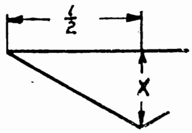
Kraftplan

$$\mathrm{tg}\,\varphi = \frac{6{,}1}{11{,}65}; \quad \varphi = 27^\circ\,40'$$

$$S_{DA} = S_{CA} = \frac{P}{2 \sin 27^\circ\,40'}$$

$$= \underline{\underline{53{,}9\ \mathrm{t}}}$$

Lösung 24



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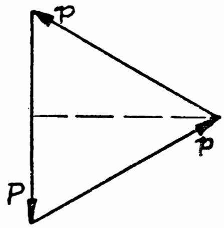


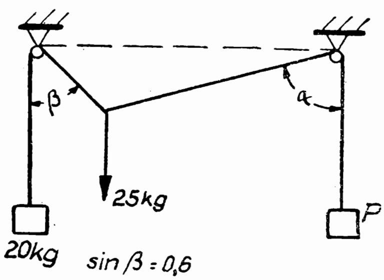
Kraftplan

Ähnlichkeit der beiden Dreiecke:

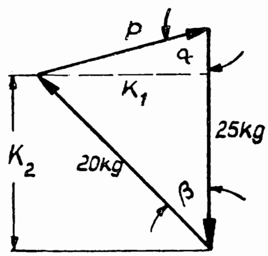
$$\frac{x}{\frac{P}{2}} = \frac{\frac{l}{2}}{\sqrt{p^2 - \frac{P^2}{4}}}$$

$$x = \underline{\underline{\frac{P \cdot l}{2\sqrt{4p^2 - P^2}}}}$$

Lösung 25



Lageplan

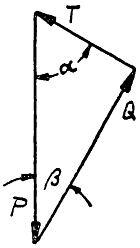


Kraftplan

$$K_1 = 20 \cdot \sin\beta = 12\ \mathrm{kg}; \quad K_2 = \sqrt{20^2 - 12^2} = 16\ \mathrm{kg}$$

$$\mathrm{tg}\,\alpha = \frac{12}{25 - 16} = 1{,}336; \quad \alpha = \underline{\underline{53^\circ\,10'}}; \quad p = \frac{K_1}{\sin\alpha} = \frac{12}{0{,}8} = \underline{\underline{15\ \mathrm{kg}}}$$

Lösung 26



Kraftplan

$$Q \cdot \sin\beta = T \sin\alpha; \quad T = Q\frac{\sin\beta}{\sin\alpha} = \underline{\underline{12{,}2\ \mathrm{kg}}}$$

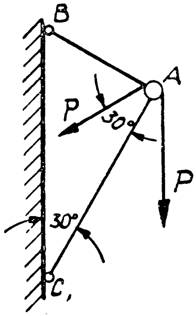
$$P = T\cos\alpha + Q\cos\beta = 13{,}7\ \mathrm{kg}$$

Lösung 27

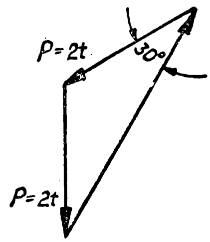
Die Resultierende der beiden Seilkräfte P liegt in Richtung des Stabes CA. Demnach ist die Stabkraft in BA:

$$\underline{\underline{Q_1 = 0}}$$

$$Q_2 = -2P \cdot \cos 30° = \underline{\underline{-3{,}46\,\mathrm{t}}}$$

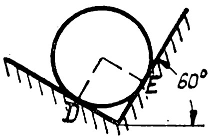


Lageplan



Kraftplan

Lösung 28



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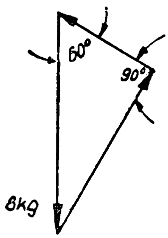
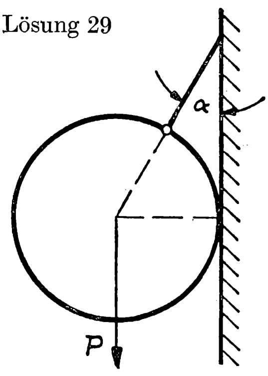


Kraftplan

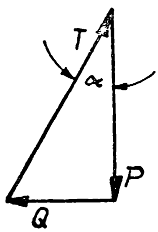
$$N_E = 6 \cdot \cos 60° = 3\ \mathrm{kg}$$

$$N_D = 6 \cdot \sin 60° = 5{,}2\ \mathrm{kg}$$

Lösung 29



Lageplan

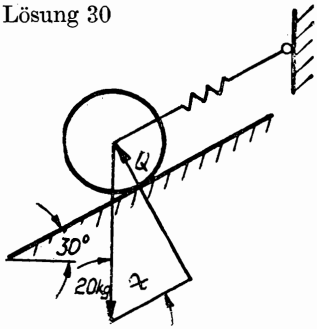


Kraftplan

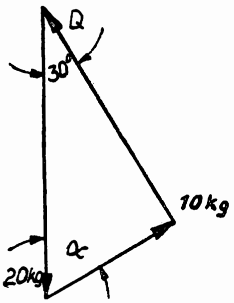
$$Q = P \cdot \mathrm{tg}\,\alpha$$

$$T = \frac{P}{\cos\alpha}$$

Lösung 30



Lageplan



Kraftplan

Kosinussatz:

$$10^2 = Q^2 + 20^2 - 2\,Q \cdot 20 \cdot \cos 30^\circ$$

$$Q^2 - 34{,}6\,Q = -300$$

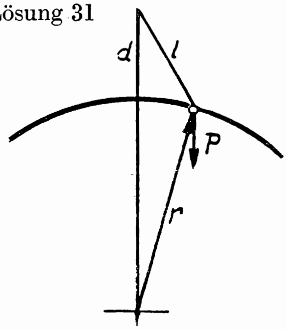
$$Q = 17{,}3 \pm \sqrt{300 - 300}$$

$$Q = \underline{\underline{17{,}3\ \text{kg}}}$$

$$\sin\alpha = \frac{Q \cdot \sin 30^\circ}{10} = 0{,}866$$

$$\underline{\underline{\alpha = 60^\circ}}$$

Lösung 31



Lageplan

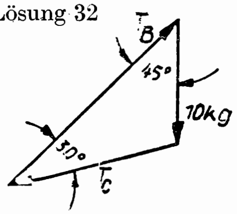
Kraftplan

Ähnlichkeit: $\dfrac{P}{Q} = \dfrac{d+r}{r}$

$$\underline{\underline{Q = P \cdot \frac{r}{d+r}}}$$

$$\frac{T}{P} = \frac{l}{d+r}; \quad \underline{\underline{T = P \cdot \frac{l}{d+r}}}$$

Lösung 32



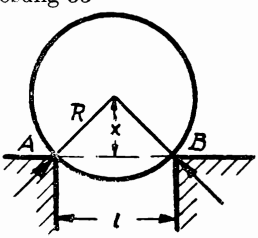
Kraftplan

$$T_B = 10 \cdot \cos 45^\circ + T_C \cdot \cos 30^\circ$$

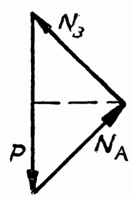
$$T_C = 10 \cdot \frac{\sin 45^\circ}{\sin 30^\circ} = \underline{\underline{14{,}1\ \text{kg}}}$$

$$T_B = 7{,}07 + 12{,}25 = \underline{\underline{19{,}3\ \text{kg}}}$$

Lösung 33



Lageplan



Kraftplan

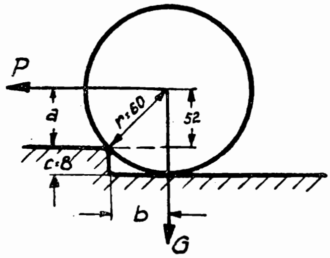
$$x = \sqrt{R^2 - \left(\frac{l}{2}\right)^2}$$

$$\frac{x}{R} = \frac{P}{2N}; \quad N = \frac{P}{2}\,\frac{R}{\sqrt{R^2 - \left(\frac{l}{2}\right)^2}}$$

$$N = 2 \cdot \frac{1}{\sqrt{1 - 0{,}64}}$$

$$N_A = N_B = N = \underline{\underline{3{,}33\ \text{t}}}$$

Lösung 34

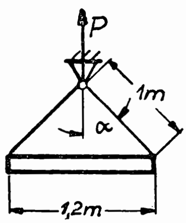


$$P \cdot a = G \cdot b$$

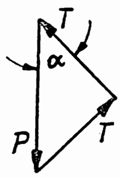
$$P = \frac{G}{a}\sqrt{r^2 - (r - c)^2}$$

$$P = \frac{2 \cdot 30}{52} = \underline{\underline{1{,}15\ \mathrm{t}}}$$

Lösung 35



Lageplan

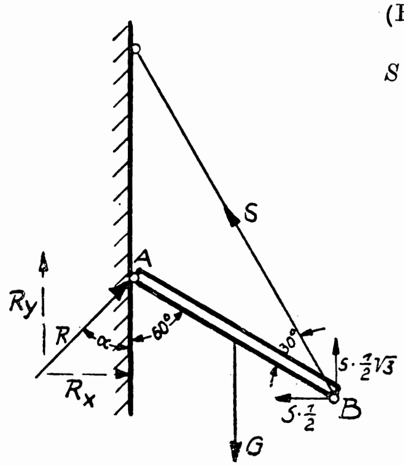


Kraftplan

$$\sin\alpha = \frac{0{,}6}{1}$$

$$T = \frac{P}{2\,\mathrm{ccs}\,\alpha} = \frac{16}{2 \cdot 0{,}8} = \underline{\underline{10\ \mathrm{kg}}}$$

Lösung 36



Momentengleichung um A:
(Brettlänge $= l$)

$$S \cdot \frac{1}{2}\sqrt{3} \cdot l \cdot \frac{1}{2}\sqrt{3} - S \cdot \frac{1}{2} \cdot \frac{1}{2}\, l - G \cdot \frac{l}{2}\,\frac{\sqrt{3}}{2} = 0$$

$$S = \frac{G\sqrt{3}}{2}$$

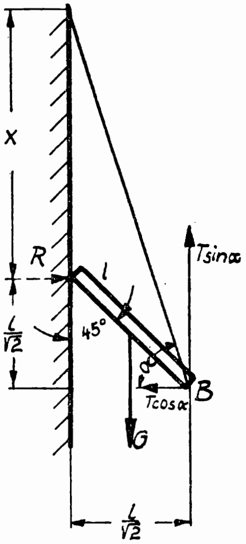
$$R_x = \frac{S}{2} = \frac{G\sqrt{3}}{4}; \qquad R_y + \frac{S}{2}\sqrt{3} - G = 0$$

$$R_y = G\left(1 - \frac{3}{4}\right) = \frac{G}{4}$$

$$R = G\sqrt{\frac{3}{16} + \frac{1}{16}} = \frac{G}{2} = \underline{\underline{1\ \mathrm{kg}}}$$

$$\mathrm{tg}\,\alpha = \frac{Rx}{Ry} = \sqrt{3}; \quad \underline{\underline{\alpha = 60^\circ}}$$

Lösung 37



Momentengleichung um B:

$$\frac{R \cdot l}{\sqrt{2}} - \frac{G}{2} \cdot \frac{l}{\sqrt{2}} = 0; \quad R = \frac{G}{2} = \underline{\underline{2{,}5\ \mathrm{kg}}}$$

$$T \cdot \cos\alpha = R = \frac{G}{2}; \quad T \cdot \sin\alpha = G$$

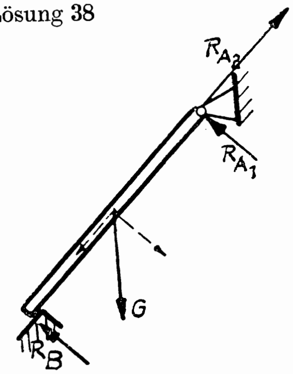
$$\operatorname{tg}\alpha = \frac{G \cdot 2}{G} = 2;$$

$$\operatorname{tg}\alpha = \frac{x + \frac{l}{\sqrt{2}}}{\frac{l}{\sqrt{2}}} = \frac{x\sqrt{2}}{l} + 1; \quad l = 2\ \mathrm{m}$$

$$x = \frac{l}{\sqrt{2}} = \underline{\underline{1{,}41\ \mathrm{m}}}$$

$$T = \sqrt{\frac{G^2}{4} + G^2} = \frac{G}{2}\sqrt{5} = \underline{\underline{5{,}6\ \mathrm{kg}}}$$

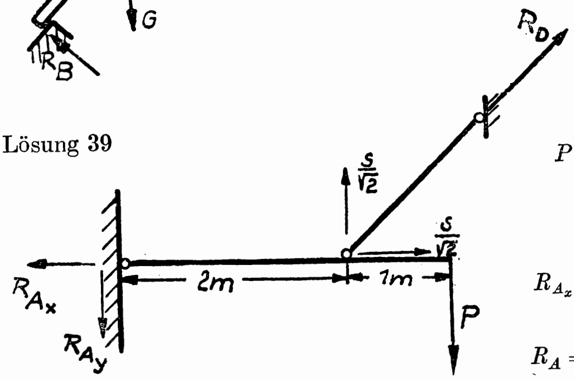
Lösung 38



$$R_{A1} = R_B = \frac{G}{2\sqrt{2}} = \underline{\underline{31{,}5\ \mathrm{kg}}}$$

$$R_{A2} = \frac{G}{\sqrt{2}}; \quad R_A = G\sqrt{\frac{1}{8} + \frac{1}{2}} = \underline{\underline{70{,}4\ \mathrm{kg}}}$$

Lösung 39



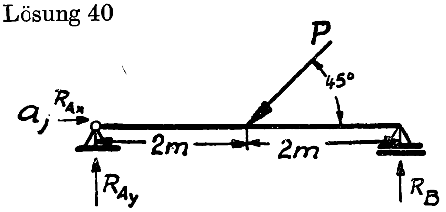
$$P \cdot 3 = \frac{S}{\sqrt{2}} \cdot 2; \quad S = R_D = \frac{3}{\sqrt{2}} \cdot P$$

$$R_D = \underline{\underline{10{,}6\ \mathrm{t}}}$$

$$R_{A_x} = \frac{R_D}{\sqrt{2}} = \frac{3}{2} \cdot P; \quad R_{A_y} = \frac{P}{2}$$

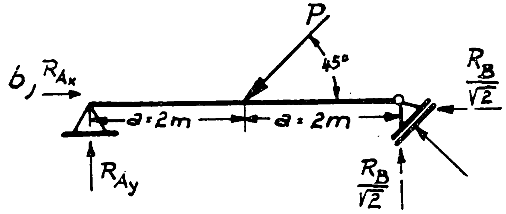
$$R_A = \sqrt{R_{A_x}^2 + R_{A_y}^2} = \frac{P}{2}\sqrt{10} = \underline{\underline{7{,}9\ \mathrm{t}}}$$

Lösung 40



$$R_B = \frac{P}{2\sqrt{2}} = \underline{\underline{0{,}71\,\mathrm{t}}}$$

$$R_A = P\sqrt{\frac{1}{2}+\frac{1}{4\cdot 2}} = \underline{\underline{1{,}58\,\mathrm{t}}}$$



$$\frac{R_B}{\sqrt{2}}\cdot 2a = \frac{P}{\sqrt{2}}\cdot a;\quad R_B = \frac{P}{2} = \underline{\underline{1\,\mathrm{t}}}$$

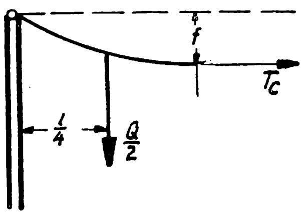
$$R_A = P\sqrt{\left(\frac{1}{\sqrt{2}}+\frac{1}{2\sqrt{2}}\right)^2+\frac{1}{4\cdot 2}} = \frac{P}{2}\sqrt{5} = \underline{\underline{2{,}24\,\mathrm{t}}}$$

Lösung 41

$$S = \frac{3}{2}\cdot F\cdot 0{,}866 = 3{,}9\,\mathrm{t}\qquad \text{a) Zug;}\quad \text{b) Druck}$$

$$Q = F\sqrt{\frac{3}{16}+\frac{1}{4}} = \frac{F}{4}\sqrt{7} = \underline{\underline{1{,}98\,\mathrm{t}}}$$

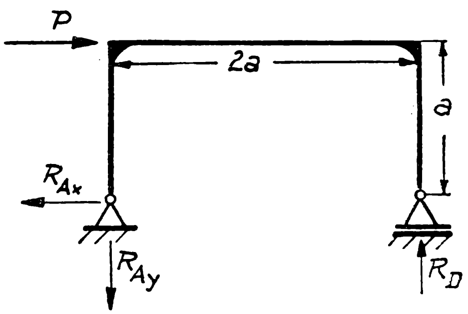
Lösung 42



$$T_C\cdot f = \frac{Q}{8}\cdot l;\qquad T_C = \frac{Ql}{8f} = \underline{\underline{200\,\mathrm{kg}}}$$

$$T_A = T_B = \sqrt{\left(\frac{Q}{2}\right)^2 + T_C^2} = \underline{\underline{201\,\mathrm{kg}}}$$

Lösung 43

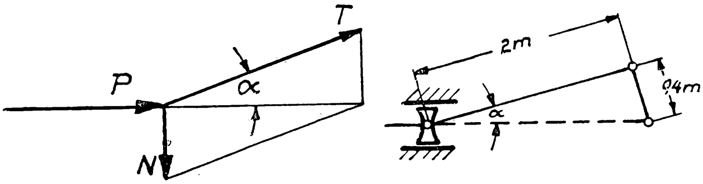


$$R_D\cdot 2a = P\cdot a;\quad R_D = \underline{\underline{\frac{P}{2}}};$$

$$R_{A_x} = P;\quad R_{A_y} = R_D$$

$$R_A = P\sqrt{\frac{1}{4}+1} = \underline{\underline{\frac{P}{2}\sqrt{5}}}$$

Lösung 44



$$\operatorname{tg}\alpha = \frac{0{,}4}{2} = 0{,}2$$

$P = (p_0 - p_1) \cdot F$;

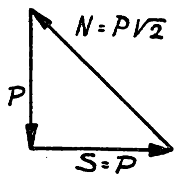
$F = 0{,}1\ \mathrm{m}^2$
$\hat{=} 1000\ \mathrm{cm}^2$

$P = 5 \cdot 1000$
$= 5000\ \mathrm{kg}$

$N = P \operatorname{tg}\alpha = \underline{\underline{1\ \mathrm{t}}}$

$T = \sqrt{P^2 + N^2}$
$= \sqrt{26} = \underline{\underline{5{,}1\ \mathrm{t}}}$

Lösung 45

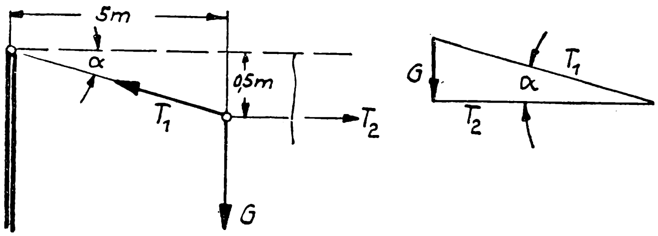


Gleichgewicht an den Punkten A; B; C; D:

$N_1 = N_2 = N_3 = N_4 = P\sqrt{2} = \underline{\underline{7{,}07\ \mathrm{t}}}$

$S_1 = S_2 = S_3 = P = \underline{\underline{5\ \mathrm{t}}}$

Lösung 46



Lageplan Kraftplan

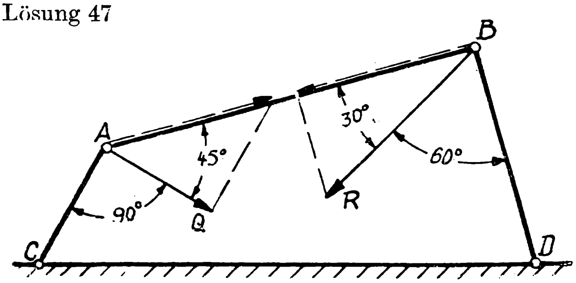
$$\operatorname{tg}\alpha = \frac{0{,}5}{5} = 0{,}1$$

$G = 40 \cdot 0{,}75 = 30\ \mathrm{kg}$

$$T_2 = \frac{G}{\operatorname{tg}\alpha} = 300\ \mathrm{kg}$$

$T_1 = \sqrt{G^2 + T^2}$
$= \underline{\underline{301{,}5\ \mathrm{kg}}} = T_3$

Lösung 47

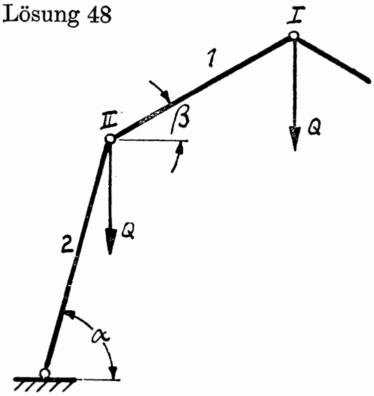


Es müssen die Kraftkomponenten von Q und R entlang AB gleich sein:

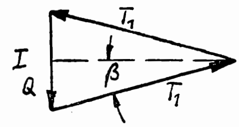
$$Q\sqrt{2} = R \cdot \frac{\sqrt{3}}{2};$$

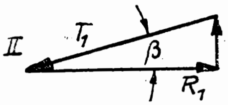
$$R = \frac{Q \cdot 2 \cdot \sqrt{2}}{\sqrt{3}} = \underline{\underline{16{,}3\ \mathrm{kg}}}$$

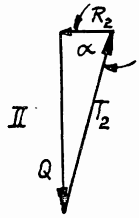
Lösung 48



Die Horizontalkomponenten der beiden Belastungen müssen sich im Gelenk II aufheben.







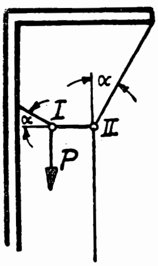
$$R_1 = \frac{Q}{2\,\mathrm{tg}\,\beta}$$

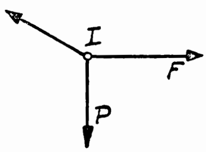
$$R_2 = \frac{Q}{\mathrm{tg}\,\alpha}; \quad R_1 = R_2;$$

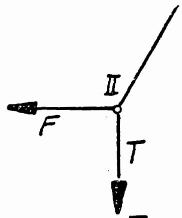
$$\alpha = 60^\circ$$

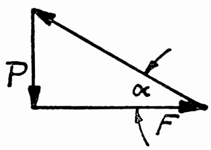
$$\mathrm{tg}\,\beta = \frac{\mathrm{tg}\,\alpha}{2}; \quad \underline{\underline{\beta = 30^\circ}}$$

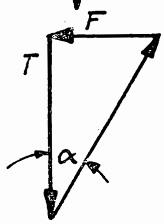
Lösung 49











$$\mathrm{ctg}\,\alpha = 14{,}3$$

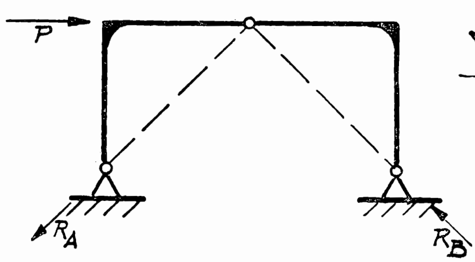
$$T = F\,\mathrm{ctg}\,\alpha$$

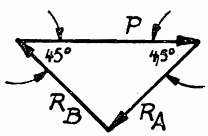
$$F = P\,\mathrm{ctg}\,\alpha$$

$$T = P\,\mathrm{ctg}^2\,\alpha$$

$$T = \underline{\underline{16{,}4\ \mathrm{t}}}$$

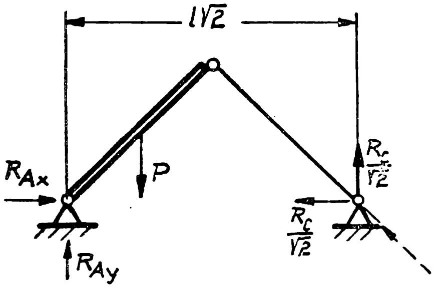
Lösung 50





$$R_A = R_B = \underline{\underline{P\,\frac{\sqrt{2}}{2}}}$$

Lösung 51

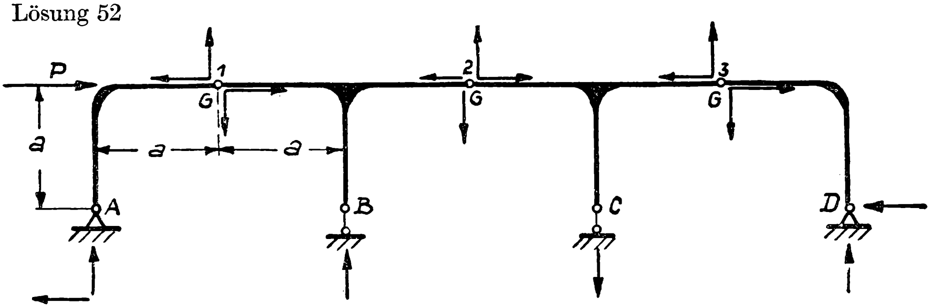


$$\frac{R_C}{\sqrt{2}} \cdot l\sqrt{2} = \frac{P \cdot l}{2\sqrt{2}}; \quad R_C = P\frac{\sqrt{2}}{4}$$

$$R_{Ay} = P\left(1 - \frac{1}{4}\right); \quad R_{Ax} = \frac{P}{4}$$

$$R_A = P\sqrt{\frac{9}{16} + \frac{1}{16}} = \frac{P}{4}\sqrt{10}$$

Lösung 52



$$P \cdot a - G_{1x} \cdot a - G_{1y} \cdot a = 0;$$

$$P = 2G; \quad G = \frac{P}{2}$$

$$R_B = P$$

$$R_C = P$$

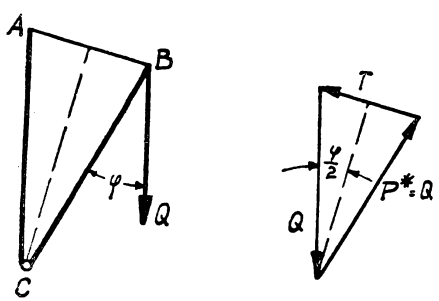
$$R_D = \sqrt{2\left(\frac{P}{2}\right)^2} = \frac{P}{2}\sqrt{2} = R_A$$

$$G_{1x} \cdot a = G_{2x} \cdot a; \quad G_{1x} = G_{2x} = G_{3x} = G_x$$

$$G_{1y} \cdot a = G_{2y} \cdot a; \quad G_{1y} = G_{2y} = G_{3y} = G_y$$

$$G_x \cdot a = G_y \cdot a; \quad G_x = G_y = G$$

Lösung 53



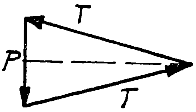
$$\frac{T}{2} = Q \sin\frac{\varphi}{2}$$

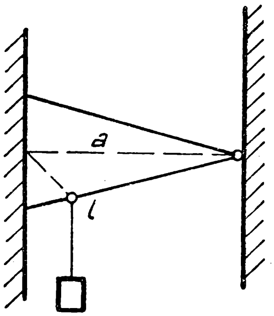
$$T = 2Q \sin\frac{\varphi}{2}$$

$$P = P^* + \text{Seilkraft}$$

$$P = P^* + Q = 2Q$$

Lösung 54





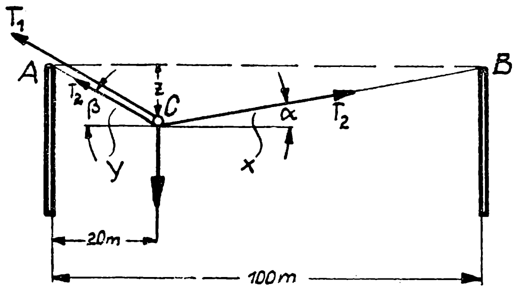
Ähnlichkeit:

$$\frac{T}{\frac{P}{2}} = \frac{l}{\sqrt{l^2-a^2}}$$

$$T = \frac{P}{2} \cdot \frac{l}{\sqrt{l^2-a^2}}$$

$$T = \underline{\underline{15 \text{ kg}}}$$

Lösung 55



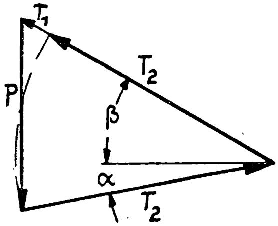
Geometrischer Teil:

$$x + y = 102$$
$$z^2 + y^2 = 20^2$$
$$z^2 + x^2 = 80^2$$
$$x^2 - y^2 = 80^2 - 20^2$$
$$x = 102 - y;$$
$$x^2 = 102^2 - 2 \cdot 102 \cdot y + y^2$$
$$102^2 - 204 y = 80^2 - 20^2$$
$$x = 80{,}4; \quad y = 21{,}6$$

$$\cos\alpha = \frac{80}{80{,}4} = 0{,}995; \quad \alpha = 5°40'$$

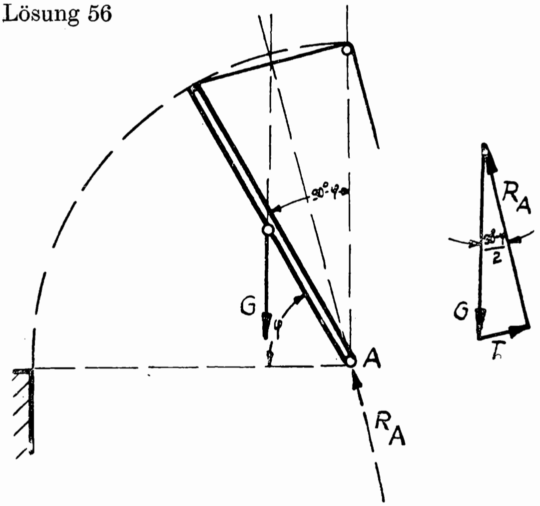
$$\cos\beta = \frac{20}{21{,}6} = 0{,}925; \quad \beta = 22°20'$$

Beim Aufstellen des Kraftplanes ist zu beachten, daß der Seilzug in den beiden Strängen des Seiles ACB gleich groß ist.



$$T_1 + T_2 = T_s$$
$$T_s \cdot \sin\beta + T_2 \sin\alpha = P$$
$$T_s \cdot \cos\beta = T_2 \cos\alpha$$
$$T_2 \cdot \sin\beta \cdot \frac{\cos\alpha}{\cos\beta} + T_2 \sin\alpha = P$$
$$T_2 = \frac{P}{\operatorname{tg}\beta \cos\alpha + \sin\alpha}; \quad T_s = T_2 \cdot \frac{\cos\alpha}{\cos\beta}$$
$$T_2 = \frac{5}{0{,}411 \cdot 0{,}995 + 0{,}116} = \underline{\underline{9{,}56 \text{ t}}} = T_{CB} = T_{CA}$$
$$T_s = 10{,}31; \quad T_1 = T_{CAD} = \underline{\underline{0{,}75}}$$

Lösung 56

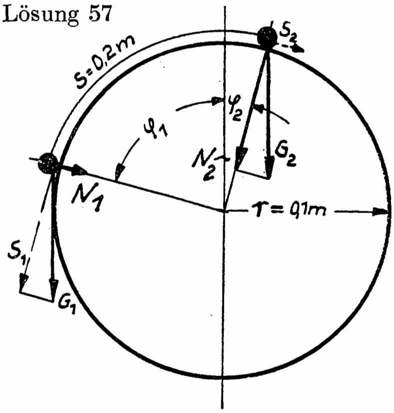


$$T = G \cdot \sin\left(45° - \frac{\varphi}{2}\right)$$

$$T_{\max;\ \varphi=0} = G \sin 45° = \underline{\underline{70{,}7\ \mathrm{kg}}}$$

$$T_{\min;\ \varphi=90°} = \underline{\underline{0}}$$

Lösung 57



$$r(\varphi_1 + \varphi_2) = s; \quad \varphi_1 = 2 - \varphi_2$$

$$S_1 = G_1 \cdot \sin \varphi_1 \qquad S_1 = S_2$$

$$S_2 = G_2 \cdot \sin \varphi_2$$

$$\frac{G_2}{G_1} = \frac{\sin 2 \cos \varphi_2 - \cos 2 \sin \varphi_2}{\sin \varphi_2}$$

$$2 = \sin 2 \operatorname{ctg} \varphi_2 - \cos 2$$

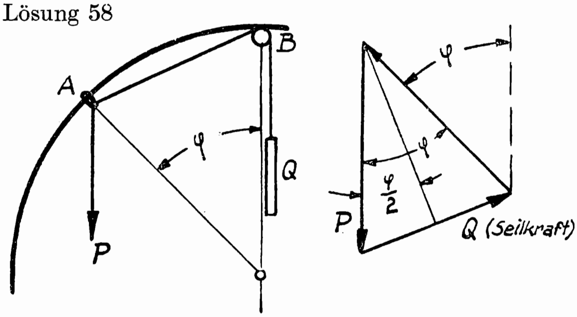
$$\operatorname{tg} \varphi_2 = \frac{\sin 2}{2 + \cos 2} = \frac{0{,}91}{2 - 0{,}415}$$

$$\varphi_2 = \underline{\underline{29° 50'}}; \quad \varphi_1 = \underline{\underline{84° 45'}}$$

$$N_1 = G_1 \cdot \cos \varphi_1 = \underline{\underline{0{,}0092\ \mathrm{kg}}}$$

$$N_2 = G_2 \cdot \cos \varphi_2 = \underline{\underline{0{,}173\ \mathrm{kg}}}$$

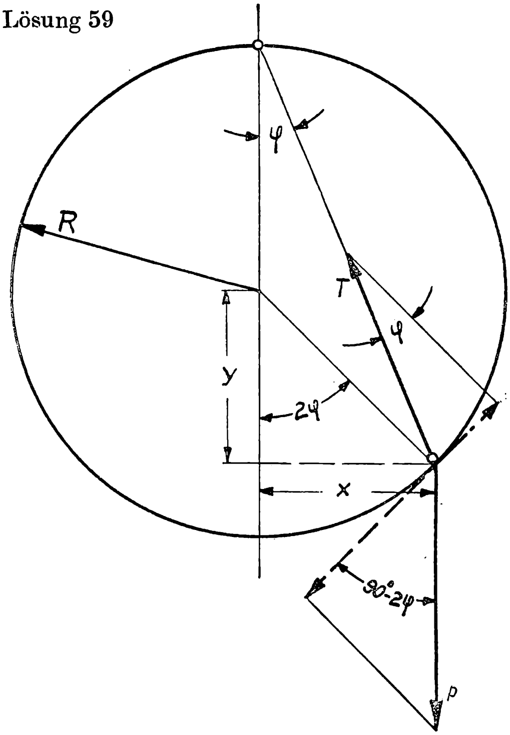
Lösung 58



$$\frac{Q}{2} = P \cdot \sin \frac{\varphi}{2}; \quad \underline{\underline{\sin \frac{\varphi}{2} = \frac{Q}{2P}}}$$

Gleichgewicht ist möglich bei $Q < 2P$; für $\varphi = \pi$ herrscht Gleichgewicht bei beliebigem P

Lösung 59



$$T = k \cdot \frac{\Delta l}{l}$$

$$p \cdot \cos(90° - 2\varphi) = T \sin\varphi$$

$$p \cdot \sin 2\varphi = T \sin\varphi$$

$$2p \cdot \sin\varphi \cos\varphi = T \sin\varphi$$

$$\Delta l = \frac{\cos\varphi \cdot l \cdot 2 \cdot p}{k}$$

$$(\Delta l + l)\sin\varphi = x$$

$$x = R \sin 2\varphi$$

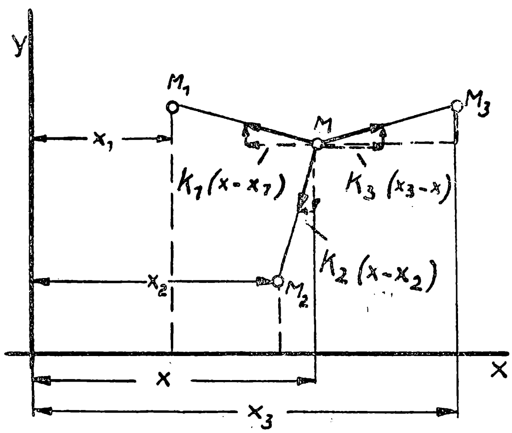
$$\cos\frac{2p}{k} \cdot l + l = 2R\cos\varphi$$

$$2\cos\varphi\left(R - \frac{p}{k} \cdot l\right) = l$$

$$\underline{\underline{\cos\varphi = \frac{1}{2} \cdot \frac{kl}{kR - l \cdot p}}}$$

Bei $k < \dfrac{2p \cdot l}{2R - l}$ wird $\varphi = 0$

Lösung 60



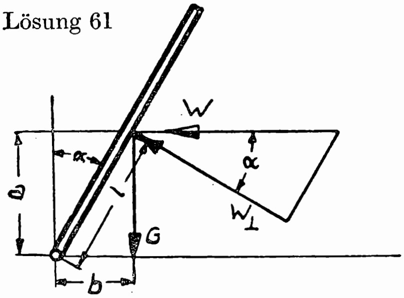
$$-k_1(x - x_1) - k_2(x - x_2) + k_3(x_3 - x) = 0$$

$$k_1(x_1 - x) + k_2(x_2 - x) + k_3(x_3 - x) = 0$$

$$\underline{\underline{x = \frac{k_1 x_1 + k_2 x_2 + k_3 x_3}{k_1 + k_2 + k_3}}}$$

$$\underline{\underline{y = \frac{k_1 y_1 + k_2 y_2 + k_3 y_3}{k_1 + k_2 + k_3}}}$$

Lösung 61

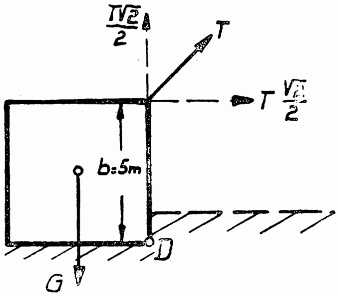


$$W \cdot a - G \cdot b = 0; \quad a = l \cdot \cos\alpha$$
$$b = l \cdot \sin\alpha$$

$$W = \frac{W_\perp}{\cos\alpha}; \quad \frac{W_\perp}{\cos\alpha} \cdot l \cos\alpha = G \cdot l \cdot \sin\alpha$$

$$\underline{\underline{W_\perp = G \cdot \sin\alpha}} \qquad \alpha = 18°; \quad G = 5\,\text{kg}$$
$$\underline{\underline{W_\perp = 1{,}55\,\text{kg}}}$$

Lösung 62

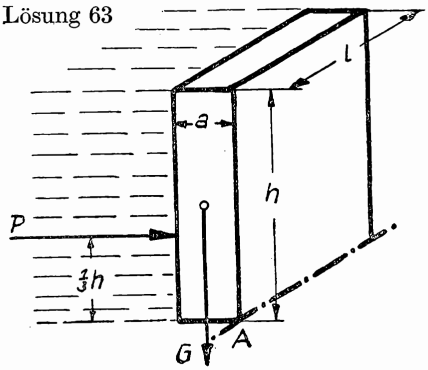


$$G = \gamma \cdot a \cdot b^2$$

$$G \cdot \frac{b}{2} = \frac{T\sqrt{2}}{2} \cdot b$$

$$a = \frac{T\sqrt{2}}{\gamma \cdot b^2} = \frac{100 \cdot \sqrt{2}}{2{,}5 \cdot 25}; \qquad \underline{\underline{a \geqq 2{,}3\,\text{m}}}$$

Lösung 63

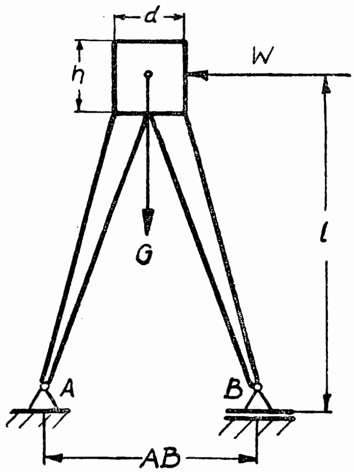


$$P \cdot \frac{h}{3} = G \cdot \frac{a}{2}; \qquad P = q \cdot l$$
$$G = h \cdot a \cdot l \cdot \gamma$$

$$q \cdot \frac{l \cdot h}{3} = \frac{h \cdot l \cdot \gamma \cdot a^2}{2}$$

$$\underline{\underline{a = \sqrt{\frac{2}{3} \cdot \frac{q}{\gamma}}}} \qquad q = 6\,\text{t/m}; \quad \gamma = 2\,\text{t/m}^3$$
$$\underline{\underline{a = \sqrt{2}\,\text{m}}}$$

Lösung 64



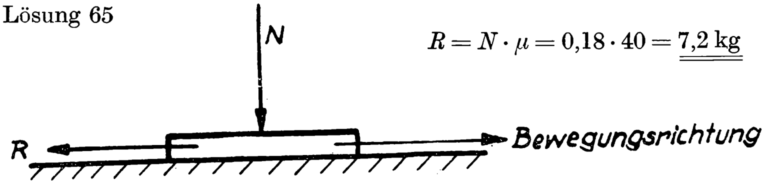
$$W \cdot l = G \cdot \frac{AB}{2}$$

$$AB = \frac{W \cdot l \cdot 2}{G}; \qquad W = p \cdot h \cdot d; \qquad p = 125\,\text{kg/m}^2$$
$$h = 6\,\text{m}$$
$$d = 4\,\text{m}$$

$$W = 3000\,\text{kg} \triangleq 3\,\text{t}; \qquad l = 20\,\text{m}$$

$$\underline{\underline{AB \geqq 15\,\text{m}}}$$

Lösung 65

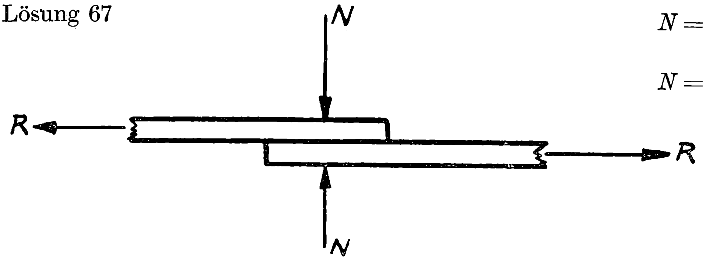


$$R = N \cdot \mu = 0{,}18 \cdot 40 = \underline{\underline{7{,}2\,\text{kg}}}$$

Lösung 66

$$K = G \cdot \mu = 50 \cdot 0{,}15 = \underline{\underline{7{,}5\,\text{kg}}}$$

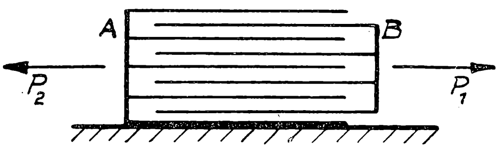
Lösung 67



$$N = \frac{R}{\mu}$$

$$N = \frac{2000}{0{,}2} = \underline{\underline{10000\,\text{kg}}}$$

Lösung 68



Um B aus A zu ziehen, ist die Kraft P_1 erforderlich:

$$P_1 = \sum_{z=1}^{z=199} z \cdot \mu \cdot G$$

$$\sum_{q=1}^{q=n} q = \frac{n}{2}(n+1)$$

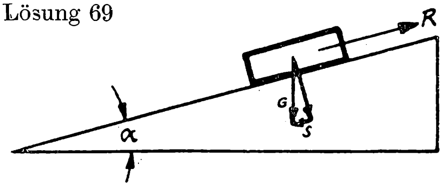
$$\sum_{z=1}^{z=199} z = 19900; \quad \mu = 0{,}2; \quad G = 0{,}006\,\text{kg}$$

$$P_1 = \underline{\underline{23{,}88\,\text{kg}}}$$

Um A aus B zu ziehen, ist die Kraft P_2 erforderlich:

$$P_2 = \sum_{z=1}^{z=200} z \cdot \mu \cdot G; \quad \sum_{z=1}^{z=200} z = 20100; \quad P_2 = \underline{\underline{24{,}12\,\text{kg}}}$$

Lösung 69



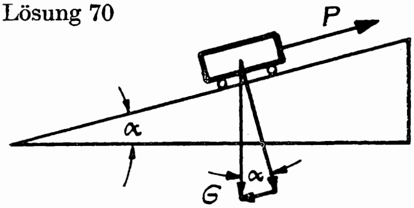
$R - S = 0$; $S = G \cdot \sin\alpha$

für kleinen Winkel α gilt: $\sin\alpha = \operatorname{tg}\alpha$

$R = G \cdot 0{,}008$

$R = \underline{\underline{80\,\text{kg}}}$

Lösung 70

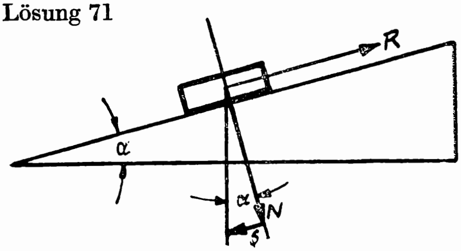


$$P = G \cdot \cos\alpha \cdot \mu + G \cdot \sin\alpha$$

$$\left.\begin{array}{l}\alpha = \operatorname{tg}\alpha = \sin\alpha = 0{,}008 \\ \cos\alpha = 1\end{array}\right\} \text{Kleiner Winkel}$$

$$P = 180\,(0{,}005 + 0{,}008) = \underline{\underline{2{,}34\ \mathrm{t}}}$$

Lösung 71

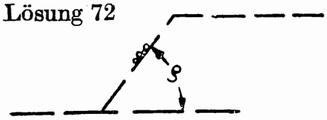


$$R = S = N \cdot \mu$$

$$\frac{S}{N} = \mu = \operatorname{tg}\alpha$$

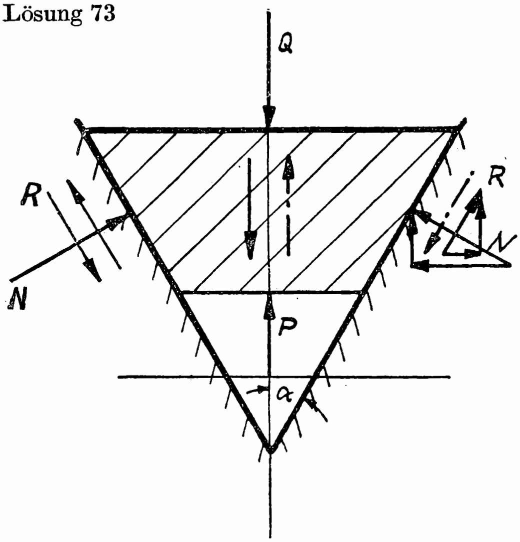
$$\underline{\underline{\mu = \operatorname{tg}\alpha}}$$

Lösung 72



Nach Aufgabe 71 gilt:
$\mu = \operatorname{tg}\varrho = 0{,}8;\quad \underline{\underline{\varrho = 38°40'}}$

Lösung 73



Keilbewegung nach unten:

$$\frac{1}{2}\,Q - R\cos\alpha - N\sin\alpha = 0$$

$$R = \mu \cdot N$$

$$\frac{1}{2}\,Q - (\mu\cos\alpha + \sin\alpha)\,N = 0$$

$$N = \frac{Q}{2\,(\sin\alpha + \mu\cos\alpha)} \qquad \begin{array}{l}\operatorname{tg}\alpha = 0{,}05 \\ \cos\alpha \approx 1\end{array}$$

$$N = \frac{Q}{2\cos\alpha\,(\mu + \operatorname{tg}\alpha)}$$

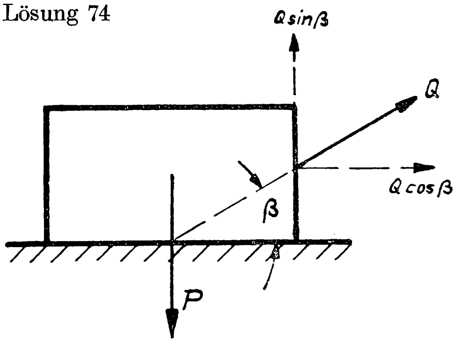
$$N = \frac{6}{2 \cdot 0{,}15} = \underline{\underline{20\ \mathrm{t}}}$$

Bewegung nach oben
(Lösen des Keiles)

$$\frac{P}{2} + N\sin\alpha - \mu \cdot N\cos\alpha = 0$$

$$\begin{aligned}P &= 2\,N\,(\mu\cos\alpha - \sin\alpha) \\ &= 40\,(0{,}1 - 0{,}05) = \underline{\underline{2\ \mathrm{t}}}\end{aligned}$$

Lösung 74



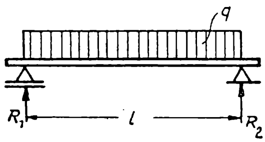
$$Q \cdot \cos\beta = (P - Q\sin\beta)\cdot\mu$$
$$Q(\cos\beta + \sin\beta\cdot\mu) = P\cdot\mu$$
$$Q = \frac{P\cdot\mu}{(\cos\beta + \sin\beta\,\mu)} = \frac{P\cdot\mu}{K}$$

Minimalbedingung:

$$\frac{dK}{d\beta} = -\sin\beta + \mu\cos\beta = 0$$
$$\underline{\underline{\mu = \operatorname{tg}\beta}}$$
$$\cos\beta = \frac{1}{\sqrt{1+\mu^2}}; \quad \sin\beta = \frac{\mu}{\sqrt{1+\mu^2}}$$
$$Q_{\min} = \frac{P\cdot\mu}{\sqrt{1+\mu^2}}$$

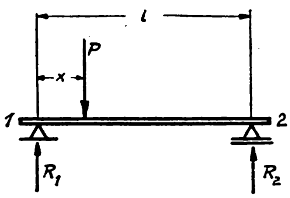
3. Parallele Kräfte und Momente

Lösung 75



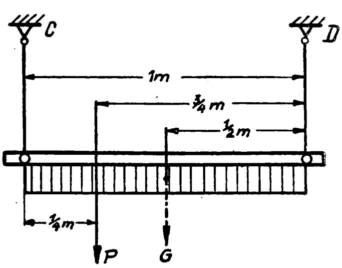
$$\underline{\underline{R_1 = R_2 = \frac{p\cdot l}{2}}}$$

Lösung 76



$$\sum M_1 = 0: \quad R_2\cdot l - P\cdot x = 0$$
$$\underline{\underline{R_2 = P\cdot\frac{x}{l}}}$$
$$\sum P_y = 0: \quad R_2 + R_1 - P = 0$$
$$P\left(\frac{x}{l} - 1\right) + R_1 = 0$$
$$\underline{\underline{R_1 = P\frac{l-x}{l}}}$$

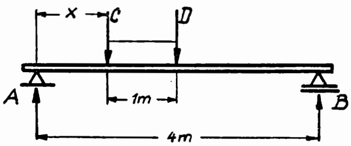
Lösung 77



$G =$ Stangengewicht

$$T_C = P\cdot\frac{3}{4} + G\cdot\frac{1}{2} = 9 + 1 = \underline{\underline{10\,\mathrm{kg}}}$$
$$T_D = P\cdot\frac{1}{4} + G\cdot\frac{1}{2} = 3 + 1 = \underline{\underline{4\,\mathrm{kg}}}$$

Lösung 78



$$B \cdot 4 - D(x+1) - C \cdot x = 0$$

$$B = \frac{x(D+C)+D}{4}$$

$$A \cdot 4 - C(4-x) - D(3-x) = 0$$

$$A = \frac{4C+3D-x(D+C)}{4}$$

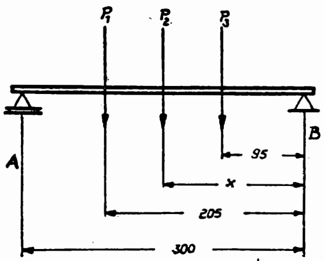
Bedingung: $A = 2B$

$$\frac{4C+3D-x(D+C)}{4} = \frac{2x(D+C)+2D}{4}$$

$$3x(D+C) = 4C+D; \quad x = \frac{4C+D}{3(D+C)}$$

$$x = \frac{800+100}{3(100+200)} = \underline{\underline{1\ \mathrm{m}}}$$

Lösung 79



$$A \cdot 300 - P_1 \cdot 205 - P_2 \cdot x - P_3 \cdot 95 = 0$$

$$B \cdot 300 - P_3 \cdot 205 - P_2(300-x) - P_1 \cdot 95 = 0$$

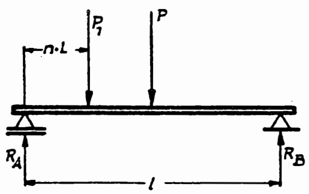
Bedingung: $A = B$

$$P_1 \cdot 205 + P_2 \cdot x + P_3 \cdot 95$$
$$= P_3 \cdot 205 + P_2 \cdot 300 - P_2 \cdot x + P_1 \cdot 95$$

$$x = \frac{P_2 \cdot 300 + P_3 \cdot 110 - P_1 \cdot 110}{2P_2}$$

$$\underline{\underline{x = 139\ \mathrm{cm}}}$$

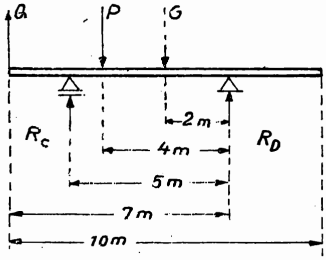
Lösung 80



$$R_B \cdot l = \frac{P}{2} \cdot l + P_1 \cdot l \cdot n; \quad \underline{\underline{R_B = (3+4n)\ \mathrm{t}}}$$

$$R_A = P_1 + P - R_B; \quad \underline{\underline{R_A = (7-4n)\ \mathrm{t}}}$$

Lösung 81



$$\sum M_D = 0: Q \cdot 7 + R_C \cdot 5 - P \cdot 4 - 6 \cdot 2 = 0$$

$$R_C = \frac{P \cdot 4 + 6 \cdot 2 - Q \cdot 7}{5}$$

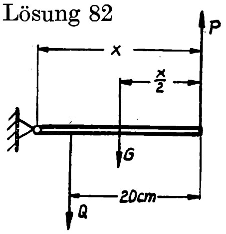
$$\underline{\underline{R_C = 300\ \mathrm{kg}}}$$

$$\sum P_y = 0: Q + R_C - P - G + R_D = 0$$

$$R_D = 800 + 200 - 300 - 300$$

$$\underline{\underline{R_D = 400\ \mathrm{kg}}}$$

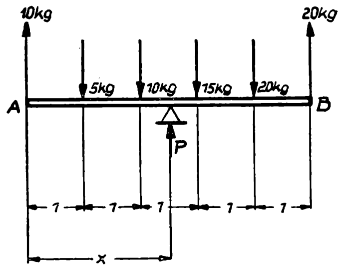
Lösung 82



$$P \cdot x - G \cdot \frac{x}{2} - Q(x-20) = 0$$

$$x = \frac{Q \cdot 20}{Q - P + \frac{G}{2}} = \frac{500 \cdot 20}{400} = \underline{\underline{25\ \text{cm}}}$$

Lösung 83



$\sum M_A = 0$:

$$5 \cdot 1 + 10 \cdot 2 + 15 \cdot 3 + 20 \cdot 4 - 20 \cdot 5 - P \cdot x = 0$$

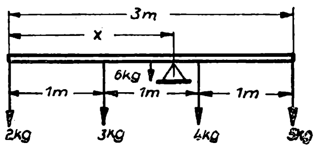
$\sum P_y = 0$:

$$P = 5 + 10 + 15 r 20 - 20 - 10 = 20\ \text{kg}$$

$$x = \frac{5 + 20 + 45 + 80 - 100}{P} = 2{,}5\ \text{m}$$

Die Stange muß in der Mitte gestützt werden

Lösung 84

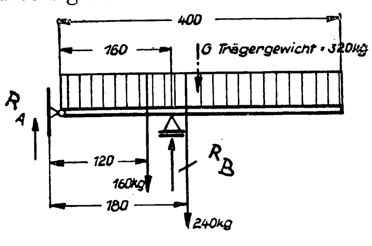


$\sum M_A = 0$:

$$3 \cdot 1 + 4 \cdot 2 + 5 \cdot 3 - (2+3+4+5+6) \cdot x + 6 \cdot 1{,}5 = 0$$

$$x = \frac{3 + 8 + 15 + 9}{2 + 3 + 4 + 5 + 6} = \underline{\underline{1{,}75\ \text{m}}}$$

Lösung 85



$\sum M_A = 0$:

$$320 \cdot 2{,}00 + 240 \cdot 1{,}80 + 160 \cdot 1{,}20 - R_B \cdot 1{,}60 = 0$$

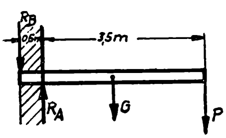
$$R_B = \frac{640 + 432 + 192}{160}$$

$R_B = \underline{\underline{790\ \text{kg}}}$ nach oben gerichtet

$\sum P_y = 0$: $R_A + R_B - 160 - 320 - 240 = 0$

$R_A = -\underline{\underline{70\ \text{kg}}}$ nach unten gerichtet

Lösung 86



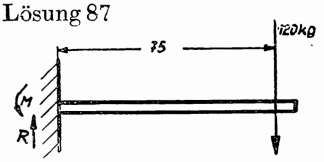
$\sum M_B = 0$: $R_A \cdot 0{,}5 - G \cdot 2 - P \cdot 4 = 0$

$$R_A = G \cdot 4 + P \cdot 8 = \underline{\underline{34\ \text{t}}}$$

$\sum P_y = 0$: $R_B + G + P = R_A$

$$R_B = R_A - G - P = \underline{\underline{29{,}5\ \text{t}}}$$

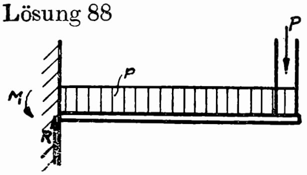
Lösung 87



$M = 75 \cdot 120 = 9000\ \text{cmkg} \triangleq \underline{\underline{90\ \text{mkg}}}$

$\underline{\underline{R = 120\ \text{kg}}}$

Lösung 88

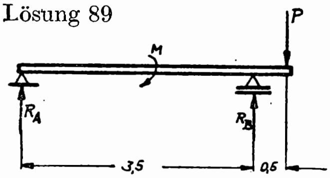


$$M = \frac{p \cdot l^2}{2} + P \cdot l = \frac{200 \cdot 2{,}25}{2} + 200 \cdot 1{,}5$$

$$M = \underline{\underline{525\ \text{mkg}}}$$

$$R = p \cdot l + P = 200 \cdot 1{,}5 + 200 = \underline{\underline{500\ \text{kg}}}$$

Lösung 89



$\sum M_A = 0;\qquad M - R_B \cdot 3{,}5 + P \cdot 4 = 0$

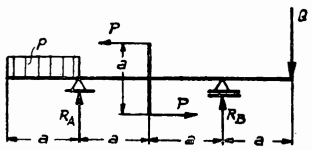
$$R_B = \frac{M + 4P}{3{,}5} = \underline{\underline{4\,\text{t}}}$$

$\sum Py = 0:\qquad R_A + R_B - P = 0$

$$R_A = P - R_B = \underline{\underline{-2\,\text{t}}}$$

R_A wirkt also entgegen der angenommenen Richtung

Lösung 90



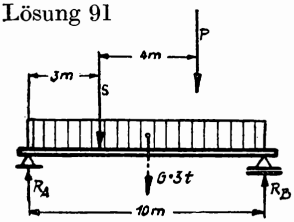
$\sum M_A = 0:\quad \frac{p \cdot a^2}{2} + P \cdot a + R_B \cdot 2a - Q \cdot 3a = 0$

$$R_B = \frac{3Q - P - \frac{p\,a}{2}}{2} = \underline{\underline{2{,}1\ \text{t}}}$$

$\sum Py = 0:\qquad R_A = Q + p \cdot a - R_B$

$$= 2 + 1{,}6 - 2{,}1 = \underline{\underline{1{,}5\ \text{t}}}$$

Lösung 91



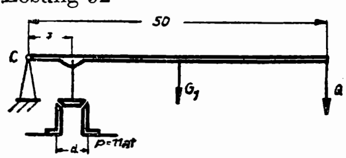
$\sum M_A = 0:\ -G \cdot 5 + R_B \cdot 10 - P \cdot 7 - S \cdot 3 = 0$

$$R_B = \frac{7 \cdot 1 + 3 \cdot 5 + 3 \cdot 5}{10} = \underline{\underline{3{,}7\ \text{t}}}$$

$\sum Py = 0:\qquad R_A + R_B - S - G - P = 0$

$$R_A = 5 + 3 + 1 - 3{,}7 = \underline{\underline{5{,}3\ \text{t}}}$$

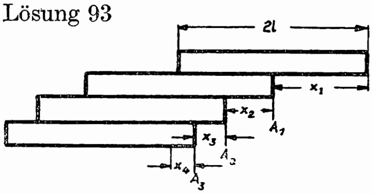
Lösung 92



$\sum M_C = 0:\qquad \frac{d^2\pi}{4} \cdot p \cdot 7 = G_1 \cdot 25 + Q \cdot 50$

$$Q = \frac{d^2\pi}{4} \cdot p \cdot \frac{7}{50} - G_1\,\frac{25}{50} = \underline{\underline{43\ \text{kg}}}$$

Lösung 93



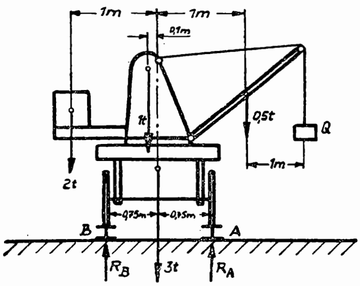
Der Schwerpunkt der n Platten, die auf der $(n+1)$-ten Platte liegen, muß über der jeweiligen Kippkante A liegen.

1. Schwerpunktsabstand: $x_1 = l$

2. Schwerpunktsabstand: $x_2 = \frac{l}{2}$

3. Schwerpunktsabstand: $x_3 = \frac{l}{3}$

Lösung 94



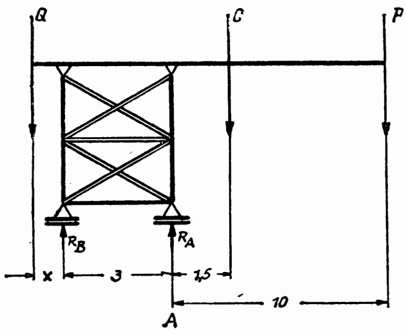
$\sum M_A = 0$: Bedingung: $R_B = 0$

$$Q \cdot 1{,}25 + 0{,}5 \cdot 0{,}25 = 3 \cdot 0{,}75 + 1 \cdot 0{,}85 + 2 \cdot 1{,}75$$

$$Q = \frac{2{,}25 + 0{,}85 + 3{,}5 - 0{,}125}{1{,}25}$$

$$Q = \underline{\underline{5{,}18\,\mathrm{t}}}$$

Lösung 95



Unbelastet: $R_A = 0$; $P = 0$

$\sum M_B = 0$:

$$Q \cdot x - 4{,}5 \cdot C = 0 \qquad \mathrm{I}$$

$\sum P_y = 0$:

$$Q + C - R_B = 0 \qquad \mathrm{II}$$

Belastet: $R_B = 0$

$\sum M_B = 0$:

$$Q \cdot x - 4{,}5 \cdot C + 3 R_A - 13 P = 0 \qquad \mathrm{III}$$

$\sum P_y = 0$:

$$Q + C + P - R_A = 0 \qquad \mathrm{IV}$$

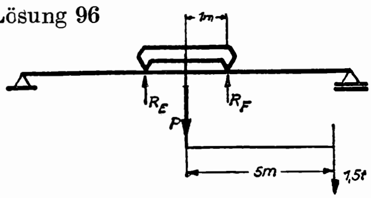
Aus I und III: $R_A = \frac{13}{3} p = 108{,}5\,\mathrm{t}$

„ IV : $Q = \underline{\underline{\frac{100}{3}\,\mathrm{t}}}$

„ I $x = \underline{\underline{6{,}75\,\mathrm{m}}}$

„ II : $R_B = \frac{10}{3} P = 83{,}3\,\mathrm{t}$

Lösung 96

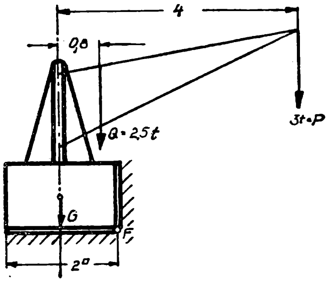


$R_E = 0$; $\Sigma M_F = 0$:

$$1{,}5\,(5 - 1) = 1 \cdot P$$

$$P = 1{,}5 \cdot 4 = \underline{\underline{6\,\mathrm{t}}}$$

Lösung 97



$$\sum M_F = 0:$$

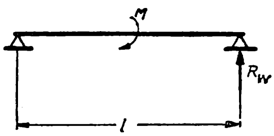
$$G \cdot 1 + Q \cdot 0{,}2 - P \cdot 3 = 0$$

$$G = 8{,}5\,\mathrm{t}$$

$$G = V \cdot \gamma = F \cdot h \cdot \gamma; \quad h = \frac{G}{F \cdot \gamma}$$

$$h = \frac{8{,}5}{4 \cdot 2} = \underline{\underline{1{,}06\,\mathrm{m}}}$$

Lösung 98



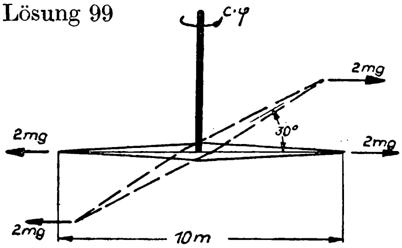
Reaktionsmoment $= M$

$$M - R_W \cdot l = 0;$$

$$R_W = (640 - 460) = 180\,\mathrm{kg}$$

$$M = 180 \cdot 2{,}5 = \underline{\underline{450\,\mathrm{mkg}}}$$

Lösung 99



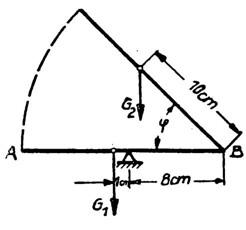
$$M = 2 \cdot 2 \cdot \frac{10}{2} \cdot \sin 30° = c \cdot \varphi$$

$$c = 5\,\frac{\mathrm{mg\,cm}}{\mathrm{Grad}}$$

$$\varphi = \frac{20}{5} \cdot \frac{1}{2} = 2°$$

Der Draht wird zur Aufnahme des Momentes um 2° gedrillt, muß also, um die Nadel 30° zu wenden, um 32° gedreht werden.

Lösung 100

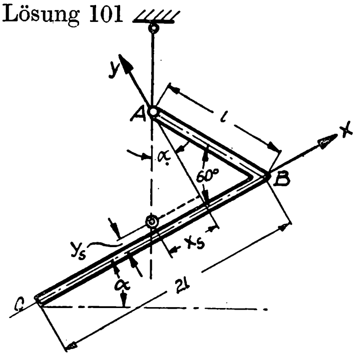


$$G_1 \cdot 1 = G_2 (8 - 10 \cdot \cos \varphi)$$

$$\cos \varphi = \frac{G_2 \cdot 8 - G_1}{G_2 \cdot 10} = \frac{12 \cdot 8 - 16}{12 \cdot 10} = \frac{2}{3}$$

$$\varphi = \arccos \frac{2}{3} = \underline{\underline{48°\ 10'}}$$

Lösung 101



Der Schwerpunkt muß auf der Wirkungslinie der Fadenkraft liegen.

Schwerpunktslage:

$$x_S \cdot 3l = l \cdot \frac{l}{2} \cos 60° - 2l(l - l \cos 60°)$$

$$x_S = -\frac{l\left(\frac{1}{4} - 2 + 1\right)}{3} = -l\frac{3}{12}$$

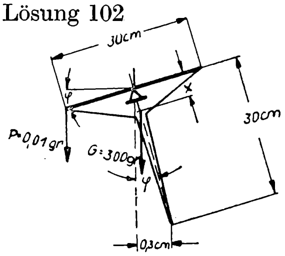
$$y_S \cdot 3l = l\frac{l}{2} \cdot \sin 60°$$

$$y_S = \frac{\sqrt{3}}{12} \cdot l$$

$$\operatorname{tg}\alpha = \left|\frac{x_S}{l \sin 60° - y_S}\right| = \frac{3}{12\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{12}\right)}; \quad \operatorname{tg}\alpha = \frac{1}{5}\sqrt{3}$$

$$\underline{\underline{\alpha = 19° 5'}}$$

Lösung 102



$$G \cdot x \cdot \sin\varphi = P \cdot 15 \cdot \cos\varphi$$

$$\varphi = \text{kleiner Winkel:} \quad \varphi \cong \sin\varphi \cong \operatorname{tg}\varphi = 0{,}01$$

$$\cos\varphi \cong 1$$

$$x = \frac{P \cdot 15}{G \cdot 0{,}01} = \underline{\underline{0{,}05 \text{ cm}}}$$

Lösung 103

$$P_2 = P_1$$

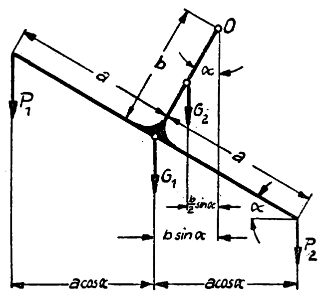
$$G_1 = 4p \cdot a; \quad G_2 = 2pb$$

$$\sum M_0 = 0:$$

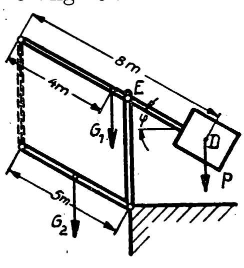
$$P_1 (a \cos\alpha + b \sin\alpha) + 4pab \sin\alpha + 2pb\frac{b}{2}\sin\alpha$$

$$- P_2(a\cos\alpha - b\sin\alpha) = 0 \quad |:\cos\alpha$$

$$\operatorname{tg}\alpha = \underline{\underline{\frac{a(P_2 - P_1)}{b[P_1 + P_2 + p(4a + b)]}}}$$



Lösung 104



$$\sum M_E = 0:$$

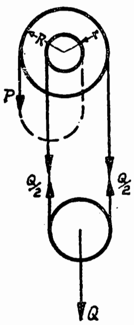
$$P \cdot 3\text{m} \cdot \cos\varphi = G_1 \cdot 1\,\text{m} \cos\varphi + G_2 \cdot 2{,}5\,\text{m} \cos\varphi$$

$$P = \frac{G_1 \cdot 1 + G_2 \cdot 2{,}5}{3}; \quad G_1 = 0{,}4\,\text{t}$$

$$G_2 = \frac{3}{2}\,\text{t}$$

$$P = 1{,}383\,\text{t} \triangleq \underline{\underline{1383\,\text{kg}}}$$

Lösung 105



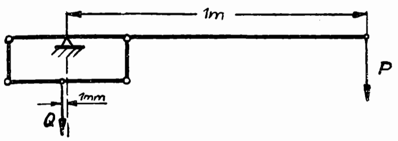
$$P \cdot R + \frac{Q}{2} \cdot r = \frac{Q}{2} \cdot R$$

$$P = \frac{Q}{2}\left(1 - \frac{r}{R}\right)$$

$$P = \frac{500}{2}\left(1 - \frac{24}{25}\right)$$

$$P = \underline{\underline{10\,\text{kg}}}$$

Lösung 106

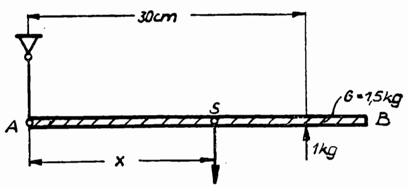


$$P \cdot 1\,\text{m} = Q \cdot 1\,\text{mm}$$

$$P = Q \cdot \frac{1}{1000}$$

$$P = \underline{\underline{1\,\text{kg}}}$$

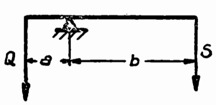
Lösung 107



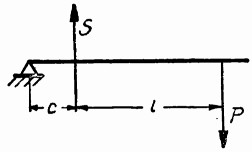
$$30\,\text{cm} \cdot 1\,\text{kg} = x \cdot 1{,}5\,\text{kg}$$

$$x = \frac{30}{1{,}5} = \underline{\underline{20\,\text{cm}}}$$

Lösung 108



$\varDelta Q = 1000\,\text{kg}$



Vor der Verschiebung von P herrscht Gleichgewicht bei:

$$Q \cdot a = S \cdot b; \quad S = Q \cdot \frac{a}{b}$$

$$S \cdot c = P\,(l + c)$$

$$Q = \frac{P\,(l + c) \cdot b}{a \cdot c}$$

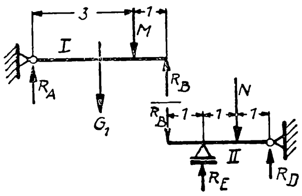
Nach der Verschiebung von P um x herrscht Gleichgewicht bei:

$$Q + \varDelta Q = \frac{P\,(l + x + c) \cdot b}{a \cdot c}$$

$$\varDelta Q = \frac{P \cdot x}{a \cdot c} \cdot b; \quad x = \frac{\varDelta Q \cdot a \cdot c}{P \cdot b}$$

$$x = \frac{1000 \cdot 3{,}3 \cdot 50}{12{,}5 \cdot 600} = 20\,\text{mm} \mathrel{\hat{=}} \underline{\underline{2\,\text{cm}}}$$

Lösung 109



Teil I: $\sum M_A = 0$

$$R_B = \frac{3}{4} \cdot M + G_1 \cdot \frac{2}{4} = \underline{\underline{160\,\text{kg}}}$$

$$R_A = \frac{1}{4} M + \frac{2}{4} G_1 = \underline{\underline{120\,\text{kg}}}$$

Teil II: $\Sigma M_D = 0$

$$R_B \cdot 3 - R_E \cdot 2 + G_2 \cdot 1{,}5 + N \cdot 1 = 0$$

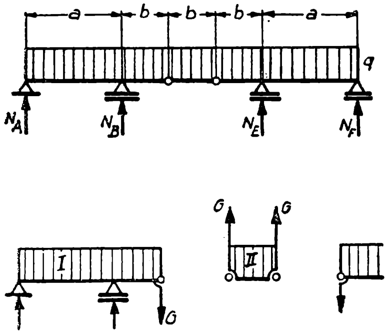
$$R_E = \frac{480 + 240 + 80}{2} = \underline{\underline{400\,\text{kg}}}$$

$\sum P_y = 0$:

$$R_D - R_B + R_E - G_2 - N = 0$$

$$R_D = 160 - 400 + 160 + 80 = \underline{\underline{0}}$$

Lösung 110



Teil II: $q \cdot b = 2 \cdot G$

$$G = \frac{q \cdot b}{2}$$

Teil I: $N_A + N_B = q\,(a + b) + \frac{q \cdot b}{2}$

$$N_B \cdot a = \frac{q\,(a+b)^2}{2} + G\,(a+b)$$

$$N_B = \frac{q\,(a+b)\,[a + 2b]}{2a}$$

$a = 50\,\text{m}$

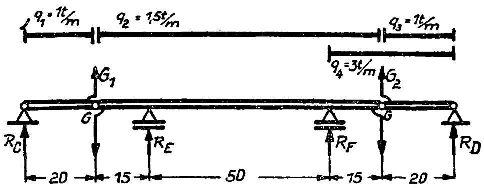
$b = 20\,\text{m}$

$q = 6\,\text{t/m}$

$$N_B = \underline{\underline{378\,\text{t}}}$$

$$N_A = q\,(a+b) + \frac{q \cdot b}{2} - N_B = \underline{\underline{102\,\text{t}}}$$

Lösung 111



Wegen Symmetrie:

$$R_D = G_2 = \frac{1}{2}\,(q_3 \cdot 20 + q_4 \cdot 20)$$

$$R_D = \underline{\underline{40\,\text{t}}}$$

$$R_C = G_1 = \frac{1}{2} \cdot q_1 \cdot 20$$

$$R_C = \underline{\underline{10\,\text{t}}}$$

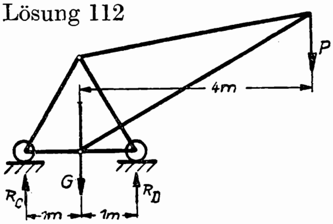
$\sum M_E = 0$: $\quad G_2 \cdot 65 - R_F \cdot 50 - G_1 \cdot 15 + q_2 \cdot 80 \cdot 25 + q_4 \cdot 15 \cdot 57{,}5 = 0$

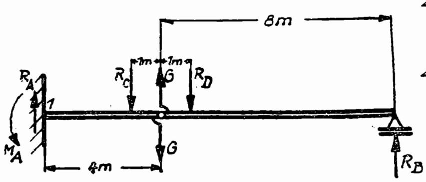
$$R_F = \frac{40 \cdot 65 - 10 \cdot 15 + 1{,}5 \cdot 80 \cdot 25 + 3 \cdot 15 \cdot 57{,}5}{50} = \underline{\underline{160{,}75\,\text{t}}}$$

$\sum P_y = 0$: $\quad R_E = G_1 + G_2 + q_2 \cdot 80 + q_4 \cdot 15 - R_F$

$$R_E = 10 + 40 + 120 + 45 - 160{,}75 = \underline{\underline{54{,}25\,\text{t}}}$$

Lösung 112





Auflagerreaktionen des Kranes:

$$P \cdot 5 + G \cdot 1 = R_D \cdot 2$$

$$R_D = \frac{1 \cdot 5 + 5 \cdot 1}{2} = \underline{\underline{5\,\text{t}}}; \quad R_C = P + G - R_D = \underline{\underline{1\,\text{t}}}$$

$\sum M_G = 0$:

$$R_B \cdot 8 = R_D \cdot 1; \quad R_B = \frac{R_D}{8} = \underline{\underline{0{,}625\,\text{t}}}$$

$\sum P_y = 0$:

$$R_A = R_C + R_D - R_B = \underline{\underline{5{,}375\,\text{t}}}$$

$\sum M_1 = 0$:

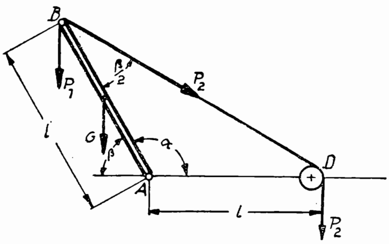
$$M_A = R_C \cdot 3 + R_D \cdot 5 - R_B \cdot 12$$

$$M_A = 3 + 25 - 7{,}5$$

$$M_A = \underline{\underline{20{,}5\,\text{mt}}}$$

4. Willkürliches ebenes Kräftesystem

Lösung 113



$\sum M_A = 0$:

$$P_1 \cdot l \cos\beta + G \frac{l}{2} \cos\beta - P_2 \cdot l \sin\frac{\beta}{2} = 0$$

Daraus mit $P_1 = 1\,\text{kg}$; $P_2 = 2\,\text{kg}$; $G = 2\,\text{kg}$;

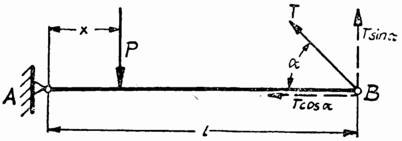
$$2\cos^2\beta + \cos\beta = 1$$

$$\cos\beta_{1,2} = -\frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{1}{2}}$$

$$\cos\beta_1 = -1; \quad \cos\beta_2 = \frac{1}{2}$$

$$\underline{\underline{\alpha = 120^\circ}}$$

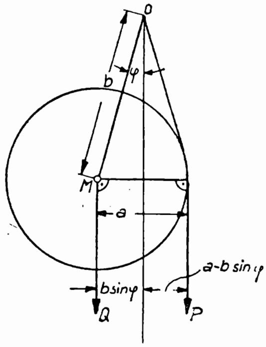
Lösung 114



$$T \sin\alpha \cdot l = P \cdot x$$

$$T = \underline{\underline{\frac{P \cdot x}{l \sin\alpha}}}$$

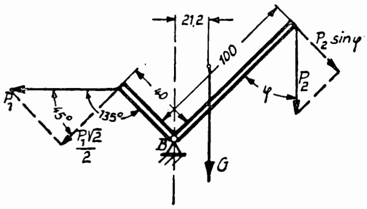
Lösung 115



$$Q \cdot b \cdot \sin\varphi = P(a - b\sin\varphi)$$

$$\sin\varphi = \underline{\underline{\frac{a}{b}\,\frac{P}{P+Q}}}$$

Lösung 116



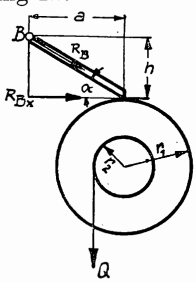
$$\sum M_B = 0:$$

$$\frac{P_1\sqrt{2}}{2}\cdot 40 = G\cdot 21{,}2 + P_2 \sin\varphi\cdot 100$$

$$\sin\varphi = \frac{P_1\frac{\sqrt{2}}{2}\cdot 40 - G\cdot 21{,}2}{P_2\cdot 100}$$

$$\sin\varphi = 0{,}707;\quad \underline{\underline{\varphi = 45°}}$$

Lösung 117



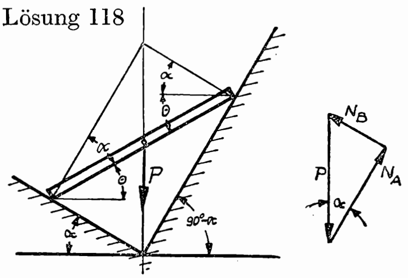
$$Q\cdot r_2 = R_x\cdot r_1$$

$$\cos\alpha = \frac{R_x}{R} = \frac{a}{\sqrt{a^2+h^2}}$$

$$R = Q\,\frac{r_2}{r_1}\,\frac{\sqrt{a^2+h^2}}{a}$$

$$R = 50\cdot\frac{240}{420}\cdot\frac{\sqrt{120^2+50^2}}{120} = \underline{\underline{31\ \text{kg}}}$$

Lösung 118



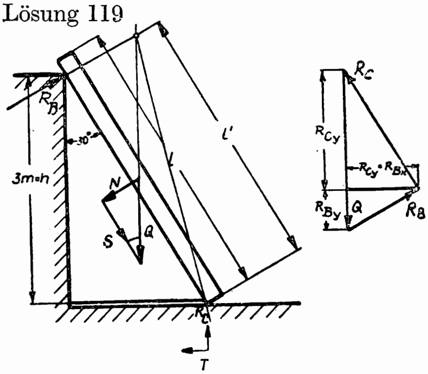
Der Träger ist nur im Gleichgewicht, wenn sich N_A, N_B und P in einem Punkte schneiden.

$$90° + \Theta + 2\alpha = 180°$$

$$\underline{\underline{\Theta = 90° - 2\alpha}}$$

$$\underline{\underline{N_A = P\cos\alpha}};\quad \underline{\underline{N_B = P\sin\alpha}}$$

Lösung 119



$$N = Q\cdot\sin 30°;\quad l' = \frac{h}{\cos 30°}$$

$$S = Q\cdot\cos 30°;$$

$$\sum M_C = 0:\quad R_B\cdot l' - N\cdot\frac{l}{2} = 0$$

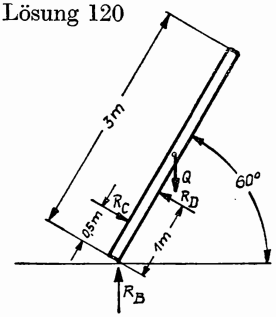
$$R_B = \frac{Q\sin 30°\cdot l\cdot\cos 30°}{2h} = \underline{\underline{17{,}32\ \text{kg}}}$$

$$R_{B_x} - T = 0;\quad T = R_B\cdot\cos 30° = \underline{\underline{15\ \text{kg}}}$$

$$R_C + R_{B_y} - Q = 0$$

$$R_C = Q - R_B\cdot\sin 30° = \underline{\underline{51{,}34\ \text{kg}}}$$

Lösung 120



Da der Balken in B reibungsfrei aufliegt:

$\sum P_x = 0$: $R_{C_x} = R_{D_x}$

Somit auch: $R_C = R_D$

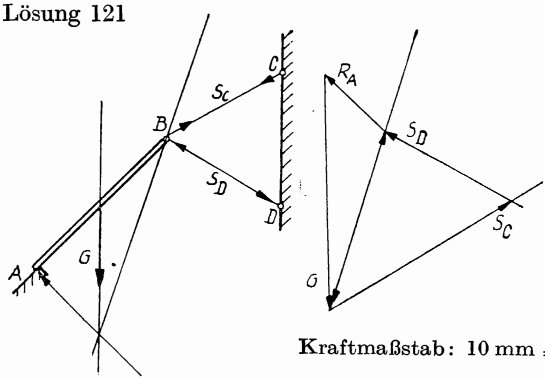
$\sum M_B = 0$: $R_D \cdot 1 - R_C \cdot 0{,}5 - Q \cdot 1{,}5 \cdot \cos 60° = 0$

$R_D = R_C = 1{,}5\,Q = \underline{\underline{30\,\text{kg}}}$

$\sum P_y = 0$: $R_B - R_{Cy} + R_{Dy} - Q = 0$

$R_B = Q = \underline{\underline{20\,\text{kg}}}$

Lösung 121



Graphische Lösung:

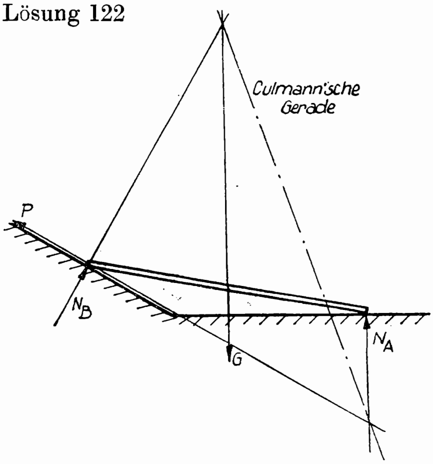
$R_A = 35{,}4$ kg

$S_C = 89{,}5$ kg

$S_D = -60{,}6$ kg

Kraftmaßstab: 10 mm ≙ 30 kg

Lösung 122



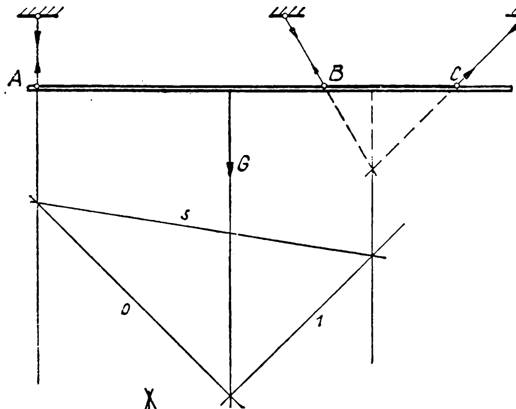
$N_B = 43{,}3$ kg

$N_A = 50$ kg

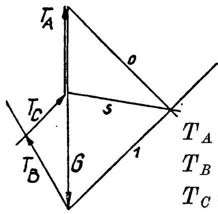
$P = 25$ kg

Kraftmaßstab: 10 mm ≙ 30 kg

Lösung 123

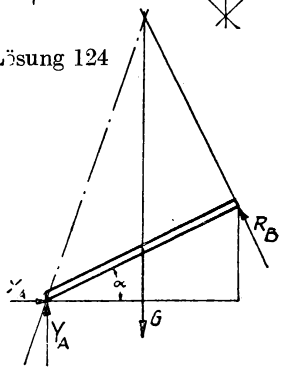


Kraftmaßstab:
10 mm ≙ 1500 kg

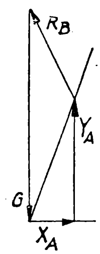


$T_A = 1800$ kg
$T_B = 1757$ kg
$T_C = 1243$ kg

Lösung 124



tg $\alpha = 0{,}5$

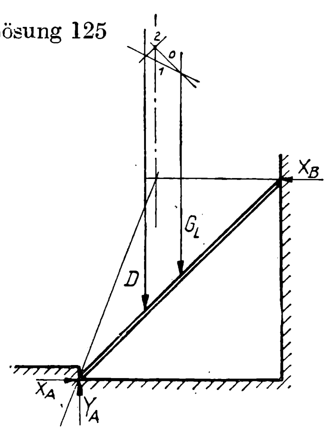


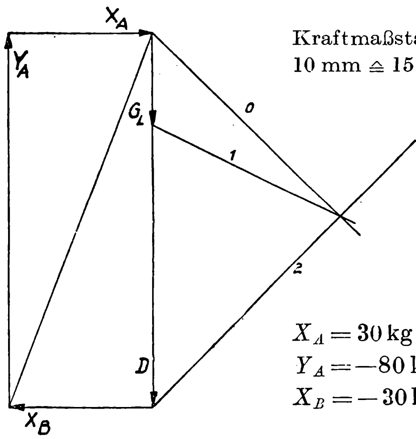
Kraftmaßstab:
10 mm ≙ 300 kg

$X_A = 180$ kg
$Y_A = 540$ kg
$R_B = 402$ kg

Diese Werte ergeben sich unter Beachtung des abgegebenen Koordinatensystems für den Leiterdruck (Aktion).

Lösung 125

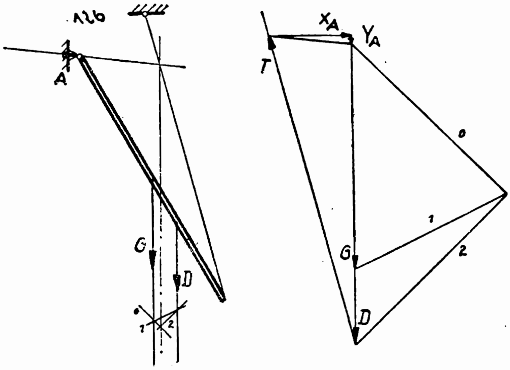




Kraftmaßstab:
10 mm ≙ 15 kg

$X_A = 30$ kg
$Y_A = -80$ kg
$X_B = -30$ kg

Lösung 126

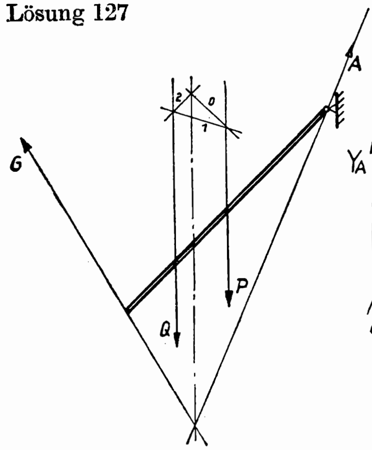


Kraftmaßstab: 10 mm ≙ 75 kg

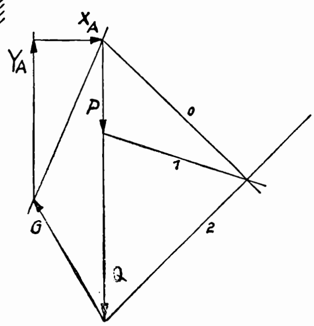
$T = 335$ kg
$X_A = 86{,}7$ kg
$Y_A = -3{,}4$ kg

Die Vorzeichen entsprechen dem angegebenem Koordinatensystem (Reaktion).

Lösung 127

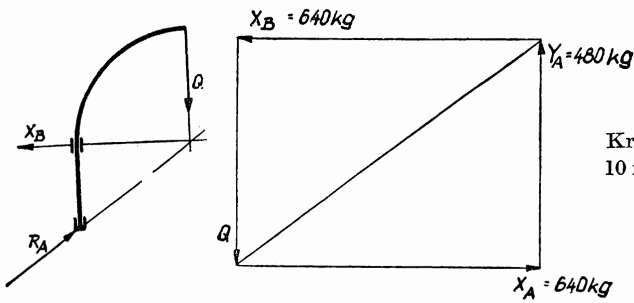


Kraftmaßstab: 10 mm = 75 kg



$G = 146$ kg
$X_A = 73$ kg
$Y_A = 173$ kg

Lösung 128

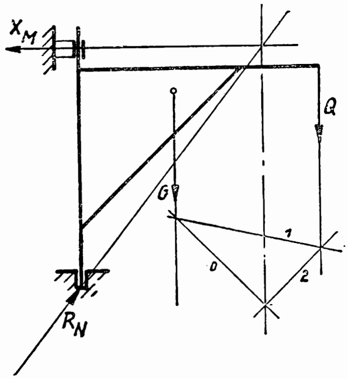


Kraftmaßstab:
10 mm = 150 kg

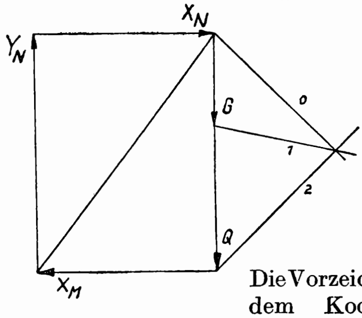
Entsprechend dem angegebenen Koordinatensystem wirken die Aktionskräfte:

$X_A = -640$ kg
$X_B = 640$ kg
$Y_A = -480$ kg

Lösung 129



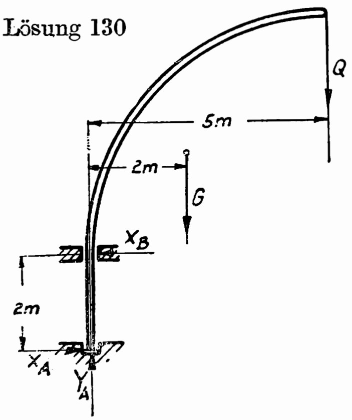
Kraftmaßstab: 10 mm ≙ 1,5 t



$$X_M = -3{,}8\,\mathrm{t}$$
$$X_N = 3{,}8\,\mathrm{t}$$
$$Y_N = 5{,}0\,\mathrm{t}$$

Die Vorzeichen entsprechen dem Koordinatensystem.

Lösung 130



Analytische Lösung:

$\sum M_A = 0$:

$$Q \cdot 5 + G \cdot 2 - X_B \cdot 2 = 0$$
$$X_B = \frac{Q \cdot 5 + G \cdot 2}{2} = \underline{\underline{12\,\mathrm{t}}}$$

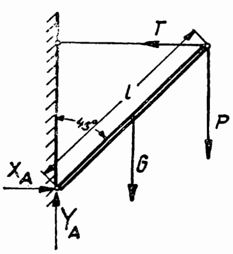
$\sum P_x = 0$:

$$X_B - X_A = 0; \quad X_A = X_B = \underline{\underline{12\,\mathrm{t}}}$$

$\sum P_y = 0$:

$$Y_A - G - Q = 0$$
$$Y_A = G + Q = \underline{\underline{6\,\mathrm{t}}}$$

Lösung 131



$\sum M_A = 0$:

$$\frac{Tl}{\sqrt{2}} - \frac{P \cdot l}{\sqrt{2}} - \frac{G \cdot l}{2\sqrt{2}} = 0$$
$$T = P + \frac{G}{2} = \underline{\underline{250\,\mathrm{kg}}}$$

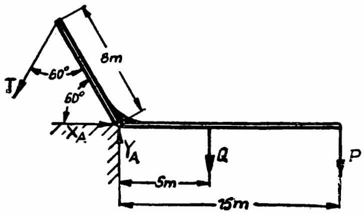
$\sum P_y = 0$:

$$Y_A - G - P = 0; \quad Y_A = G + P = \underline{\underline{300\,\mathrm{kg}}}$$

$\sum P_x = 0$: $\quad T - X_A = 0; \quad X_A = \underline{\underline{250\,\mathrm{kg}}}$

Die angegebenen Auflagerkräfte sind Reaktionen, sie unterscheiden sich von den entsprechenden Aktionen nur durch das Vorzeichen.

Lösung 132



$$\sum M_A = 0: \quad T \sin 60° \cdot 8 - Q \cdot 5 - P \cdot 15 = 0$$

$$T = \frac{12 \cdot 5 + 20 \cdot 15}{8 \frac{\sqrt{3}}{2}} = \underline{\underline{52\,\text{t}}}$$

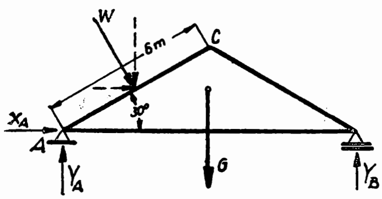
$$\sum P_x = 0: \quad T \cdot \cos 60° - X_A = 0$$

$$X_A = 52 \cdot \frac{1}{2} = \underline{\underline{26\,\text{t}}}$$

$$\sum P_y = 0: \quad Y_A - T \cos 30° - P - Q = 0$$

$$Y_A = 52 \frac{\sqrt{3}}{2} + 20 + 12 = \underline{\underline{77\,\text{t}}}$$

Lösung 133



$$\sum M_A = 0:$$

$$G \cdot 6 \cdot \cos 30° - Y_B \cdot 2 \cdot 6 \cdot \cos 30° + W \cdot \frac{6}{2} = 0$$

$$Y_B = \frac{G}{2} + \frac{W}{\sqrt{3} \cdot 2} = 5 + \frac{0{,}8}{\sqrt{3} \cdot 2} = \underline{\underline{5{,}23\,\text{t}}}$$

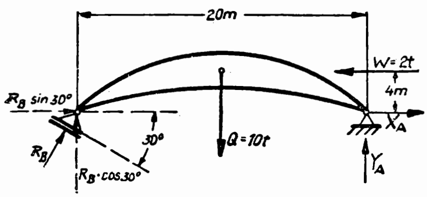
$$\sum P_x = 0: \quad X_A + W \sin 30° = 0$$

$$X_A = -\frac{0{,}8}{2} = \underline{\underline{-0{,}4\,\text{t}}}$$

$$\sum P_y = 0: \quad Y_A + Y_B - G - W \cos 60° = 0$$

$$Y_A = 10 + 0{,}8 \frac{\sqrt{3}}{2} - 5{,}23 = \underline{\underline{5{,}46\,\text{t}}}$$

Lösung 134



$$\sum M_A = 0:$$

$$Q \cdot 10 + W \cdot 4 - R_B \cdot \cos 30° \cdot 20 = 0$$

$$R_B = \frac{10 \cdot 10 + 2 \cdot 4}{20 \cdot \frac{\sqrt{3}}{2}} = \underline{\underline{6{,}24\,\text{t}}}$$

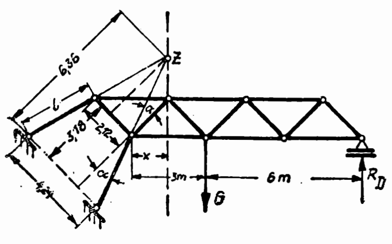
$$\sum P_y = 0: \quad R_B \cdot \cos 30° + Y_A - Q = 0$$

$$Y_A = 10 - 6{,}24 \frac{\sqrt{3}}{2} = \underline{\underline{4{,}6\,\text{t}}}$$

$$\sum P_x = 0: \quad R_B \cdot \sin 30° + X_A - W = 0$$

$$X_A = 2 - 6{,}24 \cdot \frac{1}{2} = \underline{\underline{-1{,}12\,\text{t}}}$$

Lösung 135



Analytische Lösung:

$$\sum M_Z = 0:$$

$$\operatorname{tg} \alpha = \frac{2{,}12}{2 \cdot 3{,}18} = 0{,}33; \quad l = \frac{3{,}18}{\cos \alpha}$$

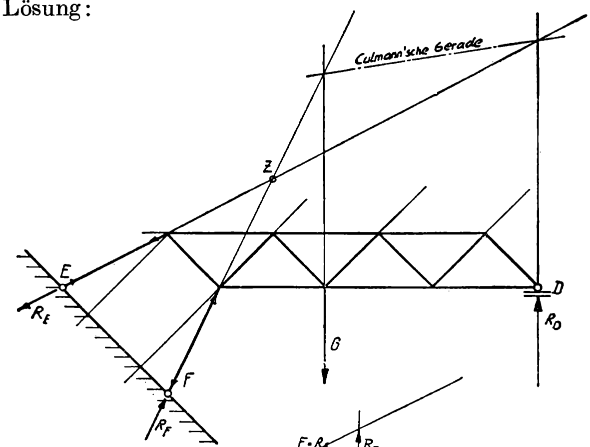
$$x = l \cdot \cos(\alpha + 45°) = \frac{3{,}18 \cdot \sqrt{2}}{2} [1 - \operatorname{tg} \alpha]$$

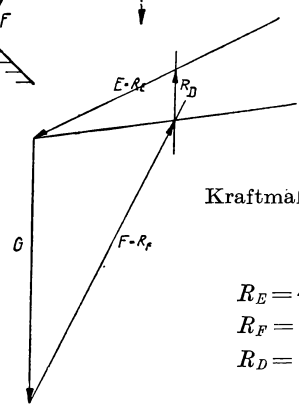
$$x = 1{,}5\,\text{m}$$

$$G \cdot 1{,}5 - R_D \cdot 7{,}5 = 0;$$

$$R_D = G \cdot \frac{1{,}5}{7{,}5} = \underline{\underline{1{,}5\,\text{t}}}$$

Graphische Lösung:

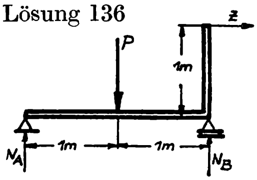




Kraftmaßstab: 10 mm ≙ 200 kg

$R_E = 4500\,\text{kg}$
$R_F = 9000\,\text{kg}$
$R_D = 1500\,\text{kg}$

Lösung 136

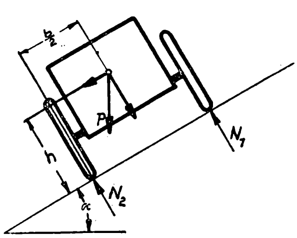


$\sum M_B = 0$: $\quad P \cdot 1 - N_A \cdot 2 - Z \cdot 1 = 0$

$$N_A = \frac{20 \cdot 1 - 2 \cdot 1}{2} = \underline{\underline{9\,\text{t}}}$$

$\sum P_y = 0$: $\quad N_A + N_B - P = 0; \quad N_B = 20 - 9 = \underline{\underline{11\,\text{t}}}$

Lösung 137



$\sum M_{N_2} = 0$:

$$N_1 b + P \cdot \sin\alpha \cdot h - P \cdot \cos\alpha \cdot \frac{b}{2} = 0$$

$$N_1 = \frac{P}{b}\left(\frac{b}{2}\cos\alpha - h \cdot \sin\alpha\right)$$

$$N_1 = \underline{\underline{448{,}29\,\text{kg}}}$$

$\sum P_N = 0$:

$$N_2 + N_1 - P \cdot \cos\alpha = 0$$

$$N_2 = \underline{\underline{548\,\text{kg}}}$$

Lösung 138

Abbildung vgl. Aufgabenstellung

$$\sum P_x = 0: \quad F - Q = 0; \quad Q = F = \underline{\underline{400\text{ kg}}}$$

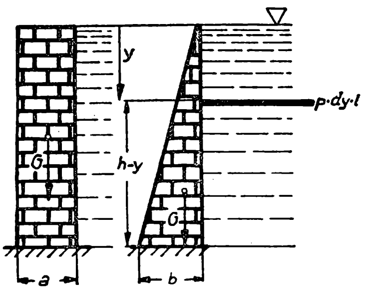
$$\sum P_y = 0: \quad P - G + P_0 = 0$$

$$\sum M_{Q,P_0} = 0: \quad G \cdot l - P(a + l) - F(b + c) = 0$$

$$P = \frac{3000 \cdot 5{,}0 - 400\,(0{,}1 + 0{,}05)}{5{,}2} = \underline{\underline{2873\text{ kg}}}$$

$$P_0 = G - P = \underline{\underline{127\text{ kg}}}$$

Lösung 139



Kippmoment:

$$dM_K = (h - y) \cdot \gamma \cdot y \cdot l \cdot dy$$

$l =$ Länge der Mauer

$$M_K = \int_0^h dM_K = \frac{\gamma \cdot h^3 \cdot l}{6}$$

Widerstandsmoment:

Fall 1: $\quad M_{W_1} = G \cdot \frac{a}{2} = \gamma_1 \cdot a \cdot h \cdot l \cdot \frac{a}{2}$

Fall 2: $\quad M_{W_2} = G \frac{2}{3} b = \gamma_1 \cdot \frac{1}{2} b \cdot h \cdot l \cdot \frac{2}{3} \cdot b$

Standfestigkeit: $\frac{M_W}{M_K} = 2$

Fall 1:

$$\gamma_1 \cdot \frac{a^2}{2} \cdot h \cdot l = 2 \cdot \frac{\gamma \cdot h^3 \cdot l}{6};$$

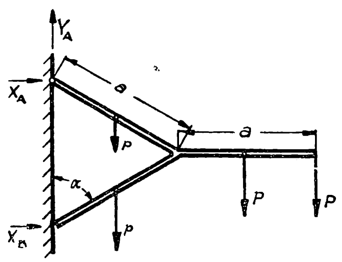
$$a = h\sqrt{\frac{2}{3}\,\frac{\gamma}{\gamma_1}} = \underline{\underline{2{,}75\text{ m}}}$$

Fall 2:

$$\gamma_1 \cdot \frac{b^2}{3} \cdot h \cdot l = 2 \cdot \frac{\gamma \cdot h^3 \cdot l}{6}$$

$$b = h\sqrt{\frac{\gamma}{\gamma_1}} = \underline{\underline{3{,}37\text{ m}}}$$

Lösung 140



$$\sum P_y = 0: \quad Y_A - 3p - P = 0$$

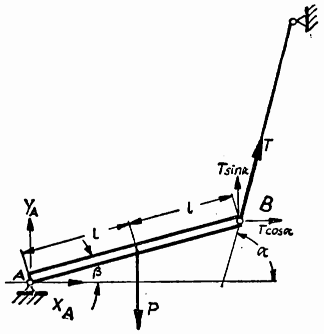
$$\underline{\underline{Y_A = 3p + P}}$$

$$\sum M_A = 0: \quad X_B \cdot 2 \cdot a\cos\alpha - P(1 + \sin\alpha)\,a$$

$$-2 \cdot p \cdot \frac{a}{2}\sin\alpha - p \cdot a\left(\frac{1}{2} + \sin\alpha\right) = 0$$

$$-X_A = X_B = \underline{\underline{\frac{2\,(P + 2p)\sin\alpha + p + 2P}{4\cos\alpha}}}$$

Lösung 141



$$\sum M_B = 0:\quad Y_A \cdot 2l\cos\beta - X_A \cdot 2l \cdot \sin\beta - P \cdot l\cos\beta = 0$$

$$\sum P_x = 0: \qquad X_A + T\cos\alpha = 0$$

$$\sum P_y = 0: \qquad Y_A + T\sin\alpha - P = 0$$

$$2P\cos\beta - 2T\cos\beta\sin\alpha + 2T\cos\alpha\sin\beta - P\cos\beta = 0$$

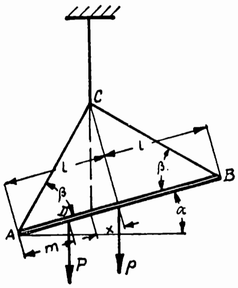
$$P\cos\beta - 2T(\sin\alpha\cos\beta - \cos\alpha\sin\beta) = 0$$

$$\underline{\underline{T = P\,\frac{\cos\beta}{2\sin(\alpha-\beta)}}}$$

$$Y_A = P - T\sin\alpha = \underline{\underline{P\left[1 - \frac{\sin\alpha\cos\beta}{2\sin(\alpha-\beta)}\right]}}$$

$$X_A = -T\cos\alpha = \underline{\underline{-P\cdot\frac{\cos\alpha\cos\beta}{2\sin(\alpha-\beta)}}}$$

Lösung 142

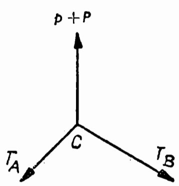


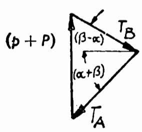
$$\sum M_C = 0: \qquad P(l - x - m)\cos\alpha = p\cdot x\cdot\cos\alpha$$

$$x = \operatorname{tg}\beta\cdot l\cdot\operatorname{tg}\alpha$$

$$P(l - \operatorname{tg}\beta\cdot l\cdot\operatorname{tg}\alpha - m) = p\cdot\operatorname{tg}\beta\cdot l\cdot\operatorname{tg}\alpha$$

$$\underline{\underline{\operatorname{tg}\alpha = \frac{P(l-m)}{l(P+p)}\operatorname{ctg}\beta}}$$





$$(p+P) = T_B\sin(\beta-\alpha) + T_A\cdot\sin(\alpha+\beta)$$

$$T_B\cdot\cos(\beta-\alpha) = T_A\cos(\alpha+\beta)$$

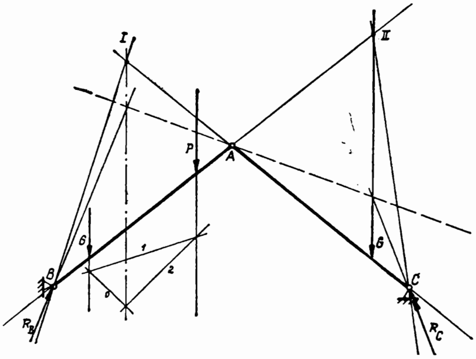
$$(p+P) = T_B\sin(\beta-\alpha) + T_B\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}\cdot\cos(\beta-\alpha)$$

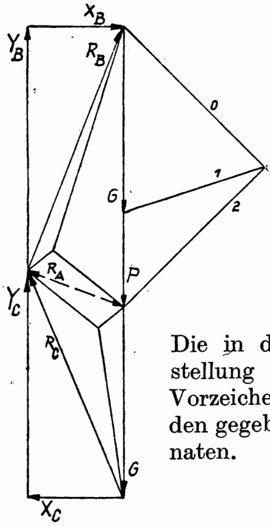
$$\underline{\underline{T_B = \frac{\cos(\alpha-\beta)}{\sin 2\beta}(P+p)}}$$

$$\underline{\underline{T_A = \frac{\cos(\alpha+\beta)}{\sin 2\beta}(P+p)}}$$

Lösung 143 Dreigelenkbogen. Graphische Lösung:

Kraftmaßstab: 10 mm $\triangleq$ 2 t

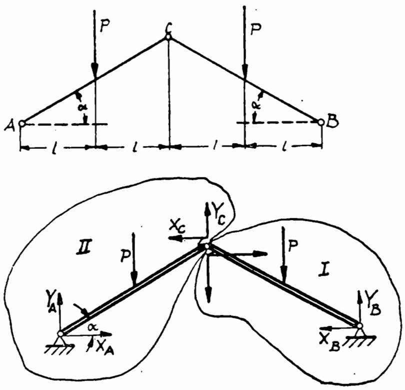




$X_A = 2\,\text{t}$
$Y_A = 0{,}8\,\text{t}$
$X_B = 2\,\text{t}$
$Y_B = 5{,}2\,\text{t}$
$X_C = 2\,\text{t}$
$Y_C = 4{,}8\,\text{t}$

Die in der Aufgabenstellung angegebenen Vorzeichen entsprechen den gegebenen Koordinaten.

Lösung 144



Dreigelenkbogen. Analytische Lösung:

Gesamtsystem:

$$\sum M_B = 0:\quad P\cdot l + P\cdot 3l - Y_A\cdot 4l = 0$$
$$Y_A = P = \underline{\underline{900\,\text{kg}}}$$

Teil I:

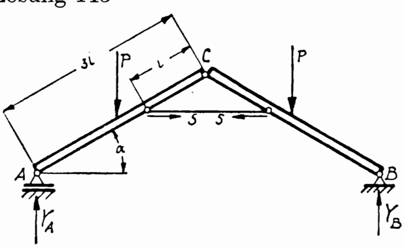
$$\sum M_B = 0:\quad P\cdot l - X_C\, 2l\cdot \operatorname{tg}\alpha + Y_C\cdot 2l = 0$$

Teil II:

$$\sum P_y = 0 \qquad -P + Y_A + Y_C = 0$$
$$Y_C = P - Y_A = \underline{\underline{0}}$$
$$\sum P_x = 0 \qquad X_A - X_C = 0$$
$$X_C = \frac{P}{2\operatorname{tg}\alpha} = \underline{\underline{900\,\text{kg}}}$$
$$X_A = \underline{\underline{900\,\text{kg}}}$$

Die Aktionskräfte haben entgegengesetzte Vorzeichen.

Lösung 145



Aus Symmetriegründen gilt für die Aktionskräfte:

$$-Y_A = -Y_B = P = \underline{\underline{800\,\text{kg}}}$$

$\sum M_C = 0$:

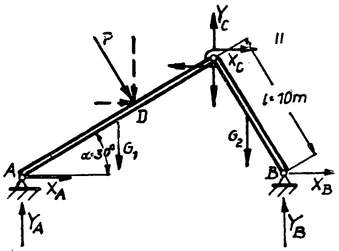
$$Y_A\cdot 3l\cos\alpha - P\,\frac{3}{2}\,l\cos\alpha - S\sin\alpha\cdot l = 0$$
$$P\left(3 - \frac{3}{2}\right) = S\operatorname{tg}\alpha$$
$$S = \frac{3}{2}\,\frac{P}{\operatorname{tg}\alpha} = \underline{\underline{2400\,\text{kg}}}$$

Lösung 146

$$\frac{AD}{DC}=\frac{3}{2};\quad AD+DC=\frac{l}{\operatorname{tg}\alpha}$$

$$AD=\frac{3}{2}DC=\frac{l}{\operatorname{tg}\alpha}-DC;\quad DC=\frac{2l}{5\operatorname{tg}\alpha}$$

$$AD=\frac{3l}{5\operatorname{tg}\alpha}$$



Gesamtsystem:

Teil I:

$\sum M_A=0$:

$$Y_B\cdot\frac{l}{\sin\alpha}-G_2\left(\frac{l}{\sin\alpha}-\frac{l}{2}\sin\alpha\right)-P\cdot\frac{3}{5}\,\frac{l}{\operatorname{tg}\alpha}$$

$$-G_1\cdot\frac{l}{2\operatorname{tg}\alpha}\cdot\cos\alpha=0$$

$$2Y_B-\frac{7}{4}G_2-\frac{3\sqrt{3}}{5}P-\frac{3}{4}G_1=0;\quad G_1=\frac{l\cdot q}{\operatorname{tg}\alpha}=1735\,\mathrm{kg}$$

$$G_2=l\cdot q=1000\,\mathrm{kg}$$

$$Y_B=\frac{7}{8}\cdot 1000+\frac{3\sqrt{3}}{10}\cdot 800+\frac{3}{8}\,1735=\underline{\underline{1940\,\mathrm{kg}}}$$

$\sum P_y=0$:

$$Y_B+Y_A-G_1-G_2-P\cdot\cos\alpha=0$$

$$Y_A=1735+1000+692-1940=\underline{\underline{1487\,\mathrm{kg}}}$$

$\sum P_x=0$: $\quad X_A=X_B+P\sin\alpha=0;$

Teil II:

$\sum M_C=0$: $\quad Y_B\cdot l\sin\alpha+X_B\cdot l\cdot\cos\alpha-G_2\cdot\frac{l}{2}\sin\alpha=0$

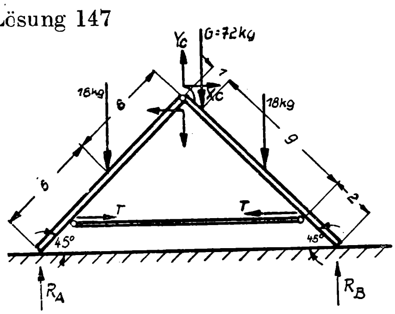
$$X_B=\operatorname{tg}\alpha\left(\frac{G_2}{2}-Y_B\right)=0{,}577\,(500-1940)=\underline{\underline{-831\,\mathrm{kg}}}$$

$$X_A=-X_B-P\sin\alpha=831-400=\underline{\underline{431\,\mathrm{kg}}}$$

$\sum P_y=0$: $\quad -Y_C+Y_B-G_2=0;\quad Y_C=1940-1000=\underline{\underline{\pm\,940\,\mathrm{kg}}}$

$\sum P_x=0$: $\quad X_C-X_B=0;\quad X_C=\underline{\underline{\mp\,831\,\mathrm{kg}}}$

Lösung 147



Gesamtsystem:

$\sum M_B=0$:

$$R_A\cdot 2\cdot 12\cdot\cos 45°-18\,(6+12)\cos 45°$$

$$-72\cdot 11\cdot\cos 45°-18\cdot 6\cdot\cos 45°=0$$

$$R_A=\frac{18\cdot 18}{24}+\frac{72\cdot 11}{24}+\frac{18\cdot 6}{24}=\underline{\underline{51\,\mathrm{kg}}}$$

$\sum P_y=0$:

$$R_A+R_B-2\cdot 18-72=0$$

$$R_B=108-51=\underline{\underline{57\,\mathrm{kg}}}$$

Teil AC: $\sum M_C = 0$: $R_A \cdot 12 \cdot \cos 45^\circ - 18 \cdot 6 \cdot \cos 45^\circ$

$$- T \cdot 10 \sin 45^\circ = 0$$

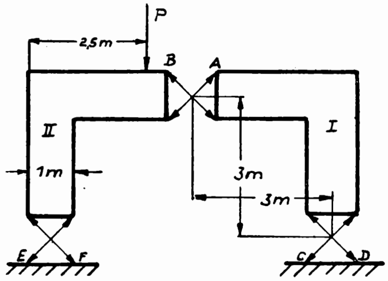
$$T = \underline{\underline{50{,}4\,\text{kg}}}$$

$\sum P_y = 0$: $R_A - Y_C - 18 = 0$

$Y_C = 51 - 18 = \underline{\underline{\pm\, 33\,\text{kg}}}$

$\sum P_x = 0$: $X_C - T = 0$; $X_C = \underline{\underline{\pm\, 50{,}4\,\text{kg}}}$

Lösung 148



Teil I:

$\sum P_{BD} = 0$: $R_D - R_B = 0$

$\sum P_{CA} = 0$: $R_A + R_C = 0$

$\sum M_{CD} = 0$: $R_A \cdot 3\sqrt{2} = 0$

$\underline{\underline{R_A = 0}}$; $\underline{\underline{R_C = 0}}$

Teil II:

$\sum P_{EA} = 0$:

$R_E - R_A - \frac{P}{\sqrt{2}} = 0$; $R_E = \underline{\underline{P\frac{\sqrt{2}}{2}}}$

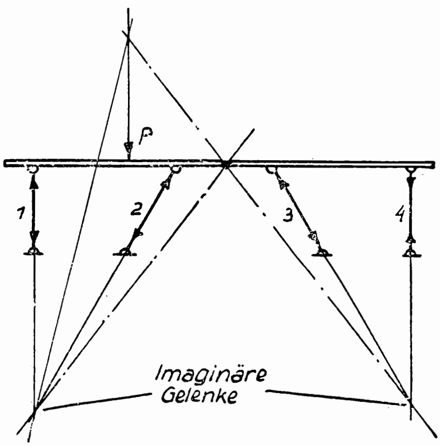
$\sum P_{FB} = 0$:

$R_F + R_B - \frac{P}{\sqrt{2}} = 0$; $R_F = \underline{\underline{P\frac{\sqrt{2}}{6}}}$

$\sum M_{EF} = 0$:

$R_B \cdot 3\sqrt{2} - P \cdot 2 = 0$; $R_B = \underline{\underline{\frac{P\sqrt{2}}{3}}}$; $R_D = \underline{\underline{\frac{P\sqrt{2}}{3}}}$

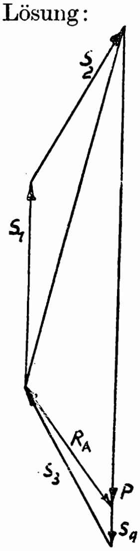
Lösung 149



Kraftmaßstab: 10 mm ≙ 2,25 t

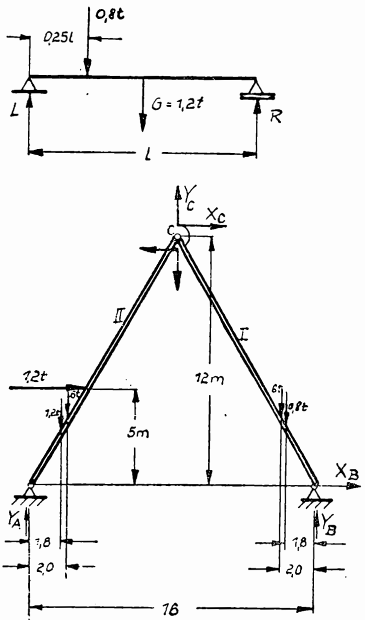
Dreigelenkbogen mit zwei imaginären Gelenken.

Graphische Lösung:



$S_1 = 6{,}25$ t	Druck	$S_4 = 1{,}25$ t	Zug
$S_2 = 5{,}77$ t	Druck	$X_A = 2{,}89$ t	
$S_3 = 5{,}77$ t	Druck	$Y_A = 3{,}75$ t	

Lösung 150



Auflagerreaktionen des Querbalkens:

$$R \cdot l - 1{,}2 \cdot \frac{1}{2} l - 0{,}8 \cdot 0{,}25 l = 0$$

$$R = 0{,}8\,\text{t}$$

$$L + R - 0{,}8 - 1{,}2 = 0; \quad L = 1{,}2\,\text{t}$$

Teil I:

1. $\sum M_B = 0: \quad Y_C \cdot 8 + X_C \cdot 12 - 6 \cdot 2 - 0{,}8 \cdot 1{,}8 = 0$
2. $\sum P_y = 0: \quad Y_C + Y_B - 6 - 0{,}8 = 0$
3. $\sum P_x = 0: \quad X_C + X_B = 0$

Teil II:

4. $\sum M_A = 0: \quad X_C \cdot 12 - Y_C \cdot 8 - 1{,}2 \cdot 5 - 6 \cdot 2 - 1{,}2 \cdot 1{,}8 = 0$
5. $\sum P_y = 0: \quad Y_A - Y_C - 6 - 1{,}2 = 0$
6. $\sum P_x = 0: \quad X_A - X_C + 1{,}2 = 0$

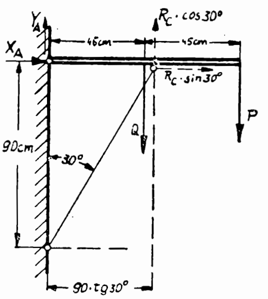
$$\begin{array}{ll} 1. & Y_C \cdot 8 + X_C \cdot 12 - 13{,}64 = 0 \\ 4. & \underline{-Y_C \cdot 8 + X_C \cdot 12 - 20{,}16 = 0} \\ & 2 \cdot X_C \cdot 10 - 33{,}8 = 0 \end{array}$$

$$X_C = \underline{\underline{\pm 1{,}4\,\text{t}}}; \quad Y_C = \underline{\underline{\mp 0{,}42\,\text{t}}}$$

Somit aus Gl. 2, 3, 5, 6: $X_A = \underline{\underline{0{,}2\,\text{t}}}$; $X_B = \underline{\underline{-1{,}4\,\text{t}}}$

$Y_A = \underline{\underline{6{,}78\,\text{t}}}$; $Y_B = \underline{\underline{7{,}22\,\text{t}}}$

Lösung 151



$$\sum M_A = 0: \quad R_C \cdot \cos 30° \cdot 90 \cdot \text{tg}\, 30° - Q \cdot 45 - P \cdot 90 = 0$$

$$R_C = Q + 2P = \underline{\underline{60\,\text{kg}}}$$

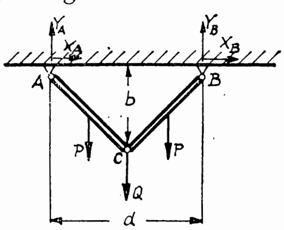
$$\sum P_y = 0: \quad Y_A + R_C \cdot \cos 30° - Q - P = 0$$

$$Y_A = 10 + 25 - 52 = \underline{\underline{-17\,\text{kg}}}$$

$$\sum P_x = 0: \quad X_A + R_C \cdot \sin 30° = 0$$

$$X_A = \underline{\underline{-30\,\text{kg}}}$$

Lösung 152



Aus Symmetriegründen:

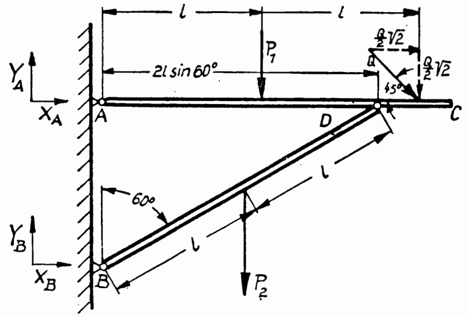
$$Y_A = Y_B = \underline{\underline{\frac{Q}{2} + P}}$$

$$\sum M_C = 0: \quad \left(\frac{Q}{2} + P\right) \frac{d}{2} - P \frac{d}{4} - X_B \cdot b = 0$$

$$X_B = \underline{\underline{\frac{Q + P}{4} \cdot \frac{d}{b}}}$$

$$\sum P_x = 0: \quad \underline{\underline{X_A = -X_B}}$$

Lösung 153



Gesamtsystem:

$\sum M_A = 0$:

$$X_B \cdot 2 \cdot l \cos 60° - P_2 \cdot l \cdot \sin 60° - P_1 \cdot l - \frac{Q}{2}\sqrt{2} \cdot 2l = 0$$

$\sum P_y = 0$:

$$Y_A + Y_B - P_1 - \frac{Q}{2}\sqrt{2} - P_2 = 0$$

$\sum P_x = 0$:

$$X_A + X_B + \frac{Q}{2}\sqrt{2} = 0$$

Teil AC:

$\sum M_D = 0$:

$$Y_A \cdot 2l \cdot \sin 60° - P_1 l(2\sin 60° - 1) + \frac{Q}{2}\sqrt{2} \cdot 2l(1 - \sin 60°) = 0$$

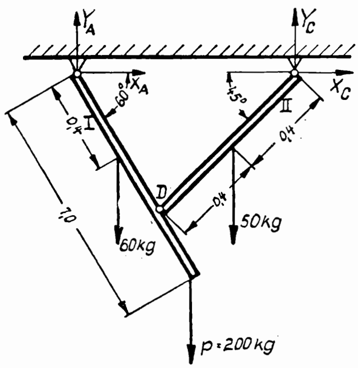
$$Y_A = \frac{P_1 \cdot 0{,}366 - Q \cdot 0{,}707 \cdot 0{,}134}{0{,}866} = \underline{\underline{6 \text{ kg}}}$$

$$X_B = \frac{P_2 \cdot 0{,}866 + P_1 + Q \cdot 1{,}41}{2 \cdot 0{,}5} = \underline{\underline{216 \text{ kg}}}$$

$$Y_B = 40 + 40 + 100 \cdot 0{,}707 - 6 = \underline{\underline{145 \text{ kg}}}$$

$$X_A = -210 - 100 \cdot 0{,}707 = \underline{\underline{-287 \text{ kg}}}$$

Lösung 154



$\sum P_y = 0$:

$$Y_A + Y_C - 60 - 50 - 200 = 0 \qquad (1)$$

$\sum P_x = 0$:

$$X_A + X_C = 0 \qquad (2)$$

$$AD = DC\frac{\sin 45°}{\sin 60°} = 0{,}65$$

$\sum M_D = 0$:

Teil I:

$$-0{,}65 \cdot Y_A \cdot \cos 60° - 0{,}65 X_A \sin 60° + 60 \cdot 0{,}25 \cdot \cos 60° - 0{,}35 \cdot 200 \cdot \cos 60° = 0 \qquad (3)$$

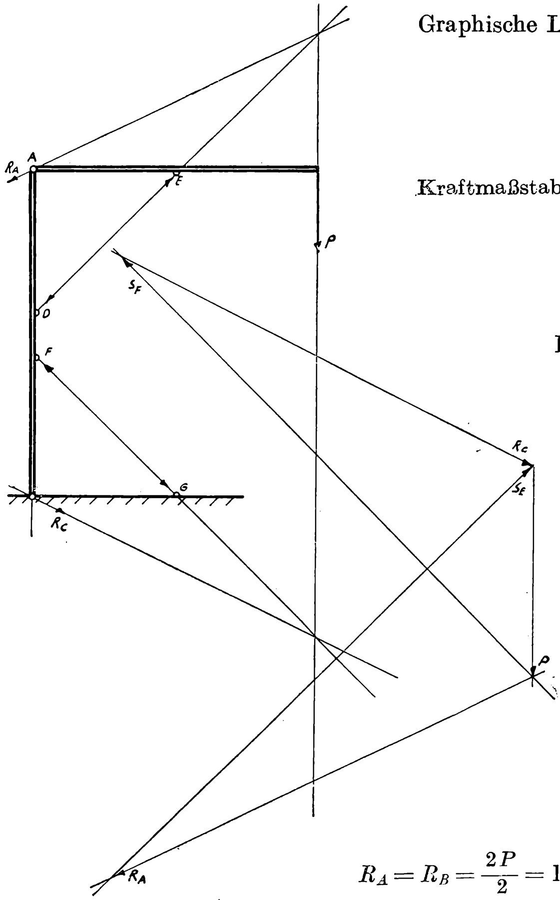
Teil II:

$$Y_C \cdot 0{,}8 \cdot \cos 45° - X_C \cdot 0{,}8 \cdot \sin 45° - 0{,}4 \cdot 50 \cos 45° = 0 \qquad (4)$$

1. $Y_A + Y_C = 310$
3. $-0{,}325 Y_A - 0{,}56 X_A = 27{,}5$
4., 2. $\underline{+\ 0{,}56 Y_C + 0{,}56 X_A = 14{,}1}$
5. $-0{,}325 Y_A + 0{,}56 Y_C = 41{,}6$

Aus 1 u. 5: $Y_C = \underline{\underline{160 \text{ kg}}}$; $Y_A = \underline{\underline{150 \text{ kg}}}$; $-X_A = X_C = \underline{\underline{135 \text{ kg}}}$

Lösung 155



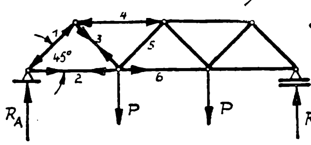
Graphische Lösung:

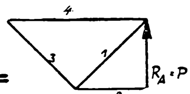
Kraftmaßstab: 10 mm $\triangleq$ 200 kg

Unter Beachtung des angegebenen Koordinatensystems ergibt sich:

$$S_E = -1410\ \text{kg}$$
$$S_F = -1410\ \text{kg}$$
$$X_c = 1000\ \text{kg}$$
$$Y_c = -500\ \text{kg}$$

Lösung 156

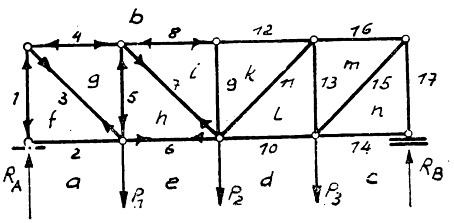




$$R_A = R_B = \frac{2P}{2} = 10\,\text{t}$$

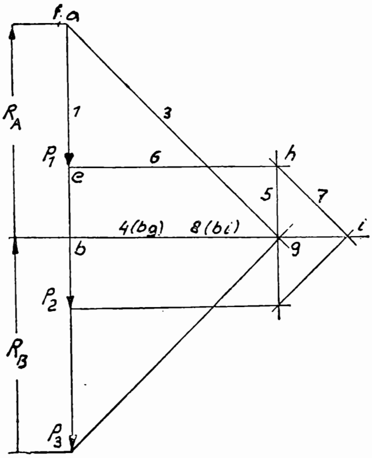
$$S_1 = -P\sqrt{2} = -14{,}1\,\text{t} \quad S_4 = -2P = -20\,\text{t}$$
$$S_2 = +P \quad = +10\,\text{t} \quad S_5 = \quad 0$$
$$S_3 = +P\sqrt{2} = +14{,}1\,\text{t} \quad S_6 = \quad 2P = \quad 20\,\text{t}$$

Lösung 157

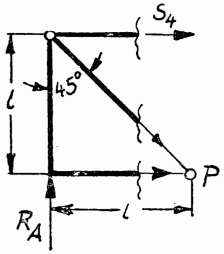


Graphische Lösung:

Kraftmaßstab: 10 mm ≙ 7,5 t



$S_1 = -15$ t
$S_2 = \quad 0$
$S_3 = +21{,}2$ t
$S_4 = -15$ t
$S_5 = -\ 5$ t
$S_6 = +15$ t
$S_7 = +7{,}1$ t
$S_8 = -20$ t
$S_9 = \quad 0$



Analytische Lösung:

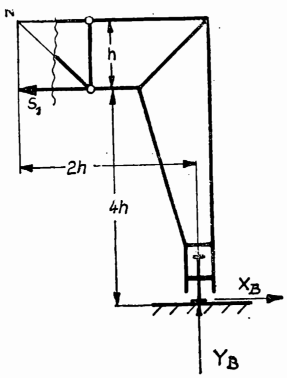
Ritterscher Schnitt.

Der Trennschnitt ist jeweils so zu legen, daß er drei Stäbe, die sich nicht in einem Punkt schneiden, zerlegt.

Z. B.: $\sum M_P = 0$: $\quad R_A \cdot l + S_4 - l = 0$

$$\underline{\underline{S_4 = -R_A = -15\,\text{t}}}$$

Lösung 158



Last greift unter dem Winkel α an:

$$X_B = H; \quad H = P \cdot \mathrm{tg}\,20° = 1{,}82\,\text{t}$$

$\sum M_A = 0$:

$$Y_B \cdot 4h + H \cdot 4h - P \cdot 2h = 0; \quad Y_B = \frac{P}{2} - H$$

Betrachten des abgeschnittenen rechten Teiles:

$\sum M_N = 0$: $\quad Y_B \cdot 2h + X_B \cdot 5h - S_1 \cdot h = 0$

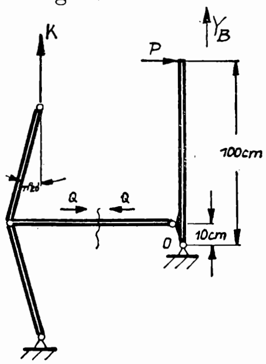
$$S_1 = P - 2H + 5H = \underline{\underline{10{,}46\,\text{t}}}$$

Last greift senkrecht an:

$\sum M_N = 0$: $\quad S_2 \cdot h - \frac{P}{2} \cdot 2h = 0$

$$S_2 = P = \underline{\underline{5\,\text{t}}}$$

Lösung 159



$$\sum M_0 = 0: \quad Q \cdot 10 = P \cdot 100; \quad Q = 10\,P$$

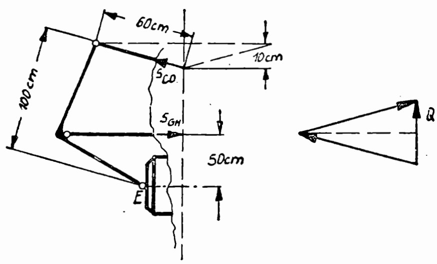
$$\operatorname{tg} 11^\circ 20' = \frac{Q}{2K}; \quad K = \frac{10\,P}{2 \cdot 0{,}2} = \underline{\underline{500\,\mathrm{kg}}}$$

Lösung 160

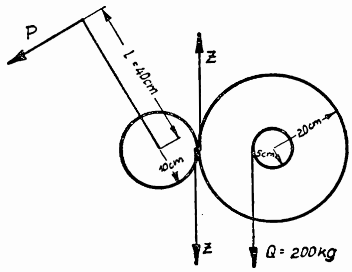
$$\frac{S_{C0}}{\frac{Q}{2}} = \frac{60}{10}$$

$$\sum M_E = 0: \quad S_{C0} \cdot 1 - S_{GH} \cdot 0{,}5 = 0$$

$$S_{GH} = 2 S_{C0} = 6\,Q = \underline{\underline{6\,\mathrm{t}}}$$



Lösung 161



$$Z \cdot 20 = 5 \cdot Q; \quad Z = \frac{5}{20}\,Q$$

$$40 \cdot P = 10 \cdot Z; \quad P = \frac{1}{4}\,Z = \frac{5}{4 \cdot 20}\,Q = \underline{\underline{12{,}5\,\mathrm{kg}}}$$

Lösung 162

$$\sum M_{0_2} = 0:$$

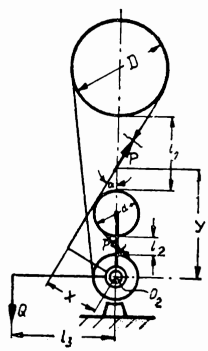
$$Q l_3 - P \cdot x - P \cdot \frac{d}{2} = 0$$

$$x = y \cdot \sin\alpha$$

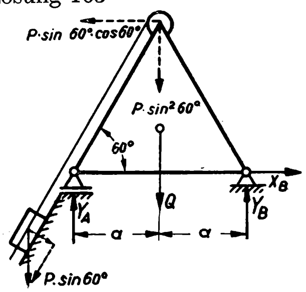
$$\sin\alpha = \frac{D+d}{2} : \frac{D+d}{2} + l_1 = \frac{1}{2}$$

$$y = \frac{d}{2} + l_2 + \frac{d}{2} + \frac{d}{2\sin\alpha} = 45\,\mathrm{cm}$$

$$Q = \frac{P\left(\frac{d}{2} + x\right)}{l_3} = \frac{18\left(\frac{15}{2} + \frac{45}{2}\right)}{45} = \underline{\underline{12\,\mathrm{kg}}}$$



Lösung 163



$\sum M_B = 0$:

$$P \cdot \sin 60° \cos 60° \cdot 2a \sin 60° + P \cdot a \sin^2 60° + Q \cdot a - Y_A \cdot 2a = 0$$

$$Y_A = \frac{3}{4} P + \frac{Q}{2} = \underline{\underline{480\,\text{kg}}}$$

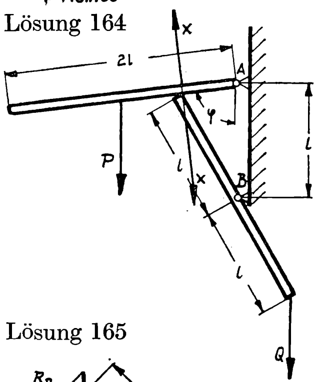
$\sum P_x = 0$: $\quad P \cdot \sin 60° \cdot \cos 60° - X_B = 0$;

$$X_B = \underline{\underline{208\,\text{kg}}}$$

$\sum P_y = 0$: $\quad P \sin^2 60° + Q - Y_A - Y_B = 0$;

$$Y_B = \underline{\underline{120\,\text{kg}}}$$

Lösung 164



$\sum M_B = 0$: $\quad X \cdot l \cdot \sin(90 - \varphi) = Q \cdot l \cdot \sin(180 - 2\varphi)$

$$X = Q \cdot \frac{\sin 2\varphi}{\cos \varphi}$$

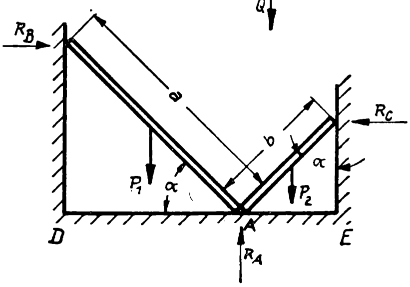
$\sum M_A = 0$: $\quad P \cdot l \cdot \sin \varphi = X \cdot 2 \cdot l \cdot \cos \varphi$

$$= Q \frac{\sin 2\varphi}{\cos \varphi} \cdot 2l \cdot \cos \varphi$$

$$Q = 2P$$

$$\cos \varphi = \frac{1}{8}; \quad \underline{\underline{\varphi = 82°\,50'}}$$

Lösung 165



$\sum M_A = 0$:

$$R_B \cdot \sin \alpha - P_1 \frac{\cos \alpha}{2} = 0$$

$$R_C \cdot \cos \alpha - P_2 \cdot \frac{\sin \alpha}{2} = 0$$

$\sum P_x = 0$: $\quad R_B - R_C = 0$

$\sum P_y = 0$: $\quad R_A - P_1 - P_2 = 0$

$$\operatorname{tg}^2 \alpha = \frac{P_1}{P_2}$$

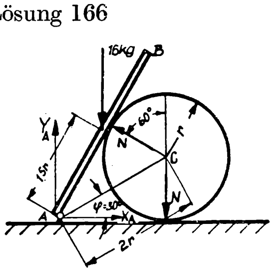
$$\sin \alpha = \frac{\operatorname{tg} \alpha}{\sqrt{1 + \operatorname{tg}^2 \alpha}}$$

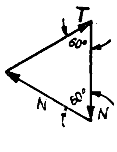
$$\cos \alpha = \frac{1}{\sqrt{1 + \operatorname{tg}^2 \alpha}}$$

$$DE = a \cos \alpha + b \sin \alpha$$

$$\underline{\underline{DE = \frac{a\sqrt{P_2} + b\sqrt{P_1}}{\sqrt{P_1 + P_2}}}}$$

Lösung 166





$$\sin \varphi = \frac{r}{2r} = \frac{1}{2}; \quad \varphi = 30°; \quad 2\varphi = 60°$$

$\sum M_A = 0$:

$$16 \cdot 1{,}5r \cdot \cos 60° - N \cdot 2r \cos 60° = 0$$

$$N = 12 \cdot \frac{\cos 60°}{\cos 30°} = 6{,}9\,\text{kg}$$

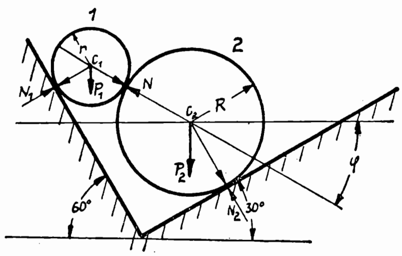
Gleichgewicht am Punkt C: $\underline{\underline{T = N}}$

$$X_A = N \cdot \sin 60° = \underline{\underline{6\,\text{kg}}}$$

$$Y_A = 16 - N \sin 30° = \underline{\underline{12{,}5\,\text{kg}}}$$

Die Aktionskräfte haben entgegengesetzte Vorzeichen.

Lösung 167



$\sum M_{C2} = 0:$

$$P_1(r+R)\cdot\cos\varphi - N_1\cos 60^\circ(r+R)\cos\varphi + N_1\sin 60^\circ(r+R)\sin\varphi = 0$$

$\sum M_{C1} = 0:$

$$P_2(r+R)\cos\varphi + N_2\sin 30^\circ(r+R)\sin\varphi - N_2\cos 30^\circ(r+R)\cos\varphi = 0$$

$\sum P_x = 0:\quad N_1\cdot\sin 60^\circ - N_2\cdot\sin 30^\circ = 0$

$$N_2 = \sqrt{3}\,N_1$$

$$P_1\cos\varphi = N_1\left(\frac{1}{2}\cos\varphi - \frac{\sqrt{3}}{2}\sin\varphi\right)$$

$$P_2\cos\varphi = N_1\left(\frac{3}{2}\cos\varphi - \frac{\sqrt{3}}{2}\sin\varphi\right)$$

$$\frac{P_1}{P_2} = \frac{\frac{1}{2}\cos\varphi - \frac{\sqrt{3}}{2}\sin\varphi}{\frac{3}{2}\cos\varphi - \frac{\sqrt{3}}{2}\sin\varphi}$$

$$\underline{\underline{\operatorname{tg}\varphi = \frac{1}{3}\sqrt{3}\cdot\frac{3P_1 - P_2}{P_1 - P_2}}}$$

Mit: $P_1 = 10\,\text{kg};\quad P_2 = 30\,\text{kg}:\quad \operatorname{tg}\varphi = 0$

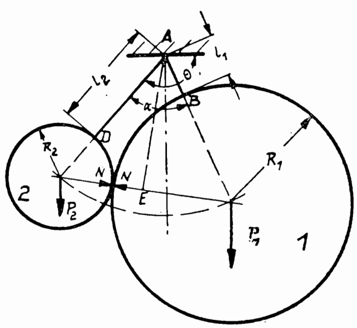
$$\underline{\underline{\varphi = 0}}$$

$$N_1 = \frac{P_1}{\cos 60^\circ} = \underline{\underline{20\,\text{kg}}}$$

$$N_2 = \frac{P_2}{\cos 30^\circ} = \underline{\underline{34{,}6\,\text{kg}}}$$

$$N = N_1\cdot\sin 60^\circ = \underline{\underline{17{,}3\,\text{kg}}}$$

Lösung 168



$\sum M_A = 0:$

1. $P_1(l_1+R_1)\sin(\alpha-\theta+90^\circ) = N(l_1+R_1)\cos\frac{\alpha}{2}$
2. $P_2(l_2+R_2)\sin(\theta-90^\circ) = N(l_2+R_2)\cos\frac{\alpha}{2}$

$$\underline{\underline{N = \pm\frac{P_2\cos\theta}{\cos\frac{\alpha}{2}}}}$$

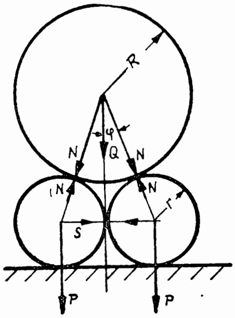
$$P_1\sin(\alpha-\theta+90^\circ) = P_2\sin(\theta-90^\circ);$$

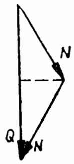
$$\underline{\underline{\operatorname{tg}\theta = -\frac{P_2 + P_1\cos\alpha}{P_1\sin\alpha}}}$$

Durch Projektion von T u. P auf die Gerade EA:

$$\underline{\underline{T_1 = P_1\frac{\sin\left(\theta-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}}}};\quad \underline{\underline{T_2 = P_2\cdot\frac{\sin\left(\theta-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}}}}$$

Lösung 169





Druck jedes Zylinders auf die Fläche: $P+\frac{Q}{2}$

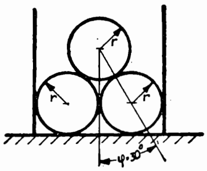
$$\sin\varphi=\frac{r}{r+R}\,; \qquad \cos\varphi=\frac{Q}{2N}$$

$$N_x=S=N\sin\varphi$$

$$N=\frac{Q}{2\cos\varphi}=\frac{Q}{2}\cdot\frac{1}{\sqrt{1-\left(\frac{r}{r+R}\right)^2}}=\underline{\underline{\frac{Q\,(R+r)}{2\sqrt{R^2+2rR}}}}$$

$$S=\underline{\underline{\frac{Q\cdot r}{2\sqrt{R^2+2rR}}}}$$

Lösung 170



$$\sin\varphi=\frac{1}{2}\,; \quad \varphi=30^\circ$$

Horizontaldruck: $H=\frac{P}{2}\,\mathrm{tg}\,30^\circ=\underline{\underline{34{,}6\ \mathrm{kg}}}$

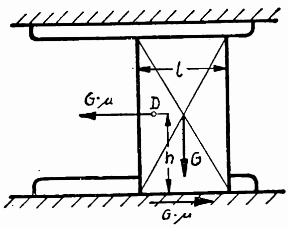
Vertikaldruck aus Symmetriegründen:

$$V=\frac{3}{2}P=\underline{\underline{180\,\mathrm{kg}}}$$

Lösung 171

$$M_{\mathrm{Reib}}=2\cdot Q\cdot\mu\cdot r=M_{\mathrm{Antr}} \qquad Q=\frac{M}{2\mu\cdot r}=\underline{\underline{800\ \mathrm{kg}}}$$

Lösung 172



$$\sum M_D=0: \qquad G\cdot\mu\cdot h=G\frac{l}{2}$$

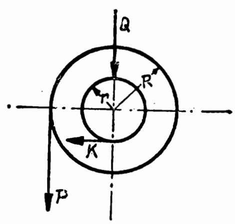
$$h=\frac{l}{2\mu}=\underline{\underline{0{,}8\ \mathrm{m}}}$$

Lösung 173

$$K=\mu\,(P+Q); \quad M_{\mathrm{Reib}}=K\cdot r; \quad r=\frac{R}{2}$$

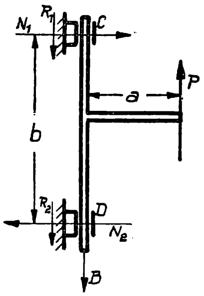
$$K\cdot r=P\cdot R; \quad (P+Q)\,\mu=2P$$

$$\underline{\underline{\frac{Q}{P}=\frac{2}{\mu}-1=39}}$$



Lösung 174 Bolzenkraft: $K=P\cdot\frac{a}{b}+\frac{P}{\mu}$; $\mu<\frac{a}{b}$; $K\cong\frac{P}{\mu}=\underline{\underline{2000\,\mathrm{kg}}}$

Lösung 175



$$P = R_1 + R_2 + B; \quad R = N \cdot \mu$$

$$\sum M_D = 0: \quad P \cdot a - N_1 \cdot b = 0$$

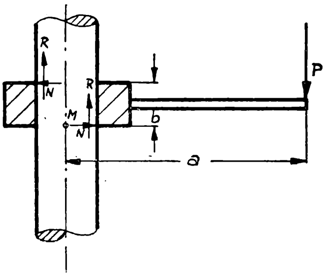
$$\sum M_C = 0: \quad P \cdot a - N_2 \cdot b = 0$$

$$N_1 = N_2 = \frac{P \cdot a}{b}$$

$$P = \mu \cdot \frac{2 P a}{b} + B$$

$$P = \frac{B \cdot b}{b - 2 a \mu} = \underline{\underline{186\,\text{kg}}}$$

Lösung 176



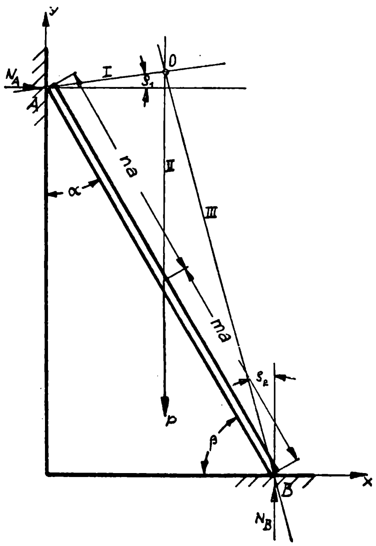
$$\sum M_M = 0: \quad P \cdot a - N \cdot b - R \cdot \frac{d}{2} + R \cdot \frac{d}{2} = 0$$

$$N = P \frac{a}{b}$$

$$\sum P_y = 0: \quad 2R = P = 2N \cdot \mu = 2P \frac{a}{b} \cdot \mu$$

$$a = \frac{b}{2\mu} = \underline{\underline{10\,\text{cm}}}$$

Lösung 177



Gleichungen der Geraden:

I. $\quad y_{\mathrm{I}} = a(m+n)\sin\beta + \operatorname{tg}\varrho_1 x_{\mathrm{I}}$

II. $\quad x_{\mathrm{II}} = an\cos\beta$

III. $\quad y_{\mathrm{III}} = \dfrac{a(m+n)\cos\beta - x_{\mathrm{III}}}{\operatorname{tg}\varrho_2}$

Im Punkt 0 müssen sich die drei Kraftwirkungslinien schneiden, damit die zugehörigen Kräfte im Gleichgewicht sind, d. h.:

$$x_{\mathrm{I}} = x_{\mathrm{II}} = x_{\mathrm{III}} = an\cos\beta$$

$$y_{\mathrm{I}} = y_{\mathrm{III}} \qquad\qquad \mu = \operatorname{tg}\varrho$$

Daraus folgt:

$$\operatorname{tg}\beta = \frac{m - n\mu_1\mu_2}{(m+n)\mu_2}; \quad \operatorname{tg}\alpha = \frac{1}{\operatorname{tg}\beta}$$

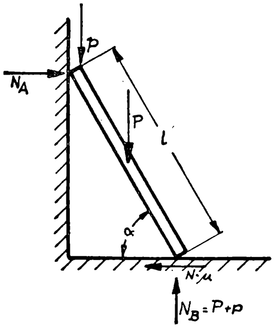
$$\underline{\underline{\operatorname{tg}\alpha = \frac{(m+n)\mu_2}{m - n\mu_1\mu_2}}}$$

Die Drücke N_A und N_B erhält man aus den Momentengleichungen um A und B mit $R = \mu \cdot N$

$$\underline{\underline{N_A = \frac{p \cdot \mu_2}{1 + \mu_1\mu_2}}}$$

$$\underline{\underline{N_B = \frac{p}{1 + \mu_1\mu_2}}}$$

Lösung 178

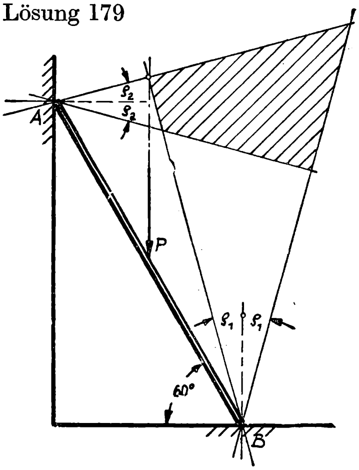


$$\sum M_A = 0:$$

$$(P + p)\, l \cos\alpha - (P + p)\, l \sin\alpha \cdot \mu - P \cdot \frac{l}{2} \cos\alpha = 0$$

$$\underline{\underline{\operatorname{tg}\alpha \geqq \frac{P + 2p}{2\mu\,(P + p)}}}$$

Lösung 179

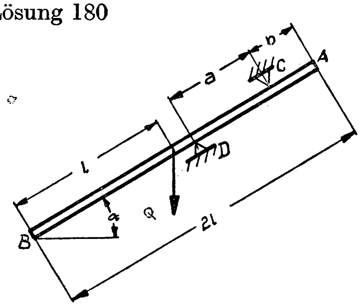


Graphische Lösung:

Die Wirkungslinie von P muß noch im Reibungsfeld liegen

$$\underline{\underline{BP = \frac{1}{2} AB}}$$

Lösung 180



$$\sum P_{AB} = 0: \quad (C + D)\,\mu = Q \sin\alpha$$

$$\sum M_C = 0: \quad D = Q \cos\alpha \cdot \frac{l - b}{a}$$

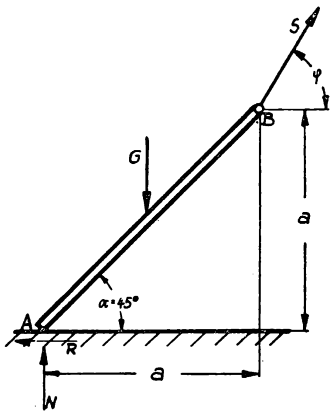
$$\sum M_D = 0: \quad C = Q \cos\alpha \cdot \frac{l - a - b}{a}$$

Somit: $2l \geqq 2b + a\left(1 + \frac{\operatorname{tg}\alpha}{\mu}\right); \quad l > a + b$

Falls $\operatorname{tg}\alpha = \mu$: $l = b + a$

Für $l < b + a$ herrscht kein Gleichgewicht, da in C und D nur Druckkräfte übertragen werden können.

Lösung 181



$$\sum M_B = 0: \quad -N \cdot a - R \cdot a + G \cdot \frac{a}{2} = 0$$

$$R = \mu \cdot N$$

$$N = \frac{G}{2(1+\mu)}$$

$$\sum P_x = 0: \quad -N \cdot \mu + S \cos\varphi = 0$$

$$\sum P_y = 0: \quad N - G + S \sin\varphi = 0$$

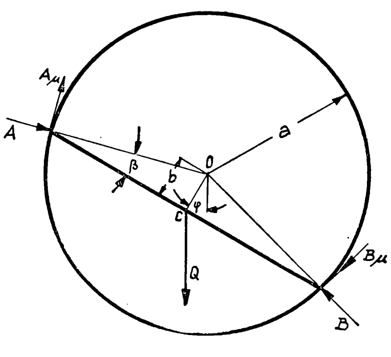
$$\frac{G \cdot \mu}{\cos\varphi \cdot 2(1+\mu)} = S$$

$$\frac{-G + G \cdot 2(1+\mu)}{2(1+\mu)\sin\varphi} = S = \frac{G(2\mu+1)}{2(1+\mu)\sin\varphi}$$

$$\frac{G \cdot \mu}{\cos\varphi \cdot 2(1+\mu)} = \frac{G(2\mu+1)}{2(1+\mu)\sin\varphi}$$

$$\operatorname{tg}\varphi = \frac{1+2\mu}{\mu}; \quad \underline{\underline{\operatorname{tg}\varphi = 2 + \frac{1}{\mu}}}$$

Lösung 182



$$\sum M_B = 0:$$

$$Q \cdot \frac{l}{2} \cos\varphi - A \cdot \mu \cdot l \cos\beta - A \cdot l \cdot \sin\beta = 0$$

$$A = \frac{Q}{2} \cdot \frac{\cos\varphi}{\sin\beta + \mu\cos\beta}$$

$$\sum M_A = 0:$$

$$Q \cdot \frac{l}{2} \cos\varphi + B \cdot \mu \cdot l \cos\beta - Bl \cdot \sin\beta = 0$$

$$B = \frac{Q}{2} \cdot \frac{\cos\varphi}{\sin\beta - \mu\cos\beta}$$

$$\sum M_0 = 0:$$

$$\mu(A+B)a = Q \cdot b \cdot \sin\varphi$$

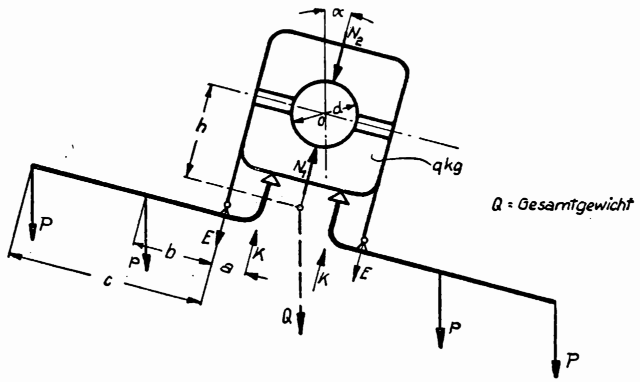
$$\frac{\mu \cdot a}{2} \cos\varphi \left(\frac{1}{\sin\beta + \mu\cos\beta} + \frac{1}{\sin\beta - \mu\cos\beta}\right)$$

$$= b \sin\varphi$$

$$\sin\beta = \frac{b}{a}; \quad \cos\beta = \sqrt{1 - \sin^2\beta}$$

$$\underline{\underline{\operatorname{ctg}\varphi = \left(\frac{b}{a}\right)^2 \cdot \frac{1+\mu^2}{\mu} - \mu}}$$

Lösung 183



$$N_1 = 2K - q \cdot \cos\alpha; \quad N_2 = 2E + \big(Q - 2(P+p) - q\big) \cdot \cos\alpha$$

$$\sum M_E = 0: \quad K \cdot a = p \cdot \cos\alpha \cdot b + P \cdot \cos\alpha \cdot c; \qquad K = \frac{p \cdot b \cdot \cos\alpha + P \cdot c \cdot \cos\alpha}{a}$$

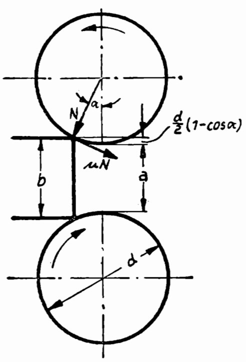
$$\sum M_K = 0: \quad E \cdot a = p \cdot \cos\alpha (a+b) + P \cos\alpha (a+c); \quad E = \frac{\cos\alpha}{a}[p(a+b) + P(a+c)]$$

$$\sum M_0 = 0: \quad M_{\text{Reib}} = Q \cdot h \cdot \sin\alpha = (N_1 + N_2) \cdot \mu \cdot \frac{d}{2}$$

$$Q \cdot h \cdot \operatorname{tg}\alpha = \mu \cdot \frac{d}{2}\left[2\left\{p\frac{b}{a} + P\frac{c}{a}\right\} - q + 2\left\{p\frac{a+b}{a} + P\frac{a+b}{a}\right\} + \big(Q - 2(P+p) - q\big)\right]$$

$$\underline{\underline{Q \cdot h \cdot \operatorname{tg}\alpha = \mu \cdot \frac{d}{2}\left[4\left\{p\frac{b}{a} + P\frac{c}{a}\right\} + Q - 2q\right]}}; \quad \underline{\underline{\mu = 0{,}0057}}$$

Lösung 184



$$\text{Grenzfall:} \ \operatorname{tg}\alpha = \frac{\mu N}{N} = \mu$$

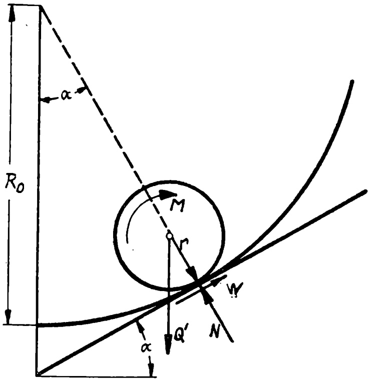
$$b = a + 2\frac{d}{2}(1 - \cos\alpha)$$

Da α klein ist, kann die Reihenentwicklung von $\cos\alpha$ nach dem zweiten Glied abgebrochen werden.

$$b = a + d\left(1 - 1 + \frac{\alpha^2}{2}\right) = a + d \cdot \frac{\mu^2}{2}$$

$$b \leqq 0{,}5 + \frac{50}{200} = \underline{\underline{0{,}75 \text{ cm}}}$$

Lösung 185



$$M = R\,(P - P_1); \quad Q' = Q + P_1 + P$$

1. $M - Q' r \sin\alpha = 0 \qquad (\sum M_N = 0)$

$$M - W \cdot r = 0 \qquad (\sum M_0 = 0)$$

$$W = R \cdot \mu = Q' \cdot \cos\alpha \cdot \mu$$

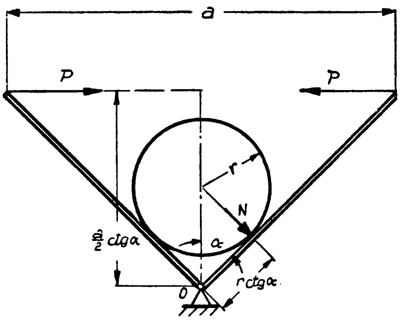
$$Q' \cdot r \sin\alpha = Q' \cos\alpha \cdot \mu \cdot r$$

2. $\underline{\operatorname{tg}\alpha = \mu}$

1., 2. $R\,(P - P_1) - (Q + P_1 + P) \cdot r \cdot \dfrac{\mu}{\sqrt{1+\mu^2}} = 0$

$$\underline{\underline{P_1 = \frac{P\,[R\sqrt{1+\mu^2} - \mu \cdot r] - Q \cdot r \cdot \mu}{R\sqrt{1+\mu^2} + \mu \cdot r}}}$$

Lösung 186



$$\sum M_0 = 0: \quad P \cdot \frac{a}{2} \operatorname{ctg}\alpha = N \cdot r \cdot \operatorname{tg}\alpha$$

$$\sum P_y = 0: \quad Q = 2N\,(\sin\alpha + \mu\cos\alpha)$$

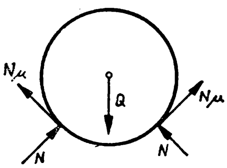
$$Q = \frac{P \cdot a}{r}\,(\sin\alpha + \mu\cos\alpha)$$

Für Abwärtsbewegung:

$$P = \frac{Q\,r}{a} \cdot \frac{1}{\sin\alpha + \mu\cos\alpha}$$

Bei Aufwärtsbewegung wirkt $N \cdot \mu$ entgegengesetzt, also:

$$P = \frac{Q\,r}{a} \cdot \frac{1}{\sin\alpha - \mu\cos\alpha}$$

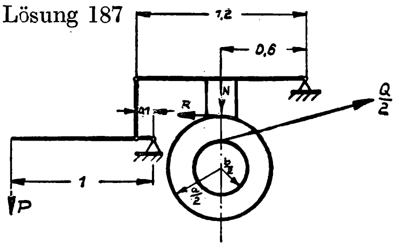


Gesamt:

$$\frac{r}{a} \cdot \frac{Q}{\sin\alpha + \mu\cos\alpha} \leqq P \leqq \frac{Q}{\sin\alpha - \mu\cos\alpha} \cdot \frac{r}{a}$$

gilt für $\operatorname{tg}\alpha > \mu$, da sonst die obere Grenze negativ wird. Für $\operatorname{tg}\alpha \leqq \mu$ fällt also die obere Grenze weg. P kann dann unendlich groß werden.

Lösung 187

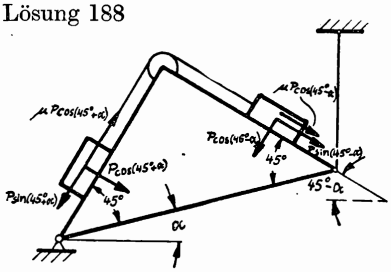


$$\frac{Q}{2} \cdot \frac{b}{2} = \frac{R \cdot a}{2}; \quad R = \mu \cdot N$$

$$P \cdot 1 = \frac{N \cdot 0{,}6}{1{,}2}\,0{,}1; \quad N = 20\,P$$

$$P = \frac{Q \cdot b}{40 \cdot a \cdot \mu} = \underline{\underline{20\ \text{kg}}}$$

Lösung 188



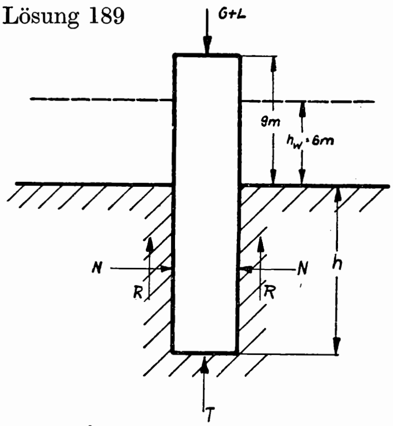
Gleichgewicht am Seil:

$$P\{\sin(45^\circ+\alpha)-\mu\cos(45^\circ+\alpha)\}$$
$$=P\{\sin(45^\circ-\alpha)+\mu\cos(45^\circ-\alpha)\}$$

Anwendung des Additionstheorems:

$$\frac{\sqrt{2}}{2}\cos\alpha+\frac{\sqrt{2}}{2}\sin\alpha-\mu\frac{\sqrt{2}}{2}\cos\alpha+\mu\frac{\sqrt{2}}{2}\sin\alpha$$
$$=\frac{\sqrt{2}}{2}\cos\alpha-\frac{\sqrt{2}}{2}\sin\alpha+\mu\frac{\sqrt{2}}{2}\cos\alpha+\mu\frac{\sqrt{2}}{2}\sin\alpha$$
$$\sin\alpha=\mu\cos\alpha;\qquad \underline{\underline{\operatorname{tg}\alpha=\mu}}$$

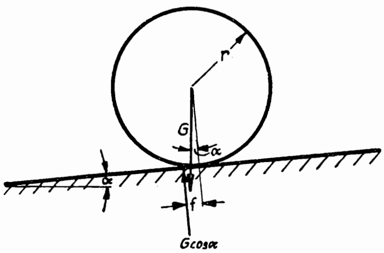
Lösung 189



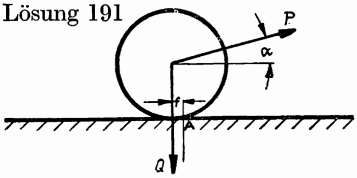
$$G+L=R+T$$
$$G+L=(9+h)\cdot 8+150$$
$$=8h+222$$
$$T=(6+1{,}8h)\cdot 3{,}5$$
$$=6{,}3h+21$$
$$R=(6+0{,}9h)\cdot\mu\cdot 7h$$
$$=1{,}134h^2+7{,}56h$$
$$1{,}134h^2+5{,}86h=201$$
$$\underline{\underline{h=11\text{ m}}}$$

Lösung 190

$$\sum M_A=0:\qquad G\cdot r\cdot\sin\alpha=f\cdot G\cdot\cos\alpha$$
$$\operatorname{tg}\alpha=\frac{f}{r}=0{,}001$$
$$\underline{\underline{\alpha=3'26''}}$$



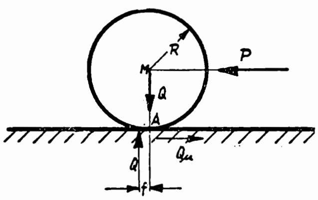
Lösung 191



$$\sum M_A=0:\qquad (Q-P\sin\alpha)\cdot f=P\cdot r\cdot\cos\alpha$$
$$P=\frac{Q\cdot f}{r\cos\alpha+f\sin\alpha}=\underline{\underline{5{,}72\text{ kg}}}$$

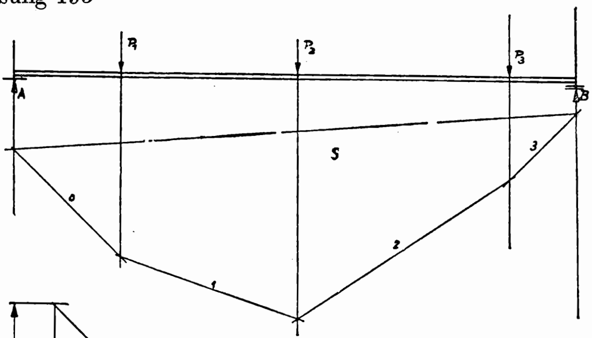
Lösung 192

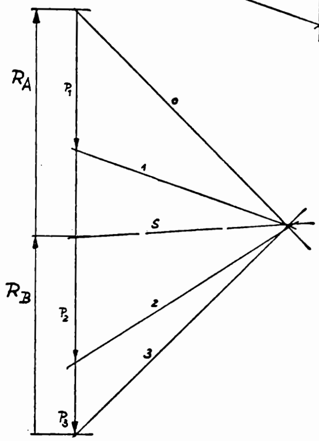
$$\sum M_M=0:\quad Q\cdot f<Q\cdot\mu\cdot R$$
$$\frac{f}{R}<\mu$$
$$\sum M_A=0:\quad P\cdot R=Q\cdot f;\qquad \underline{\underline{P=Q\cdot\frac{f}{R}}}$$



5. Graphische Statik

Lösung 193

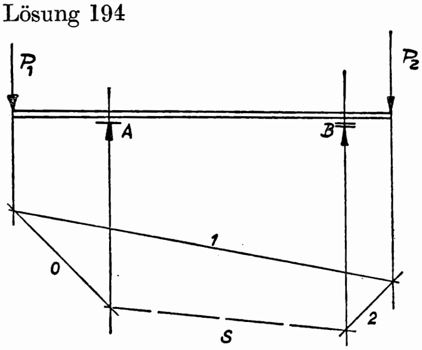


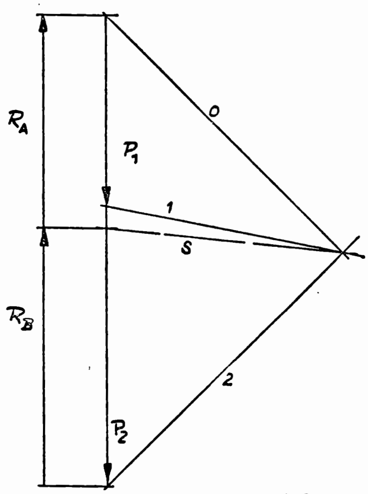


Kraftmaßstab:
10 mm ≙ 1 t

$R_A = 3{,}25$ t
$R_B = 2{,}75$ t

Lösung 194

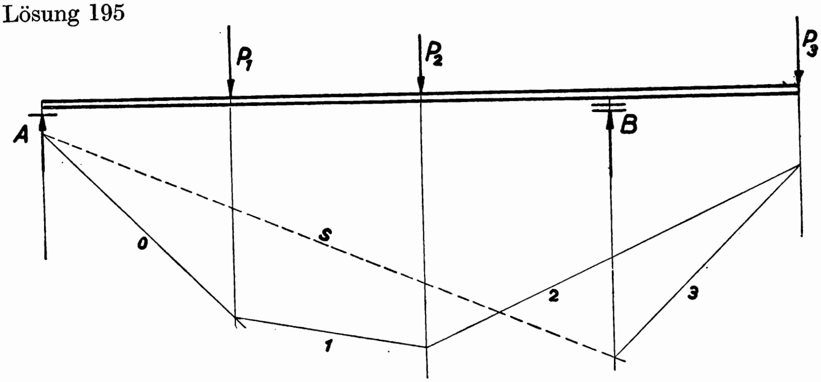




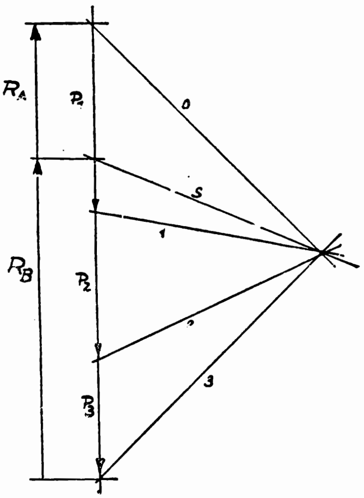
Kraftmaßstab:
10 mm ≙ 0,75 t

$R_A = 2{,}2$ t
$R_B = 2{,}8$ t

Lösung 195

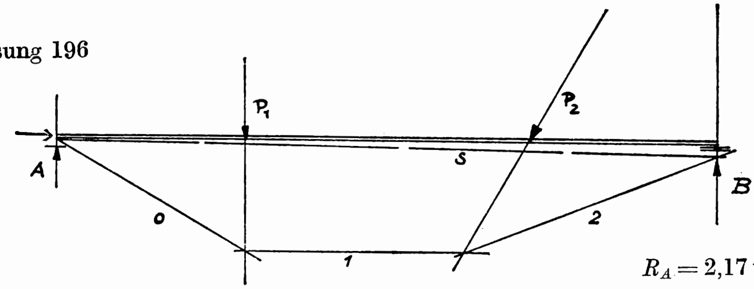


Kraftmaßstab: 10 mm ≙ 375 kg



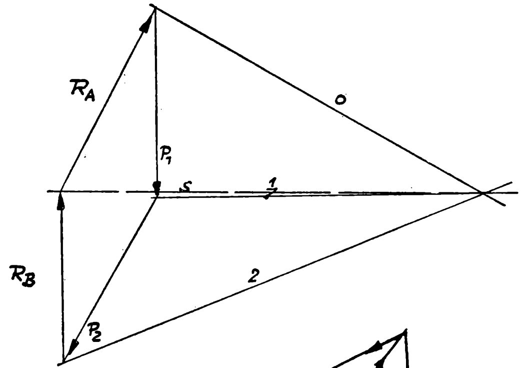
$R_A = 0{,}73$ t
$R_B = 1{,}67$ t

Lösung 196

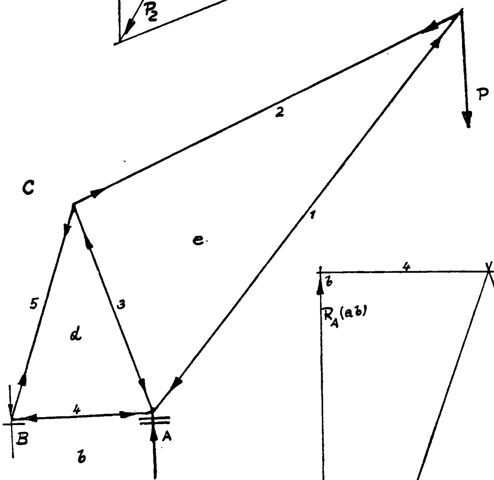


$R_A = 2{,}17$ t
$R_B = 1{,}81$ t

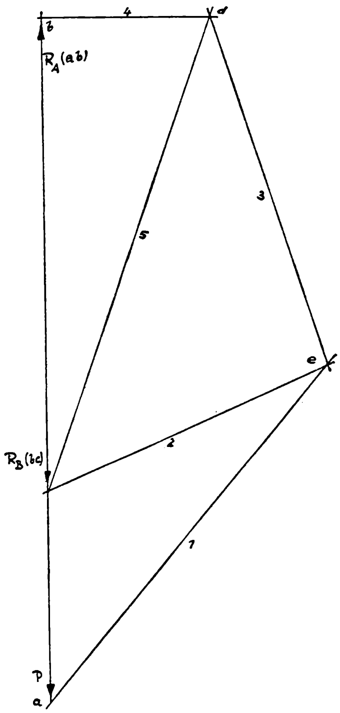
Kraftmaßstab:
10 mm ≙ 0,75 t



Lösung 197



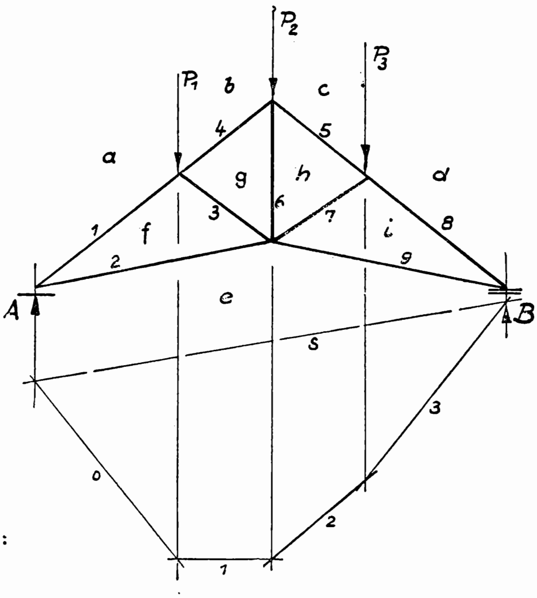
Kraftmaßstab:
10 mm ≙ 6 t



$S_1 = -16{,}4$ t
$S_2 = +11{,}5$ t
$S_3 = -14{,}3$ t
$S_4 = -\ \ 6$ t
$S_5 = +19$ t

$R_A = 26$ t
$R_B = 18$ t

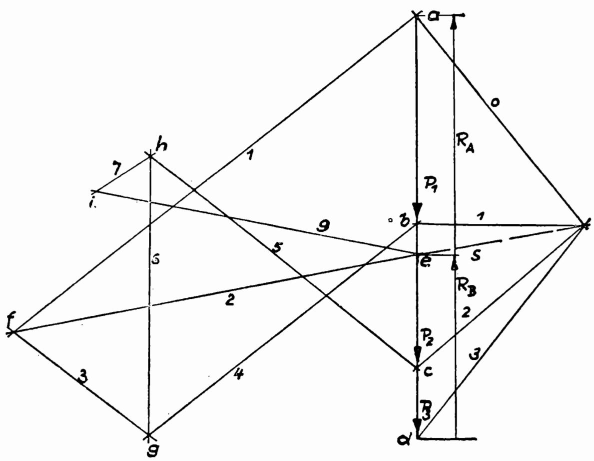
Lösung 198



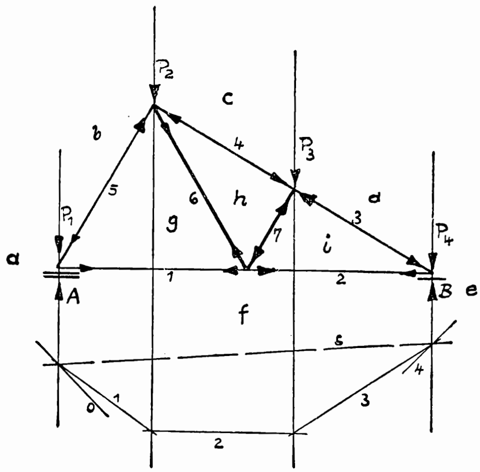
Kraftmaßstab:
10 mm ≙ 1 t

$S_1 = -7{,}3$ t
$S_2 = +5{,}8$ t
$S_3 = -2{,}44$ t
$S_4 = -4{,}7$ t
$S_5 = -4{,}7$ t
$S_6 = +3{,}9$ t
$S_7 = -0{,}81$ t
$S_8 = -5{,}5$ t
$S_9 = +4{,}4$ t

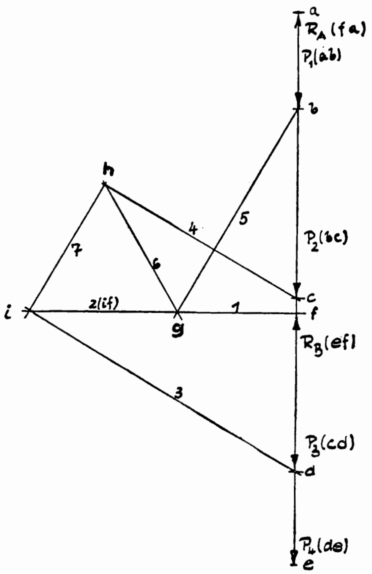
$R_A = 3{,}4$ t
$R_B = 2{,}6$ t



Lösung 199



Kraftmaßstab: 10 mm ≙ 750 kg



$S_1 = +1{,}3\text{ t}$

$S_2 = +3{,}03\text{ t}$

$S_3 = -3{,}5\text{ t}$

$S_4 = -2{,}5\text{ t}$

$S_5 = -2{,}6\text{ t}$

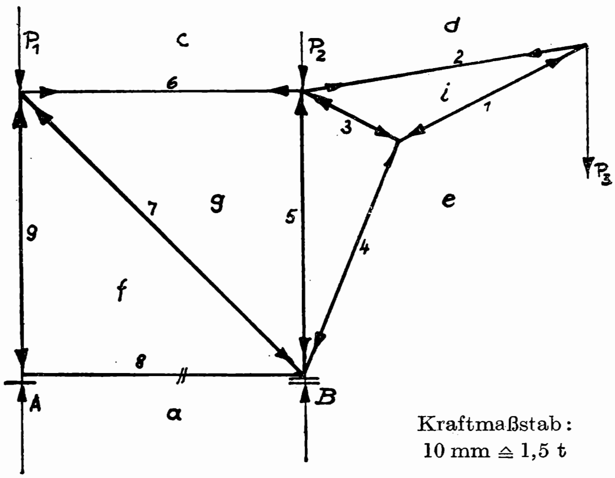
$S_6 = +1{,}73\text{ t}$

$S_7 = -1{,}73\text{ t}$

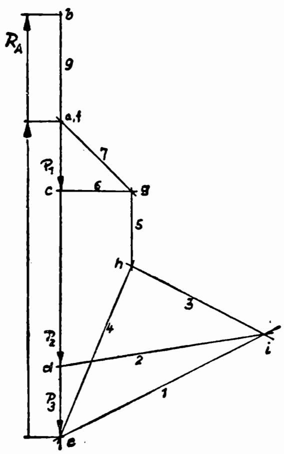
$R_A = 3{,}25\text{ t}$

$R_B = 2{,}75\text{ t}$

Lösung 200



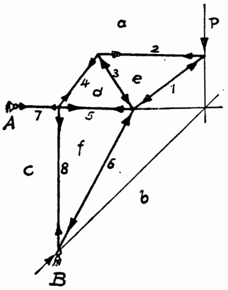
Kraftmaßstab:
10 mm ≙ 1,5 t



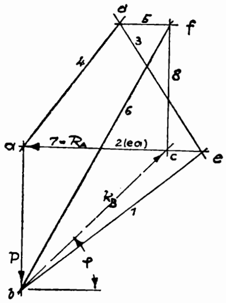
$S_1 = -6{,}0$ t
$S_2 = +5{,}1$ t
$S_3 = -3{,}13$ t
$S_4 = -5{,}4$ t
$S_5 = -2{,}0$ t
$S_6 = +2{,}0$ t
$S_7 = -2{,}83$ t
$S_8 = \quad 0$ t
$S_9 = -3{,}0$ t

$R_A = 3$ t
$R_B = 9$ t

Lösung 201



Kraftmaßstab:
10 mm ≙ 1 t



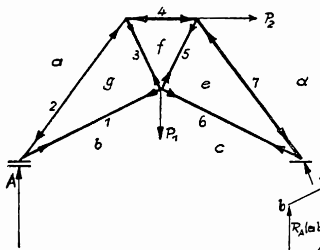
$S_1 = -3{,}33$ t $\quad S_5 = +0{,}67$ t

$S_2 = +2{,}67$ t $\quad S_6 = -4{,}47$ t $\quad R_A = 2$ t

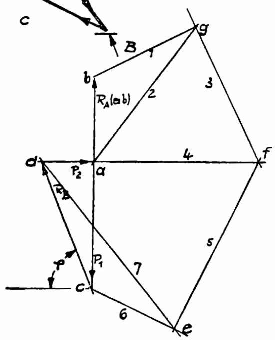
$S_3 = -2{,}4$ t $\quad S_7 = +2$ t $\quad R_B = 2{,}83$ t

$S_4 = +2{,}4$ t $\quad S_8 = +2$ t $\quad \varphi = 45°$

Lösung 202



Kraftmaßstab:
10 mm ≙ 1,33 t



$S_1 = +2$ t $\quad S_5 = +3{,}6$ t $\quad R_A = 1{,}5$ t

$S_2 = -3$ t $\quad S_6 = +1{,}57$ t $\quad R_B = 2{,}7$ t

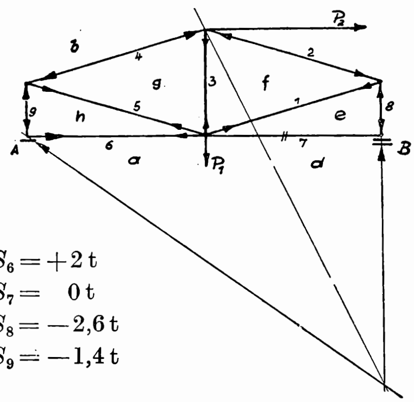
$S_3 = +2{,}7$ t $\quad S_7 = -4$ t $\quad \varphi = 68°$

$S_4 = -3$ t

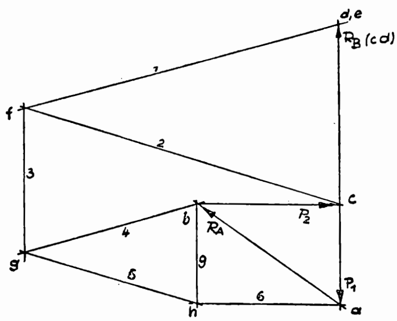
Lösung 203

$X_A = -2$ t
$Y_A = 1{,}4$ t
$Y_B = 2{,}6$ t

$S_1 = +4{,}5$ t	$S_6 = +2$ t
$S_2 = -4{,}5$ t	$S_7 = 0$ t
$S_3 = +2$ t	$S_8 = -2{,}6$ t
$S_4 = -2{,}44$ t	$S_9 = -1{,}4$ t
$S_5 = +2{,}44$ t	



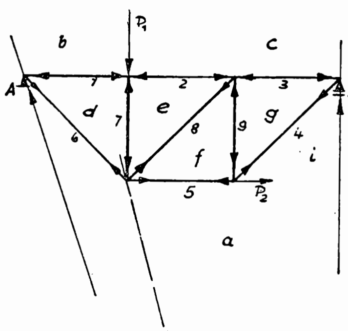
Kraftmaßstab:
10 mm ≙ 1 t

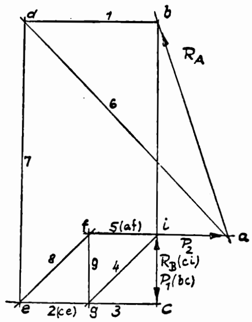


Lösung 204

$X_A = -1$ t
$Y_A = 3$ t
$Y_B = 1$ t

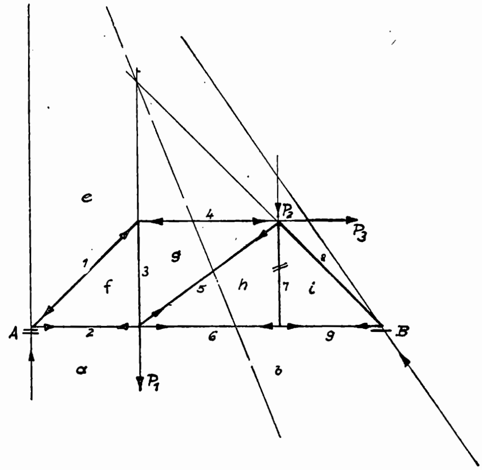
$S_1 = -2$ t
$S_2 = -2$ t
$S_3 = -1$ t
$S_4 = +1{,}41$ t
$S_5 = +2$ t
$S_6 = +4{,}24$ t
$S_7 = -4$ t
$S_8 = +1{,}41$ t
$S_9 = -1$ t





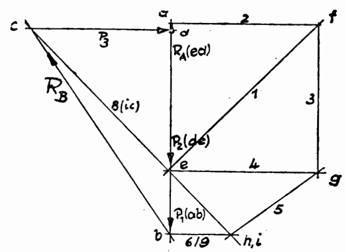
Kraftmaßstab: 10 mm ≙ 1 t

Lösung 205



$Y_A = 2{,}1\ \mathrm{t}$
$X_B = -2\ \mathrm{t}$
$Y_B = 2{,}9\ \mathrm{t}$

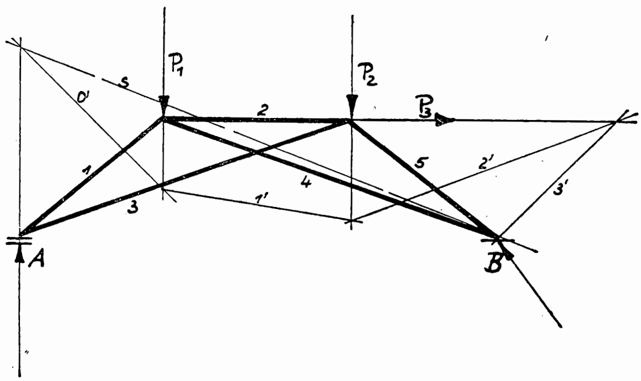
Kraftmaßstab: 10 mm ≙ 1 t



$S_1 = -2{,}97\ \mathrm{t}$
$S_2 = +2{,}1\ \mathrm{t}$
$S_3 = +2{,}1\ \mathrm{t}$
$S_4 = -2{,}1\ \mathrm{t}$
$S_5 = +1{,}5\ \mathrm{t}$

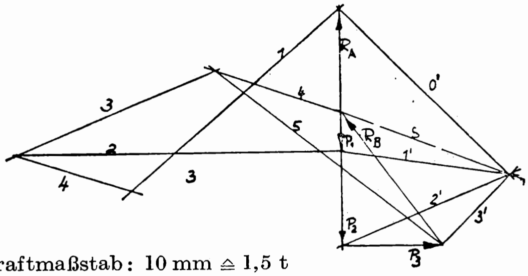
$S_6 = +0{,}9\ \mathrm{t}$
$S_7 = \quad 0\ \mathrm{t}$
$S_8 = -4{,}1\ \mathrm{t}$
$S_9 = +0{,}9\ \mathrm{t}$

Lösung 206



$Y_A = 2{,}2\ \mathrm{t}$
$X_B = -2\ \mathrm{t}$
$Y_B = 2{,}8\ \mathrm{t}$

$S_1 = -6$ t
$S_2 = -7$ t
$S_3 = +4{,}9$ t
$S_4 = +2{,}53$ t
$S_5 = -5{,}7$ t



Kraftmaßstab: 10 mm ≙ 1,5 t

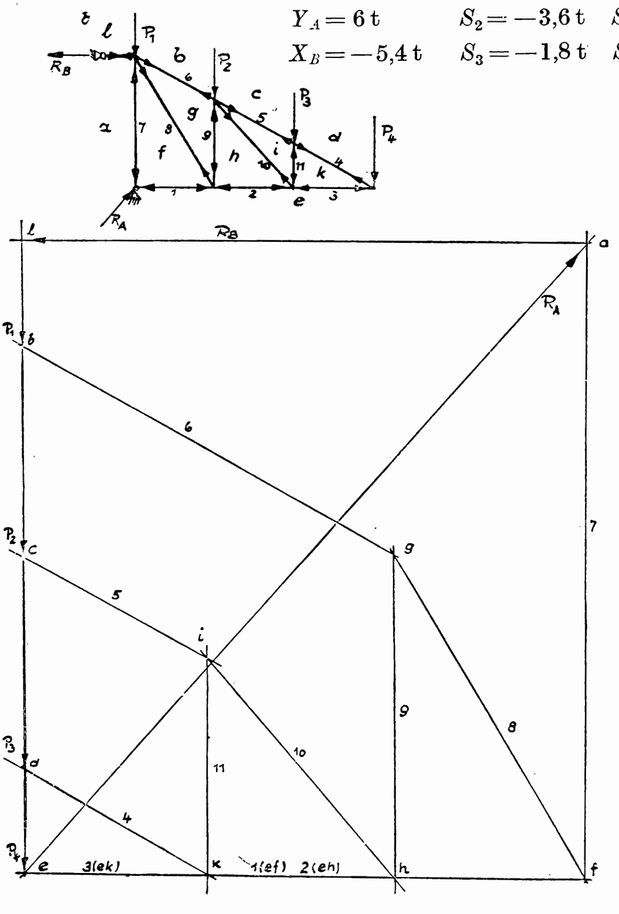
Lösung 207

$X_A = 5{,}4$ t
$Y_A = 6$ t
$X_B = -5{,}4$ t

$S_1 = -5{,}4$ t
$S_2 = -3{,}6$ t
$S_3 = -1{,}8$ t

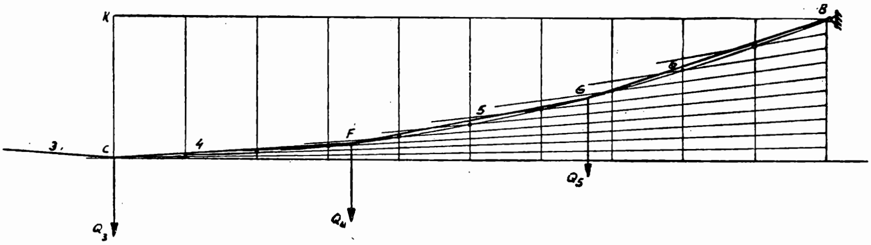
$S_4 = +2{,}06$ t
$S_5 = +2{,}06$ t
$S_6 = +4{,}1$ t

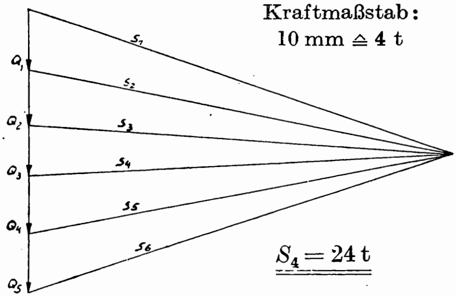
$S_7 = -\ 6$ t
$S_8 = +3{,}5$ t
$S_9 = -\ 3$ t
$S_{10} = +2{,}7$ t
$S_{11} = -2$ t



Kraftmaßstab: 10 mm ≙ 0,67 t

Lösung 208



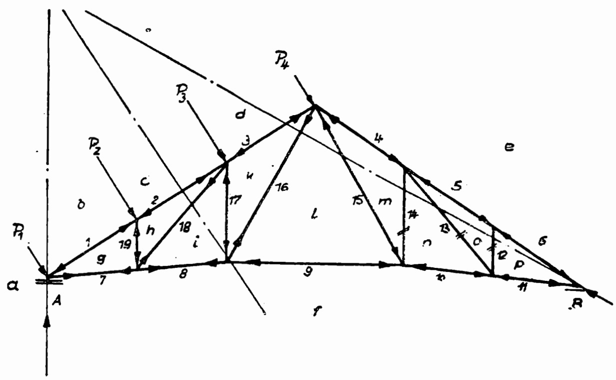


Kraftmaßstab:
10 mm $\triangleq$ **4** t

$\underline{\underline{S_4 = 24\text{ t}}}$

Der Seilzug setzt sich aus Parabelsehnen zusammen. Der Horizontalzug ergibt sich somit aus der Gesamtbelastung eines Tragseiles und der Neigung der beiden Endsehnen. Der Horizontalzug ist für das ganze Tragseil konstant.

Lösung 209

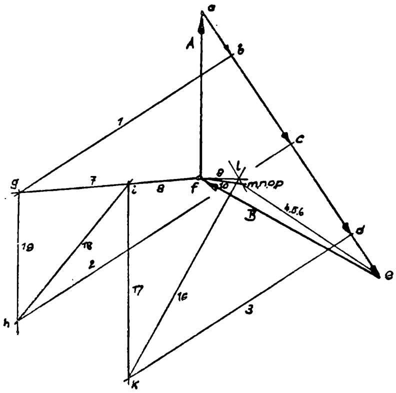


$Y_A = 997$ kg; $X_B = 1040$ kg; $Y_B = 563$ kg;

$S_1 = -1525\,\mathrm{kg}$
$S_2 = -1940\,\mathrm{kg}$
$S_3 = -1560\,\mathrm{kg}$
$S_4 = -970\,\mathrm{kg}$
$S_5 = -970\,\mathrm{kg}$
$S_6 = -970\,\mathrm{kg}$
$S_7 = +1100\,\mathrm{kg}$
$S_8 = +440\,\mathrm{kg}$
$S_9 = -215\,\mathrm{kg}$
$S_{10} = -230\,\mathrm{kg}$
$S_{11} = -230\,\mathrm{kg}$
$S_{12} = 0$

$S_{13} = 0$
$S_{14} = 0$
$S_{15} = -26\,\mathrm{kg}$
$S_{16} = +1340\,\mathrm{kg}$
$S_{17} = -1130\,\mathrm{kg}$
$S_{18} = +1050\,\mathrm{kg}$
$S_{19} = -750\,\mathrm{kg}$

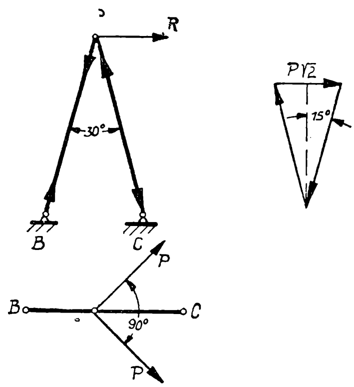
Kraftmaßstab:
10 mm ≙ 390,6 t



II. Räumliches Kräftesystem

6. Kräfte, deren Wirkungslinien sich in einem Punkt schneiden

Lösung 210

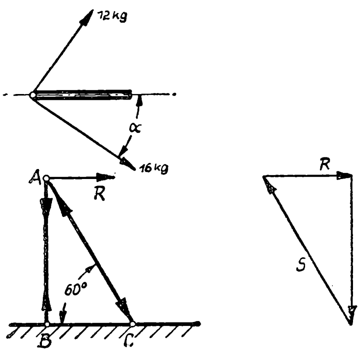


Resultierende Kraft in der Ebene BCA

$$R = P\sqrt{2}$$

$$S_B = \frac{R}{2 \sin 15^\circ} = -S_C = \underline{\underline{273\,\mathrm{kg}}}$$

Lösung 211



Um kein Biegemoment zu übertragen, muß die Resultierende der beiden Kräfte in der Ebene BCA liegen.

$$16 \sin\alpha = 12 \sin(90^\circ - \alpha)$$

$$\operatorname{tg}\alpha = \frac{12}{16} = \frac{3}{4}$$

$$\underline{\underline{\alpha = 36^\circ\,50'}}$$

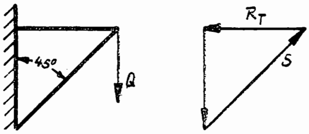
Resultierende: $R = 16 \cos\alpha + 12 \sin\alpha$

$$R = 20\,\mathrm{kg}$$

$$S = \frac{R}{\cos 60^\circ} = \underline{\underline{40\,\mathrm{kg}}}\ \text{Druck}$$

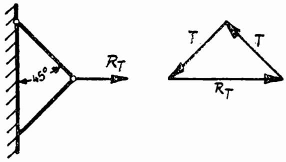
Lösung 212

Ebene AOQ:



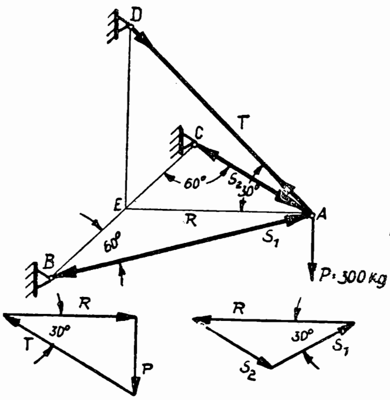
$$S = Q\sqrt{2} = \underline{\underline{141\,\text{kg}}}\ \text{Druck}$$
$$R_T = Q$$

Ebene BCO:



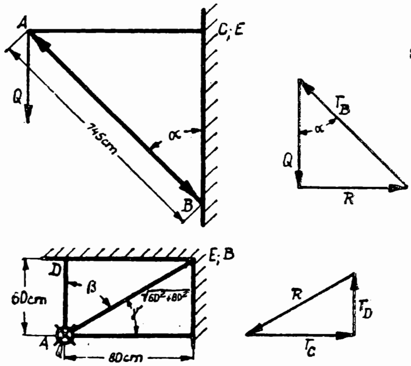
$$T = \frac{Q\sqrt{2}}{2} = \underline{\underline{71\,\text{kg}}}$$

Lösung 213



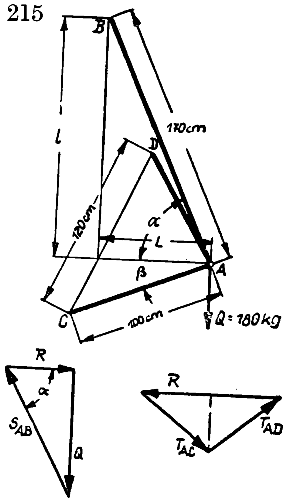
$$T = \frac{P}{\sin 30^\circ} = \underline{\underline{600\,\text{kg}}}$$
$$R = P\cdot\text{ctg}\,30^\circ$$
$$S_1 = S_2 = \frac{R}{2\cos 30^\circ} = \frac{P}{2\cdot\sin 30^\circ} = \underline{\underline{300\,\text{kg}}}\ \text{Druck}$$

Lösung 214



$$\sin\alpha = \frac{\sqrt{60^2+80^2}}{145} = 0{,}69$$
$$\alpha = 43^\circ\,40'$$
$$T_B = \frac{Q}{\cos\alpha} = \underline{\underline{58\,\text{kg Druck}}}$$
$$R = Q\cdot\text{tg}\,\alpha = 39\,\text{kg}$$
$$\frac{T_C}{R} = \frac{80}{100};\quad T_C = R\cdot\frac{80}{100} = \underline{\underline{32\,\text{kg}}}$$
$$\frac{T_D}{R} = \frac{60}{100};\quad T_D = R\cdot\frac{60}{100} = \underline{\underline{24\,\text{kg}}}$$

Lösung 215



$$L = 100^2 - 60^2 = 80$$

$$l = 170^2 - 80^2 = 150$$

$$\frac{Q}{S_{BA}} = \frac{l}{170}; \quad S_{AB} = \frac{Q \cdot 170}{150}$$

$$\underline{\underline{S_{AB} = 204\,\text{kg}}}$$

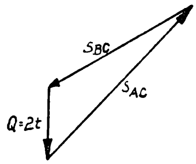
$$\frac{R}{Q} = \frac{L}{l}; \quad R = Q \cdot \frac{80}{150} = 96\,\text{kg}$$

$$T_{AC} = T_{AD} = \frac{R \cdot 100}{2 \cdot 80} = \underline{\underline{60\,\text{kg}}}\ \text{Druck}$$

Lösung 216

I. Zerlegen von Q in S_{BC} und S_{AC}:

$$\frac{Q}{S_{BC}} = \frac{BA}{BC}: \quad S_{BC} = \frac{Q \cdot 5}{2 \cdot \sin 60°} \cong \underline{\underline{5{,}8\,\text{t} = P_2}}$$



II. Zerlegen von S_{BC} in S_{BD} und S_{BA}:

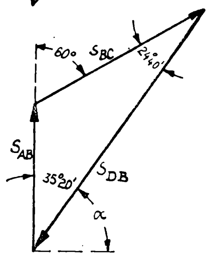
$$\operatorname{tg}\alpha = \frac{BA}{DA} = \sqrt{2}; \quad \alpha = 54°40'$$

$$S_{BC} \cdot \cos 30° = S_{DB} \cdot \sin 35°20'$$

$$S_{DB} = S_{BC} \cdot \frac{\cos 30°}{\sin 35°20'} = 5{,}8 \cdot \frac{0{,}866}{0{,}578} \cong 8{,}7\,\text{t}$$

$$S_{AB} = S_{DB} \cdot \cos 35°20' - S_{BC} \cdot \cos 60°$$

$$= 8{,}7 \cdot 0{,}816 - 5{,}8 \cdot 0{,}5 \cong \underline{\underline{4{,}2\,\text{t} = P_1}}$$

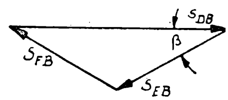


III. Zerlegen von S_{DB} in S_{FB} und S_{EB}:

$$AE = 2$$

$$DF = DA = \sqrt{2}: \quad BF = 2\sqrt{2}; \quad \sin\beta = \frac{\sqrt{2}}{2\sqrt{2}}; \quad \beta = 30°$$

$$S_{EB} = S_{FB} = P_3 = P_4 = \frac{S_{DB}}{2\cos 30°} \cong \underline{\underline{5\,\text{t}}}$$



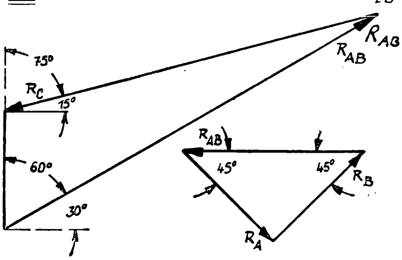
Lösung 217

$$R_{AB} \cdot \cos 60° = Q + R_C \cos 75°$$

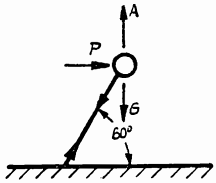
$$R_C = R_{AB} \cdot \frac{\sin 60°}{\sin 75°}$$

$$R_{AB} = 3{,}73\,\text{t}; \quad \underline{\underline{R_C = 3{,}35\,\text{t}}}$$

$$R_A = R_B = \frac{R_{AB}}{\sqrt{2}} = \underline{\underline{2{,}64\,\text{t}}}$$



Lösung 218



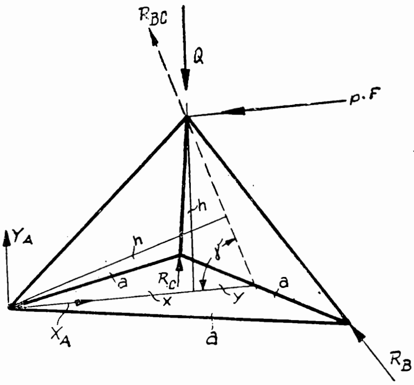
$$A - G = 215{,}4 \cdot 1{,}3 - 250 = 30\ \text{kg}$$

$$P = (A - G)\ \text{ctg}\ 60° = \underline{\underline{17{,}3\ \text{kg}}}$$

$$R = \frac{(A - G)}{\sin 30°} = 34{,}6\ \text{kg}$$

$$T_1 = T_2 = \frac{R}{\sqrt{2}} = \underline{\underline{24{,}5\ \text{kg}}}$$

Lösung 219



Aus Symmetriegründen ist:

$$R_B = R_C; \quad R_{BC} = 2R_B \cdot \cos 30°$$

$\sum M_A = 0$:

$$Q \cdot x - p \cdot F \cdot h - R_{BC} \cdot h = 0 \qquad x = \frac{a}{3}\sqrt{3}$$

$$R_{BC} = \frac{Q - p\pi r^2 \cdot \sqrt{2}}{\sqrt{2}} \qquad y = \frac{a}{6}\sqrt{3}$$

$$h = \frac{a}{3}\sqrt{2}\sqrt{3}$$

$$R_B = R_C = \frac{Q - p\pi r^2 \cdot \sqrt{2}}{2\sqrt{2}\cos 30°} = \underline{\underline{60\ \text{t}}}$$

$\sum M_{\overline{BC}} = 0$:

$$Q \cdot y + p \cdot F \cdot h - Y_A \cdot h = 0$$

$$Y_A = Q \cdot \frac{1}{2\sqrt{2}} + p \cdot F$$

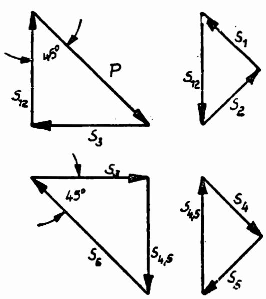
$\sum P_x = 0$:

$$X_A - R_{BC} \cdot \cos\gamma - p \cdot F = 0;$$

$$X_A = \frac{R_{BC}}{3} + p \cdot \pi \cdot r^2 \qquad \cos\gamma = \frac{y}{x+y}$$

$$R_A = \sqrt{X_A^2 + Y_A^2} = \underline{\underline{125\ \text{t}}}$$

Lösung 220



Punkt A:

$$S_3 = S_{1,2} = \frac{P}{\sqrt{2}} = \underline{\underline{0{,}707\ \text{t}}}\ \text{Druck}$$

$$S_1 = S_2 = \frac{S_{1,2}}{\sqrt{2}} = \underline{\underline{0{,}5\ \text{t}}}\ \text{Druck}$$

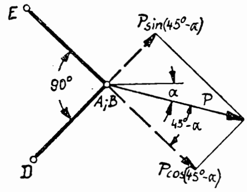
Punkt B:

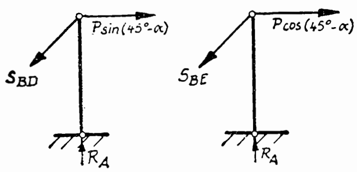
$$S_{4,5} = S_3 = 0{,}707\ \text{t Zug}$$

$$S_6 = S_3 \cdot \sqrt{2} = \underline{\underline{1\ \text{t}}}\ \text{Druck}$$

$$S_4 = S_5 = \frac{S_{4,5}}{\sqrt{2}} = \underline{\underline{0{,}5\ \text{t}}}\ \text{Zug}$$

Lösung 221





$$S_{BC} = P$$

$$\sum M_A = 0:\ S_{BE} \cdot a\frac{\sqrt{2}}{2} = P \cdot a \cdot \cos(45° - \alpha)$$

$$S_{BD} \cdot a\frac{\sqrt{2}}{2} = P \cdot a \cdot \sin(45° - \alpha)$$

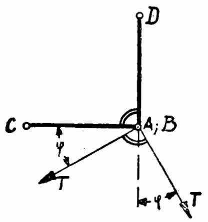
$$\underline{\underline{S_{BE} = P(\sin\alpha + \cos\alpha)}}$$

$$\underline{\underline{S_{DB} = P(\cos\alpha - \sin\alpha)}}$$

$$\sum P_y = 0:\ R_A - S_{BE} \cdot \cos 45° - S_{BD} \cdot \cos 45° = 0$$

$$R_A = S_{AB} = \underline{\underline{P\sqrt{2} \cdot \cos\alpha}} \text{ Druck}$$

Lösung 222



$\sum P_{DB} = 0:$

$$S_{AD} \cdot \cos 60° = T\sin\varphi + T\cos\varphi$$

$$\underline{\underline{S_{AD} = 2T(\sin\varphi + \cos\varphi)}}$$

$\sum P_{CB} = 0:$

$$S_{CA} \cdot \cos 60° = T\sin\varphi - T\cos\varphi$$

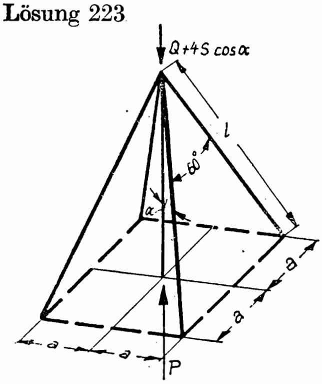
$$\underline{\underline{S_{AC} = 2T(\sin\varphi - \cos\varphi)}}$$

$\sum M_{\overline{CD}} = 0:$

$$S_{AB} \cdot \operatorname{tg} 30° \cdot \cos 45° = T[\sin(\varphi - 45°) + \cos(\varphi - 45°)]$$

$$\underline{\underline{S_{AB} = -2\sqrt{3}\, T\sin\varphi}}$$

Lösung 223



$$\sin\alpha = \frac{a\sqrt{2}}{l}; \quad \sin 30° = \frac{a}{l}$$

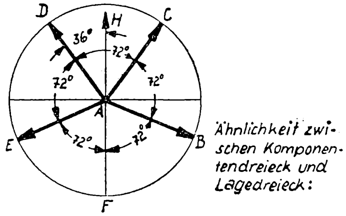
$$\sin\alpha = \frac{a\sqrt{2}}{a} \cdot \sin 30° = \frac{\sqrt{2}}{2}; \quad \alpha = 45°$$

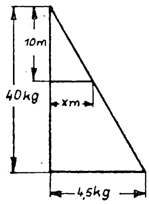
$$\sum P_y = 0: \quad 4S \cdot \cos a + Q - P = 0$$

$$P = 400 \cdot \frac{\sqrt{2}}{2} + 200$$

$$P = \underline{\underline{483 \text{ kg}}}$$

Lösung 224





Komponente in Richtung AO:

$V = 4 \cdot 10 = 40$ kg

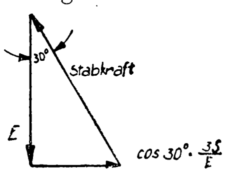
Horizontalkomponente H:

$$H = 2\,(4{,}5 \cdot \cos 36^\circ - 4{,}5 \cos 72^\circ)$$
$$= 9\,(0{,}809 - 0{,}309) = 4{,}5$$
$$R = \sqrt{H^2 + V^2} = \sqrt{1620{,}5}$$
$$= \underline{\underline{40{,}25\ \text{kg}}}$$

Durchstoßpunkt:

$$\frac{40}{4{,}5} = \frac{10}{x}; \quad x = \frac{4{,}5}{4} = \underline{\underline{1{,}125\ \text{m}}}$$

Lösung 225



$\sum P_y = 0$:

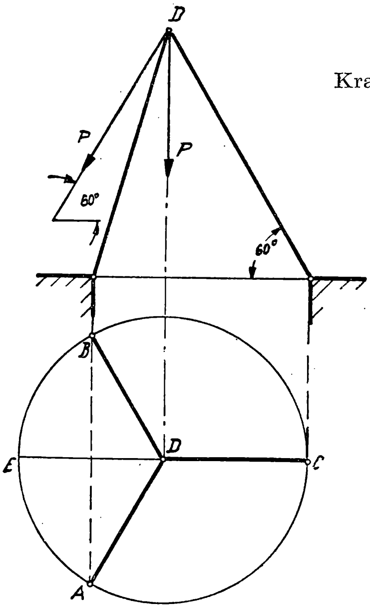
$$E = 3 \cdot S \cdot \cos 30^\circ$$
$$S = \frac{10}{3 \cdot 0{,}866} = \underline{\underline{3{,}85\ \text{kg}}}$$

Lösung 226

Durch die Seilführung wird die Rolle D mit $2 \cdot 3$ t belastet. Der Vertikaldruck einer Stütze beträgt auf Grund der dreiseitig symmetrischen Anordnung: 2 t

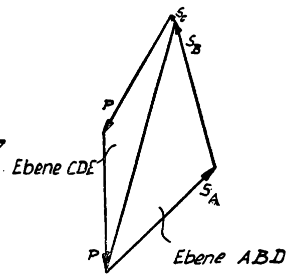
$$S = \frac{2}{\cos 30^\circ} = \frac{4}{\sqrt{3}} = \underline{\underline{2{,}3\ \text{t}}}$$

Lösung 227



Graphische Lösung:

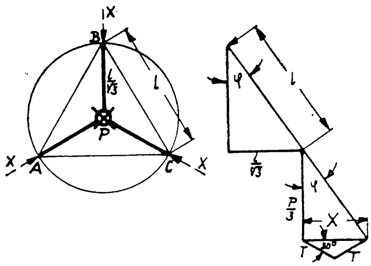
Kraftmaßstab: 10 mm ≙ 1,5 t



$S_C = 0.15$ t Druck

$S_B = S_A = 3{,}15$ t Druck

Lösung 228



Der Fußboden kann nur senkrechte Reaktionen übertragen
$$\underline{\underline{R=\frac{1}{3}P}}$$

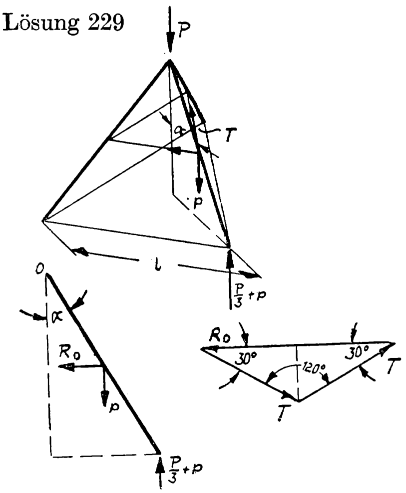
Der Faden muß die Spreizreaktionen X aufnehmen:

$$\sin\varphi=\frac{l}{l\sqrt{3}};\quad X=\frac{P}{3}\operatorname{tg}\varphi=\frac{P}{3}\,\frac{\sin\varphi}{\sqrt{1-\sin^2\varphi}}$$

$$X=\frac{P}{3}\,\frac{1}{\sqrt{3}\sqrt{1-\frac{1}{3}}}=\frac{P}{3\sqrt{2}}$$

$$T=\frac{X}{2\cos 30^\circ}=\frac{P\cdot 2}{2\sqrt{2}\cdot 3\cdot\sqrt{3}}=\underline{\underline{\frac{P}{3\sqrt{6}}}}$$

Lösung 229



$$R=\frac{1}{3}P+p;\qquad \sin\alpha=\sqrt{\frac{1}{3}};\qquad \cos\alpha=\sqrt{\frac{2}{3}}$$

$$\sum M_0=0:$$

$$\left(\frac{P}{3}+p\right)l\sin\alpha-R_0\cdot\frac{l}{2}\cos\alpha-p\cdot\frac{l}{2}\sin\alpha=0$$

$$2\left(\frac{P}{3}+p\right)\operatorname{tg}\alpha-R_0-p\cdot\operatorname{tg}\alpha=0$$

$$\frac{P}{3}\sqrt{2}+\frac{p}{2}\sqrt{2}=R_0$$

$$\cos 30^\circ=\frac{R_0}{2T}$$

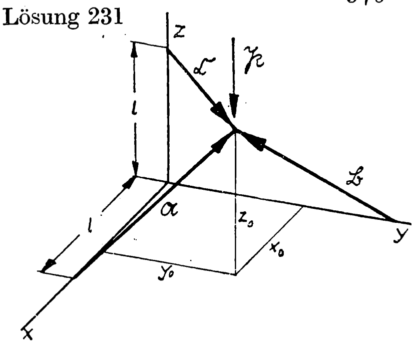
$$T=\frac{R_0}{2\cdot\frac{1}{2}\sqrt{3}};$$

$$\underline{\underline{T=\frac{2P+3p}{18}\sqrt{6}}}$$

Lösung 230 Die Kugeln bilden ein regelmäßiges, in seiner Spitze durch $P=10$ kg belastetes Tetraeder. Nach Aufgabe 228 gilt:

$$T=\frac{P}{3\sqrt{6}}=\frac{10}{3\cdot 2{,}45}=\underline{\underline{1{,}36\text{ kg}}}$$

Lösung 231



$$\mathfrak{P}=-Q\cdot k;\ x=y=z=x_0=\frac{1}{3}\left(l-\sqrt{3L^2-2l^2}\right)$$

$$\mathfrak{A}=T_A[-(l-x_0)\cdot i+x_0\cdot j+x_0\cdot k]\cdot\frac{1}{\alpha}$$

$$\mathfrak{B}=T_B[x_0\cdot i-(l-x_0)\,j+x_0\cdot k]\cdot\frac{1}{\alpha}$$

$$\mathfrak{C}=T_C[x_0\cdot i+x_0\cdot j-(l-x_0)\,k]\cdot\frac{1}{\alpha}$$

$$\alpha=\sqrt{2x_0^2+(x_0-l)^2}$$

$$\mathfrak{A}+\mathfrak{B}+\mathfrak{C}+\mathfrak{P}=0$$

$$\begin{aligned} T_A(x_0-l)+T_B x_0 \quad &+T_C\cdot x_0 &&=0;\\ T_A\cdot x_0 \quad +T_B(x_0-l)&+T_C x_0 &&=0;\\ T_A\cdot x_0 \quad +T_B\cdot x_0 \quad &+T_C(x_0-l)&&=Q\cdot\alpha; \end{aligned}$$

$$\Delta=\begin{vmatrix}(x_0-l) & x_0 & x_0\\ x_0 & (x_0-l) & x_0\\ x_0 & x_0 & (x_0-l)\end{vmatrix}=l^2(3x_0-l)$$

$$T_A=\frac{\begin{vmatrix}0 & x_0 & x_0\\ 0 & x_0-l & x_0\\ Q\cdot\alpha & x_0 & x_0-l\end{vmatrix}}{\Delta}=Q\cdot\frac{\alpha\cdot x_0}{(3x_0-l)\cdot l}$$

$$T_B=\frac{\begin{vmatrix}x_0-l & 0 & x_0\\ x_0 & 0 & x_0\\ x_0 & Q\cdot\alpha & x_0-l\end{vmatrix}}{\Delta}=Q\cdot\frac{\alpha\cdot x_0}{(3x_0-l)\cdot l}$$

$$T_C=\frac{\begin{vmatrix}x_0-l & x_0 & 0\\ x_0 & x_0-l & 0\\ x_0 & x_0 & Q\cdot\alpha\end{vmatrix}}{\Delta}=Q\cdot\frac{\alpha(l-2x_0)}{(3x_0-l)\cdot l}$$

$$\underline{\underline{T_A=T_B=\frac{l-\sqrt{3L^2-2l^2}}{3l\sqrt{3L^2-2l^2}}\cdot L\cdot Q}}$$

$$\underline{\underline{T_C=\frac{l+2\sqrt{3L^2-2l^2}}{3l\sqrt{3L^2-2l^2}}\cdot L\cdot Q}}$$

7. Reduktion von Kräftesystemen

Lösung 232 Gleichgewichtsbedingungen: $\sum\mathfrak{P}_i=0$; $\sum\mathfrak{M}_i=0$;

$\sum\mathfrak{P}_i=0$: $F_4=F_1$; $F_5=F_3$; $F_6=F_2$

I $\sum M_x=0$: $F_4-F_6+F_1-F_2=0$

II $\sum M_y=0$: $F_5-F_4+F_1-F_3=0$ $x; y; z=$ Schwerpunktskoordinaten

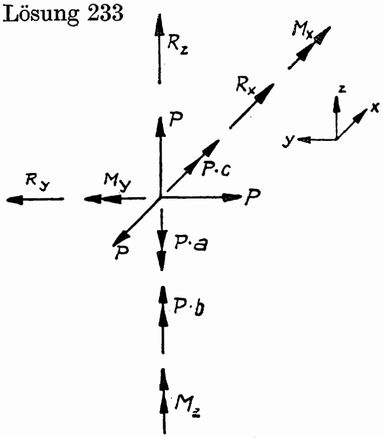
III $\sum M_z=0$: $F_6-F_5-F_3+F_2=0$

In I $F_4=F_1$; $F_2=F_6$ eingesetzt: $F_1=F_6$

In II $F_3=F_5$; $F_2=F_6$ eingesetzt: $F_5=F_6$

Somit: $\underline{\underline{F_1=F_2=F_3=F_4=F_5=F_6}}$

Lösung 233



Die Kräfte werden an einen Punkt verschoben. Dadurch entstehen die folgenden Momente, die gleich den Momenten sein müssen, die durch die Komponenten der Resultierenden hervorgerufen werden.

$$\begin{aligned} &M_z+P\cdot b-P\cdot a=0; && R_x-P=0\\ &M_x+P\cdot c=0; && R_y-P=0\\ &M_y=0: && R_z+P=0 \end{aligned}$$

$$\begin{aligned} M_z&=R_y\cdot x-R_x\cdot y\\ M_x&=-R_y\cdot z+R_z\cdot y\\ M_y&=-R_z\cdot x+R_x\cdot z \end{aligned}$$

Diese drei Gleichungssysteme ineinander eingesetzt:

$$\begin{aligned} b-a+x-y&=0\\ c-z-y&=0\\ x+z&=0 \end{aligned}$$

Daraus: $c-b+a=0$; $\underline{\underline{a=b-c}}$

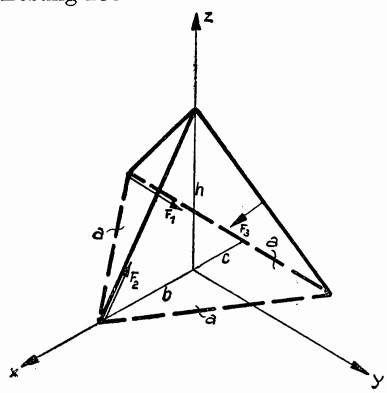
Lösung 234 Die Vektordarstellungen der eingezeichneten Kräfte lauten:

$$\mathfrak{P}_1 = \frac{P}{2}\sqrt{2}\,(\mathfrak{i}+\mathfrak{j}); \quad \mathfrak{P}_2 = \frac{P}{2}\sqrt{2}\,(\mathfrak{j}-\mathfrak{i}); \quad \mathfrak{P}_3 = \frac{P}{2}\sqrt{2}\,(\mathfrak{k}-\mathfrak{j}); \quad \mathfrak{P}_4 = \frac{P}{2}\sqrt{2}\,(\mathfrak{j}+\mathfrak{k})$$

$$\mathfrak{R} = \mathfrak{P}_1 + \mathfrak{P}_2 + \mathfrak{P}_3 + \mathfrak{P}_4 = P\sqrt{2}\,(\mathfrak{j}+\mathfrak{k}) = \underline{\underline{2\,\mathfrak{P}_4}}$$

Die Resultierende hat also die Größe $2P$ und die Richtung von P_4.

Lösung 235



$$h = \frac{a}{3}\sqrt{2}\sqrt{3}$$

$$b = \frac{a}{3}\sqrt{3}$$

$$c = \frac{a}{6}\sqrt{3}$$

$$\mathfrak{F}_1 = F_1 \cdot \mathfrak{j}$$

$$\mathfrak{F}_2 = \frac{F_2\sqrt{3}}{3}\left(-\mathfrak{i} + \sqrt{2}\,\mathfrak{k}\right)$$

$$\mathfrak{F}_3 = F_2\left(\frac{5}{6}\sqrt{3}\,\mathfrak{i} - \frac{1}{2}\,\mathfrak{j} - \sqrt{\frac{2}{3}}\,\mathfrak{k}\right)$$

Resultierende $\mathfrak{V} = \mathfrak{F}_1 + \mathfrak{F}_2 + \mathfrak{F}_3$

$$\mathfrak{V} = F_2\frac{\sqrt{3}}{2}\,\mathfrak{i} - F_2\left(\frac{1}{2} - \frac{F_1}{F_2}\right)\mathfrak{j}$$

Somit; $V_x = F_2 \cdot \frac{\sqrt{3}}{2}$; $V_y = F_1 - 0{,}5 F_2$; $V_z = 0$

Momentkomponenten der Resultierenden;

$$M_x = V_z \cdot y - V_y \cdot z$$
$$M_y = V_z \cdot x + V_x \cdot z$$
$$M_z = V_y \cdot x - V_x \cdot y$$

Momentkomponenten der Einzelkräfte:

$$M_x = F_{3z} \cdot y_3 - F_{3y} \cdot z_3$$
$$= -F_2 \cdot \sqrt{\frac{2}{3}} \cdot \frac{a}{4} + F_2 \cdot \frac{1}{2} \cdot \frac{a\sqrt{2}\sqrt{3}}{6}$$
$$M_x = 0$$
$$M_y = F_{3z} \cdot \frac{c}{2} - F_{3x} \cdot \frac{h}{2} + F_{2z} \cdot b$$
$$M_y = 0$$
$$M_z = F_{1y} \cdot x_1 + F_{3y} \cdot x_3 - F_{3x}\,\frac{1}{3}$$
$$= -F_1\frac{a\sqrt{3}}{6} + \frac{F_2 a\sqrt{3}}{2 \cdot 12} - \frac{F_2 \cdot 5\sqrt{3} \cdot a}{6 \cdot 4}$$
$$M_z = -\frac{a\sqrt{3}}{6}\,(F_1 + F_2)$$

Aus der Bedingung, daß die jeweiligen Momentkomponenten gleich sein müssen ergibt sich der Durchstoßpunkt der Resultierenden in der xz-Ebene ($y = 0$)

$$-\frac{a\sqrt{3}}{6}\,(F_1 + F_2) = F_2\left(\frac{F_1}{F_2} - \frac{1}{2}\right) \cdot x; \qquad \underline{\underline{x = -\frac{a\sqrt{3}\,(F_1 + F_2)}{6\left(F_1 - \frac{F_2}{2}\right)}; \quad z = 0}}$$

Lösung 236

Resultierende Kraft $= 0$

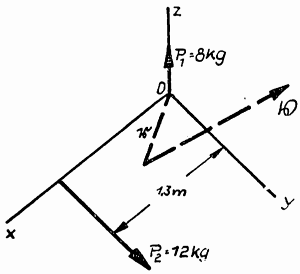
Resultierendes Moment:
$$M_x = F_4 \cdot \frac{a}{2} + F_1 \cdot \frac{a}{2} + F_6 \cdot \frac{a}{2} + F_3 \cdot \frac{a}{2}$$
$$M_y = -F_1 \cdot \frac{a}{2} - F_2 \cdot \frac{a}{2} - F_4 \cdot \frac{a}{2} - F_5 \cdot \frac{a}{2} \qquad a = 5 \text{ cm}, \quad F_i = 2 \text{ kg}$$
$$M_z = F_2 \cdot \frac{a}{2} + F_5 \cdot \frac{a}{2} + F_6 \cdot \frac{a}{2} + F_3 \cdot \frac{a}{2}$$

$$\mathfrak{M} = 20(\mathfrak{i} - \mathfrak{j} + \mathfrak{k}) = \underline{\underline{20\sqrt{3} \cdot \left(\frac{\mathfrak{i} - \mathfrak{j} + \mathfrak{k}}{\sqrt{3}}\right)}}; \quad |\mathfrak{M}| = 20\sqrt{3} \text{ kgcm}$$

$$\cos\alpha = -\cos\beta = \cos\gamma = \frac{1}{3}\sqrt{3}$$

Lösung 237

$$\mathfrak{V} = \mathfrak{P}_1 + \mathfrak{P}_2 = P_1 \cdot \mathfrak{k} + P_2 \cdot \mathfrak{j}$$
$$|\mathfrak{V}| = V = \sqrt{P_1^2 + P_2^2} = \underline{\underline{14{,}4 \text{ kg}}}$$



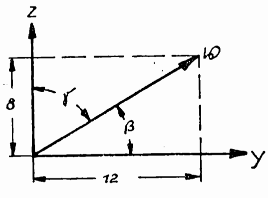
Drehpunkt 0: $\mathfrak{M}_{ges} = (\mathfrak{r} \times \mathfrak{V}) + \mathfrak{M}$
$$\mathfrak{r} = x \cdot \mathfrak{i} + y \cdot \mathfrak{j}$$

Bedingung der Kraftschraubenachse: $\frac{\mathfrak{V}}{V} = \frac{\mathfrak{M}}{M}$;
$$\mathfrak{M} = \frac{M}{V} \cdot \mathfrak{V}$$

$$\mathfrak{M}_{ges} = -P_1 \cdot x \cdot \mathfrak{j} + P_2 \cdot x \cdot \mathfrak{k} + P_1 \cdot y \cdot \mathfrak{i} + \frac{M}{V}(P_1 \mathfrak{k} + P_2 \cdot \mathfrak{j})$$

$\mathfrak{M}_{ges}$ muß gleich sein dem Momentenvektor, der sich aus den gezeichneten Kräften bildet.

$$M_x = 0 = P_1 \cdot y \qquad \text{Daraus:} \quad y = 0$$
$$M_y = 0 = -P_1 \cdot x + \frac{M}{V} \cdot P_2 \qquad x = \frac{M}{V} \cdot \frac{P_2}{P_1}$$
$$M_z = P_2 \cdot 1{,}3 \text{ m} = P_2 \cdot x + \frac{M}{V} \cdot P_1 \qquad P_2 \cdot 1{,}3 \text{ m} = \frac{M}{V}\left(\frac{P_2^2}{P_1} + P_1\right) = \frac{M \cdot V}{P_1}$$
$$x = \frac{P_2^2 \cdot 1{,}3 \text{ m}}{P_1^2 + P_2^2} = \underline{\underline{0{,}9 \text{ m}}} \qquad M = \frac{P_2 \cdot 1{,}3 \text{ m} \cdot P_1}{\sqrt{P_1^2 + P_2^2}} = \underline{\underline{8{,}65 \text{ kgm}}}$$



Aus der Lage von P_1 und P_2 folgt, daß $\alpha = 90°$ beträgt.

$$\beta = \operatorname{arc\,tg} \frac{2}{3}; \quad \gamma = \operatorname{arc\,tg} \frac{3}{2}$$

Lösung 238

a) $\mathfrak{V} = P_1\mathfrak{i} + P_2\mathfrak{j} + P_3\mathfrak{k}$; $\mathfrak{V}$ schneidet die x, y-Ebene in $\mathfrak{r} = \mathfrak{i}x + \mathfrak{j}y$

$\mathfrak{M} = (\mathfrak{r} \times \mathfrak{V}) = P_2 \cdot x \cdot \mathfrak{k} - P_3 \cdot x \cdot \mathfrak{j} - P_1 \cdot y \cdot \mathfrak{k} + P_3 \cdot y \cdot \mathfrak{i}$

Moment aus den Komponenten: $M_x = P_3 \cdot b$

$M_y = P_1 \cdot c$

$M_z = P_2 \cdot a$

Da beide Momente gleich sein müssen, folgt: $P_3 \cdot b = P_3 \cdot y$; $\quad y = b$

$P_1 \cdot c = -P_3 \cdot x$; $\quad x = -\frac{P_1}{P_3} \cdot c$

$P_2 \cdot a = P_2 \cdot x - P_1 \cdot y$;

$$P_2 \cdot a = -\frac{P_1 P_2}{P_3} \cdot c - P_1 \cdot b; \qquad \underline{\underline{\frac{a}{P_1} + \frac{b}{P_2} + \frac{c}{P_3} = 0}}$$

b) $\mathfrak{r} = 0$; $\quad \mathfrak{M}_{\text{ges}} = \frac{M}{V}\mathfrak{V} = \frac{M}{V}(P_1\mathfrak{i} + P_2 \cdot \mathfrak{j} + P_3\mathfrak{k})$

$$P_3 \cdot b = \frac{M}{V} \cdot P_1$$

$$P_1 \cdot c = \frac{M}{V} \cdot P_2 \qquad \underline{\underline{\frac{V}{M} = \frac{P_1}{P_3 \cdot b} = \frac{P_2}{P_1 \cdot c} = \frac{P_3}{P_2 \cdot a}}}$$

$$P_2 \cdot a = \frac{M}{V} \cdot P_3$$

Lösung 239

Geometrische Abmessungen vgl. Aufgabe 235.

$\mathfrak{V} = \mathfrak{F}_1 + \mathfrak{F}_2$; $\quad \mathfrak{F}_1 = F_1 \cdot \mathfrak{j}$

$$\frac{\mathfrak{M}}{M} = \frac{\mathfrak{V}}{V} \qquad \mathfrak{F}_2 = F_2 \frac{\sqrt{3}}{3}(-\mathfrak{i} + \sqrt{2}\,\mathfrak{k})$$

$$\mathfrak{V} = F_1 \cdot \mathfrak{j} - F_2 \frac{\sqrt{3}}{3}\mathfrak{i} + F_2 \cdot \sqrt{\frac{2}{3}}\,\mathfrak{k}$$

Momentenkomponenten der Einzelkräfte = Komponenten von $\mathfrak{M}_{\text{ges}}$

$M_x = 0$;

$M_y = -F_{2\,z} \cdot \frac{a}{3}\sqrt{3}$; $\qquad \mathfrak{M}_{\text{ges}} = (\mathfrak{r} \times \mathfrak{V}) + \mathfrak{M}$; $\qquad \mathfrak{r} = x\mathfrak{i} + y\mathfrak{j}$

$M_z = -F_1 \cdot \frac{a}{6}\sqrt{3}$;

$$\mathfrak{M}_{\text{ges}} = F_1 \cdot x \cdot \mathfrak{k} - F_2\sqrt{\frac{2}{3}}\,x \cdot \mathfrak{j} + F_2 \frac{\sqrt{3}}{3} \cdot \mathfrak{k} \cdot y$$

$$+ F_2\sqrt{\frac{2}{3}}\,y i + \frac{M}{V} \cdot \mathfrak{V}$$

Daraus: $0 = F_2\sqrt{\frac{2}{3}}\,y - \frac{M}{V}F_2\frac{\sqrt{3}}{3}$; $\qquad \frac{M}{V} = y\sqrt{2}$

$$-F_2\frac{\sqrt{2}}{3}a = -F_2\sqrt{\frac{2}{3}}\,x + \frac{M}{V}\cdot F_1;$$

$$-F_1\frac{a}{6}\sqrt{3} = F_1\cdot x + F_2\frac{\sqrt{3}}{3}\cdot y + \frac{M}{V}\cdot F_2\sqrt{\frac{2}{3}};$$

$$-F_2 a\frac{\sqrt{2}}{3} = -F_2\sqrt{\frac{2}{3}}\,x + y\cdot F_1\sqrt{2}$$

$$-F_1\frac{a}{6}\sqrt{3} = F_1 x + F_2 y\frac{\sqrt{3}}{3} + F_2\frac{2}{\sqrt{3}}y$$

$$\underline{\underline{x = \frac{a\sqrt{3}}{6}\,\frac{2F_2^2 - F_1^2}{F_1^2 + F_2^2}}}$$

$$\underline{\underline{y = -\frac{a}{2}\,\frac{F_1F_2}{F_1^2+F_2^2}}}$$

Lösung 240

Resultierende: $\mathfrak{V} = 4P\mathfrak{k} - 2P\mathfrak{i} + 2P\mathfrak{j}$

$$\underline{\mathfrak{V} = 2P\sqrt{6}\left(\frac{-\mathfrak{i}+\mathfrak{j}+2\mathfrak{k}}{\sqrt{6}}\right)}$$

Der Vektor der Resultierenden hat also die Größe:

$$\underline{\underline{V = 2P\sqrt{6}}}$$

und die Richtung:

$$\underline{\underline{\cos\alpha = -\frac{\sqrt{6}}{6}}}$$

$$\underline{\underline{\cos\beta = +\frac{\sqrt{6}}{6}}}$$

$$\underline{\underline{\cos\gamma = \frac{\sqrt{6}}{3}}}$$

Die Richtung der Kraftschraubenachse ist gleich der Richtung der Resultierenden.

Momentenkomponenten der Einzelkräfte = Komponenten von $\mathfrak{M}_{\text{ges}}$ (bezogen auf den Koordinatenursprung)

$M_x = 2Pa$;
$M_y = -2Pa$;
$M_z = 4Pa$;

$$\mathfrak{M}_{\text{ges}} = (\mathfrak{r}\times\mathfrak{V}) + \mathfrak{M}; \qquad \frac{\mathfrak{M}}{M} = \frac{\mathfrak{V}}{V}; \quad \mathfrak{r} = x\mathfrak{i} + y\mathfrak{i}$$

$$\mathfrak{M}_{\text{ges}} = -4Px\mathfrak{j} + 2Px\mathfrak{k} + 4Py\mathfrak{i} + 2Py\mathfrak{k} + \frac{M}{V}(4P\mathfrak{k} - 2P\mathfrak{i} + 2P\mathfrak{j})$$

$$\left.\begin{aligned} 2Pa &= 4Py - \frac{M}{V}\cdot 2P \\ -2Pa &= -4Px + \frac{M}{V}\cdot 2P \end{aligned}\right\} \quad x = y = \frac{1}{2}\left(\frac{M}{V} + a\right); \qquad \frac{M}{V} = 2x - a$$

$$4Pa = 2Px + 2Py + 4\frac{M}{V}\cdot P \;\Big\}\; 2a = 2x + 4x - 2a; \qquad \underline{\underline{x = y = \frac{2}{3}a}}$$

$$M = (2x - a)\cdot 2P\sqrt{6}; \quad \underline{\underline{M = \frac{2}{3}P\cdot a\sqrt{6}}}$$

Lösung 241 Lösungsweg vgl. vorhergehende Aufgabe.

$\mathfrak{P} = (P_2 - P_5)\,\mathfrak{i} + (P_1 - P_4)\,\mathfrak{j} + (P_6 - P_3)\,\mathfrak{k}$; $\quad \mathfrak{P} = 2\mathfrak{j} + 5\mathfrak{k}$

$\mathfrak{P} = \sqrt{29}\left(\dfrac{2\mathfrak{j} + 5\mathfrak{k}}{\sqrt{29}}\right)$ Also $\underline{\underline{V = \sqrt{29} = 5{,}4\,\text{kg}}}$;

$\cos\alpha = 0$

$\cos\beta = \dfrac{2}{5{,}4} = 0{,}37$

$\cos\gamma = \dfrac{5}{5{,}4} = 0{,}93$

$M_x = -P_1 \cdot 5 - P_3 \cdot 10 = -50\,\text{kgm}$; $\quad \mathfrak{M}_{\text{ges}} = (\mathfrak{r} \times \mathfrak{P}) + \mathfrak{M}$

$M_y = +P_3 \cdot 4 - P_2 \cdot 5 = -42\,\text{kgm}$; $\quad = 2x\mathfrak{k} - 5x\mathfrak{j} + 5y\mathfrak{i} + \dfrac{M}{V}(2\mathfrak{j} + 5\mathfrak{k})$

$M_z = +P_4 \cdot 4 + P_2 \cdot 10 = 68\,\text{kgm}$;

$-50 = 5y$; $\quad \underline{\underline{y = -10\,\text{m}}}$;

$-42 = -5x + 2\dfrac{M}{V}$; $\quad \dfrac{M}{V} = 2{,}5x - 21$

$68 = 2x + 5\dfrac{M}{V}$; $\quad 68 = 14{,}5x - 105$; $\quad x = \dfrac{173}{14{,}5} = \underline{\underline{11{,}9\,\text{m}}}$

$\dfrac{M}{V} = 2{,}5 \cdot 11{,}9 - 21 = 8{,}8$; $\quad M = 8{,}8 \cdot 5{,}4 = \underline{\underline{47{,}5\,\text{kgm}}}$

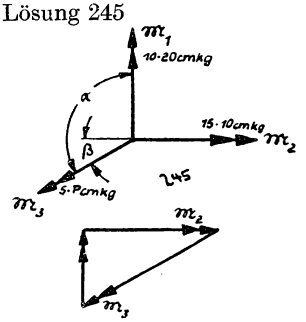
8. Gleichgewicht beliebiger Kräftesysteme

Lösung 242 $\quad M = 3 \cdot 400 \cdot \sin 20° = \underline{\underline{410\,\text{kg m}}}$

Lösung 243 $\quad M = 4 \cdot 100 \cdot \sin 15° \cdot 3 = \underline{\underline{311\,\text{kg m}}}$

Lösung 244 $\quad Q = \dfrac{60\,000\,\text{cm kg}}{60\,\text{cm}} = 1000\,\text{kg} = \underline{\underline{1\,\text{t}}}$

Lösung 245



$\text{tg}\,\beta = \dfrac{\mathfrak{M}_1}{\mathfrak{M}_2} = \dfrac{200}{150} = 1{,}33$

$\beta = -(90 - \alpha)$

$\text{tg}\,\beta = \text{tg}\,[-(90 - \alpha)] = -\text{ctg}\,\alpha$

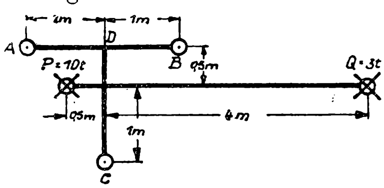
$\text{ctg}\,\beta = -\text{tg}\,\alpha$; $\quad \underline{\underline{\alpha = -\text{arc tg}\,0{,}75 = 143°10'}}$

$|\mathfrak{M}_3| = \sqrt{|\mathfrak{M}_1|^2 + |\mathfrak{M}_2|^2} = \sqrt{400 + 225} \cdot 10$

$= 250\,\text{kg m}$

$P = \dfrac{|\mathfrak{M}_3|}{5} = \underline{\underline{50\,\text{kg}}}$

Lösung 246



$\sum M_{\overline{AB}} = 0$: $\quad N_C \cdot 1{,}5 = (P + Q) \cdot 0{,}5$

$\underline{\underline{N_C = 4\tfrac{1}{3}\,\text{t}}}$

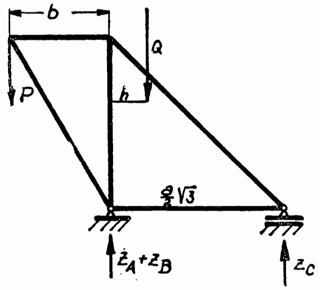
$\sum M_{\overline{CD}} = 0$:

$N_A \cdot 1 - N_B \cdot 1 + Q \cdot 4 - P \cdot 0{,}5 = 0$

$\sum P_y = 0$: $\quad N_A + N_B + N_C = Q + P = 13\,\text{t}$

$\underline{\underline{N_A = \tfrac{5}{6}\,\text{t}}}$; $\quad \underline{\underline{N_B = 7\tfrac{5}{6}\,\text{t}}}$

Lösung 247



$$\sum M_{\overline{AB}} = 0:$$

$$Z_C \cdot \frac{a}{2}\sqrt{3} - Q \cdot \mathrm{h} + P \cdot b = 0$$

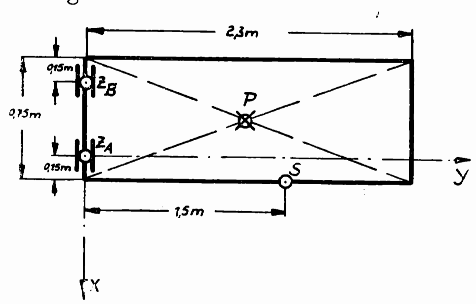
$$Z_C = \frac{(Q \cdot h - P \cdot b) \cdot 2}{a\sqrt{3}} = -\frac{2100}{\sqrt{3}} = \underline{\underline{-1212 \text{ kg}}}$$

Symmetrie: $\underline{\underline{Z_A = Z_B}}$

$$\sum P_z = 0: \quad P + Q - 2 Z_A - Z_C = 0$$

$$Z_A = \underline{\underline{1506 \text{ kg}}}$$

Lösung 248



$$\sum M_x = 0: \quad S \cdot 1{,}5 - P \cdot 1{,}15 = 0$$

$$S = \underline{\underline{138 \text{ kg}}}$$

$$\sum M_y = 0: \quad S \cdot 0{,}15 + P\left(\frac{0{,}75}{2} - 0{,}15\right)$$

$$- Z_B \cdot (0{,}75 - 2 \cdot 0{,}15) = 0$$

$$Z_B = \frac{S \cdot 0{,}15 + P \cdot 0{,}225}{0{,}45}$$

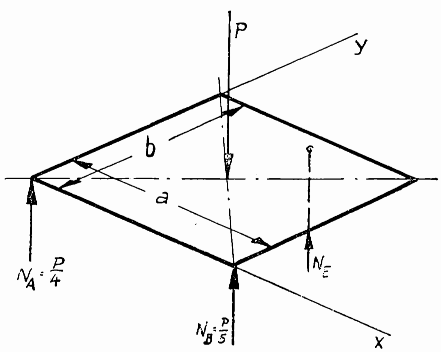
$$Z_B = \underline{\underline{136 \text{ kg}}}$$

$$\sum P_z = 0: \quad S + Z_A + Z_B - P = 0$$

$$Z_A = 180 - 136 - 138$$

$$= \underline{\underline{-94 \text{ kg}}}$$

Lösung 249



$$\sum M_x = 0: \quad R_E \cdot y - P \cdot \frac{b}{2} = 0$$

$$N_E = \frac{P \cdot b}{2y}$$

$$\sum M_y = 0: \quad \frac{P}{5} \cdot a + N_E \cdot x - P\,\frac{a}{2} = 0$$

$$N_E = \frac{3}{10} \cdot \frac{Pa}{x}$$

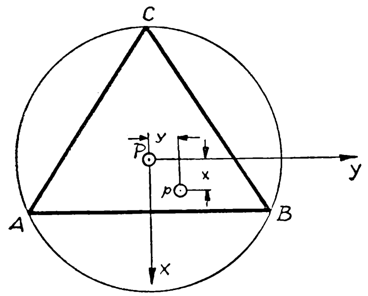
$$\sum P_z = 0: \quad P - \frac{P}{4} - \frac{P}{5} - N_E = 0$$

$$N_E = P\left(1 - \frac{5}{20} - \frac{4}{20}\right) = \underline{\underline{\frac{11}{20} P}}$$

$$x = \frac{3 \cdot P \cdot a \cdot 20}{10 \cdot P \cdot 11} = \underline{\underline{\frac{6a}{11}}}$$

$$y = \frac{P \cdot b \cdot 20}{2 \cdot P \cdot 11} = \underline{\underline{\frac{10}{11} b}}$$

Lösung 250



$$\sum P_z = 0:\quad N_A + N_B + N_C = P + p$$

$$\text{Für } p = 0:\quad \bar{N}_A = \bar{N}_B = \bar{N}_C = \frac{P}{3}$$

Für $P = 0$:

$$\sum M_{\overline{AB}} = 0:\quad \bar{\bar{N}}_C\, a\frac{\sqrt{3}}{2} - p\left(a\frac{\sqrt{3}}{6} - x\right) = 0$$

$$\sum M_y = 0:\quad (\bar{\bar{N}}_A + \bar{\bar{N}}_B)\frac{a\sqrt{3}}{6} - \bar{\bar{N}}_C \cdot \frac{a\sqrt{3}}{3} - p\cdot x = 0$$

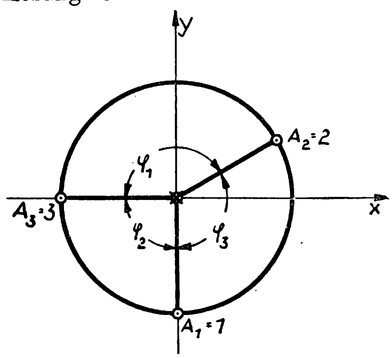
$$\sum M_x = 0:\quad (\bar{\bar{N}}_A - \bar{\bar{N}}_B)\frac{a}{2} + p\cdot y = 0$$

$$N_A = \bar{N}_A + \bar{\bar{N}}_A = \frac{P+p}{3} + p\left(\frac{x}{a}\,\frac{\sqrt{3}}{3} - \frac{y}{a}\right)$$

$$N_B = \bar{N}_B + \bar{\bar{N}}_B = \frac{P+p}{3} + p\left(\frac{x}{a}\,\frac{\sqrt{3}}{3} + \frac{y}{a}\right)$$

$$N_C = \bar{N}_C + \bar{\bar{N}}_C = \frac{P+p}{3} - \frac{2}{3}\cdot\frac{x}{a}\sqrt{3}\cdot p$$

Lösung 251



$$1\cdot r - 3r\cdot\cos(180 - \varphi_2) - 2r\cos(180 - \varphi_3) = 0$$
$$3\cdot r\cdot\sin(180 - \varphi_2) = 2r\sin(180 - \varphi_3)$$
$$3\cos(180 - \varphi_2) + 2\cos(180 - \varphi_3) = 1$$
$$3\sin(180 - \varphi_2) - 2\sin(180 - \varphi_3) = 0$$

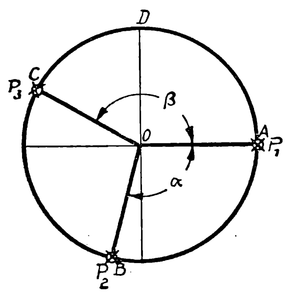
Durch Anwendung des Additionstheorems:

$$3\cos\varphi_2 + 2\cos\varphi_3 = 1$$
$$-3\sin\varphi_2 + 2\sin\varphi_3 = 0$$

Daraus: $\underline{\underline{\varphi_2 = 90^\circ}}$; $\underline{\underline{\varphi_3 = 120^\circ}}$

$$\varphi_1 = 360 - \varphi_2 - \varphi_3 = \underline{\underline{150^\circ}}$$

Lösung 252



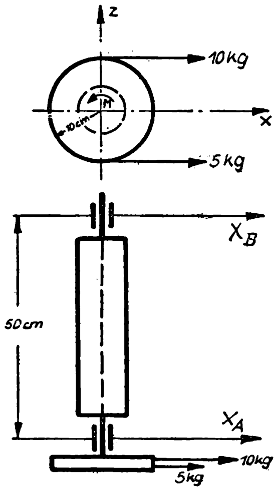
$\sum M_{\overline{OA}} = 0$:

$$P_3\cos(\beta - 90^\circ) = P_2\cos(\alpha - 90^\circ)$$

$\sum M_{\overline{OD}} = 0$:

$$P_1 = P_2\sin(\alpha - 90^\circ) + P_3\sin(\beta - 90^\circ)$$
$$2\sin\beta - \sin\alpha = 0$$
$$4\cos\beta + 2\cos\alpha = -3$$
$$\sin\alpha = 2\sin\beta$$
$$2\cos\alpha = -3 - 4\cos\beta$$
$$4(1 - 4\sin^2\beta) = 9 + 24\cos\beta + 16\cos^2\beta$$
$$0 = 21 + 24\cos\beta$$
$$\cos\beta = -\frac{7}{8};\quad \underline{\underline{\beta = 151^\circ}}$$
$$\cos\alpha = \frac{1}{4};\quad \underline{\underline{\alpha = 75^\circ\,30'}}$$

Lösung 253



$$M = (10 - 5) \cdot 10 = \underline{\underline{50 \text{ kg cm}}}$$

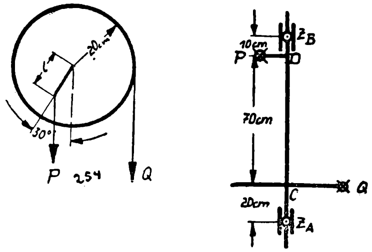
$$\sum M_{Bz} = 0: \quad (10 + 5) \cdot 60 + X_A \cdot 50 = 0$$

$$\underline{\underline{X_A = -18 \text{ kg}}}$$

$$\sum P_x = 0: \quad 15 - 18 + X_B = 0$$

$$\underline{\underline{X_B = 3 \text{ kg}}}$$

Lösung 254



$$\sum M_{\overline{AB}} = 0: \quad Q \cdot 20 = P \cdot l \sin 30^\circ$$

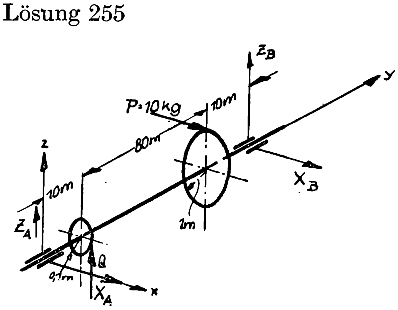
$$l = \frac{25 \cdot 20}{100 \cdot 0{,}5} = \underline{\underline{10 \text{ cm}}}$$

$$\sum M_A = 0: \quad Z_B \cdot 100 - P \cdot 90 - Q \cdot 20 = 0$$

$$Z_B = \frac{100 \cdot 90 + 25 \cdot 20}{100} = \underline{\underline{95 \text{ kg}}}$$

$$\sum P_z = 0: \quad Z_A = Q + P - Z_B = \underline{\underline{30 \text{ kg}}}$$

Lösung 255



$$Q \cdot 0{,}1 = P \cdot 1{,}0; \quad Q = P \cdot 10 = \underline{\underline{100 \text{ kg}}}$$

$$\sum M_{Bz} = 0: \quad X_A \cdot 100 + P \cdot 10 = 0$$

$$X_A = \underline{\underline{-1 \text{ kg}}}$$

$$\sum P_x = 0: \quad X_A + X_B + P = 0$$

$$X_B = \underline{\underline{-9 \text{ kg}}}$$

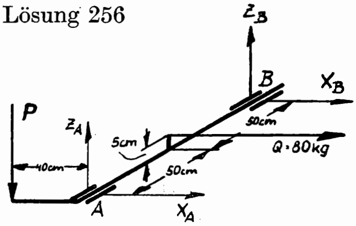
$$\sum M_{Bx} = 0: \quad Z_A \cdot 100 + Q \cdot 90 = 0$$

$$Z_A = \underline{\underline{-90 \text{ kg}}}$$

$$\sum P_z = 0: \quad Z_A + Q + Z_B = 0$$

$$Z_B = \underline{\underline{-10 \text{ kg}}}$$

Lösung 256

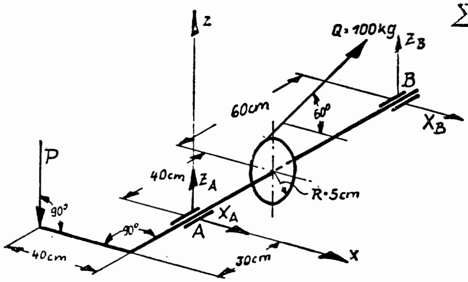


$$\sum M_{\overline{AB}} = 0: \quad P \cdot 40 = Q \cdot 5$$
$$P = \underline{\underline{10\,\text{kg}}}$$
$$\text{Symmetrie:} \quad X_A = X_B = -\frac{Q}{2} = \underline{\underline{-40\,\text{kg}}}$$
$$\sum M_{B_x} = 0: \quad P \cdot 100 = Z_A \cdot 100$$
$$Z_A = \underline{\underline{10\,\text{kg}}}$$
$$\sum P_z = 0: \quad Z_A + Z_B - P = 0; \quad Z_B = \underline{\underline{0}}$$

Die entsprechenden Aktionskräfte haben entgegengesetzte Vorzeichen.

Lösung 257

$$Q \cdot R = P \cdot 40$$
$$P = Q \cdot \frac{5}{40} = \underline{\underline{12{,}5\,\text{kg}}}$$

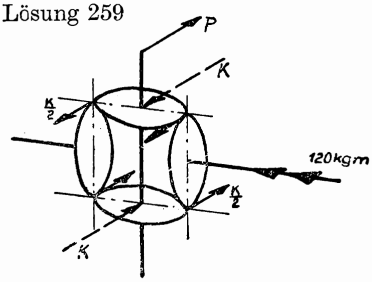


$$\sum M_{B_z} = 0: \quad X_A \cdot 100 + Q \cdot 60 \cdot \cos 60° = 0$$
$$X_A = -\frac{100 \cdot 60 \cdot 1}{100 \cdot 2}$$
$$= \underline{\underline{-30\,\text{kg}}}$$
$$\sum P_x = 0: \quad Q \cdot \cos 60° + X_A + X_B = 0$$
$$X_B = \underline{\underline{-20\,\text{kg}}}$$
$$\sum M_{B_x} = 0: \quad Z_A \cdot 100 - P \cdot 130 + Q \cdot \sin 60° \cdot 60 = 0$$
$$Z_A = \underline{\underline{-35{,}7\,\text{kg}}}$$
$$\sum P_z = 0: \quad Z_A + Z_B - P + Q \sin 60° = 0$$
$$Z_B = \underline{\underline{-38{,}4\,\text{kg}}}$$

Lösung 258

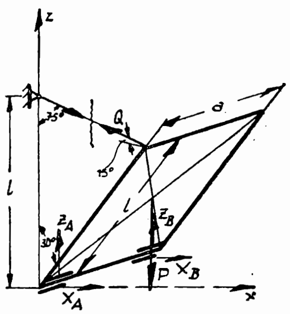
$$\sum M_{\overline{BC}} = 0: \quad Q = 6P = \underline{\underline{36\,\text{kg}}}$$
$$\sum M_{A_z} = 0: \quad P \cdot \cos\alpha \cdot 0{,}5 = X_B \cdot 1{,}5; \quad X_B = \frac{6}{3}\,\frac{\sqrt{3}}{2} = \underline{\underline{1{,}73\,\text{kg}}}$$
$$\sum P_x = 0: \quad P \cdot \cos\alpha - X_B - X_A = 0; \quad X_A = \underline{\underline{-6{,}93\,\text{kg}}}$$
$$\sum M_{A_x} = 0: \quad P \cdot \sin\alpha \cdot 0{,}5 = Q \cdot 1 - Z_B \cdot 1{,}5; \quad Z_B = \frac{-\frac{6}{2}\,0{,}5 + 36 \cdot 1}{1{,}5} = \underline{\underline{23\,\text{kg}}}$$
$$\sum P_z = 0: \quad P \sin\alpha + Q - Z_A - Z_B = 0; \quad Z_A = \underline{\underline{16\,\text{kg}}}$$

Lösung 259



$$\frac{2}{2}\,K = \frac{1200}{r} = \underline{\underline{120\,\text{kg}}} \quad N_E = N_F = \underline{\underline{K}}$$
$$K \cdot 10 + K \cdot 10 - P \cdot 60 = 0$$
$$P = \frac{2\,K \cdot 10}{60} = \underline{\underline{40\,\text{kg}}}$$

Lösung 260



$$\sum M_{\overline{AB}} = 0{:}\quad P \cdot \frac{l}{2} \sin 30^\circ = Q l \cos 15^\circ$$

$$Q = \frac{P}{2} \cdot \frac{1}{2 \cdot 0{,}96} = \underline{\underline{10{,}4\ \mathrm{kg}}}$$

$$\sum M_{A_x} = 0{:}\quad P \cdot \frac{a}{2} = Z_B \cdot a; \quad Z_B = \underline{\underline{20\ \mathrm{kg}}}$$

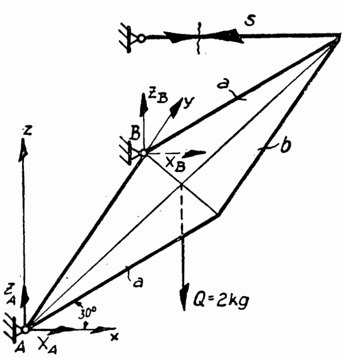
$$\sum M_{A_z} = 0{:}\quad \underline{\underline{X_B = 0}}$$

$$\sum P_x = 0{:}\quad X_A = Q \cdot \cos 15^\circ = \underline{\underline{10\ \mathrm{kg}}}$$

$$\sum P_z = 0{:}\quad Z_B + Z_A + Q \cos 75^\circ - P = 0$$

$$Z_A = \underline{\underline{17{,}3\ \mathrm{kg}}}$$

Lösung 261



$$\sum M_{\overline{AB}} = 0{:}\quad Q \cdot \frac{a}{2} \cdot \cos 30^\circ - S \cdot a \cdot \sin 30^\circ = 0$$

$$S = \underline{\underline{1{,}73\ \mathrm{kg}}}$$

$$\sum M_{A_x} = 0{:}\quad Z_B \cdot b - Q \cdot \frac{b}{2} = 0$$

$$Z_B = \frac{Q}{2} = 1\ \mathrm{kg}$$

$$\sum P_z = 0{:}\quad Z_A + Z_B - Q = 0$$

$$Z_A = \underline{\underline{1\ \mathrm{kg}}}$$

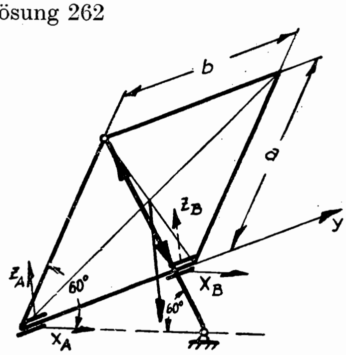
$$\sum M_{A_z} = 0{:}\quad X_B \cdot b - S \cdot b = 0$$

$$X_B = \underline{\underline{1{,}73\ \mathrm{kg}}}$$

$$\sum P_x = 0{:}\quad X_B + X_A - S = 0;$$

$$X_A = \underline{\underline{0}}$$

Lösung 262



$$\sum M_{\overline{AB}} = 0{:}\quad Q \cdot \frac{a}{2} \cos 60^\circ = S \cdot a \cdot \cos 30^\circ$$

$$S = \underline{\underline{3{,}45\ \mathrm{kg}}}$$

$$\sum M_{A_x} = 0{:}\quad Q \cdot \frac{b}{2} - Z_B \cdot b = 0$$

$$Z_B = \underline{\underline{6\ \mathrm{kg}}}$$

$$\sum P_z = 0{:}\quad Z_A + Z_B + S \cdot \cos 30^\circ - P = 0$$

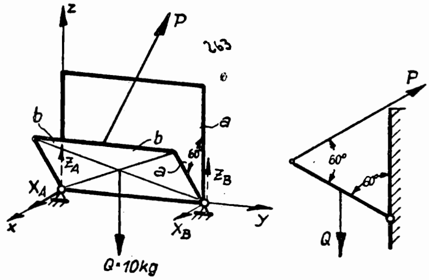
$$Z_A = \underline{\underline{3\ \mathrm{kg}}}$$

$$\sum M_{A_z} = 0{:}\quad X_B = \underline{\underline{0}}$$

$$\sum P_x = 0{:}\quad S \cdot \cos 60^\circ - X_A - X_B = 0$$

$$X_A = \underline{\underline{1{,}73\ \mathrm{kg}}}$$

Lösung 263

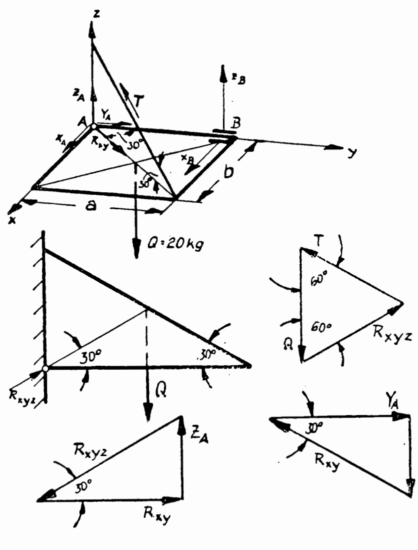


$$\sum M_{\overline{AB}} = 0: \quad Q \cdot \frac{a}{2} \cos 30° - P \cdot \frac{a}{2} \sqrt{3} = 0$$
$$P = \underline{\underline{5\,\text{kg}}}$$
$$\sum M_{A_z} = 0: \quad P \cdot \cos 30° \cdot b - X_B \cdot 2b = 0$$
$$X_B = \underline{\underline{2{,}17\,\text{kg}}}$$
$$\sum P_x = 0: \quad X_A + X_B - P \cos 30° = 0$$
$$X_A = \underline{\underline{2{,}17\,\text{kg}}}$$
$$\sum M_{A_x} = 0: \quad Q \cdot \frac{b}{2} - P \cdot \sin 30° \cdot \frac{b}{2} - Z_B \cdot b = 0$$
$$Z_B = \underline{\underline{3{,}75\,\text{kg}}}$$
$$\sum P_z = 0: \quad Z_A + Z_B + P \cdot \sin 30° - Q = 0$$
$$Z_A = \underline{\underline{3{,}75\,\text{kg}}}$$

Lösung 264 Die Kraftrichtungen werden entsprechend dem Koordinatensystem als positiv definiert.

$$\sum M_{\overline{AB}} = 0: \quad T \cdot 1 - Q \cdot \frac{1}{2} \cdot \cos 60° = 0; \qquad T = \underline{\underline{375\,\text{kg}}}$$
$$\sum M_{B_z} = 0: \quad Y_A = \underline{\underline{0}}$$
$$\sum P_y = 0: \quad T \cdot \cos 30° + Y_B = 0; \quad Y_B = \underline{\underline{-325\,\text{kg}}}$$
$$\sum M_{B_y} = 0: \quad Z_A \cdot 1 - Q \cdot \frac{1}{2} = 0; \quad Z_A = \underline{\underline{750\,\text{kg}}}$$
$$\sum P_z = 0: \quad Z_A + Z_B + T \sin 30° - Q = 0; \quad Z_B = \underline{\underline{562{,}5\,\text{kg}}}$$

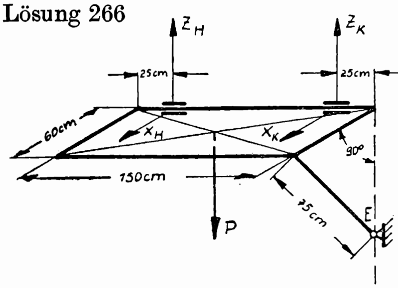
Lösung 265



Da Q; T; A in einer Ebene liegen, wird $X_B = Z_B = 0$

$$Q = T = R_{xyz} = \underline{\underline{20\,\text{kg}}}$$
$$Z_A = R_{xyz} \cdot \sin 30° = \underline{\underline{10\,\text{kg}}}$$
$$R_{xy} = R_{xyz} \cdot \cos 30°$$
$$Y_A = R_{xy} \cdot \cos 30° = R_{xyz} \cdot \cos^2 30° = \underline{\underline{15\,\text{kg}}}$$
$$X_A = R_{xy} \cdot \sin 30° = \underline{\underline{8{,}66\,\text{kg}}}$$

Lösung 266



$$\overline{AE} = \sqrt{75^2 - 60^2} = 45 \text{ cm}$$

$$\sum M_{\overline{AB}} = 0: \quad P \cdot 30 - S \cdot \frac{45}{75} \cdot 60 = 0$$

$$S = \underline{\underline{66\frac{2}{3} \text{ kg}}}$$

$$\sum M_{H_x} = 0:$$

$$P \cdot 50 - Z_K \cdot 100 - S \cdot 125 \cdot \frac{45}{75} = 0$$

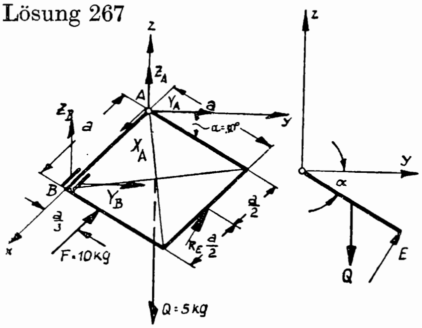
$$Z_K = \underline{\underline{-10 \text{ kg}}}$$

$$\sum M_{H_z} = 0: \quad X_K \cdot 100 + S \cdot \frac{60}{75} \cdot 125 = 0 \qquad X_K = \underline{\underline{-66\frac{2}{3} \text{ kg}}}$$

$$\sum P_x = 0: \quad X_H + X_K + S \cdot \frac{60}{75} = 0 \qquad X_H = \underline{\underline{+13\frac{1}{3} \text{ kg}}}$$

$$\sum P_z = 0: \quad Z_K + Z_H + S \cdot \frac{45}{75} - P = 0 \qquad Z_H = \underline{\underline{50 \text{ kg}}}$$

Lösung 267



$$\sum M_{\overline{BA}} = 0: \quad R_E \cdot a - Q \cdot \frac{\cos 30° \cdot a}{2} = 0$$

$$R_E = \underline{\underline{2{,}17 \text{ kg}}}$$

$$\sum P_x = 0: \quad X_A - F = 0;$$

$$X_A = \underline{\underline{10 \text{ kg}}}$$

$$\sum M_{A_z} = 0:$$

$$Y_B \cdot a + F \frac{a}{3} \cos 30° + R_E \cdot \sin 30° \cdot \frac{a}{2} = 0$$

$$Y_B = \underline{\underline{-3{,}43 \text{ kg}}}$$

$$\sum P_y = 0: \quad Y_B + Y_A + R_E \cdot \sin 30° = 0$$

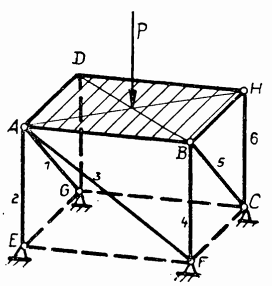
$$Y_A = \underline{\underline{2{,}35 \text{ kg}}}$$

$$\sum M_{A_y} = 0: \quad Z_B \cdot a - F \cdot \frac{a}{3} \sin 30° - Q \cdot \frac{a}{2} \cos 30° + R_E \cdot \frac{a}{2} \cos 30° = 0$$

$$Z_B = \underline{\underline{3{,}23 \text{ kg}}}$$

$$\sum P_z = 0: \quad Z_A + Z_B - Q + R_E \cos 30° = 0; \quad Z_A = \underline{\underline{0{,}11 \text{ kg}}}$$

Lösung 268



$$\sum M_{\overline{AC}} = 0: \quad S_4 = 0$$

$$\sum M_{\overline{AB}} = 0: \quad S_6 = -\frac{P}{2}$$

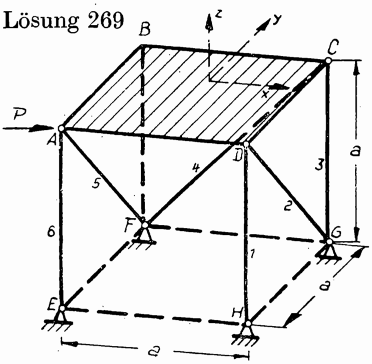
$$\sum P_{EF} = 0: \quad S_3 = 0$$

$$\sum M_{\overline{CG}} = 0: \quad S_2 = -\frac{P}{2}$$

$$\sum M_{\overline{BE}} = 0: \quad S_1 = 0$$

$$\sum P_{CE} = 0: \quad S_5 = 0$$

Lösung 269



Alle Stäbe werden als Zugstäbe angenommen, ihre Berechnung erfolgt auf Grund von Schnittbetrachtungen.

$$\sum P_x = 0: \quad P - S_4 \frac{\sqrt{2}}{2} = 0; \quad S_4 = P \cdot \sqrt{2}$$

$$\sum M_{\overline{GF}} = 0: \quad S_1 \cdot a + S_6 \cdot a = 0$$

$$\sum M_{\overline{DC}} = 0: \quad S_6 \frac{\sqrt{2}}{2} \cdot a + S_5 \cdot a = 0$$

$$\sum M_{\overline{HD}} = 0: \quad S_5 \frac{\sqrt{2}}{2} a - S_4 \frac{\sqrt{2}}{2} a = 0;$$

$$S_5 = P\sqrt{2} \quad S_6 = -P; \quad S_1 = P$$

$$\sum P_z = 0: \quad S_6 + S_1 + S_3 + S_5 \frac{\sqrt{2}}{2} + S_4 \frac{\sqrt{2}}{2} + S_2 \frac{\sqrt{2}}{2} = 0$$

$$S_3 + 2P + S_2 \frac{\sqrt{2}}{2} = 0$$

$$\sum P_y = 0: \quad S_5 \frac{\sqrt{2}}{2} + S_2 \frac{\sqrt{2}}{2} = 0; \quad S_2 = -P\sqrt{2}; \quad S_3 = -P$$

Lösung 270

Die Kraftrichtungen werden entsprechend dem Koordinatensystem als positiv definiert.

$$\sum M_z = 0: \quad T \cdot a \cdot \sin 60° = P \cdot a \cdot \cos 30°; \quad T = P = 32\,\text{kg}$$

$$\sum P_z = 0: \quad Z_B = 64\,\text{kg}$$

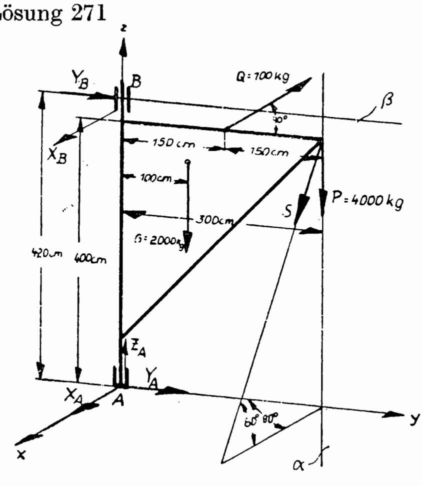
$$\sum M_x = 0: \quad 64 \cdot 90 \cdot \sin 30° + Y_A \cdot 240 + P \cdot \cos 60° \cdot 240 = 0; \qquad Y_A = -28\,\text{kg}$$

$$\sum P_y = 0: \quad Y_A + Y_B + P \cos 60° - T = 0; \quad Y_B = 44\,\text{kg}$$

$$\sum M_y = 0: \quad 64 \cdot 90 \cdot \sin 60° - X_A \cdot 240 - P \cdot 240 \cdot \cos 30° = 0: \qquad X_A = 6{,}9\,\text{kg}$$

$$\sum P_x = 0: \quad X_A + X_B + P \cos 30° = 0; \quad X_B = 20{,}8\,\text{kg}$$

Lösung 271



$$S \cdot 300 \cdot \sin 30° = Q \cdot 150$$

$$S = Q = 100\,\text{kg}$$

$$\sum M_x = 0: \quad P \cdot 300 + S \cos 30° \cdot 300 + G \cdot 100 + Y_B \cdot 420 = 0$$

$$Y_B = -3395\,\text{kg}$$

$$\sum P_y = 0: \quad Y_A = -Y_B$$

$$\sum M_\alpha = 0:$$

$$-Q \cdot 150 + (X_B + X_A) \cdot 300 = 0$$

$$X_B + X_A = 50\,\text{kg}$$

$$\sum M_\beta = 0:$$

$$-(Q - S \sin 30°) \cdot 20 + X_A \cdot 420 = 0$$

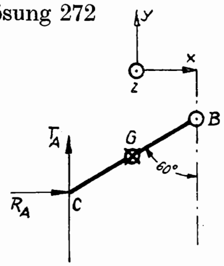
$$X_A = 2{,}4\,\text{kg}$$

$$X_B = 47{,}6\,\text{kg}$$

$$\sum P_z = 0:$$

$$Z_A = 4000 + 2000 + 86{,}6 = 6087\,\text{kg}$$

Lösung 272



$\sum P_z = 0$: $\quad R_B = G = \underline{\underline{8\,\text{kg}}}$

Momentengleichung um eine Achse parallel zu CE durch B:

$$R_A \cdot l \cdot \cos 30° - G \cdot \frac{l}{2} \cdot \cos 60° \sin 60° = 0$$

$$R_A = \underline{\underline{2\,\text{kg}}}$$

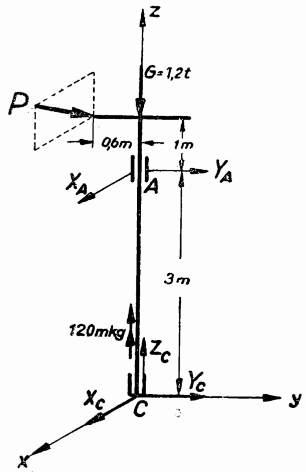
$\sum M_{\overline{CB}} = 0$: $\quad R_A \cdot l \cos 60° = T_A \cdot l \cdot \cos 30°$

$$T_A = \underline{\underline{1{,}15\,\text{kg}}}$$

$\sum P_y = 0$: $\quad T_A - T_B \cdot \cos 60° = 0$

$$T_B = \underline{\underline{2{,}3\,\text{kg}}}$$

Lösung 273



$$P \cos 15° \cdot 0{,}6 = M = 120\,\text{mkg}$$

$$P = \underline{\underline{208\,\text{kg}}}$$

$\sum P_z = 0$: $\quad 1{,}2\,\text{t} + P \cdot \sin 15°\,\text{kg} = Z_C$

$$Z_C = \underline{\underline{1254\,\text{kg}}}$$

$\sum M_x = 0$: $\quad Y_A \cdot 3 - P \sin 15° \cdot 0{,}6 = 0$

$$Y_A = \underline{\underline{10{,}8\,\text{kg}}}$$

$\sum P_y = 0$: $\quad Y_A + Y_C = 0; \quad Y_C = \underline{\underline{-10{,}8\,\text{kg}}}$

$\sum M_y = 0$: $\quad X_A \cdot 3 - P \cos 15° \cdot 4 = 0$

$\quad X_A = \underline{\underline{267\,\text{kg}}}$

$\sum P_x = 0$: $\quad X_C + X_A - P \cos 15° = 0$

$\quad X_C = \underline{\underline{-67\,\text{kg}}}$

Lösung 274 Kraft auf einen Flügel: 120 kg. Davon $120 \cdot \cos 30° = 60\sqrt{3}$ kg in Achsrichtung und $120 \cdot \sin 30° = 60$ kg senkrecht zur Achsrichtung.

1) $\sum M_y = 0$: $\quad 4 \cdot 60 \cdot 200 = P \cdot 120; \quad P = \underline{\underline{400\,\text{kg}}}$

$\sum P_y = 0$: $\quad Y_C + 4 \cdot 60 \cdot \sqrt{3} = 0; \quad Y_C = \underline{\underline{-416\,\text{kg}}}$

$\sum M_{A\,x} = 0$: $\quad P \cdot 100 - Z_C \cdot 150 = 0; \quad Z_C = 266{,}6\,\text{kg}$

$\sum M_{C\,x} = 0$: $\quad P \cdot 50 - Z_A \cdot 150 = 0; \quad Z_A = \underline{\underline{133\,\text{kg}}}$

$\sum P_x = 0$: $\quad X_A = X_C = 0$

2) $\sum M_y = 0$: $\quad 3 \cdot 60 \cdot 200 = P \cdot 120; \quad P = \underline{\underline{300\,\text{kg}}}$

$\sum P_y = 0$: $\quad Y_C + 3 \cdot 60\sqrt{3} = 0; \quad Y_C = \underline{\underline{-312\,\text{kg}}}$

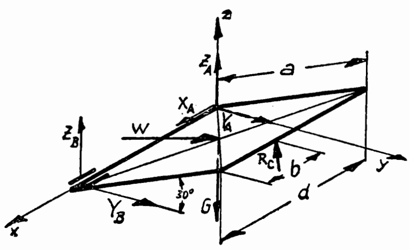
$\sum M_{A\,x} = 0$: $\quad -60\sqrt{3} \cdot 200 - P \cdot 100 + Z_C \cdot 150 = 0; \quad Z_C = \underline{\underline{339\,\text{kg}}}$

$\sum M_{C\,x} = 0$: $\quad -60\sqrt{3} \cdot 200 + P \cdot 50 - Z_A \cdot 150 = 0; \quad Z_A = \underline{\underline{-38{,}6\,\text{kg}}}$

$\sum M_{A\,z} = 0$: $\quad X_C \cdot 150 + 60 \cdot 50 = 0; \quad X_C = \underline{\underline{-20\,\text{kg}}}$

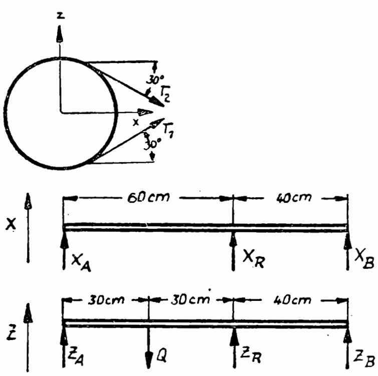
$\sum M_{C\,z} = 0$: $\quad -X_A \cdot 150 + 60 \cdot 200 = 0; \quad X_A = \underline{\underline{80\,\text{kg}}}$

Lösung 275



$$G = 20 \cdot 3 \cdot 6 = 360\,\text{kg}$$
$$W_x = 900 \cdot \cos 15° \sin 30° = 435\,\text{kg}$$
$$W_y = 900 \cdot \cos 15° \cos 30° = 754\,\text{kg}$$
$$W_z = 900 \cdot \sin 15° \qquad = 234\,\text{kg}$$
$$\sum M_x = 0:\ R_C \cdot a - G\frac{a}{2}\cos 30° - W_z \cdot \frac{a}{2}\cos 30° - W_y\frac{a}{2}\sin 30° = 0$$
$$R_C = \underline{\underline{445\,\text{kg}}}$$
$$\sum P_x = 0:\ X_A - W_x = 0: \quad X_A = \underline{\underline{435\,\text{kg}}}$$
$$\sum M_y = 0:\ Z_B \cdot d - G\frac{d}{2} + R_C(d-b)\cdot\cos 30° + W_x \cdot \frac{a}{2}\cdot \sin 30° - W_z \cdot \frac{d}{2} = 0$$
$$Z_B = \underline{\underline{-14{,}8\,\text{kg}}}$$
$$\sum P_z = 0:\quad Z_B + Z_A - G + R_C\cos 30° - W_z = 0; \quad Z_A = \underline{\underline{222\,\text{kg}}}$$
$$\sum M_z = 0:\quad Y_B \cdot d - R_C \cdot \sin 30°\,(d-b) + W_x \cdot \frac{a}{2}\cos 30° + W_y \cdot \frac{d}{2} = 0;$$
$$Y_B = \underline{\underline{-323\,\text{kg}}}$$
$$\sum P_y = 0:\quad Y_B + Y_A - R_C \cdot \sin 30° + W_y = 0; \quad Y_A = \underline{\underline{-208\,\text{kg}}}$$

Lösung 276

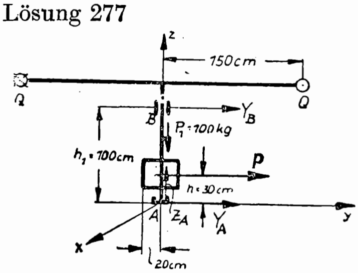


$$\sum M_y = 0:\quad Q \cdot r = T \cdot R; \quad T = T_1 - T_2$$
$$T_1 = 2\,T_2; \quad T = T_2$$
$$T_2 = \underline{\underline{0{,}5\,\text{t}}}; \quad T_1 = \underline{\underline{1\,\text{t}}}$$

Resultierende Riemenkraft:

$$X_R = (T_1 + T_2)\cos 30° = 0{,}75\sqrt{3}\,\text{t}$$
$$Z_R = (T_1 - T_2)\sin 30° = 0{,}25\,\text{t}$$
$$X_B = -0{,}75\sqrt{3}\cdot\frac{60}{100} = \underline{\underline{-0{,}78\,\text{t}}}$$
$$X_A = -0{,}75\sqrt{3}\cdot\frac{40}{100} = \underline{\underline{-0{,}52\,\text{t}}}$$
$$Z_B = Q \cdot 0{,}3 - Z_R \cdot 0{,}6 = \underline{\underline{0{,}15\,\text{t}}}$$
$$Z_A = Q \cdot 0{,}7 - Z_R \cdot 0{,}4 = \underline{\underline{0{,}60\,\text{t}}}$$

Lösung 277



$$\sum M_z = 0:\quad P \cdot 20 = 2 \cdot Q \cdot 150; \quad Q = \underline{\underline{20\,\text{kg}}}$$
$$\sum M_x = 0:\quad Y_B \cdot h_1 + P \cdot h = 0; \quad Y_B = \underline{\underline{-90\,\text{kg}}}$$
$$\sum M_y = 0:\quad X_B = 0; \quad \sum P_x = 0; \quad X_A = 0$$
$$\sum P_y = 0:\quad P + Y_B + Y_A = 0; \quad Y_A = \underline{\underline{-210\,\text{kg}}}$$
$$\sum P_z = 0:\quad Z_A = \underline{\underline{100\,\text{kg}}}$$

Lösung 278

$$\sum M_z = 0: \quad 4P \cdot l = Q \cdot \frac{d}{2} \cdot \sin 30^\circ; \qquad P = \underline{\underline{15\,\text{kg}}}$$

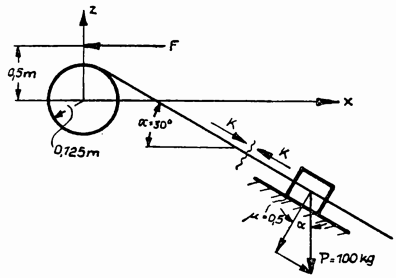
$$\sum P_z = 0: \quad Z_A - q = 0; \quad Z_A = \underline{\underline{100\,\text{kg}}}$$

$$\sum M_x = 0: \quad Q \sin 30^\circ \cdot 1{,}5 + Y_B \cdot 2 = 0; \qquad Y_B = -\underline{\underline{375\,\text{kg}}}$$

$$\sum P_y = 0: \quad Y_B + Y_A + Q \cdot \sin 30^\circ = 0; \qquad Y_A = -\underline{\underline{125\,\text{kg}}}$$

$$\sum M_y = 0: \quad X_B = 0; \quad \sum P_x = 0: \quad X_A = 0$$

Lösung 279



$$K = P(\sin\alpha + \mu\cos\alpha)$$

$$\sum M_y = 0:$$

$$K \cdot 0{,}125 = F \cdot 0{,}5; \quad F = \underline{\underline{23{,}3\,\text{kg}}}$$

$$\sum M_{\overline{BZ}} = 0:$$

$$X_A \cdot 1{,}5 - F \cdot 1{,}5 + K \cdot \cos 30^\circ \cdot 0{,}5 = 0$$

$$X_A = -\underline{\underline{3{,}61\,\text{kg}}}$$

$$\sum P_x = 0:$$

$$X_A + X_B - F + K\cos 30^\circ = 0$$

$$X_B = -\underline{\underline{53{,}9\,\text{kg}}}$$

$$\sum M_{B_x} = 0: \quad Z_A \cdot 1{,}5 - K \cdot \sin 30^\circ \cdot 0{,}5 - Q \cdot 0{,}75 = 0; \qquad Z_A = \underline{\underline{30{,}6\,\text{kg}}}$$

$$\sum P_z = 0: \quad Z_A + Z_B - K \cdot \sin 30^\circ - Q = 0; \qquad Z_B = \underline{\underline{46{,}1\,\text{kg}}}$$

Lösung 280

$$(T_1 - t_1)\,r_1 = (T_2 - t_2)\,r_2; \quad T_1 = 2t_1; \quad T_2 = 2t_2; \quad t_1 \cdot r_1 = t_2 \cdot r_2$$

$$t_2 = \underline{\underline{200\,\text{kg}}}; \quad T_2 = \underline{\underline{400\,\text{kg}}}$$

$$\sum M_z = 0: \quad 3t_1 \cdot a + 3t_2(a+c)\sin\alpha + X_B(a+c+b) = 0; \qquad X_B = -\underline{\underline{412{,}5\,\text{kg}}}$$

$$\sum P_x = 0: \quad X_A + 3t_1 + 3t_2 \sin 30^\circ + X_B = 0; \quad X_A = -\underline{\underline{637{,}5\,\text{kg}}}$$

$$\sum M_x = 0: \quad Z_B(a+b+c) - 3t_2\cos\alpha\,(a+c) = 0; \quad Z_B = \underline{\underline{390\,\text{kg}}}$$

$$\sum P_z = 0: \quad Z_A + Z_B - 3t_2\cos\alpha = 0; \qquad Z_A = \underline{\underline{130\,\text{kg}}}$$

Lösung 281

$$\sum M_{\overline{AB}} = 0: \quad (T - t) \cdot 100 - P \cdot \cos 20^\circ \cdot 12{,}5 = 0;$$

$$T + t = 750 \qquad T = \underline{\underline{492\,\text{kg}}}; \quad t = \underline{\underline{258\,\text{kg}}}$$

$$\sum M_{B_z} = 0: \quad X_A(n+m) + P\cos 10^\circ \cdot m - (T+t) \cdot \cos 30^\circ \cdot l = 0$$

$$X_A = -\underline{\underline{571\,\text{kg}}}$$

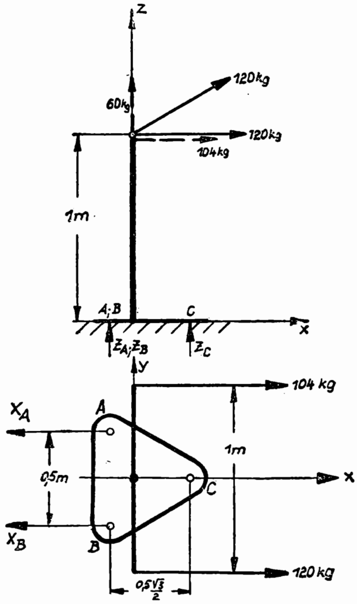
$$\sum P_x = 0: \quad X_A + X_B + P\cos 10^\circ + (T+t)\cos 30^\circ = 0; \quad X_B = -\underline{\underline{2048\,\text{kg}}}$$

$$\sum M_{B_x} = 0: \quad Z_A(m+n) + P \cdot m \sin 10^\circ - (T+t)\sin 30^\circ \cdot l + Q \cdot l = 0$$

$$Z_A = \underline{\underline{-447\,\text{kg}}}$$

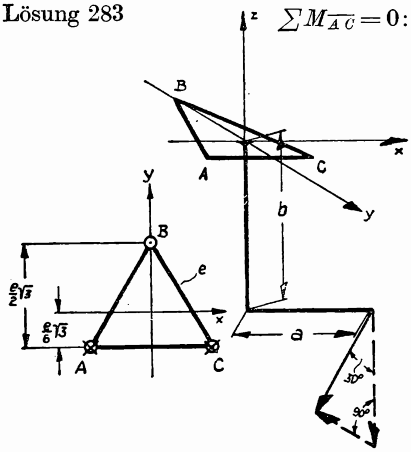
$$\sum P_z = 0: \quad Z_A + Z_B + P\sin 10^\circ + (T+t)\sin 30^\circ - Q = 0; \quad Z_B = \underline{\underline{1025\,\text{kg}}}$$

Lösung 282 In y-Richtung treten keine Kräfte auf.



$$\sum M_{\overline{AB}} = 0:$$
$$(120 + 104)\cdot 1 - 60\cdot 0{,}144 - Z_C\cdot 0{,}432 + 300\cdot 0{,}144 = 0$$
$$Z_C = \underline{\underline{597\ \mathrm{kg}}}$$
$$\sum M_{Cz} = 0:$$
$$(120 - 104)\cdot 0{,}5 + (X_A - X_B)\cdot 0{,}25 = 0$$
$$\sum P_x = 0:$$
$$(120 + 104) - (X_A + X_B) = 0$$
$$X_A - X_B + 32 = 0$$
$$X_A + X_B - 224 = 0$$
$$X_A = \underline{\underline{96\ \mathrm{kg}}}; \quad X_B = \underline{\underline{128\ \mathrm{kg}}}$$
$$\sum M_{Cx} = 0: \quad 60\cdot 0{,}5 + Z_A\cdot 0{,}25 - Z_B\cdot 0{,}25 = 0$$
$$\sum P_z = 0: \quad Z_A + Z_B + Z_C - (300 - 60) = 0$$
$$Z_A + Z_B = -357$$
$$Z_A - Z_B = -120$$
$$Z_A = \underline{\underline{-239\ \mathrm{kg}}}$$
$$Z_B = \underline{\underline{-119\ \mathrm{kg}}}$$

Lösung 283

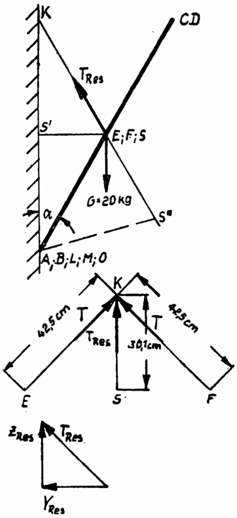


$$\sum M_{\overline{AC}} = 0: \quad Z_B\cdot\frac{e}{2}\sqrt{3} + 2P\cos 30°\,\frac{e}{6}\sqrt{3} - 2Pb\sin 30° = 0$$
$$Z_B = \underline{\underline{115\ \mathrm{kg}}}$$
$$X_B = Y_B = 0$$
$$\sum M_{\overline{AB}} = 0:$$
$$2P\cos 30°\,\frac{e}{6}\sqrt{3} + 2Pa\cos^2 30° + 2Pb\sin^2 30° + Z_C\cdot\frac{e}{2}\sqrt{3} = 0$$
$$Z_C = \underline{\underline{-508\ \mathrm{kg}}}$$
$$\sum P_z = 0: \quad Z_B + Z_C + Z_A + 2P\cos 30° = 0$$
$$Z_A = \underline{\underline{185\ \mathrm{kg}}}$$
$$\sum M_{Cz} = 0: \quad Y_A\cdot e - 2P\sin 30°\left(a - \frac{e}{2}\right) = 0$$
$$Y_A = \underline{\underline{140\ \mathrm{kg}}}$$
$$\sum P_y = 0: \quad Y_A + Y_C + 2P\sin 30° = 0$$
$$Y_C = \underline{\underline{-260\ \mathrm{kg}}}$$

Lösung 284 $(SK)^2 = (EK)^2 - (ES)^2 = (42{,}5)^2 - (30)^2 = 906{,}25\ \mathrm{cm}^2$

$SK = 30{,}1\ \mathrm{cm}$

$OS = 37{,}5\ \mathrm{cm}; \quad \sin\alpha = \dfrac{\operatorname{tg}\alpha}{\sqrt{1 + \operatorname{tg}^2\alpha}} = \dfrac{3}{5}$



$$SS' = OS\sin\alpha = 22{,}5\ \mathrm{cm}$$

$$(KS')^2 = (KS)^2 - (S'S)^2 = 906{,}25 - 506{,}25 = 400\ \mathrm{cm}^2$$

$$KS' = 20\ \mathrm{cm}$$

$$\sum M_{S'} = 0: \quad G\cdot 22{,}5 - T_{\mathrm{Res}}\cdot 22{,}5\cdot\frac{50}{30{,}1} = 0$$

$$T_{\mathrm{Res}} = 12\ \mathrm{kg}$$

$$T = \frac{T_{\mathrm{Res}}}{2}\cdot\frac{42{,}5}{30{,}1} = \underline{\underline{8{,}5\ \mathrm{kg}}}$$

$$Z_{T\,\mathrm{Res}} = T_{\mathrm{Res}}\cdot\frac{20}{30{,}1} = 8\ \mathrm{kg}$$

$$Y_{T\,\mathrm{Res}} = T_{\mathrm{Res}}\cdot\frac{22{,}5}{30{,}1} = -9\ \mathrm{kg}$$

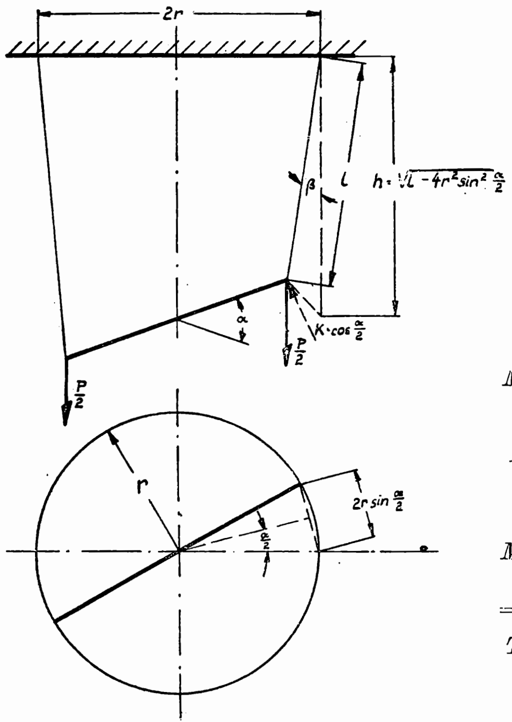
Symmetrie: $Z_L = Z_M$; $Y_L = Y_M$

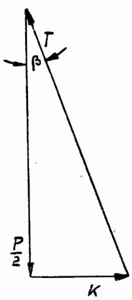
$$\sum P_z = 0: \quad Z_L + Z_M + 20 - Z_{T\,\mathrm{Res}} = 0$$

$$Z_L = Z_M = \underline{\underline{-6\ \mathrm{kg}}}$$

$$\sum P_y = 0: \quad Y_L = Y_M = \frac{Y_{T\,\mathrm{Res}}}{2} = \underline{\underline{-4{,}5\ \mathrm{kg}}}$$

Lösung 285





$$M = 2r\cdot K\cdot\cos\frac{\alpha}{2}$$

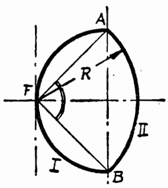
$$K = \frac{P}{2}\,\mathrm{tg}\,\beta = \frac{P}{2}\cdot\frac{2r\sin\frac{\alpha}{2}}{\sqrt{l^2 - 4r^2\sin^2\frac{\alpha}{2}}}$$

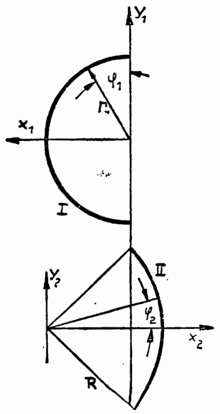
$$\underline{\underline{M = \frac{P\cdot r^2\sin\alpha}{\sqrt{l^2 - 4r^2\sin^2\frac{\alpha}{2}}}}}$$

$$T = \frac{P}{2\cos\beta}; \quad \underline{\underline{T = \frac{P}{2}\cdot\frac{l}{\sqrt{l^2 - 4r^2\sin^2\frac{\alpha}{2}}}}}$$

9. Schwerpunkt

Lösung 286





$$x_{S_1} = \frac{\int x_1 \cdot ds_1}{\int ds_1} = \frac{2r^2 \int\limits_0^{\frac{\pi}{2}} \sin\varphi_1 \, d\varphi_1}{r \cdot \pi} = \frac{2r}{\pi}$$

$$x_{S_2} = \frac{\int x_2 \cdot ds_2}{\int ds_2} = \frac{2R^2 \int\limits_0^{\frac{\pi}{4}} \cos\varphi_2 \, d\varphi_2}{R \cdot \frac{\pi}{2}} = \frac{2R\sqrt{2}}{\pi}$$

$$r = \frac{R\sqrt{2}}{2}$$

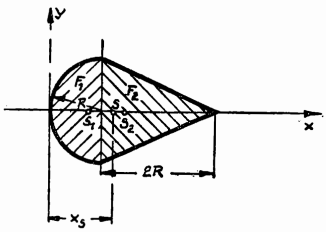
$$\sum M_F = 0:$$

$$R\sqrt{2}\left(\frac{1}{2} - \frac{1}{\pi}\right) R \frac{\sqrt{2}\pi}{2} + \frac{2R^2\sqrt{2}}{\pi} \cdot \frac{\pi}{2}$$

$$= x_S \cdot R\left(\frac{\pi}{2} + \frac{\pi\sqrt{2}}{2}\right)$$

$$x_S = CF = \underline{\underline{0{,}524\,R}}$$

Lösung 287

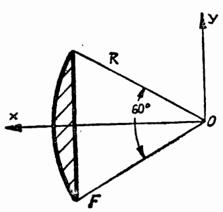


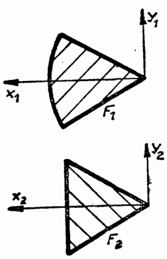
$$x_S = \frac{1}{F} \sum_{i=1}^{2} \cdot x_i \cdot F_i; \quad y_S = 0$$

$$x_S = \frac{\frac{\pi R^2}{2}\left(R - \frac{4R}{3\pi}\right) + 2R^2\left(R + \frac{2}{3}R\right)}{\frac{\pi R^2}{2} + 2R^2}$$

$$x_S = 0C = \frac{3\pi + 16}{3\pi + 12} \cdot R = \underline{\underline{1{,}19\,R}}$$

Lösung 288





$$F_1 x_{S_1} = \int x_1 \cdot dF_1 = 2\int\limits_0^R \int\limits_0^{\frac{\pi}{6}} r^2 \cdot \cos\varphi \, dr \, d\varphi$$

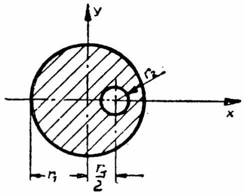
$$= \frac{R^3}{3}$$

$$F_2 x_{S_2} = R^2 \frac{\sqrt{3}}{4} \cdot \frac{2}{3} R \frac{\sqrt{3}}{2} = \frac{R^3}{4}$$

$$x_{S_1} \cdot F_1 - x_{S_2} \cdot F_2 = x_S \cdot F$$

$$x_S = \frac{R\left(\frac{1}{3} - \frac{1}{4}\right)}{\frac{\pi}{6} - \frac{\sqrt{3}}{4}} = 0{,}92\,R = \underline{\underline{27{,}7\text{ cm}}} = 0C$$

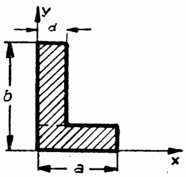
Lösung 289



$$x_S = \frac{\pi r_1^2 \cdot 0 - \frac{r_1}{2} \cdot \pi r_2^2}{\pi r_1^2 - \pi r_2^2}$$

$$x_S = -\underline{\underline{\frac{r_1 r_2^2}{2(r_1^2 - r_2^2)}}}$$

Lösung 290

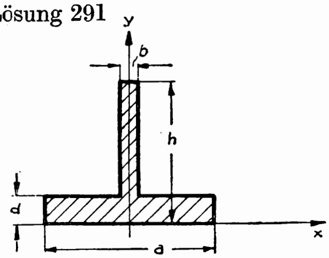


$$\sum M_y = 0: \quad x_s = \frac{(b-d) \cdot d \cdot \frac{d}{2} + a \cdot d \cdot \frac{a}{2}}{ad + d(b-d)}$$

$$x_s = \underline{\underline{\frac{a^2 + bd - d^2}{2(a+b-d)}}}$$

$$\sum M_x = 0: \quad y_s = \frac{(a-d)\frac{d}{2} + b \cdot d \cdot \frac{b}{2}}{a \cdot d + d(b-d)} = \underline{\underline{\frac{b^2 + ad - d^2}{2(a+b-d)}}}$$

Lösung 291

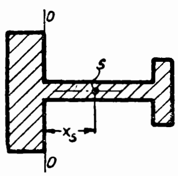


$$x_S = 0$$

$$y_S = \frac{a \cdot d \frac{d}{2} + b(h-d)\left(\frac{h-d}{2} + d\right)}{a \cdot d + b(h-d)}$$

$$y_S = \underline{\underline{\frac{ad^2 + b(h^2 - d^2)}{2[ad + b(h-d)]}}}$$

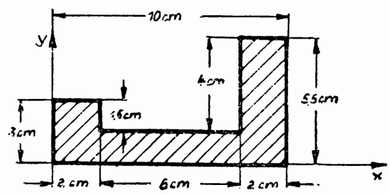
Lösung 292



$$x_s = \frac{20 \cdot 2 \cdot 10 + 1{,}5 \cdot 2 \cdot 21 - 20 \cdot 2 \cdot 1}{20 \cdot 2 + 20 \cdot 2 + 1{,}5 \cdot 2}$$

$$x_s = \underline{\underline{9 \text{ cm}}}$$

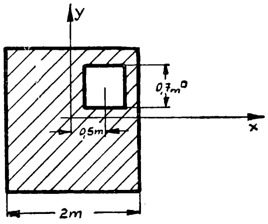
Lösung 293



$$x_S = \frac{6 \cdot 1 + 9 \cdot 5 + 11 \cdot 9}{26} = \frac{150}{26} = \underline{\underline{5\frac{10}{13} \text{ cm}}}$$

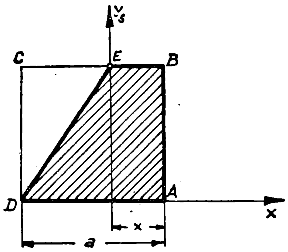
$$y_S = \frac{6 \cdot 1{,}5 + 9 \cdot 0{,}75 + 11 \cdot 2{,}75}{6 + 9 + 11} = \underline{\underline{1\frac{10}{13} \text{ cm}}}$$

Lösung 294



$$x_S = y_S = \frac{-0{,}7^2 \cdot 0{,}5}{(2^2 - 0{,}7^2)} = \underline{\underline{-0{,}07\ \text{m}}}$$

Lösung 295



Bedingung: $x_S = 0$

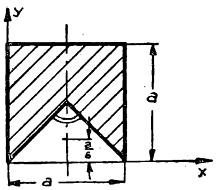
$$x_S = 0 = \frac{F_\triangle \cdot \frac{x-a}{3} + F_\square \cdot \frac{x}{2}}{F_\triangle + F_\square}$$

$$F_\triangle = \frac{a-x}{2} \cdot h; \quad F_\square = x \cdot h$$

$$\frac{(a-x)^2}{2 \cdot 3} - \frac{x^2}{2} = 0$$

$$x^2 + a x = \frac{a^2}{2}; \quad x = \frac{a}{2}(\sqrt{3} - 1) = \underline{\underline{0{,}366\,a}}$$

Lösung 296

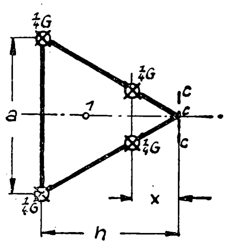


$$y_S = \frac{a^2 \cdot \frac{a}{2} - \frac{a^2}{4} \cdot \frac{a}{6}}{\frac{3}{4}a^2} = \frac{11}{18}a; \quad y_S = \underline{\underline{0{,}61\,a}}$$

Die Symmetrieachse bleibt erhalten, deshalb:

$$\underline{\underline{x_S = \frac{a}{2}}}$$

Lösung 297



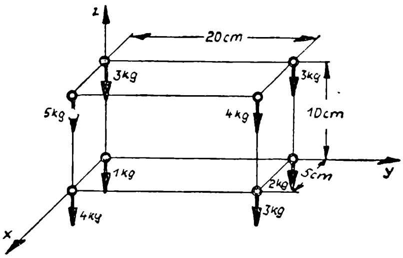
Gewicht der Platte: G

$$\sum M_C = 0: \quad \frac{2}{4} \cdot G \cdot h - G \cdot \frac{2}{3} h + \frac{2}{4} G \cdot x = 0$$

$$x = \left(\frac{4}{3} - 1\right) \cdot h; \quad x = \frac{1}{3} h$$

Nach dem Strahlensatz entspricht dies auch $^1/_3$ der Kantenlänge.

Lösung 298



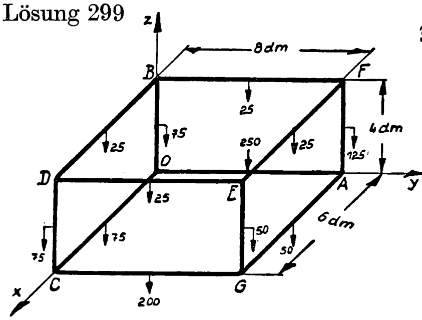
$$\sum M_y = 0: \quad x_C = \frac{4+4+5+3}{25} \cdot 5 = \underline{\underline{3{,}2\ \text{cm}}}$$

$$\sum M_x = 0: \quad y_C = \frac{2+3+4+3}{25} \cdot 20 = \underline{\underline{9{,}6\ \text{cm}}}$$

Drehung des Koordinatensystems um die y-Achse um $\frac{\pi}{2}$:

$$\sum M_y = 0: \quad z_C = \frac{3+5+3+4}{25} \cdot 10 = \underline{\underline{6\ \text{cm}}}$$

Lösung 299



$$y_S \cdot \sum G = 200 \cdot 4 + 25 \cdot 4 + 25 \cdot 4 + 250 \cdot 4 + (50 + 125)\,8 + (50 + 25)\,8$$

$$y_S = \underline{\underline{4\ \text{dm}}}$$

$$x_S \cdot \sum G = (75 + 50 + 25 + 25) \cdot 3 + (200 + 50 + 25) \cdot 6 + 75 \cdot 6$$

$$x_S = \underline{\underline{2{,}625\ \text{dm}}}$$

$$z_S \cdot \sum G = 4 \cdot 25 \cdot 4 + (75 + 125 + 50 + 75) \cdot 2$$

$$z_S = \underline{\underline{1{,}05\ \text{dm}}}$$

Lösung 300

$$G_1 = G_2 = G_3 = \ldots . G_n; \quad a = 44\ \text{cm}$$

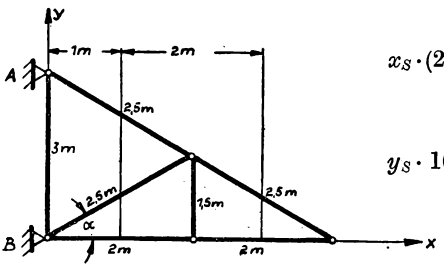
$$(G_1 + G_2 + G_3 + G_4) \cdot \frac{a}{2} - (G_9 + G_{10} + 2 \cdot G_{11}) \cdot \frac{a}{2} = 0$$

Demnach: $z_S = \underline{\underline{0}}$

Aus Symmetrie: $x_S = -\frac{a}{2} = \underline{\underline{-22\ \text{cm}}}$

$$y_S = \frac{a\,(G_4 + G_3 + G_7) + \frac{a}{2}\,(G_8 + G_6)}{\Sigma G} = \frac{a}{2} \cdot \frac{8\,G}{11\,G} = \underline{\underline{16\ \text{cm}}}$$

Lösung 301



$$x_S \cdot (2 + 2 + 1{,}5 + 3 \cdot 2{,}5 + 3) = 2 \cdot 1 + 2 \cdot 3 + 1{,}5 \cdot 2 + 2 \cdot 2{,}5 \cdot 1 + 2{,}5 \cdot 3$$

$$x_S = \underline{\underline{1{,}47\ \text{m}}}$$

$$y_S \cdot 16 = 3 \cdot 1{,}5 + 2 \cdot 2{,}5 \cdot 0{,}75 + 2{,}5 \cdot 2{,}25 + 1{,}5 \cdot 0{,}75$$

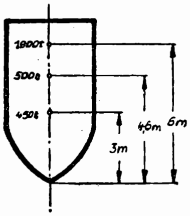
$$y_S = \underline{\underline{0{,}94\ \text{m}}}$$

Lösung 302

Symmetrie: $x_S = 0;\quad z_S = 0$

$$y_S = \frac{d \cdot l^2 \left(b + \frac{d}{2}\right) + \frac{a\,b^2 c}{2}}{a\,b\,c + d\,l^2} = \frac{360 \cdot 28 + 640 \cdot 9}{1440 + 360} = \underline{\underline{8{,}8 \text{ cm}}}$$

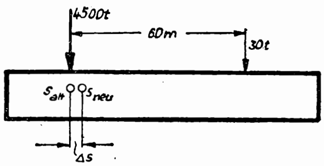
Lösung 303



$$y_S = \frac{1900 \cdot 6 + 500 \cdot 4{,}6 + 450 \cdot 3}{1900 + 500 + 450}$$

$$y_S = \underline{\underline{5{,}28 \text{ m}}}$$

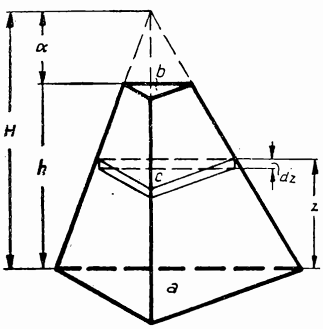
Lösung 304



$\sum M_{S_{\text{Alt}}} = 0{:}\quad 30 \cdot 60 = 4500 \cdot \Delta s$

$$\Delta s = \underline{\underline{0{,}4 \text{ m}}}$$

Lösung 305



$$\frac{H^2}{\alpha^2} = \frac{a}{b};\quad \alpha = H\sqrt{\frac{b}{a}}$$

$$H - h = \alpha = H\sqrt{\frac{a}{b}};\quad H = \frac{h}{1 - \sqrt{\frac{b}{a}}}$$

$$(H - z)^2 = H\,\frac{c}{a};\quad c = (H^2 - 2\,H_z + z^2) \cdot \frac{a}{}$$

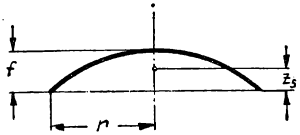
$$z_S = \frac{\int z \cdot dV}{\int dV} = \frac{\int_0^h (H^2 - 2\,H_z + z^2) \cdot z \cdot dz}{\int_0^h (H^2 - 2\,H_z + z^2)\,dz}$$

$$z_S = \frac{\frac{H^2 h^2}{2} - \frac{2\,H\,h^3}{3} + \frac{h^4}{4}}{H^2 h - \frac{2\,H\,h^2}{2} + \frac{h^3}{3}} = \frac{\left[6 - 8\left(1 - \sqrt{\frac{b}{a}}\right) + 3\left(-1\sqrt{\frac{b}{a}}\right)^2\right]h}{12 - 12\left(1 - \sqrt{\frac{b}{a}}\right) + 4\left(1 - \sqrt{\frac{b}{a}}\right)^2}$$

$$z_S = \frac{h\left[1 + 2\sqrt{\frac{b}{a}} + 3\,\frac{b}{a}\right]}{4 + 4\sqrt{\frac{b}{a}} + 4\,\frac{b}{a}};$$

$$z_S = \underline{\underline{\frac{h}{4} \cdot \frac{a + 2\sqrt{b \cdot a} + 3\,b}{a + \sqrt{b \cdot a} + 3\,b}}}$$

Lösung 306



Wegen Symmetrie: $x_S = y_S = 0$

$$z_S = \frac{F_1 \cdot z_1 + F_2 \cdot z_2 + F_3 \cdot z_3}{F_1 + F_2 + F_3}$$

$z_1 = 2{,}45 r;\quad F_1 = 1{,}25\,\pi r^2$

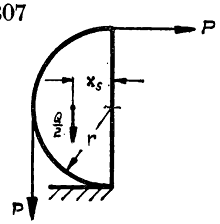
$z_2 = 1{,}2\ r;\quad F_2 = 4\,\pi r^2$

$z_3 = 0{,}1\ r;\quad F_3 = 1{,}04\,\pi r^2$

$$z_S = \frac{2{,}45 \cdot 1{,}25 + 1{,}2 \cdot 4 + 0{,}1 \cdot 1{,}04}{1{,}25 + 4 + 1{,}04} \cdot r$$

$$z_S = 1{,}267 \cdot r = \underline{\underline{0{,}507\ \text{m}}}$$

Lösung 307

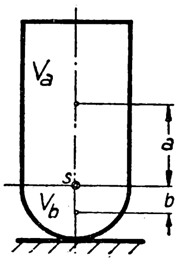


$$P \cdot r + \frac{Q}{2} \cdot x_S - P \cdot 2r = 0$$

$$P = \frac{Q \cdot x_S}{2r}$$

$$x_S = \frac{4r}{3\pi};\qquad \underline{\underline{P = Q \cdot \frac{2}{3\pi}}}$$

Lösung 308



$$V_a \cdot a = V_b \cdot b$$

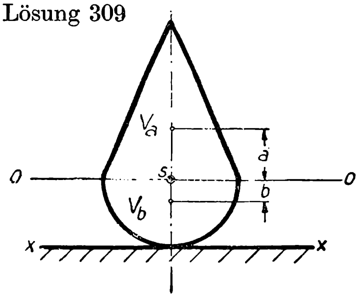
$$a = \frac{h}{2};\qquad b = \frac{3}{8} r$$

$$V_a = \pi r^2 h;\qquad V_b = \frac{2}{3} \pi r^3$$

$$\frac{\pi r^2 h^2}{2} = \frac{1}{4} \pi r^4$$

$$h^2 = \frac{1}{2} r^2;\qquad \underline{\underline{h = \frac{r}{\sqrt{2}}}}$$

Lösung 309



Gesamtschwerpunkt muß auf $\overline{00}$ liegen
Bezugsachse $\overline{xx}$:

$$\left(r + \frac{h}{4}\right) \cdot \frac{1}{3} \pi r^2 \cdot h + \frac{5}{8} r \cdot \frac{2}{3} \pi r^3$$

$$= r \left(\frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3\right)$$

$$\frac{r^2 h^2}{4} + \frac{5 r^4}{4} - \frac{8 r^4}{4} = 0$$

$$h^2 = 3 r^2$$

$$\underline{\underline{h = r \sqrt{3}}}$$

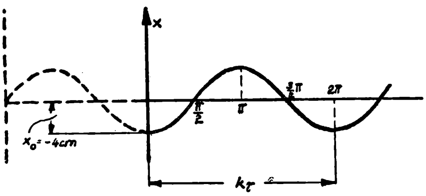
Zweiter Teil

Kinematik

III. Punktbewegung

10. Bewegungsbahn und Bewegungsgleichungen der Punktbewegung

Lösung 310



Bewegungsgleichung:

$$x = a\sin\left(kt + \frac{3\pi}{2}\right)$$

Anfangsbedingung:

$$t = 0: \quad x = x_0 = -4\ \text{cm}$$

$$-4 = a \cdot \sin\frac{3\pi}{2}; \quad \underline{\underline{a = 4\ \text{cm}}}$$

Zeit einer vollen Schwingung: $\tau = 0{,}4$ sek

$$k \cdot \tau = 2\pi; \quad k = \frac{2\pi}{\tau} = \underline{\underline{5\pi \frac{1}{\text{sek}}}}$$

Lösung 311

1. $x = 20\,t^2 + 5;$ $\quad t^2 = \dfrac{x-5}{20} = \dfrac{y-3}{15}$ $\quad \underline{\underline{3x - 4y = 3}}$ Gerade

   $y = 15\,t^2 + 3;$

2. $x = 4\,t - 2\,t^2;$ $\quad t^2 - 2\,t = \dfrac{x}{2}$ $\quad t = 1 \pm \sqrt{1 + \dfrac{x}{2}};$ $\quad 1 + \dfrac{x}{2} > 0$

   $y = 3\,t - 1{,}5\,t^2;$ $\quad t^2 - 2\,t = \dfrac{y}{1{,}5}$ $\quad t = 1 \pm \sqrt{1 + \dfrac{y}{1{,}5}};$ $\quad 1 + \dfrac{y}{1{,}5} > 0$

   $$\frac{x}{2} - \frac{y}{1{,}5} = 0 \qquad -\infty < x \leqq 2$$

   Gerade: $\underline{\underline{3x - 4y = 0}}$ $\qquad -\infty < y \leqq 1{,}5$

3. $x = 5 + 3\cos t;$ $\quad \left(\dfrac{x-5}{3}\right)^2 = \cos^2 t$

   $y = 4\sin t;$ $\quad \left(\dfrac{y}{4}\right)^2 = \sin^2 t$

   $$\left(\frac{x-5}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$

   Ellipse: $\underline{\underline{\dfrac{(x-5)^2}{9} + \dfrac{y^2}{16} = 1}}$

4. $x = at^2;\quad t^2 = \frac{x}{a} = \frac{y^2}{b^2};$ Parabel: $\underline{\underline{ay^2 - b^2 \cdot x = 0}}$
$y = bt;$

5. $x = 5\sin\frac{\pi}{2}t;\quad \frac{x^2}{25} = \sin^2\frac{\pi}{2}t$
$y = 4\cos\frac{\pi}{2}t;\quad \frac{y^2}{16} = \cos^2\frac{\pi}{2}t$
$\frac{x^2}{25} + \frac{y^2}{16} = 1;$ Ellipse: $\underline{\underline{16x^2 + 25y^2 - 400 = 0}}$

6. $x = 5\cos t;\quad \frac{x^2}{25} = \cos^2 t$
$y = 3 - 5\sin t;\quad \frac{(y-3)^2}{25} = \sin^2 t$
$\frac{x^2}{25} + \frac{(y-3)^2}{25} = 1$ Kreis: $\underline{\underline{x^2 + (y-3)^2 = 25}}$

7. $x = 3 + 4\cos t;\quad \frac{(x-3)^2}{16} = \cos^2 t$
$y = 2 + 5\sin t;\quad \frac{(y-2)^2}{25} = \sin^2 t$ Ellipse: $\underline{\underline{\frac{(x-3)^2}{16} + \frac{(y-2)^2}{25} = 1}}$

8. $x = 3\cos\left(\frac{\pi}{8} + \pi t\right);\quad \cos\left(\frac{\pi}{8} + \pi t\right) = \frac{x}{3}$
$y = 4\sin\left(\frac{\pi}{4} + \pi t\right);\quad \sin\left(\frac{\pi}{8} + \pi t\right)\cdot\cos\frac{\pi}{8} + \sin\frac{\pi}{8}\cos\left(\frac{\pi}{8} + \pi t\right) = \frac{y}{4}$

$$\sqrt{1 - \frac{x^2}{9}}\cdot\cos\frac{\pi}{8} + \sin\frac{\pi}{8}\cdot\frac{x}{3} = \frac{y}{4}$$
$$\left(1 - \frac{x^2}{9}\right)\cos^2\frac{\pi}{8} = \frac{y^2}{16} - 2\frac{xy}{12}\sin\frac{\pi}{8} + \frac{x^2}{9}\sin^2\frac{\pi}{8}$$
Ellipse: $\underline{\underline{\frac{x^2}{9} + \frac{y^2}{16} - \frac{xy}{6}\sin\frac{\pi}{8} - \cos^2\frac{\pi}{8} = 0}}$

Lösung 312

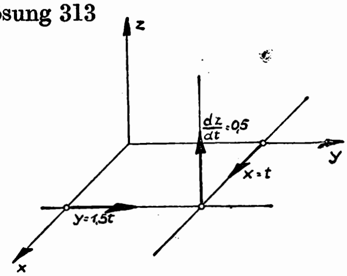
1. $x = 3t^2;\quad 4x - 3y = 0;\quad dx = 3\cdot 2t\cdot dt\quad ds = \sqrt{(dx)^2 + (dy)^2}$
$y = 4t^2;\quad dy = 4\cdot 2t\cdot dt\quad ds = \sqrt{(36 + 64)t^2(dt)^2} = 10t\,dt$
$\underline{\underline{s = 5t^2}}$

2. $x = 3\sin t;\quad x^2 + y^2 = 9;\quad dx = 3\cos t\cdot dt;\quad ds = \sqrt{9(\cos^2 t + \sin^2 t)(td)^2} = 3dt$
$y = 3\cos t;\quad dy = -3\sin t\cdot dt\quad \underline{\underline{s = 3t}}$

3. $x = a\cos^2 t;\quad x + y = a;\quad dx = -a\cdot 2\cos t\sin t\,dt;\quad ds = \sqrt{2}\cdot a\cdot 2\sin t\cos t\cdot dt$
$y = a\sin^2 t;\quad dy = a\cdot 2\sin t\cos t\,dt;\quad \underline{\underline{s = a\cdot\sqrt{2}\cdot\sin^2 t}}$

4. $x = 5\cos 5t^2;\quad \left(\frac{x}{5}\right)^2 + \left(\frac{y}{5}\right)^2 = 1;\quad x^2 + y^2 = 25;\quad dx = -50t\sin 5t^2 dt$
$y = 5\sin 5t^2;\quad dy = 50t\cos 5t^2 dt$
$ds = 50t\,dt;\quad \underline{\underline{s = 25t^2}}$

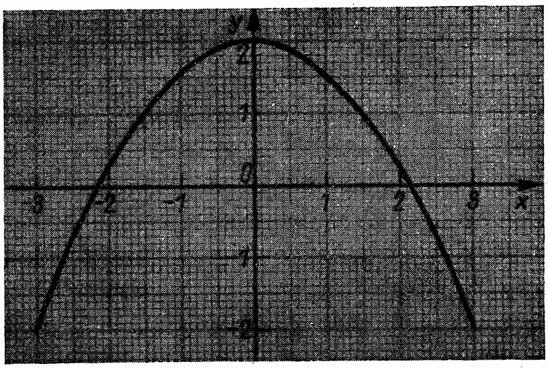
Lösung 313



$$z = 0{,}5t; \quad y = 1{,}5t; \quad x = t$$

$$\underline{\underline{y = 1{,}5x}}; \quad \underline{\underline{z = 0{,}5x}}$$

Lösung 314



$$x = 3\sin t$$

$$y = 2\cos 2t$$

$$\cos 2t = \cos^2 t - \sin^2 t$$

$$= 1 - 2\sin^2 t$$

$$\frac{y}{2} = 1 - \frac{2x^2}{9}$$

$$\underline{4x^2 + 9y = 18}$$

Wegen $|\sin t| \leqq 1$; $|\cos 2t| \leqq 1$ gilt nur der Bereich:

$$|x| \leqq 3; \quad |y| \leqq 2$$

Schnittpunkt mit der Abszisse $y = 0$:

$$\cos 2t_0 = 0; \quad 2t_0 = \frac{\pi}{2};$$

$$\underline{\underline{t_0 = \frac{\pi}{4}}} \text{ sek}$$

Lösung 315

$$\frac{x}{a} = \sin(kt + \alpha + \beta - \beta)$$

$$\frac{y}{b} = \sin(kt + \beta)$$

$$\sin[kt + \beta + (\alpha - \beta)] = \sin(kt + \beta)\cos(\alpha - \beta) + \cos(kt + \beta)\sin(\alpha - \beta)$$

$$\frac{x}{a} = \frac{y}{b} \cdot \cos(\alpha - \beta) + \sqrt{1 - \frac{y^2}{b^2}} \cdot \sin(\alpha - \beta)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \cdot \cos^2(\alpha - \beta) - \frac{2xy}{ab} \cdot \cos(\alpha - \beta) = \left(1 - \frac{y^2}{b^2}\right)\sin^2(\alpha - \beta)$$

$$\underline{\underline{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos(\alpha - \beta) = \sin^2(\alpha - \beta)}}$$

Lösung 316

1. $x = a \sin 2\omega t;$ $\quad y = a \sin \omega t;$ $\quad \sin 2\omega t = 2 \sin \omega t \cos \omega t;$

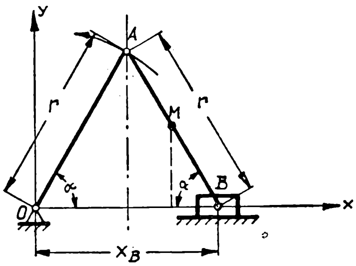
$$\frac{x}{a} = 2\frac{y}{a} \cdot \sqrt{1 - \frac{y^2}{a^2}}; \quad \underline{\underline{x^2 a^2 = 4y^2(a^2 - y^2);}} \quad \text{mit } |x| \leqq a; \quad |y| \leqq a;$$

(vergl. Aufg. 314)

2. $x = a \cos 2\omega t;$ $\quad y = a \cos \omega t;$ $\quad \cos 2\omega t = -1 + 2\cos^2 \omega t;$

$$\frac{x}{a} = -1 + 2\frac{y^2}{a^2}; \quad \underline{\underline{a x = -a^2 + 2y^2;}} \quad \text{mit } |x| \leqq a; \quad |y| \leqq a$$

Lösung 317



Gleitstück B: $x_B = 2r \cdot \cos\alpha; \quad \alpha = \omega t$

$$\underline{\underline{x_B = 160 \cdot \cos 10t}}$$

Punkt M: $x_M = 2r \cos \omega t - \frac{r}{2} \cos \omega t$

$$x_M = \frac{3}{2} r \cos \omega t = \underline{\underline{120 \cos 10t}}$$

$$y_M = \frac{1}{2} r \sin \omega t = \underline{\underline{40 \sin 10t}}$$

$$\frac{x_M^2}{\frac{9}{4} \cdot 80^2} = \cos^2 \omega t; \quad \frac{y_M^2}{\frac{1}{4} \cdot 80^2} = \sin^2 \omega t$$

$$\underline{\underline{\frac{x^2}{120^2} + \frac{y^2}{40^2} = 1}}$$

Lösung 318

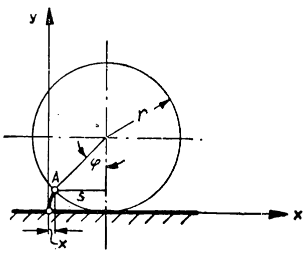
$x = a(kt - \sin kt)$ $\quad$ 1. $y = 0$: $\cos kt = 1$; $\quad kt = \lambda \cdot 2\pi$; $\quad \underline{\underline{t = \frac{2\pi}{k} \cdot \lambda}}$

$y = a(1 - \cos kt)$ $\quad$ 2. $x = a$: $\cos kt = 0$; $\quad kt = \left(\frac{\pi}{2} + 2\pi\lambda\right)$; $\quad \underline{\underline{t = \left(\frac{\pi}{2} + 2\pi\lambda\right) \cdot \frac{1}{k}}}$

3. $y = 2a$: $+\cos kt = -1$; $\quad kt = (\pi + 2\pi\lambda)$;

$$\underline{\underline{t = (\pi + 2\pi\lambda) \cdot \frac{1}{k}}} \quad \text{mit} \quad \lambda = 0;\ 1;\ 2;\ 3;\ \ldots$$

Lösung 319



$$x = r \cdot \varphi - s;$$

$$x = r \cdot \varphi - r \sin \varphi; \quad y = r - r \cos \varphi$$

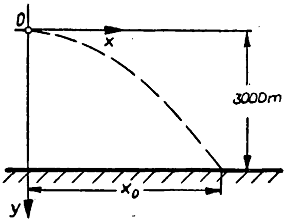
$$x = r(\varphi - \sin \varphi); \quad y = r(1 - \cos \varphi)$$

$$\omega \cdot t = \varphi; \quad \omega = \frac{v}{r} = 20\,\frac{1}{\text{sek}}; \quad r = 1\,\text{m}$$

$$\underline{\underline{x = 20t - \sin 20t;}} \quad \underline{\underline{y = 1 - \cos 20t}}$$

Der Punkt A bewegt sich auf einer Zykloide.

Lösung 320



$$x = 40t; \quad y = 4{,}9\,t^2$$

$$x = v_0 \cdot t; \quad y = \frac{1}{2}\,g\,t^2$$

$$y = 4{,}9 \cdot \left(\frac{x}{40}\right)^2; \quad \underline{\underline{y = 0{,}00306\,x^2}}$$

$$y_0 = 4{,}9 \cdot t_0^2; \quad t_0 = \sqrt{\frac{y_0}{4{,}9}} = \underline{\underline{24{,}74\,\text{sek}}}$$

$$x_0 = 40 \cdot t_0 = \underline{\underline{989{,}6\,\text{m} = L}}$$

Lösung 321

$$x = 250\,t$$

$$y = 430\,t - 4{,}9\,t^2$$

Bewegungsbahn:

$$\frac{x}{250} = t; \quad y = \frac{430}{250} \cdot x - \frac{4{,}9}{(250)^2} \cdot x^2$$

$$\underline{\underline{y = 1{,}72 \cdot x - 0{,}0000784\,x^2}}$$

Flugdauer: Bedingung: $y = 0; \quad 430\,t_0 - 4{,}9\,t_0^2 = 0$

$$\frac{430}{4{,}9} = t_0; \quad t_0 = \underline{\underline{87{,}75\,\text{sek}}}$$

Flugweite: Bedingung: Bewegungsbahn $y = 0$

$$1{,}72 \cdot x_0 - 0{,}0000784\,x_0^2 = 0; \quad x_0 = \frac{1{,}72}{0{,}784} \cdot 10^4\,\text{m}$$

$$\underline{\underline{x_0 = 21{,}94\,\text{km} = L}}$$

11. Punktgeschwindigkeit

Lösung 322

1. $s(t) = 0{,}1\,t^2 + t$ $\quad s(t)$ bedeutet: s ist Funktion von t

$$s(t-10) = 0{,}1\,(t-10)^2 + (t-10) = 0{,}1\,t^2 - 2t + 10 + t - 10$$

$$v_m = \frac{s(t) - s(t-10)}{10} = \frac{2t}{10} = \frac{t}{5}$$

Die mittleren Geschwindigkeiten betragen also

nach 10 sek: $\frac{10}{5} = 2$ m/sek

„ 20 „ : $\frac{20}{5} = 4$ „

„ 30 „ : $\frac{30}{5} = 6$ „

usw.

2. $s(0) = 0$

$$s(60) = 360 + 60 = 420\,\text{m}; \quad v_m = \frac{s(60) - s(0)}{60} = \underline{\underline{7\,\text{m/sek}}}$$

Lösung 323

$x = e(1 - \cos\omega t)$ 1. Bedingung: $\dot{x} = 0$ $\quad \dot{x} = \frac{dx}{dt} = v_x$

$$\dot{x} = e\omega \sin\omega t = 0;$$

$$\omega t = 0;\ \pi;\ 2\pi$$ (ωt = 0 entfällt, da dies den Bewegungsbeginn darstellt.)

$$\underline{\underline{t_1 = \frac{\pi}{\omega}}};\quad \underline{\underline{t_2 = \frac{2\pi}{\omega}}}$$

2. Bedingung: $\ddot{x} = 0$

$$\ddot{x} = e\omega^2 \cos\omega t = 0;\quad \omega t = \frac{\pi}{2};\ \frac{3\pi}{2}$$

$$\underline{\underline{t = \frac{\pi}{2\omega}}}$$

3. Bedingung: $x = 0$

$$e(1 - \cos\omega t) = 0;\quad \omega t = 0;\ 2\pi$$

$$\underline{\underline{T = \frac{2\pi}{\omega}}}$$

Lösung 324

Schwingungsgleichung: $x = A\sin kt + B\cos kt$

$t_0 = 0$: $x_0 = 0$: $\quad x_0 = A\sin kt_0 + B\cos kt_0 = 0;\quad B = 0$

$v_0 = 62{,}8$: $\quad \dot{x}_0 = kA\cos kt_0 = v_0$

Schwingungszeit: $\tau = \frac{1}{2} = \frac{2\pi}{k};\quad k = 4\pi$

$$k \cdot A = 62{,}8;\quad A = \frac{62{,}8}{4\pi} = 5$$

$$\underline{\underline{x = 5\sin 4\pi t}}$$

Lösung 325

$$x = a\sin kt \qquad \dot{x} = ak\cos kt$$

$$x_1 = a\sin kt \qquad x_2 = a\sin kt$$

$$v_1 = ak\cos kt \qquad v_2 = ak\cos kt$$

$$\frac{x_1}{a} = \sin kt \qquad \frac{x_2}{a} = \sin kt$$

$$\frac{v_1}{ak} = \cos kt \qquad \frac{v_2}{ak} = \cos kt$$

$$\frac{x_1^2}{a^2} + \frac{v_1^2}{a^2k^2} = 1 \qquad \frac{x_2^2}{a^2} + \frac{v_2^2}{a^2k^2} = 1$$

$$x_1^2 + \frac{v_1^2}{k^2} = x_2^2 + \frac{v_2^2}{k^2}$$

$$\underline{\underline{k = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}}}$$

$$\frac{1}{a^2}\left(x_1^2 + \frac{v_1^2}{k^2}\right) = 1;\quad a^2 = \frac{v_1^2 + x_1^2k^2}{k^2};\quad \underline{\underline{a = \sqrt{\frac{v_1^2x_2^2 - v_2^2x_1^2}{v_1^2 - v_2^2}}}}$$

Lösung 326

$$x = 15\sin\frac{\pi}{4}\cdot t \qquad \dot{x} = \frac{15\pi}{4}\cos\frac{\pi}{4}\cdot t;$$

$$y = 15\cos\frac{\pi}{4}\cdot t \qquad \dot{y} = -\frac{15\pi}{4}\cdot\sin\frac{\pi}{4}\cdot t$$

$$\left(\frac{x}{15}\right)^2 + \left(\frac{y}{15}\right)^2 = 1$$

Für $x=0$: $y = 15\,\text{cm}$; $v_x = \frac{15\pi}{4}\,\text{cm/sek}$; $v_y = 0$

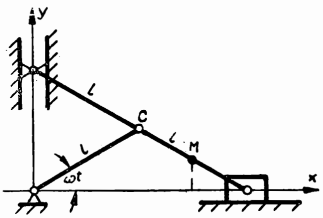
„ $x = 15\,\text{cm}$: $y = 0$; $v_x = 0$; $v_y = -\frac{15\pi}{4}\,\text{cm/sek}$

„ $x=0$: $y = -15\,\text{cm}$; $v_x = -\frac{15\pi}{4}\,\text{cm/sek}$; $v_y = 0$

„ $x = -15\,\text{cm}$: $y = 0$; $v_x = 0$; $v_y = +\frac{15\pi}{4}\,\text{cm/sek}$

Hodograph: $x_1 = \dot{x}$; $y_1 = \dot{y}$; $\underline{\underline{x_1^2 + y_1^2 = \frac{225\pi^2}{16}}}$

Lösung 327



$$x_M = \frac{3}{2}\,l\cos\omega t; \qquad \dot{x} = -\frac{3}{2}\,l\cdot\omega\sin\omega t$$

$$y_M = \frac{l}{2}\sin\omega t; \qquad \dot{y} = \frac{l}{2}\,\omega\cos\omega t$$

Bewegungsbahn

$$\frac{x^2}{\frac{9l^2}{4}} = \cos^2\omega t$$

$$\frac{y^2}{\frac{l^2}{4}} = \sin^2\omega t$$

$$\underline{\underline{\frac{x^2}{900} + \frac{y^2}{100} = 1;}}$$

Hodograph

$$\frac{\dot{x}^2}{\frac{9l^2\omega^2}{4}} = \sin^2\omega t; \quad \dot{x} = x_1$$

$$\frac{\dot{y}^2}{\frac{l^2\omega^2}{4}} = \cos^2\omega t \quad \dot{y} = y_1$$

$$\underline{\underline{\frac{x_1^2}{900\,\omega^2} + \frac{y_1^2}{100\,\omega^2} = 1}}$$

Lösung 328

$x = 2\cos t$; $\dot{x} = -2\sin t$

$y = 4\cos 2t$; $\dot{y} = -8\sin 2t$

Für $x=0$: $\cos t = 0$; $t = \frac{2\lambda+1}{2}\pi$ mit $\lambda = 0; 1; 2; 3\ldots$

$$v_x = \dot{x} = -2\sin\frac{2\lambda+1}{2}\pi = \underline{\underline{\pm 2\,\frac{\text{cm}}{\text{sek}}\;\begin{matrix}(+)\text{ bei }\lambda\text{ ungerade}\\(-)\text{ bei }\lambda\text{ gerade}\end{matrix}}}$$

$v_y = \dot{y} = 0$

Lösung 329

$x = 4\sin\frac{\pi}{2}t$; $\dot{x} = \frac{\pi}{2}\cdot 4\cdot\cos\frac{\pi}{2}t$

$y = 3\sin\frac{\pi}{2}t$; $\dot{y} = \frac{\pi}{2}\cdot 3\cdot\cos\frac{\pi}{2}t$

$t=0$: $\dot{x}=2\pi$; $\dot{y}=\frac{3}{2}\pi$; $\dot{s}=\sqrt{\dot{x}^2+\dot{y}^2}$; $\dot{s}=v$; $\dot{x}=v_x$

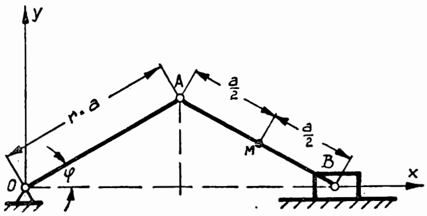
$\dot{y}=v_y$

$$\dot{s}=\pi\sqrt{4+\frac{9}{4}}=\underline{\underline{\frac{5}{2}\pi}}=v_0 \quad \cos(\dot{s},\dot{x})=\frac{2\pi\cdot 2}{5\pi}=\underline{\underline{\frac{4}{5}}}; \quad \cos(\dot{s},\dot{y})=\underline{\underline{\frac{3}{5}}}$$

$t=1$: $\dot{x}=0$; $\dot{y}=0$; $\dot{s}=0=v_1$

$t=2$: $\dot{x}=-2\pi$; $\dot{y}=-\frac{3\pi}{2}$; $\underline{\underline{\dot{s}=\frac{5}{2}\pi=v_2}}$ $\cos(\dot{s},\dot{x})=\underline{\underline{-\frac{4}{5}}}$; $\cos(\dot{s},\dot{y})=\underline{\underline{-\frac{3}{5}}}$

Lösung 330



$$x_A=a\cos\omega t; \quad x_B=2a\cos\omega t$$
$$y_A=a\sin\omega t; \quad y_B=0$$
$$\dot{x}_A=-a\omega\sin\omega t; \quad \dot{x}_B=-2a\omega\sin\omega t$$
$$\dot{y}_A=a\omega\cos\omega t; \quad \dot{y}_B=0$$
$$v_{xM}=\frac{\dot{x}_A+\dot{x}_B}{2}=-\frac{3}{2}a\omega\sin\omega t$$
$$v_{yM}=\frac{\dot{y}_A+\dot{y}_B}{2}=\frac{a\omega}{2}\cos\omega t$$
$$v_M=\sqrt{v_{xM}^2+v_{yM}^2}=\underline{\underline{\frac{a\omega}{2}\sqrt{1+8\sin^2\omega t}}}$$
$$v_B=\sqrt{\dot{x}_B^2+\dot{y}_B^2}=\underline{\underline{2a\omega\sin\omega t}}$$

Lösung 331

$$x=v_0\cdot t$$
$$y=h-g\frac{t^2}{2}$$

Bahngleichung:

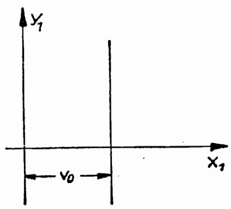
$$t^2=\frac{x^2}{v_0^2}; \quad \underline{\underline{y=h-\frac{g x^2}{v_0^2\cdot 2}}}$$

Punktgeschwindigkeit:

$$\dot{x}=v_0 \qquad \dot{s}=\sqrt{v_0^2+g^2t^2}$$
$$\dot{y}=-g\cdot t$$

Für $y=0$ gilt: $t^2=\frac{2h}{g}$; $\underline{\underline{\dot{s}=v=\sqrt{v_0^2+2gh}}}$

$$\cos(\dot{s},\dot{x})=\underline{\underline{\frac{v_0}{v}}}; \quad \cos(\dot{s},\dot{y})=\underline{\underline{-\frac{\sqrt{2gh}}{v}}}$$



Flugentfernung:

$$\underline{\underline{x=v_0\sqrt{\frac{2h}{g}}}}$$

Hodograph:

$$\dot{x}=x_1=v_0$$
$$\dot{y}=y_1=-gt; \quad \dot{y}_1=v_{y1}=-g.$$

Lösung 332

$$x = v \cdot t - a \sin \frac{v}{R} \cdot t = 10t - 0{,}5 \sin 10t$$

$$y = R - a \cos \frac{v}{R} \cdot t = 1 - 0{,}5 \cos 10t$$

$$\dot{x} = v - \frac{av}{R} \cdot \cos \frac{v}{R} \cdot t; \quad \dot{y} = \frac{av}{R} \sin \frac{v}{R} t$$

1. Horizontale Lage: $y = R$; also: $\cos \frac{v}{R} t = 0$; $\sin \frac{v}{R} t = \pm 1$

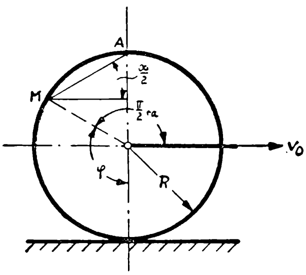
$$v_h = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{v^2 + \frac{v^2 a^2}{R^2}} = v\sqrt{1{,}25} = \underline{\underline{11{,}18 \text{ m/sek}}}$$

2. Vertikale Lage: $y = R \mp a$; also: $\cos \frac{v}{R} t = \pm 1$; $\sin \frac{v}{R} t = 0$

$$v_v = \sqrt{\dot{x}^2 + \dot{y}^2} = v \mp \frac{av}{R}; \quad v_{v_u} = \frac{1}{2} v = \underline{\underline{5 \text{ m/sek}}}$$

$$v_{v_o} = \frac{3}{2} v = \underline{\underline{15 \text{ m/sek}}}$$

Lösung 333



$$v_0 = 72 \cdot \frac{1000}{3600} = 20 \text{ m/sek}$$

$$R = 1 \text{ m}$$

$x = R(\varphi - \sin\varphi)$ vergl.
$y = R(1 - \cos\varphi)$ Aufg.: 319

$$\varphi = \omega t; \quad \omega = \frac{v_0}{R}; \quad \omega t = \pi - \alpha$$

$$\dot{x} = R(\omega - \omega \cos \omega t)$$
$$\dot{y} = R(\omega \sin \omega t)$$
$$\dot{x} = v_0(1 - \cos(\pi - \alpha))$$
$$\dot{y} = v_0 \sin(\pi - \alpha)$$

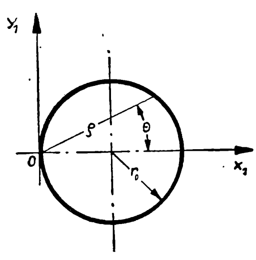
$$\dot{s} = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{1 - 2\cos(\pi - \alpha) + \cos^2(\pi - \alpha) + \sin^2(\pi - \alpha)} \cdot v_0$$

$$\dot{s} = v_0 \sqrt{2[1 - \cos(\pi - \alpha)]} = \underline{\underline{2 v_0 \cos \frac{\alpha}{2}}}; \quad \dot{s} = v = 40 \cos \frac{\alpha}{2}$$

Richtung: $\cos(\dot{s}, \dot{x}) = \frac{v_0[1 - \cos(\pi - \alpha)]}{v_0 \sqrt{2[1 - \cos(\pi - \alpha)]}} = \frac{1}{\sqrt{2}} \sqrt{1 - \cos(\pi - \alpha)} = \underline{\underline{\cos \frac{\alpha}{2}}}$

Die Geschwindigkeit hat also die Richtung der Geraden MA

Hodograph: $\dot{x} = x_1$; $\dot{y} = y_1$



$$\left.\begin{aligned} \frac{x_1}{R \cdot \omega} - 1 &= \cos \omega t \\ \frac{y_1}{R \cdot \omega} &= \sin \omega t \end{aligned}\right\} (x^2 - R\omega)^2 + y^2 = R^2 \omega^2 = v_0^2$$

Der Hodograph ist also ein Kreis vom Radius v_0, dessen Mittelpunkt um v_0 in x_1-Richtung verschoben ist

$$\varrho = 2 v_0 \cos \frac{\alpha}{2}; \quad \frac{\alpha}{2} = \Theta$$

$$v_1 = \sqrt{\dot{x}_1^2 + \dot{y}_1^2} = R\omega^2 = \underline{\underline{\frac{v_0^2}{R}}}$$

Lösung 334

$x = v \cdot t - a \cdot \sin \frac{v}{R} t$; vergl. Aufg. 319; $\dot{x} = v - \frac{a v}{R} \cdot \cos \frac{v}{R} t$

$y = R - a \cos \frac{v}{R} t$; $\dot{y} = \frac{a v}{R} \sin \frac{v}{R} t$

Extremlagen bei $\dot{y} = 0$, d. h.: $\sin \frac{v}{R} t = 0$ oder: $\cos \frac{v}{R} t = \pm 1$; $k = 0; 1; 2; \ldots$

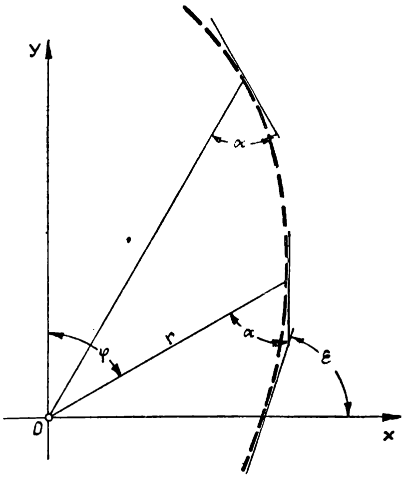
Bei $t = \frac{R}{v} \cdot 2 k \pi$; $\left(\cos \frac{v}{R} t = 1\right)$: $y = R - a$; tiefste Lage $v_y = 0$

$$v_x = v - \frac{a v}{R} = \underline{\underline{-2\ \text{m/sek}}}$$

Bei $t = \frac{R \pi}{v} \left(2 k + 1\right)$; $\left(\cos \frac{v}{R} t = -1\right)$: $y = R + a$; höchste Lage $v_y = 0$

$$v_x = v + \frac{a v}{R} = \underline{\underline{22\ \text{m/sek}}}$$

Lösung 335



$\operatorname{tg} \varepsilon = \frac{dy}{dx}$; $y = r \cos \cdot \varphi$

$x = r \sin \varphi$

$\varepsilon = 90° - (\varphi - \alpha)$; $\operatorname{tg} \varepsilon = \operatorname{ctg}(\varphi - \alpha)$

$$\operatorname{ctg}(\varphi - \alpha) = \frac{dr \cdot \cos \varphi - r \cdot d\varphi \cdot \sin \varphi}{dr \cdot \sin \varphi + r \cdot d\varphi \cdot \cos \varphi}$$

Mit $\operatorname{ctg}(\varphi - \alpha) = \frac{1 + \operatorname{ctg} \alpha \operatorname{ctg} \varphi}{\operatorname{ctg} \alpha - \operatorname{ctg} \varphi}$

ergibt sich:

$$\frac{dr}{r} = - \operatorname{ctg} \alpha \cdot d\varphi$$

$$\underline{\underline{r = C \cdot e^{-\operatorname{ctg} \alpha \cdot \varphi}}}$$

Konstantenbestimmung: $\varphi = 0$; $r = r_0$: $C = r_0$

$$r = r_0 \cdot e^{-\operatorname{ctg} \alpha \cdot \varphi}$$

Für $\alpha = \frac{\pi}{2}$ ergibt sich ein Kreis vom Radius $r = r_0$; für $\alpha = 0$ und $\alpha = \pi$ eine Gerade.

12. Punktbeschleunigung

Lösung 336

72 km/h $\triangleq$ 20 m/sek; $v = b \cdot t$; $t = \frac{v}{b} = \frac{20}{0{,}4} = \underline{\underline{50\ \text{sek}}}$

$$s = \frac{b}{2} t^2 = \frac{v \cdot t}{2} = \frac{20 \cdot 50}{2} = \underline{\underline{500\ \text{m}}}$$

Lösung 337

$$s = v \cdot t - \frac{b}{2} t^2; \quad b = \frac{v}{t}; \quad s = 6\,\text{cm}; \quad t = 0{,}02\,\text{sek}$$

$$6 = v_0 \cdot 0{,}02 - \frac{1}{2}\,\frac{v_0}{0{,}02} \cdot 0{,}02^2; \quad v_0 = \frac{12}{0{,}02}\,\text{cm/sek}; \quad v_0 = \underline{\underline{6\,\text{m/sek}}}$$

Lösung 338

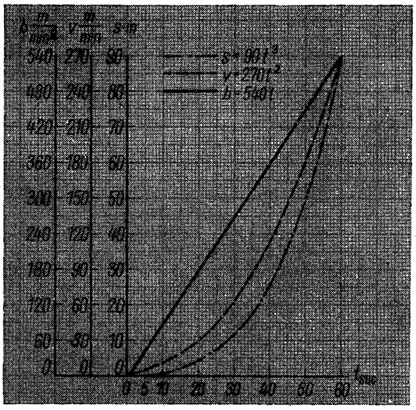
$$s_1 = \frac{g t_1^2}{2} \quad \text{mit} \quad t_1 = 1\,\text{sek}$$

$$s_2 = \frac{g t_2^2}{2} \quad \text{mit} \quad t_2 = 0{,}9\,\text{sek}$$

$$s_1 - s_2 = \frac{g}{2}(t_1 + t_2)(t_1 - t_2) = \frac{981 \cdot 0{,}19}{2}$$

$$\underline{\underline{s = 93{,}2\,\text{cm}}}$$

Lösung 339



$$s = a \cdot t^3; \quad t = 1\,\text{min}; \quad s = 90\,\text{m}$$

$$a = 90\,\text{m/min}^3$$

$$s = a t^3$$

$$\dot{s} = 3 a t^2 = v$$

$$\ddot{s} = 6 a t = b$$

$$t = 0: \quad v_0 = 0; \quad b_0 = 0$$

$$t = 5\,\text{sek} = \frac{1}{12}\,\text{min}: \quad v_5 = \underline{\underline{\frac{15}{8}\,\text{m/min}}}$$

$$b_5 = \underline{\underline{45\,\text{m/min}^2}}$$

Lösung 340

$$s = v_0 \cdot t - \frac{b}{2} t^2; \quad v = \dot{s} = v_0 - b t; \quad t = \frac{v_0 - v}{b}$$

$$s = \frac{v_0 (v_0 - v)}{b} - \frac{(v_0 - v)^2}{2b} = \frac{v_0^2 - v^2}{2b}; \quad b = \frac{v_0^2 - v^2}{2s} = \frac{225 - 25}{2 \cdot 34} = \underline{\underline{2{,}94\,\text{m/sek}^2}}$$

$$t = \frac{v_0 - v}{v_0^2 - v^2} \cdot 2s = \frac{2s}{v_0 + v} = \frac{68}{20} = \underline{\underline{3{,}4\,\text{sek}}}$$

Lösung 341

$$b = \frac{v}{t}; \quad s = \frac{b}{2} t^2; \quad \underline{\underline{b = \frac{v^2}{2s}}}; \quad b = \frac{100^2}{3{,}6^2 \cdot 2 \cdot 100} = \underline{\underline{3{,}86\,\text{m/sek}^2}}$$

Lösung 342

$$s = \frac{g}{2} t^2; \quad t_{\text{Fall}} = \sqrt{2\frac{s}{g}}; \quad t_{\text{Hub}} = 3\sqrt{2\frac{s}{g}}$$

$$t_{\text{ges}} = t_{\text{Fall}} + t_{\text{Hub}} = 4\sqrt{2\frac{s}{g}} = 4\sqrt{\frac{2 \cdot 2{,}5}{9{,}81}} = 2{,}8\,\text{sek}$$

$$z = \frac{60}{t_{\text{ges}}} = 21\,\text{Schläge/Minute}$$

Lösung 343

$$b^2 = b_t^2 + b_n^2; \qquad b_t = \text{Tangentialbeschleunigung (Bahnbeschl.)}$$
$$b_n = \text{Normalbeschleunigung (Zentripetalbeschl.)}$$

$$s = v_0 \cdot t + \frac{1}{2} b_t \cdot t^2; \qquad b_t = 2\,\frac{s - v_0 t}{t^2} = \frac{3}{9}\ \text{m/sek}^2$$

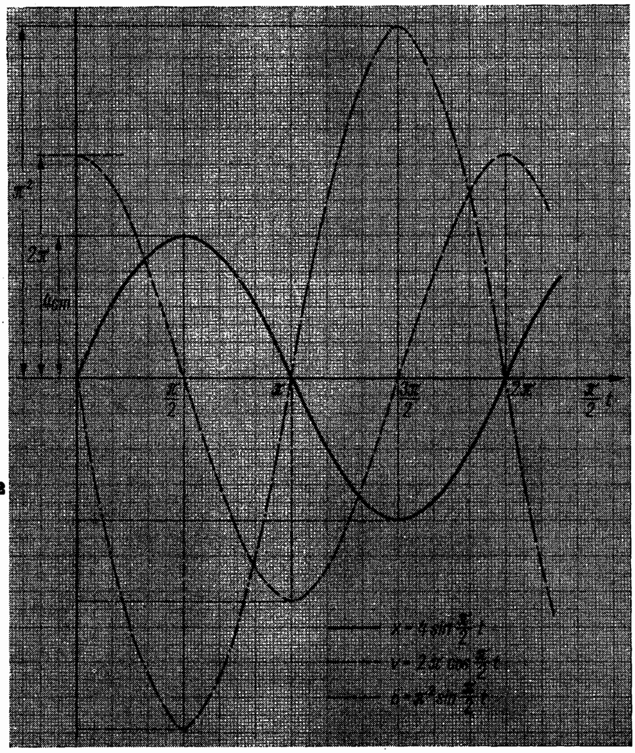
$$v = v_0 + b_t \cdot t = 15 + 30 \cdot \frac{1}{3} = \underline{\underline{25\ \text{m/sek}}}$$

$$b_n = \frac{v^2}{R} = \frac{25^2}{1000} = 0{,}625\ \text{m/sek}^2$$

$$b = \sqrt{0{,}625^2 + 0{,}33^2} = \underline{\underline{0{,}708\ \text{m/sek}^2}}$$

Lösung 344

Bewegungsdiagramme



$$b_x = -\pi^2 \sin\frac{\pi}{2} \cdot t\ \text{m/sek}^2$$

$$v_x = \int -\pi^2 \sin\frac{\pi}{2}\, t\, dt$$
$$= \frac{2\pi^2}{\pi} \cos\frac{\pi}{2}\, t + C_1$$

$$x = \int \left[2\pi \cos\frac{\pi}{2}\, t + C_1\right] dt$$
$$= 4 \sin\frac{\pi}{2}\, t + C_1 t + C_2$$

Konstantenbestimmung: $t = 0, \quad v_x = 2\pi: \quad C_1 = 0$

$t = 0, \quad x = 0: \quad C_2 = 0$

$$x = \underline{\underline{4 \sin\frac{\pi}{2} \cdot t\ \text{m}}}$$

Lösung 345

$$v_0 = 54\ \text{km/h} \mathrel{\hat{=}} 15\ \text{m/sek}; \qquad v_1 = 18\ \text{km/h} \mathrel{\hat{=}} 5\ \text{m/sek}$$

$$b_t = \frac{v_1 - v_0}{t}; \qquad s = v_0 \cdot t + \frac{b_t}{2} \cdot t^2 = v_0 t + \frac{v_1 - v_0}{2} t = \frac{t}{2}(v_1 + v_0)$$

$$t = \frac{2s}{v_1 + v_0} = \frac{1600}{20} = \underline{\underline{80\ \text{sek}}}$$

$$b_t = -\frac{10}{t} = -\frac{1}{8}\ \text{m/sek}^2; \qquad b_{n_0} = \frac{v_0^2}{R} = \frac{225}{800} = 0{,}281\ \text{m/sek}^2$$

$$b_{n_1} = \frac{v_1^2}{R} = \frac{25}{800} = 0{,}031\ \text{m/sek}^2$$

$$b_0 = \sqrt{b_t^2 + b_{n_0}^2} = \underline{\underline{0{,}308\ \text{m/sek}^2}}$$

$$b_1 = \sqrt{b_t^2 + b_{n_1}^2} = \underline{\underline{0{,}129\ \text{m/sek}^2}}$$

Lösung 346

$$v = 36\ \text{km/h} \mathrel{\hat{=}} 10\ \text{m/sek}; \qquad b_n = \frac{v^2}{\varrho}$$

$$b_{n_1} = \frac{10^2}{300} = \underline{\underline{\frac{1}{3}\ \text{m/sek}^2}}; \qquad b_{n_2} = \frac{10^2}{400} = \underline{\underline{\frac{1}{4}\ \text{m/sek}^2}}$$

Lösung 347

$s = 0{,}1 t^3$; $\quad \dot{s} = 3 \cdot 0{,}1 t^2$; $\quad$ Nach $t = \sqrt{\dfrac{\dot{s}}{0{,}3}}$ sek $\quad$ beträgt $\quad \dot{s} = v_t = 30$ m/sek

$$t = \sqrt{\frac{30}{0{,}3}} = \underline{\underline{10\ \text{sek}}}$$

$$b_n = \frac{v_t^2}{R} = \frac{30^2}{2} = \underline{\underline{450\ \text{m/sek}^2}}; \qquad b_t = \ddot{s}/_{t=10} = 0{,}6 \cdot t = \underline{\underline{6\ \text{m/sek}^2}}$$

Lösung 348

$s = 20 \sin \pi t$

$\dot{s} = v = 20\pi \cos \pi t$

$\ddot{s} = b_t = -20\pi^2 \sin \pi t$

Für $t = 5$ sek: $\quad v = -20\pi$

$$b_t = 0; \qquad b_n = b = \frac{v^2}{R} = \frac{400\pi^2}{20}$$

$$b_n = b = \underline{\underline{20\pi^2\ \text{cm/sek}^2}}$$

Die Geschwindigkeit ist also entgegen der positiven Weg-Richtung gerichtet

Lösung 349

$$s = \frac{g}{a^2}(at + e^{-at})$$

$$\dot{s} = \frac{g}{a^2}(a - a e^{-at}) = \frac{g}{a}(1 - e^{-at}) = v$$

$$\ddot{s} = \frac{g}{a^2} \cdot a^2 \cdot e^{-at} = g \cdot e^{-at} = \underline{\underline{g - av}}$$

$$t = 0: \quad \underline{\underline{v_0 = 0;}} \quad b_0 = g$$

Lösung 350

$$x = 10\cos 2\pi\frac{t}{5}; \quad y = 10\sin 2\pi\frac{t}{5}$$

$x^2 + y^2 = r^2 = 100$; Bewegungsbahn: $\underline{\underline{\text{Kreis mit } r = 10\text{ cm}}}$

$$\operatorname{tg}\varphi = \frac{y}{x} = \operatorname{tg} 2\pi\frac{t}{5}; \qquad \varphi = \frac{2\pi t}{5}; \qquad s = r\cdot\varphi = 4\pi t$$

$$\dot{s} = v = \underline{\underline{4\pi}}\text{ cm/sek}$$

$$b_n = \frac{\dot{s}^2}{r} = \frac{16\pi^2}{10} = \underline{\underline{1{,}6\pi^2}} \qquad \ddot{s} = b_t = 0$$

$b = b_n$; Die Beschleunigung ist also zum Zentrum hin gerichtet.

Lösung 351

$$x = 20t^2 + 5; \quad \dot{x} = 40t; \quad \ddot{x} = 40; \quad \ddot{s} = \sqrt{\ddot{x}^2 + \ddot{y}^2} = b = \underline{\underline{50\text{ cm/sek}^2}} = \text{const}$$

$$y = 15t^2 - 3; \quad \dot{y} = 30t; \quad \ddot{y} = 30$$

$t = 2$ sek: $\dot{x} = 80$; $\dot{s} = \sqrt{\dot{x}^2 + \dot{y}^2} = \underline{\underline{v = 100\text{ cm/sek}}}$; $\cos(v, \dot{x}) = \cos(b, \ddot{x}) = 0{,}8$

$\dot{y} = 60$ $\qquad\cos(v, \dot{y}) = \cos(b, \ddot{y}) = 0{,}6$

$t = 3$ sek: $\dot{x} = 120$; $\dot{s} = \underline{\underline{150\text{ cm/sek}}}$;

$\dot{y} = 90$;

Lösung 352

$$x = a(e^{kt} + e^{-kt}) = 2a\,\mathfrak{Cos}\,kt$$

$$y = a(e^{kt} - e^{-k}) = 2a\,\mathfrak{Sin}\,kt \qquad r = 2a\sqrt{\mathfrak{Cos}^2 kt + \mathfrak{Sin}^2 kt}$$

$(\mathfrak{Cos}\,kt)^2 - (\mathfrak{Sin}\,kt)^2 = \dfrac{x^2 - y^2}{4a^2} = 1$ oder: $\underline{\underline{x^2 - y^2 = 4a^2}}$ Hyperbel

$$v_x = \dot{x} = 2ak\,\mathfrak{Sin}\,kt;$$

$$v_y = \dot{y} = 2ak\,\mathfrak{Cos}\,kt; \qquad v = \sqrt{v_x^2 + v_y^2} = 2ak\sqrt{\mathfrak{Cos}^2 kt + \mathfrak{Sin}^2 kt} = \underline{\underline{k\cdot r}}$$

$$b_x = \ddot{x} = 2ak^2\,\mathfrak{Cos}\,kt;$$

$$b_y = \ddot{y} = 2ak^2\,\mathfrak{Sin}\,kt; \quad b = \sqrt{b_x^2 + b_y^2} = 2ak^2\sqrt{\mathfrak{Cos}^2 kt + \mathfrak{Sin}^2 kt} = \underline{\underline{k^2\cdot r}}$$

Lösung 353

$x = 2t$ Bewegungsbahn: $x^2 = 4y$

$y = t^2$ $\qquad \underline{\underline{y = \dfrac{x^2}{4}}}$ Parabel

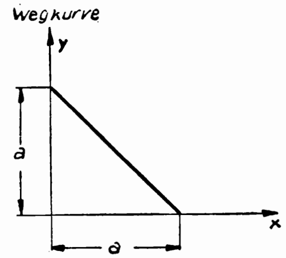
Krümmungsradius: $\varrho = \dfrac{(1 + y'^2)^{\frac{3}{2}}}{y''}$; $\quad y' = \dfrac{x}{2}$; $\quad \varrho = \dfrac{\left(1 + \dfrac{x^2}{4}\right)^{\frac{3}{2}}}{\dfrac{1}{2}}$

$$y'' = \frac{1}{2};$$

für $t = 0$; $x = 0$:

$$\underline{\underline{\varrho_0 = 2\text{ m}}}$$

Lösung 354



$x = a\cos^2 t \qquad y = a\sin^2 t$

$x + y = a$

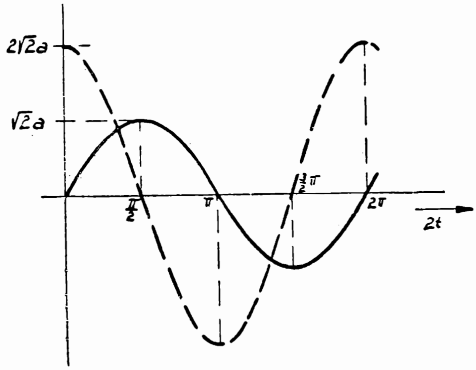
mit $|x| \leqq a \quad |y| \leqq a$

$v_x = \dot{x} = -2a\sin t\cos t$

$v_y = \dot{y} = +2a\sin t\cos t$

$$\underline{v = \sqrt{v_x^2 + v_y^2} = \sqrt{2}\; a\sin 2t\ \text{cm/sek}}$$

$$\underline{\underline{b = \frac{dv}{dt} = \dot{v} = 2a\sqrt{2}\cos 2t\ \text{cm/sek}^2}}$$



——— $v = \sqrt{2}\,a\sin 2t$

- - - - $b = 2\sqrt{2}\,a\cos 2t$

Lösung 355

$x = 75\cos 4t^2; \quad \dot{x} = -75\cdot 8t\cdot\sin 4t^2; \quad \ddot{x} = -75\cdot 64t^2\cos 4t^2$

$y = 75\sin 4t^2; \quad \dot{y} = 75\cdot 8t\cdot\cos 4t^2; \quad \ddot{y} = -75\cdot 64t^2\sin 4t^2$

Beim Anfahren ist: $\cos 4t^2 = 1; \quad \sin 4t^2 = 0$

$$\dot{x} = 0; \quad \dot{y} = \underline{\underline{v_t = 600t\ \text{cm/sek}}}$$

$$b_t = \frac{dv_t}{dt} = \underline{\underline{600\ \text{cm/sek}^2}}$$

$$b_n = \frac{v_t^2}{R} = \frac{(600\,t)^2}{75} = \underline{\underline{4\,800t^2\ \text{cm/sek}^2}}$$

Lösung 356

$$x = 4\sin\frac{\pi}{2}t; \quad \ddot{x} = -\pi^2\sin\frac{\pi}{2}t$$

$$y = 3\sin\frac{\pi}{2}t; \quad \ddot{y} = -3\frac{\pi^2}{4}\sin\frac{\pi}{2}t; \quad b = \sqrt{\ddot{x}^2 + \ddot{y}^2}$$

$\underline{\underline{b_{t=1} = \frac{5}{4}\pi^2\ \text{cm/sek}^2}}$ $\qquad \varrho = \infty$, da die Bewegungsbahn $3x = 4y$ eine Gerade ist.

Lösung 357

$$x = -a \sin 2wt; \qquad x = -a \cdot 2 \sin \omega t \cos \omega t$$

$$y = -a \sin \omega t; \qquad \cos \omega t = \sqrt{1 - \frac{y^2}{a^2}}$$

$$x = 2y \cdot \cos \omega t; \quad ax = 2y\sqrt{a^2 - y^2} \text{ Bewegungsbahn}$$

$$\varrho = \frac{(1+y'^2)^{\frac{3}{2}}}{y''}; \quad a = 2y'\sqrt{a^2-y^2} + 2y \cdot \frac{1}{2} \frac{-2yy'}{\sqrt{a^2-y^2}}$$

$$y = 0: \quad y' = \frac{1}{2}$$

$$0 = 2y''\sqrt{a^2-y^2} + 2y' \frac{(-2yy')}{2\sqrt{a^2y^2}} + \frac{(2y' \cdot 2yy' + 2y^2y'')\sqrt{a^2-y^2}}{(a^2-y^2)}$$

$$-2y^2y' \cdot \frac{1}{2} \frac{(-2yy')}{\sqrt{a^2-y^2}}$$

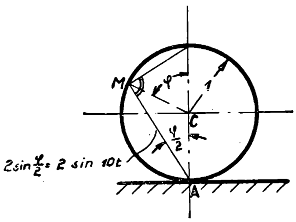
$y = 0$:
$y'' = 0$; somit $\underline{\underline{\varrho = \infty}}$

Lösung 358

$$x = 20t - \sin 20t; \quad \dot{x} = 20 - 20 \cos 20t; \quad \ddot{x} = 400 \sin 20t$$

$$y = 1 - \cos 20t; \quad \dot{y} = 20 \sin 20t; \quad \ddot{y} = 400 \cos 20t$$

$b = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \underline{\underline{400 \text{ m/sek}^2}}$ Richtung: MC, da (Zentripetalbeschl.)

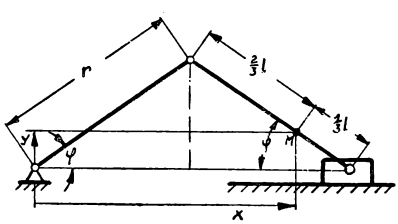


$$\varrho = \frac{\sqrt{\dot{x}^2 + \dot{y}^2}^3}{\ddot{x}\dot{y} - \dot{x}\ddot{y}} = \frac{\sqrt{800 - 800 \cos 20t}^3}{8000 - 8000 \cos 20t}$$

$$\varrho = 4 \frac{\sqrt{1 - \cos 20t}}{2} = 4 \sin 10t = \underline{\underline{2MA}}$$

$$\varrho_{t=0} = \underline{\underline{0}}$$

Lösung 359



Bewegungsbahn:

$$x = r \cos \omega t + \frac{3}{2} l \cos \omega t; \quad \varphi = \omega t; \quad \omega = 4\pi$$

$$x = \frac{5}{3} r \cos \omega t; \qquad r = l$$

$$y = \frac{1}{3} r \sin \omega t;$$

$$\frac{x^2}{\frac{5^2}{3^2} \cdot r^2} = \cos^2 \omega t;$$

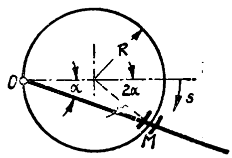
$$\frac{y^2}{\frac{r^2}{3^2}} = \sin^2 \omega t; \qquad \underline{\underline{\frac{x^2}{100^2} + \frac{y^2}{20^2} = 1}}$$

$$x = 100\cos 4\pi t; \qquad y = 20\sin 4\pi t \qquad \varphi = 0: \quad t = 0$$
$$\dot{x} = -400\pi\sin 4\pi t; \qquad \dot{y} = 80\pi\cos 4\pi t$$
$$\ddot{x} = -1600\pi^2\cos 4\pi t; \qquad \ddot{y} = -320\pi^2\sin 4\pi t \qquad \dot{x} = 0; \quad \underline{\underline{\dot{y} = 80\pi \text{ cm/sek}}}$$
$$\ddot{x} = \underline{\underline{-1600\pi^2 \text{ cm/sek}^2}}; \quad \ddot{y} = 0$$

Krümmungsradius: $\varrho = \dfrac{(1+y'^2)^{\frac{3}{2}}}{y''}$ bzw.: $\varrho = \dfrac{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}}{\dot{x}\ddot{y}-\dot{y}\ddot{x}}$

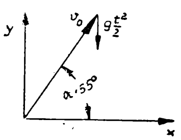
$$\varphi = 0: \quad t = 0: \quad \varrho = \frac{(6400\pi^2)^{\frac{3}{2}}}{80\pi\cdot 1600\pi^2}; \quad \underline{\underline{\varrho = 4 \text{ cm}}}$$

Lösung 360



$$\alpha = \omega t; \quad \alpha = \frac{\pi}{2} \quad \text{für} \quad t = 5 \text{ sek}$$
$$\omega = \frac{\alpha}{t} = \frac{\pi}{10}\text{ sek}^{-1}$$
$$s = 2R\alpha = 2R\omega t$$
$$v = 2R\omega = \underline{\underline{2\pi \text{ cm/sek}}}$$
$$b = \frac{v^2}{R} = \underline{\underline{0{,}4\,\pi^2 \text{ cm/sek}^2}}$$

Lösung 361



$$v_0 = 1600 \text{ m/sek} \qquad x = v_0\cdot t\cdot\cos\alpha$$
$$y = v_0\cdot t\sin\alpha - \frac{1}{2}gt^2; \quad t = \frac{x}{v_0\cos\alpha}$$
$$\underline{\underline{y = x\,\mathrm{tg}\,\alpha - \frac{g x^2}{2v_0^2\cos^2\alpha}}}$$

Schußweite: Bedingung $y = 0$: $\dfrac{2v_0^2\cos^2\alpha}{g}\cdot\mathrm{tg}\,\alpha = x$

$$x = 24{,}5\cdot 10^4 \text{ m} \mathrel{\hat{=}} \underline{\underline{245 \text{ km}}}$$

Größte Schußhöhe: $y' = 0 = \mathrm{tg}\,\alpha - \dfrac{gx}{v_0^2\cos^2\alpha}$; $x_{y\,\max} = \dfrac{v_0^2\cos^2\alpha}{g}\cdot\mathrm{tg}\,\alpha$

$$y_{\max} = 122{,}5\cdot\frac{0{,}82}{0{,}574} - \frac{9{,}81\cdot 15000\cdot 10^3}{2\cdot 256\cdot 10^4\cdot 0{,}33} \mathrel{\hat{=}} \underline{\underline{87{,}5 \text{ km}}}$$

Lösung 362

Größte Schußweite: $x_e = \dfrac{2v_0^2\sin\alpha\cos\alpha}{g} = \dfrac{v_0^2}{g}\cdot\sin 2\alpha$ (vergl. 361)

$$\alpha = 45° \pm \varepsilon: \quad x_e = \frac{v_0^2}{g}\sin(90° \pm 2\varepsilon) = \frac{v_0^2}{g}\cos 2\varepsilon$$

Bei $\varepsilon = 0$, also $\alpha = 45°$, erreicht x_e ein Maximum.

Lösung 363

$$x = v_0 \cdot t \cos\alpha_0$$
$$y = v_0 \cdot t \cdot \sin\alpha_0 - \frac{1}{2} g t^2$$
$$\underline{\underline{y = x \cdot \operatorname{tg}\alpha_0 - \frac{g x^2}{2 v_0^2 \cos\alpha_0}}}$$

Schußweite für $y = 0$: $\underline{\underline{x_{\max} = \frac{v_0^2}{2g} \cdot \sin 2\alpha_0}}$.

Schußhöhe: $\underline{\underline{y_{\max} = \frac{v_0^2}{2g} \sin^2\alpha_0}}$

} Koordinaten des höchsten Bahnpunktes.

$\dot{x} = \underline{\underline{v_x = v_0 \cdot \cos\alpha_0}}$ $\quad \dot{y} = v_y = v_0 \cdot \sin\alpha_0 - g t$

für $t = 0$: $v_y = v_0 \sin\alpha_0$

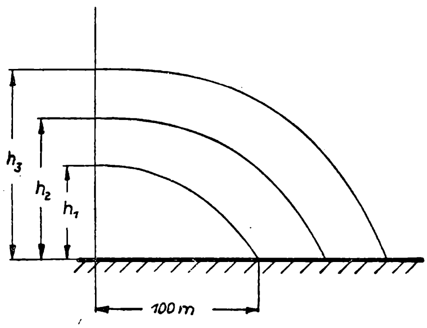
für $t = \frac{2 v_0 \sin\alpha_0}{9}$: $v_y = -v_0 \sin\alpha_0$

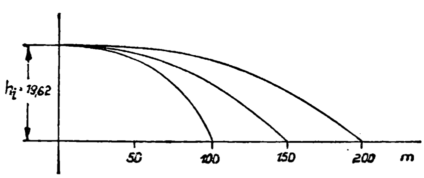
Lösung 364

Aus $x = v_0 \cdot t \cos\alpha = v_1 t$ folgt: $v_1 = v_0 \cos\alpha = \underline{\underline{10\ \text{m/sek}}}$

Nach 362 ist: $x_1 = \frac{v_0^2}{g} \cdot \sin 2\alpha = \underline{\underline{35{,}3\ \text{m}}}$

Lösung 365





Für alle drei Kugeln gilt:

$$x = v_0 \cdot t$$
$$y = -\frac{g}{2} t^2 + h_i$$

Für alle drei Kugeln ist t gleich, also gilt:

$$y = -\frac{g}{2} \frac{x^2}{v_0^2} + h_i$$

Gegeben: $x_1 = 100\ \text{m}$; $v_{0_1} = 50\ \text{m/sek}$

$$t = \frac{x_1}{v_{0_1}} = \underline{\underline{T = 2\ \text{sek}}}$$

Wurfweiten: $x_2 = t \cdot v_{0_2} = 2 \cdot 75 = 150\ \text{m}$

$x_3 = 2 \cdot 100 = 200\ \text{m}$

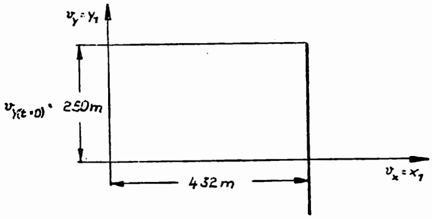
Am Aufschlagpunkt ist $y = 0$, somit

$$h_{1,2,3} = \frac{g}{2} \frac{x^2_{1,2,3}}{v^2_{1,2,3}}$$
$$\frac{x^2_{1,2,3}}{v^2_{1,2,3}} = t^2 = \text{const}$$
$$h_1 = h_2 = h_3 = \frac{g t^2}{2} = \frac{9{,}81 \cdot 4}{2} = \underline{\underline{19{,}62\ \text{m}}}$$

Aufschlaggeschwindigkeiten:

$$v_1 = \sqrt{v_{0_1}^2 + g^2 t^2} = \underline{\underline{53{,}71}}\ \text{m/sek}$$
$$v_2 = \sqrt{v_{0_2}^2 + g^2 t^2} = \underline{\underline{77{,}52}}\ \text{m/sek}$$
$$v_3 = \sqrt{v_{0_3} + g^2 t^2} = \underline{\underline{101{,}95}}\ \text{m/sek}$$

Lösung 366



$$v_x = v_0 \cos\alpha = 432 \text{ m}$$
$$v_y = v_0 \sin\alpha - gt$$
$$v_1 = \frac{d v_y}{dt} = g = \underline{\underline{9{,}81 \text{ m/sek}^2}}$$

Lösung 367

$$y = x \operatorname{tg}\alpha - \frac{g x^2}{2 v_0^2 \cdot \cos^2\alpha}$$

$$y' = \operatorname{tg}\alpha - \frac{g x}{v_0^2 \cos^2\alpha} \qquad \varrho = \frac{(1+y'^2)^{\frac{3}{2}}}{y''}; \quad \text{Für } t = 0 \text{ gilt } x = 0$$

$$y'' = -\frac{g}{v_0^2 \cdot \cos^2\alpha}; \qquad \varrho\big|_{x=0} = \frac{(1+\operatorname{tg}^2\alpha)^{\frac{3}{2}}}{-\dfrac{g}{v_0^2 \cos^2\alpha}}$$

$$|\varrho| = \frac{(1+\operatorname{tg}^2\alpha)\sqrt{1+\operatorname{tg}^2\alpha}\, v_0^2 \cos^2\alpha}{g}; \qquad \frac{1}{\sqrt{1+\operatorname{tg}^2\alpha}} = \cos\alpha$$

$|\varrho| = \dfrac{v_0^2}{g \cdot \cos\alpha}$; ϱ ist für Abschuß und Aufschlag gleich.

Lösung 368

$$x = 300\, t \qquad \dot{x} = v_x = 300 \qquad \ddot{x} = b_x = 0$$
$$y = 400\, t - 5\, t^2 \qquad \dot{y} = v_y = 400 - 10\, t \qquad \ddot{y} = b_y = -10$$

für $t = 0$ gilt: $v_0 = \sqrt{v_x^2(0) + v_y^2(0)} = \underline{\underline{500 \text{ m/sek}}}$

$$b = \sqrt{b_x^2 + b_y^2} = \underline{\underline{10 \text{ m/sek}^2}}$$

$$h = y_{(v_y=0)} = y_{(t=40)} = 8000 \text{ m}; \qquad \underline{\underline{h = 8 \text{ km}}}$$

$$s = x_{(y=0)} = x_{(t=80)} = 24000 \text{ m}; \qquad \underline{\underline{s = 24 \text{ km}}}$$

$$\varrho = \frac{\sqrt{v_x^2 + v_y^2}^{\,3}}{b_x \cdot v_y - b_y \cdot v_x} = \frac{\sqrt{250\,000 - 8000\, t + 100\, t^2}^{\,3}}{3000}$$

$$\varrho_0 = \varrho_{(t=0)} = \underline{\underline{41{,}67 \text{ km}}} \qquad \varrho_k = \varrho_{(t=40)} = \underline{\underline{9 \text{ km}}}$$

Lösung 369

Die Lösung der Aufgabe ergibt sich durch Einsetzen der gegebenen Zahlenwerte in die in Aufgabe 363 aufgestellten Gleichungen.

$$x = 500\, t; \quad y = 866\, t - 4{,}905\, t^2;$$
$$y = 1{,}732\, x - 10^{-8} \cdot 1962\, x^2;$$
$$h = 38{,}24 \text{ km}; \quad s = 88{,}3 \text{ km}$$

Lösung 370

$$x = v_0 t \cos\alpha_0; \qquad y = h + v_0 t \sin\alpha_0 - \frac{g}{2} t^2$$

Auftreffen, wenn $y = 0$

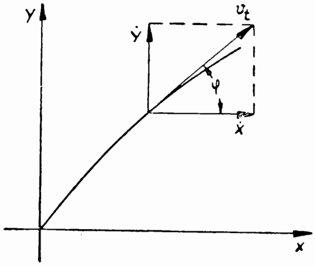
$$t_A{}^2 - \frac{2 v_0 \sin\alpha_0}{g} t_A = \frac{2h}{g}; \qquad t_A = \frac{2 v_0 \sin\alpha_0}{g} + \sqrt{\frac{v_0{}^2 \sin^2\alpha_0}{g^2} + \frac{2h}{g}}$$

$$x_A = v_0 t_A \cdot \cos\alpha_0 = \underline{\underline{102\ \text{km}}}$$

Lösung 371

$$\dot{x} = \alpha \quad; \qquad \ddot{x} = 0 \qquad\qquad v_t = \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$\dot{y} = \beta - gt; \qquad \ddot{y} = -g \qquad\qquad = \sqrt{\alpha^2 + (\beta - gt)^2}$$



$$b_t = \dot{v}_t = \frac{2\dot{x}\ddot{x} + 2\dot{y}\ddot{y}}{2\sqrt{\dot{x}^2 + \dot{y}^2}}$$

$$b_t = \frac{(\beta - gt)(-g)}{v_t} = \underline{\underline{-\frac{g(\beta - gt)}{v_t}}}$$

$$b_n = \frac{v_t^2}{\varrho}; \qquad \varrho = \frac{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}{\dot{x}\ddot{y} - \dot{y}\ddot{x}}$$

$$b_n = \frac{v_t^2(\dot{x}\ddot{y} - \dot{y}\ddot{x})}{v_t^3} = \underline{\underline{-\frac{\alpha g}{v_t}}}$$

Lösung 372

$v = b_t \cdot t$; Für $t = 180$ sek ist $v = 72$ km/h $\triangleq 20$ m/sek

Daraus $b_t = \frac{v}{t} = \underline{\underline{\frac{1}{9}\ \text{m/sek}^2}}$; $v_{(t=2\,\text{min})} = \frac{40}{3}$ m/sek

$$b_n = \frac{v^2}{R} = \underline{\underline{\frac{2}{9}\ \text{m/sek}^2}}; \qquad b = \sqrt{b_t{}^2 + b_n{}^2} = \frac{\sqrt{5}}{9}\ \text{m/sek}^2 = \underline{\underline{0{,}25\ \text{m/sek}^2}}$$

Lösung 373

Normalform der Schraubenbewegung:

$$x = a\cos t$$
$$y = a\sin t$$
$$z = ct$$

Hierfür gilt: $k = \frac{1}{\varrho} = \frac{a}{a^2 + c^2}$; Für die Aufgabe gilt analog:

$$x = 2\cos 4t; \quad a = 2$$
$$y = 2\sin 4t;$$
$$z = \frac{1}{2}\cdot(4t); \quad c = \frac{1}{2}$$

$$\varrho = \frac{4 + \frac{1}{4}}{2} = \underline{\underline{2\frac{1}{8}\ \text{m}}}$$

Lösung 374

In $r = a e^{kt}$ wird $\varphi = kt$ eingesetzt. Bahngleichung: $\underline{\underline{r = a e^{\varphi}}}$

$$v = \sqrt{\dot r^2 + r^2 \dot\varphi^2} = \sqrt{k^2 r^2 + r^2 k^2}\,; \quad \frac{dr}{dt} = \dot r\,; \quad \frac{d\varphi}{dt} = \dot\varphi$$

$$\frac{d^2 r}{dt^2} = \ddot r\,; \quad \frac{d^2\varphi}{dt^2} = \ddot\varphi$$

$$\underline{\underline{v = kr\sqrt{2}}}\,; \quad \varrho = \frac{\sqrt{\dot r^2 + r^2\dot\varphi^2}}{r\cdot\ddot r\cdot\dot\varphi - 2\dot r^2\dot\varphi - r^2\dot\varphi^3 - r\dot r\ddot\varphi}\,; \quad \underline{\underline{\varrho = r\sqrt{2}}}$$

$$b = \sqrt{b_t^2 + b_n^2}\,; \quad b_t = \dot v = k^2 r\sqrt{2}\,; \quad b_n = \frac{v^2}{\varrho} = k^2 r\sqrt{2}\,; \quad \underline{\underline{b = 2k^2 r}}$$

IV. Elementarbewegung fester Körper

13. Drehung des festen Körpers um eine starre Achse

Lösung 375

1. $\omega = \dfrac{\pi\cdot n}{30} = \dfrac{\pi\cdot 1}{30} = \underline{\underline{\dfrac{\pi}{30}}}$ 1/sek
2. $\omega = \dfrac{\pi\cdot 1}{30\cdot 60} = \dfrac{\pi}{1800}$ „
3. $\omega = \dfrac{\pi\cdot 1}{30\cdot 12\cdot 60} = \dfrac{\pi}{21\,600}$ „
4. $\omega = \dfrac{\pi\cdot 1}{30\cdot 24\cdot 60} = \dfrac{\pi}{43\,200}$ „
5. $\omega = \dfrac{\pi\cdot 1500}{30} = \pi\cdot 500$ „

Lösung 376

Ansatz: $\varphi = c\cdot t^3$

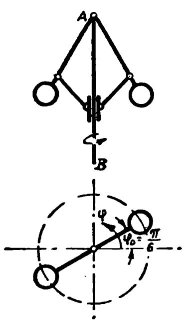
$n = 810$ U/min $\triangleq 13{,}5$ U/sek; $\omega = 2\pi n = 27\pi$ 1/sek

$\omega = \dfrac{d\varphi}{dt} = \dot\varphi = 3c\cdot t^2$; $t = 3$ sek gesetzt:

$27\pi = 3c\cdot 9$

$c = \pi$ 1/sek³; $\varphi = \underline{\underline{\pi t^3 \text{ Bg}}}$ t in sek.

Lösung 377



$$\varphi = \omega t + \varphi_0; \quad \omega = \frac{\pi n}{30} = 4\pi$$

$$\varphi_{t=\frac{1}{2}\text{sek}} = \frac{4\pi}{2} + \frac{\pi}{6} = \underline{\underline{\frac{13}{6}\pi \text{ Bg}}}$$

$$\Delta\varphi = \omega t = \underline{\underline{2\pi \text{ Bg}}}$$

Lösung 378

Ansatz: $\varphi = \frac{\varepsilon}{2} t^2$; Bei $t = 120$ sek ist $\varphi = 3600 \cdot 2\pi$

$$7200\pi = \varepsilon \frac{14400}{2}: \quad \underline{\underline{\varepsilon = \pi \text{ 1/sek}^2}}$$

Lösung 379

$$\frac{d\omega}{dt} = \varepsilon = \frac{\omega_1 - \omega_0}{t}; \quad \underline{\omega_0 = 0}$$

$\varphi = \varepsilon \frac{t^2}{2}$; Für $t = 5$ sek ist $\varphi = 12{,}5 \cdot 2\pi$

$$12{,}5 \cdot 2\pi = \varepsilon \cdot \frac{25}{2}; \quad \varepsilon = \frac{4\pi \cdot 12{,}5}{25}; \quad \omega = \varepsilon \cdot t = \frac{4\pi \cdot 12{,}5 \cdot 5}{25} = \underline{\underline{10\pi \text{ 1/sek}}}$$

Lösung 380

$\varphi = \frac{\varepsilon}{2} t^2$; $\omega = \dot{\varphi} = \varepsilon \cdot t$; Daraus: $\varphi = \frac{\omega \cdot t}{2}$

Werden die gegebenen Dimensionen direkt übertragen, so kann für $\varphi \triangleq Z$ (Anzahl der Umdrehungen) gesetzt werden.

$$Z = \frac{120 \text{ U/min} \cdot 10 \text{ min}}{2}; \quad Z = \underline{\underline{600 \text{ Umdr.}}}$$

Lösung 381

$\varphi = \dot{\varphi}_0 t - \frac{1}{2} \varepsilon t^2$; $\dot{\varphi} = -\varepsilon \cdot t + \dot{\varphi}_0$

Am Ende der Bewegung ist $\dot{\varphi} = 0$

$\varepsilon t = \dot{\varphi}_0 = 2\pi$; $\varphi = 10 \cdot 2\pi$

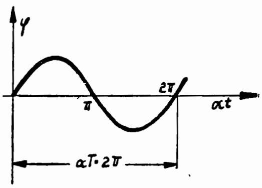
$$\varphi = \varphi_0 \cdot t - \frac{1}{2} \varphi_0 \cdot t; \quad t = 2 \cdot \frac{\varphi}{\varphi_0} = 20 \text{ sek}; \quad \varepsilon = \frac{2\pi}{20} = \underline{\underline{\frac{\pi}{10} \text{ 1/sek}^2}} \text{ Verzögerung}$$

Lösung 382

Umdrehungszahl $= Z$; $Z = \frac{n \cdot t}{2}$

Daraus: $t = \frac{2 \cdot Z}{n} = \frac{160}{1200}$ min; $t = \frac{2}{15}$ min $\triangleq \underline{\underline{8 \text{ sek}}}$

Lösung 383



$\varphi = a \cdot \sin \alpha t$; $\dot{\varphi} = a \cdot \alpha \cdot \cos \alpha t$; $\dot{\varphi}_{(t=0)} = a\alpha$

$$a = 20° \triangleq \frac{20 \cdot \pi}{180} = \frac{\pi}{9} \text{ Bg}$$

$$\alpha = 2\text{°/sek} \triangleq \frac{2 \cdot \pi}{180} \text{ 1/sek}$$

$$\dot{\varphi}_{(t=0)} = \omega = \underline{\underline{\frac{1}{810} \cdot \pi^2 \text{ 1/sek}}}$$

Bei Änderung der Bewegungsrichtung ist $\varphi = 0$, also

$$\cos \alpha t = 0; \quad \alpha t = \frac{\pi}{2}; \quad \frac{3}{2}\pi; \quad t_1 = \frac{\pi}{2} \cdot \frac{180}{2\pi} = \underline{\underline{45 \text{ sek}}}$$

$$t_2 = \frac{3}{2}\pi \cdot \frac{180}{2\pi} = \underline{\underline{135 \text{ sek}}}; \quad \alpha T = 2\pi; \quad T = \frac{2\pi}{\alpha} = \frac{2\pi \cdot 180}{2\pi} = \underline{\underline{180 \text{ sek}}}$$

Lösung 384

Ansatz:

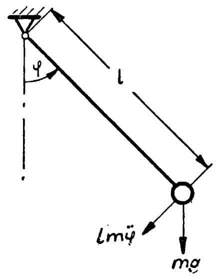
$$\varphi = \alpha \cdot \sin \omega t; \quad \alpha = \frac{\pi}{2}; \quad \omega T = 2\pi; \quad \omega = 4\pi$$

$$\varphi = \frac{\pi}{2} \sin 4\pi t$$

$$\dot\varphi = 2\pi^2 \cos 4\pi t; \quad \omega_{(t=2\,\text{sek})} = \underline{\underline{2\pi^2}}\,\frac{1}{\text{sek}}$$

$$\ddot\varphi = -8\pi^3 \sin 4\pi t; \quad \varepsilon_{(t=2\,\text{sek})} = \underline{\underline{0}}$$

Lösung 385



$$l^2 m\ddot\varphi + mgl\sin\varphi = 0; \quad \varphi = \text{kleiner Winkel: } \sin\varphi = \operatorname{tg}\varphi = \varphi$$

$$\ddot\varphi + \frac{g}{l} \cdot \varphi = 0$$

Lösungsansatz: $\varphi = A \sin kt + B \cos kt$

$$\ddot\varphi = -A k^2 \sin kt - B k^2 \cos kt$$

mit $\frac{g}{l} = k^2$ ist die Differentialgleichung erfüllt.

Anfangsbedingungen: $t = \frac{2}{3}$ sek: $\dot\varphi = 0$

$$\varphi_{\dot\varphi=0} = \alpha = \frac{\pi}{16}$$

Schwingungszeit: $T = 4t = \frac{8}{3}$ sek

$$kT = 2\pi: \quad k = \frac{3}{4}\pi$$

Konstantenbestimmung: $\dot\varphi = 0 = Ak \cdot \cos kt - Bk \sin kt; \quad t = \frac{2}{3}: \quad \sin kt = \sin\frac{\pi}{2}$

$$\varphi = \frac{\pi}{16} = A \sin kt + B \cos kt; \qquad B = 0; \quad A = \frac{\pi}{16}$$

Schwingungsgleichung: $\underline{\underline{\varphi = \frac{\pi}{16} \sin \frac{3}{4}\pi t}}$

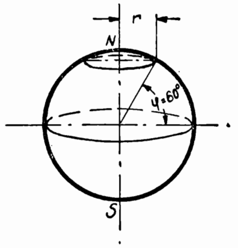
Die größte Geschwindigkeit herrscht bei $\ddot\varphi = 0$: $\ddot\varphi = -\frac{\pi}{16}\left(\frac{3\pi}{4}\right)^2 \sin\frac{3\pi}{4} t = 0$

$$\frac{3\pi}{4} t = 0; \quad t = 0$$

somit liegt $\dot\varphi_{\max}$ bei $\varphi = 0$

$$\dot\varphi_{\max} = \omega = Ak = \underline{\underline{\frac{3}{64}\pi^2 \text{ 1/sek}}}$$

Lösung 386



Abstand von der Drehachse: $r = \varrho \cdot \cos\varphi$

Winkelgeschwindigkeit: $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{1\,\text{Tag}}$

$$v = r \cdot \omega = \frac{2\pi \cdot 6370 \cdot \frac{1}{2}}{24 \cdot 3600} = \underline{\underline{0{,}232\ \text{km/sek}}}$$

$$b = r \cdot \omega^2 = \frac{4\pi^2 \cdot 6370 \cdot \frac{1}{2}}{24^2 \cdot 3600^2} = 1{,}69 \cdot 10^{-5}\ \text{km/sek}^2$$

$$b = \underline{\underline{0{,}0169\ \text{m/sek}^2}}$$

Lösung 387

$$v = \omega \cdot r; \quad \omega = \frac{\pi \cdot n}{30}; \quad v = \frac{\pi \cdot n \cdot r}{30}; \quad n = \frac{30 \cdot v}{\pi \cdot r} = \frac{30 \cdot 2}{\pi \cdot 0{,}5} = \underline{\underline{38{,}2\ \text{U/min}}}$$

Lösung 388

Es gilt wegen $v = r \cdot \omega$: $\quad \dfrac{d}{2} \cdot \omega = 50$

$$\left(\frac{d}{2} - 20\right)\omega = 10$$

Durch Subtraktion: $\quad 20\omega = 40; \quad \underline{\underline{\omega = 2\ \ 1/\text{sek}}}$

$$\frac{d}{2} \cdot 2 = 50; \quad \underline{\underline{d = 50\ \text{cm}}}$$

Lösung 389

Nach 10 sek: $\quad b_t = \dfrac{v_t}{t} = \dfrac{100}{10} = \underline{\underline{10\ \text{m/sek}^2}} = \text{const}$

Nach 15 sek: $\quad v_t = b_t \cdot t = 10 \cdot 15 = \underline{\underline{150\ \text{m/sek}}}$

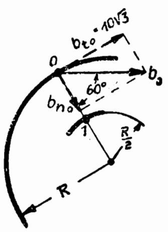
$$b_n = \frac{v_t^2}{R} = \frac{150^2}{2} = \underline{\underline{11\,250\ \text{m/sek}^2}}$$

Lösung 390

$$\frac{v^2}{R} = b = g; \quad v^2 = R \cdot g; \quad v = \sqrt{R \cdot g} = \underline{\underline{7{,}9\ \text{km/sek}}}$$

$$v \cdot T = 2\pi R; \quad T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{R}{g}}; \quad \underline{\underline{T = 1{,}4\,\text{h}}}$$

Lösung 391

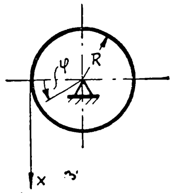


$$b_{n_1} = \frac{v_1^2}{R/2}; \quad b_{n_0} = \frac{v_0^2}{R}; \quad v_1 = \frac{v_0}{2}$$

$$b_{n_0} = \frac{v_0^2}{R} = \frac{4v_1^2}{R} = 2b_{n_1}$$

$$b_{n_0} = \frac{b_{t_0}}{\operatorname{tg} 60°}; \quad b_{n_1} = \frac{b_{t_0}}{2\operatorname{tg} 60°} = \frac{10\sqrt{3}}{2\sqrt{3}} = \underline{\underline{5\,\text{m/sek}^2}}$$

Lösung 392



$$\varphi = \frac{x}{R} = 10\,t^2$$

$$\omega = \dot\varphi = \underline{\underline{20\,t}}\ \ 1/\text{sek}$$

$$\varepsilon = \ddot\varphi = \underline{\underline{20}}\ \ 1/\text{sek}^2$$

$$b_n = \omega^2 \cdot R = 4000\,t^2$$

$$b = \sqrt{(R\cdot\varepsilon)^2 + b_n^2} = \underline{\underline{200\sqrt{1+400\,t^4}}}\ \text{cm/sek}^2$$

Lösung 393

$$b = \sqrt{b_t^2 + b_n^2}\,; \quad b_t = b_0; \quad b_n = \frac{v^2}{R}\,; \quad v = \sqrt{2 b_0 h}$$

$$b = \sqrt{b_0^2 + \frac{4 b_0^2 h^2}{R^2}}\,; \qquad \underline{\underline{b = \frac{b_0}{R}\sqrt{R^2 + 4h^2}}}$$

Lösung 394

$$\varphi = \frac{\pi}{8}\sin\frac{\pi}{2}t; \qquad \dot\varphi = \frac{\pi^2}{16}\cos\frac{\pi}{2}t; \qquad \ddot\varphi = -\frac{\pi^3}{32}\sin\frac{\pi}{2}t$$

1) $b_n = l\cdot\dot\varphi^2 = l\cdot\frac{\pi^4}{256}\cdot\cos^2\frac{\pi}{2}t;\qquad b_n = 0:\quad \cos\frac{\pi}{2}t = 0,\ \text{also}\ \frac{\pi}{2}t = \frac{\pi}{2}$

$$\underline{\underline{t = 1\ \text{sek}}}$$

2) $b_t = \ddot\varphi\cdot l = -l\frac{\pi^3}{32}\sin\frac{\pi}{2}t;\qquad b_t = 0:\quad \sin\frac{\pi}{2}t = 0;\qquad \frac{\pi}{2}t = \pi$

$$\underline{\underline{t = 2\ \text{sek}}}$$

3) $t = \frac{1}{2}\,\text{sek}:\qquad b_n = l\cdot\frac{\pi^4}{256}\cdot\cos^2\frac{\pi}{4} = l\cdot\frac{\pi^4}{512}$

$$b_t = -l\frac{\pi^3}{32}\cdot\sin\frac{\pi}{4} = -l\cdot\frac{\pi^3}{32\cdot\sqrt{2}}$$

$$b = \sqrt{b_n^2 + b_t^2} = \underline{\underline{283}}\ \text{cm/sek}^2$$

14. Übertragung von Elementarbewegungen starrer Körper

Lösung 395

$$v = D_1\cdot\frac{\pi\cdot n_1}{60} = D_2\frac{\pi n_2}{60}; \quad D_2 = D_1\cdot\frac{n_1}{n_2} = 360\cdot\frac{100}{300} = \underline{\underline{120\,\text{mm}}}$$

Lösung 396

$$k = \frac{z_1}{z_2}\cdot\frac{z_3}{z_4} = \frac{10}{60}\cdot\frac{12}{70} = \underline{\underline{\frac{1}{35}}}$$

Lösung 397

$$k = \frac{z_1}{z_2}\cdot\frac{z_3}{z_4}; \quad k = \frac{1}{60}; \quad \frac{1}{60} = \frac{8}{60}\cdot\frac{z_3}{64}; \quad \underline{\underline{z_3 = 8}}$$

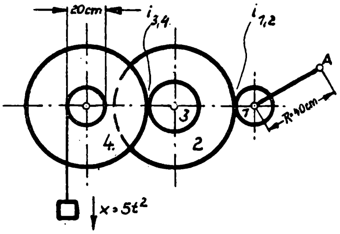
Lösung 398

$$\frac{\omega_B}{\omega_A} = \frac{r_1}{r_2} = \frac{30}{75} = 0{,}4; \quad \omega_B = \varepsilon \cdot t = 0{,}4 \cdot \pi \cdot t; \quad \omega_A = \frac{\omega_B}{0{,}4} = t\pi$$

$$n_0 = 300 \text{ U/min} \mathrel{\hat{=}} 5 \text{ U/sek}; \quad \omega_{A_0} = 2\pi \cdot 5 = 10\pi$$

$$10\pi = t \cdot \pi; \quad \underline{\underline{t = 10 \text{ sek}}}$$

Lösung 399



$$i_{1,2} = \frac{z_2}{z_1} = 3; \quad i_{3,4} = \frac{z_4}{z_3} = 7$$

$$x = 5t^2 \quad \dot{x} = 10t \quad \ddot{x} = 10$$

$$\dot{x}_{(t=2)} = 20 \text{ cm/sek}$$

$$\omega_4 = \frac{\dot{x}}{20/2} = 2 \cdot 1/\text{sek}$$

$$\omega_1 = \omega_4 \cdot i_{1,2} \cdot i_{3,4} = 2 \cdot 3 \cdot 7 = 42 \text{ 1/sek}$$

$$v_A = 40 \cdot 42 = 1680 \text{ cm/sek} \mathrel{\hat{=}} 16{,}80 \text{ m/sek}$$

$$\varepsilon_4 = \frac{\ddot{x}}{20/2} = 1 \text{ 1/sek}^2$$

$$\varepsilon_1 = 1 \cdot 3 \cdot 7 = 21 \text{ 1/sek}^2; \quad b_{t_A} = \varepsilon_1 \cdot 40 = 840 \text{ cm/sek}^2 \mathrel{\hat{=}} 8{,}4 \text{ m/sek}^2$$

$$b_{n_A} = \frac{v^2}{R} = \frac{16{,}8^2}{0{,}4} = 705 \text{ m/sek}^2; \quad b_A = \sqrt{b_{t_A}^2 + b_{n_A}^2} = \underline{\underline{707{,}6}} \text{ m/sek}^2$$

Lösung 400

$$x = a \sin kt; \quad \omega_2 = \frac{1}{r_2} \cdot \dot{x} = \frac{ak}{r_2} \cos kt; \quad \omega_4 = \frac{r_3}{r_4} \cdot \omega_2$$

$$\underline{\underline{\omega_4 = \frac{r_3}{r_2 \cdot r_4} \cdot ak \cos kt}}$$

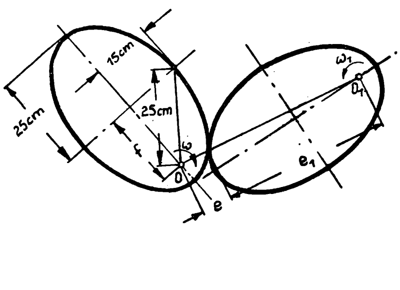
Lösung 401

Die Umfangsgeschwindigkeit von Rad 5 ist gleichzeitig die Hubgeschwindigkeit

$$i_{1,2} = \frac{z_2}{z_1} = 4; \quad i_{3,4} = \frac{z_4}{z_3} = 4; \quad n_5 = \frac{n_1}{i_{1,2} \cdot i_{3,4}} = \frac{30}{16} \text{ U/min}$$

$$v_5 = v_B = \frac{2 r_5 \cdot \pi \cdot n}{60} = \frac{2 \cdot 4 \cdot \pi \cdot 30}{60 \cdot 16} = \underline{\underline{0{,}78 \text{ cm/sek}}}; \quad v_B = 7{,}8 \text{ mm/sek}$$

Lösung 402



$$\omega = \frac{\pi \cdot n}{30} = \frac{\pi \cdot 270}{30} = 9\pi \text{ 1/sek}$$

$$\omega_1 = \frac{e}{e_1} \cdot \omega$$

$$e_1 + e = 50 \text{ cm}; \quad f = \sqrt{25^2 - 15^2} = 20 \text{ cm}$$

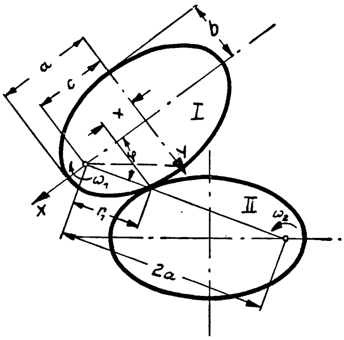
$$e_{\min} = 25 - 20 = 5 \text{ cm}$$

$$e_{\max} = 25 + 20 = 45 \text{ cm}$$

$$\omega_{1\min} = \frac{5}{45} \cdot 9\pi = \underline{\underline{\pi}} \text{ 1/sek}$$

$$\omega_{1\max} = \frac{45}{5} 9\pi = \underline{\underline{81\pi}} \text{ 1/sek}$$

Lösung 403



$$\omega_2 = \omega_1 \cdot \frac{r_1}{2a - r_1}$$

$$x = c - r_1 \cos\varphi \qquad a^2 = c^2 + b^2$$

$$y^2 = r_1^2 - (c - x)^2 = r_1^2 (1 - \cos^2\varphi)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \quad \frac{r_1^2}{b^2}(1 - \cos^2\varphi) = 1 - \frac{(c - r_1\cos\varphi)^2}{a^2}$$

$$r_1^2 a^2 - r_1^2 a^2 \cos^2\varphi$$
$$= a^2 b^2 - c^2 b^2 + 2cb^2 r_1 \cos\varphi - b^2 r_1^2 \cos^2\varphi$$

$$r_1^2 (a^2 - a^2\cos\varphi + b^2\cos^2\varphi) - 2cb^2 r_1 \cos\varphi = b^4$$

$$r_1^2 - \frac{2cb^2 r_1 \cos\varphi}{(a^2 - c^2\cos^2\varphi)} = \frac{b^4}{(a^2 - c^2\cos^2\varphi)}$$

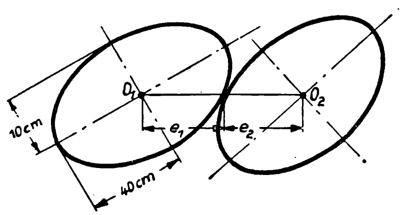
$$\left[r_1 - \frac{cb^2\cos\varphi}{(a^2 - c^2\cos^2\varphi)}\right]^2 = \frac{b^4}{(a^2 - c^2\cos^2\varphi)} + \frac{c^2 b^4 \cos^2\varphi}{(a^2 - c^2\cos^2\varphi)^2}$$

$$r_1 - \frac{cb^2\cos\varphi}{(a^2 - c^2\cos^2\varphi)} = \frac{b^2 a}{(a^2 - c^2\cos^2\varphi)}; \quad r_1 = \frac{b^2 a + cb^2\cos\varphi}{(a^2 - c^2\cos^2\varphi)} = \frac{b^2(a + c\cos\varphi)}{(a + c\cos\varphi)(a - c\cos\varphi)}$$

$$\omega_2 = \omega_1 \frac{b^2}{(a - c\cos\varphi)\left(2a - \dfrac{b^2}{a - c\cos\varphi}\right)} = \omega_1 \frac{b^2}{2a^2 - 2ac\cos\varphi - b^2}$$

$$\underline{\underline{\omega_2 = \omega_1 \frac{a^2 - c^2}{a^2 - 2ac\cos\varphi + c^2}}}$$

Lösung 404



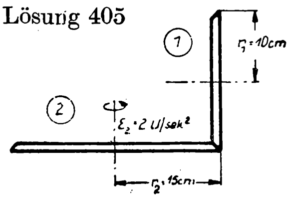
$$n_1 = 240 \text{ U/min}; \quad \omega_1 = \frac{\pi \cdot n}{30} = 8\pi \text{ 1/sek}$$

$$\omega_2 = \frac{e_1}{e_2} \cdot \omega_1; \qquad \begin{aligned} e_1 + e_2 &= 5 \text{ cm} \\ e_{1\,\min} &= 10 \text{ cm} \\ e_{1\,\max} &= 40 \text{ cm} \end{aligned}$$

$$\omega_{2\,\min} = \frac{1}{4}\,\omega_1 = \underline{\underline{2\pi \text{ 1/sek}}}$$

$$\omega_{2\,\max} = 4\,\omega_1 = \underline{\underline{32\pi \text{ 1/sek}}}$$

Lösung 405



$$\omega_1 r_1 = \omega_2 r_2; \quad \omega = \frac{\pi \cdot n}{30}; \quad n_2 = \varepsilon_2 \cdot t \cdot 60$$

$$n_1 \cdot r_1 = n_2 \cdot r_2; \quad n_1 \cdot r_1 = \varepsilon_2 \cdot t \cdot r_2 \cdot 60$$

$$t = \frac{n_1 \cdot r_1}{\varepsilon_2 \cdot r_2 \cdot 60} = \frac{4320 \cdot 10}{2 \cdot 15 \cdot 60} = \underline{\underline{24 \text{ sek.}}}$$

Lösung 406

$$n_{\mathrm{I}} = 600\ \mathrm{U/min}; \quad \omega_{\mathrm{I}} = \frac{\pi n}{30} = 20\pi\ 1/\mathrm{sek}; \quad \omega_{\mathrm{II}} = \frac{r\,\omega_{\mathrm{I}}}{d}$$

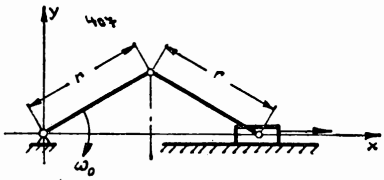
1. $$\varepsilon_{\mathrm{II}} = \frac{d\,\omega_{\mathrm{II}}}{dt} = \frac{r\,\omega_{\mathrm{I}}}{d^2}\cdot\left(-\frac{dd}{dt}\right) = 0{,}5\cdot\frac{r\,\omega_{\mathrm{I}}}{d^2} = \underline{\underline{\frac{50\,\pi}{d^2}\ 1/\mathrm{sek}^2}}$$

2. $$b = \sqrt{b_n^2 + b_t^2} = \sqrt{\omega_{\mathrm{II}}^4 R^2 + R^2\varepsilon_{\mathrm{II}}^2} = R\sqrt{\omega_{\mathrm{II}}^4 + \varepsilon_{\mathrm{II}}^2}$$

Für $d = r$: $\quad \omega_{\mathrm{II}} = \omega_{\mathrm{I}}; \quad \varepsilon_{\mathrm{II}} = 2\pi$

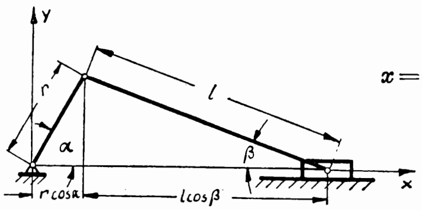
$$\underline{\underline{b = 30\pi\sqrt{40000\,\pi^2 + 1}\ \mathrm{cm/sek}^2}}$$

Lösung 407



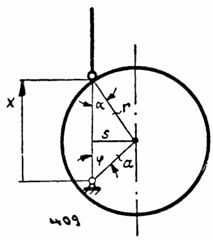
$$x = \underline{\underline{2r\cos\omega_0 t}}$$
$$y = 0$$
$$\dot{x} = v_x = \underline{\underline{-2r\,\omega_0\sin\omega_0 t}}$$
$$\ddot{x} = b_x = -2r\,\omega_0^2\cos\omega_0 t = \underline{\underline{-\omega_0^2\cdot x}}$$

Lösung 408



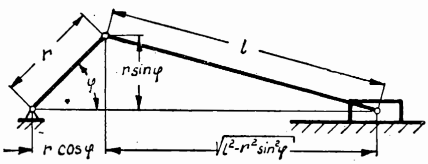
$$x = r\cdot\cos\alpha + l\cos\beta$$
$$\text{da}\ \beta \ll 1: \beta = \frac{r\sin\alpha}{l}; \quad \cos\beta = 1 - \frac{\beta^2}{2}$$
$$x = r\cos\alpha + l\left(1 - \frac{r^2\sin^2\alpha}{2l^2}\right); \quad \sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$
$$x = r\left(\cos\omega_0 t + \frac{\lambda}{4}\cos 2\,\omega_0 t\right) + 1 - \frac{\lambda}{4}\,r$$
$$v_x = \dot{x} = -r\,\omega_0\left(\sin\omega_0 t + \frac{\lambda}{2}\sin 2\,\omega_0 t\right)$$
$$b_x = \ddot{x} = -r\,\omega_0^2\,(\cos\omega_0 t + \lambda\cos 2\,\omega_0 t)$$

Lösung 409



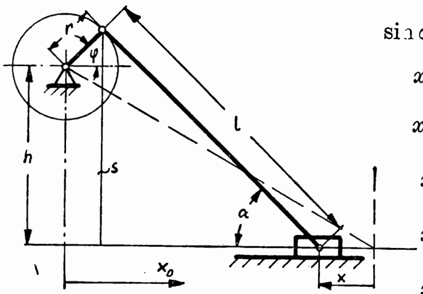
$$x = a\cos\varphi + r\cos\alpha$$
$$\sin\varphi = \frac{s}{a}; \qquad \sin\alpha = \frac{s}{r}$$
$$\sin\alpha = \frac{a}{r}\sin\varphi; \qquad \cos\alpha = \sqrt{1 - \lambda^2\sin^2\varphi}$$
$$\underline{\underline{x = a\cos\varphi + r\sqrt{1 - \lambda^2\sin^2\varphi}}}$$

Lösung 410



$$x = r\cos\varphi + \sqrt{l^2 - r^2\sin^2\varphi}$$
$$x = \underline{\underline{10\,(\cos\varphi + \sqrt{25 - \sin^2\varphi})\ \mathrm{cm}}}$$
$$x_{\max}\ \text{für}\ \varphi = 0: \quad x_{\max} = 60\ \mathrm{cm}$$
$$x_{\min}\ \text{für}\ \varphi = \pi: \quad x_{\min} = 40\ \mathrm{cm}$$
$$s = x_{\max} - x_{\min} = \underline{\underline{20\ \mathrm{cm}}}$$

Lösung 411



$$\sin\alpha = \frac{s}{l}; \quad \sin\varphi = \frac{s-h}{r}$$

$$\sin\alpha = \frac{h}{l} + \frac{r}{l}\sin\varphi$$

$$x_0 = r\cos\varphi + l\cos\alpha$$

$$x_0 = r\cos\varphi + l\sqrt{1-\left(\frac{h}{l}+\frac{r}{l}\sin\varphi\right)^2}$$

$$x = \sqrt{(r+l)^2-h^2} - x_0 \qquad \frac{l}{r} = \lambda$$

$$x = \sqrt{\left(1+\frac{l}{r}\right)^2 - \frac{h^2}{r^2}}\cdot r - x_0; \qquad \frac{h}{r} = k$$

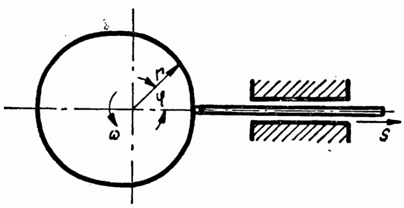
$$\underline{\underline{x = r\left[\sqrt{(1+\lambda)^2-k^2} - \sqrt{\lambda^2-(k+\sin\varphi)^2} - \cos\varphi\right]}}$$

Lösung 412

$$n = 7{,}5\ \text{U/min}; \qquad \omega = \frac{\pi n}{30} = \frac{\pi}{4}\ \text{1/sek}; \qquad \varphi = \omega t; \qquad t = \frac{\varphi}{\omega}$$

$$x = 5t + 30; \qquad \underline{\underline{r = \frac{20}{\pi}\varphi + 30}} \qquad \text{(Archimedische Spirale)}$$

Lösung 413



$$r = \frac{15}{\pi}\cdot\varphi + 25\ \text{cm}$$

$$\dot{s} = \dot{r} = \frac{15}{\pi}\cdot\dot{\varphi}; \qquad \dot{\varphi} = \omega = \frac{\pi\cdot n}{30}$$

$$v = \dot{s} = \frac{15}{\pi}\cdot\frac{\pi\cdot 20}{30} = \underline{\underline{10\ \text{cm/sek}}}$$

Lösung 414

Ansatz im ersten Drittel: $r = c\cdot\varphi + 70$; für $\varphi = \frac{2\pi}{3}$: $r = 90$; $c = \frac{30}{\pi}$

$$\underline{\underline{r = \left(\frac{30}{\pi}\varphi + 70\right)\ \text{cm}}}$$

Zweites Drittel: $\underline{\underline{r = 90\ \text{cm}}}$

Letztes Drittel: $r = 90 - c\varphi$; für $\varphi = \frac{2}{3}\pi$; $r = 70$ cm; $c = \frac{30}{\pi}$

$$\underline{\underline{r = \left(90 - \frac{30}{\pi}\varphi\right)\ \text{cm}}}$$

Lösung 415

$$x = \dot{x}t = 5\cdot 3 = 15\ \text{cm}; \quad x^2 + y^2 = r^2; \quad y^2 = r^2 - x^2$$

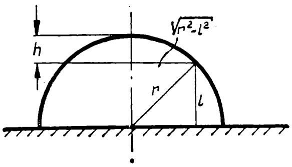
$$y^2 = 900 - 225 = 675$$

$$y = 25{,}98\ \text{cm}$$

Die Höhenabnahme beträgt also:

$$h = r - y; \qquad \underline{\underline{h = 4{,}02\ \text{cm}}}$$

Lösung 416



$$\sqrt{r^2 - l^2} = \sqrt{100 - 36} = 8 \text{ cm}$$

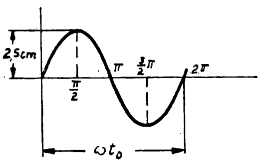
$$s = \frac{b}{2} t^2; \quad s = 8 \text{ cm}; \quad t = 4 \text{ sek}$$

$$b = \frac{2s}{t^2} = \frac{16}{16} = \underline{\underline{1 \text{ cm/sek}^2}}$$

V. Zusammensetzen und Zerlegen von Punktbewegungen

15. Bewegungsgleichung und Bewegungsbahn zusammengesetzter Punktbewegungen

Lösung 417



$$y = a \sin x; \quad y_{\max} = a = 2{,}5 \text{ cm}$$

$$x = \omega t; \quad t_0 = \frac{x_0}{v}; \quad v = 2 \text{ m/sek} \triangleq 200 \text{ cm/sek}$$

$$\omega t_0 = 2\pi; \quad \omega = \frac{2\pi}{t_0} = 50\pi$$

$$\underline{\underline{y = 2{,}5 \sin 50\pi t}}$$

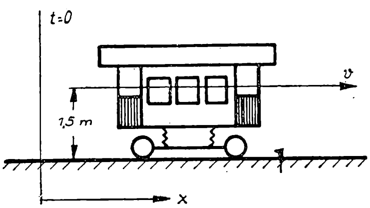
Lösung 418

$\xi;\ \eta$: Plattenkoordinaten
$x;\ y$: Raumkoordinaten

$$\xi = x - u \cdot t; \quad x = 0: \quad \underline{\underline{\xi = -u \cdot t}}$$

$$\eta = y; \quad y = \frac{g t^2}{2}: \quad \underline{\underline{\eta = \frac{g t^2}{2}}}; \quad \underline{\underline{\eta = \frac{g \xi^2}{2 u^2}}} \text{ (Parabel)}$$

Lösung 419



$$y = b + a \sin \omega t; \quad t = 0 \ y = 1{,}5 \text{ m} : b = 1{,}5 \text{ m}$$

$$x = v \cdot t;$$

$$\omega T = 2\pi; \quad \omega = \frac{2\pi}{T} = 4\pi; \qquad v = 18 \text{ km/h}, \quad v = 5 \text{ m/sek}$$

$$y = b + a \sin \frac{\omega x}{v}$$

$$\underline{\underline{y = 1{,}5 + 0{,}008 \sin 0{,}8\pi x}}$$

Lösung 420

$$x = a \sin(\omega t + \alpha) = a (\cos\alpha \sin\omega t + \sin\alpha \cos\omega t)$$

$$y = b \sin(\omega t + \beta) = b (\cos\beta \sin\omega t + \sin\beta \cos\omega t)$$

$$\frac{x}{a}\sin\beta - \frac{y}{b}\sin\alpha = \sin(\beta-\alpha)\sin\omega t$$

$$\frac{x}{a}\cos\beta - \frac{y}{b}\cos\alpha = \sin(\alpha-\beta)\cos\omega t$$

$$\left(\frac{x}{a}\sin\beta - \frac{y}{b}\sin\alpha\right)^2 + \left(\frac{x}{a}\cos\beta - \frac{y}{b}\cos\alpha\right)^2 = \sin^2(\alpha-\beta)$$

$$\underline{\underline{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos(\alpha-\beta) = \sin^2(a-\beta)\ \text{(Ellipse)}}}$$

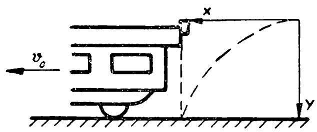
Lösung 421

$x = a\sin 2\omega t;\quad x = a\cdot 2\sin\omega t\cos\omega t;\quad x^2 = a^2 4\sin^2\omega t\cdot\cos^2\omega t$

$y = a\sin\omega t \qquad\qquad y^2 = a^2\sin^2\omega t$

$x^2 = 4y^2\cos^2\omega t;\quad x^2 = 4y^2\left(1-\dfrac{y^2}{a^2}\right);\quad \underline{\underline{a^2x^2 = 4y^2(a^2-y^2)}}$

Lösung 422



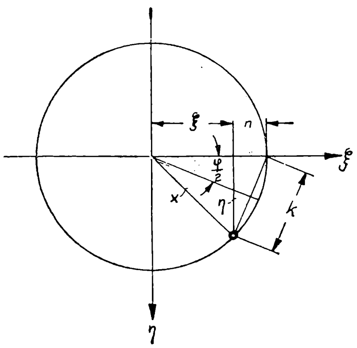
$v_0 = 30\ \text{km/h} \triangleq \dfrac{25}{3}\ \text{m/sek}$

$x = v_0 t;\quad y = \dfrac{g}{2}t^2;\quad y = \dfrac{g}{2v_0^2}\cdot x^2$

$\underline{\underline{y = 0{,}0706\,x^2;}}$

Für $y = h$: $\quad x = s = \sqrt{\dfrac{2v_0^2 h}{g}} = \dfrac{25}{3} = \underline{\underline{8\dfrac{1}{3}}}\ \text{m}$

Lösung 423



Koordinaten der Bewegungsbahn des Punktes auf der Scheibe: ξ; η

$\xi = x - n;\quad \eta = x\cdot\sin\varphi;\quad x = a\sin\omega t$

$\sin\dfrac{\varphi}{2} = \dfrac{k}{2x};\quad \sin\dfrac{\varphi}{2} = \dfrac{n}{k}$

$n = \dfrac{k^2}{2x} = 2x\cdot\sin^2\dfrac{\varphi}{2};\quad \omega\cdot t = \varphi$

$\xi = a\sin\varphi - 2x\sin^2\dfrac{\varphi}{2}$

$\xi = a\sin\varphi\cos\varphi$

$\eta = a\sin^2\varphi$

$\xi = a\sqrt{\dfrac{\eta}{a}}\sqrt{1-\dfrac{\eta}{a}}$

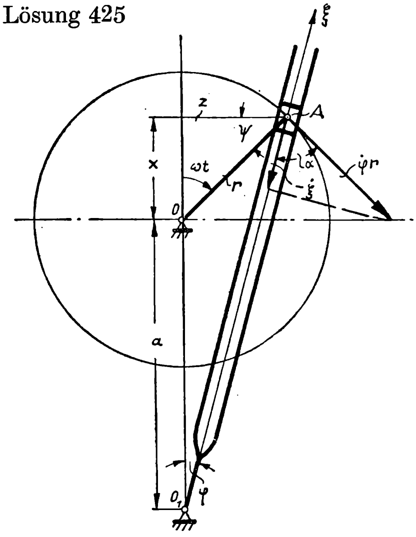
$\xi^2 = a^2\dfrac{\eta}{a}\left(1-\dfrac{\eta}{a}\right);\quad \xi^2 = a\eta - \eta^2$

$$\underline{\underline{\xi^2 + \left(\eta - \frac{a}{2}\right)^2 = \frac{a^2}{4}}}$$

Lösung 424

1) $s = \dfrac{1}{n}(h_1 + h_2) = 0{,}0045\ \text{mm}$; 2) $s = \dfrac{1}{n}(h_1 - h_2) = 0{,}0005\ \text{mm}$

Lösung 425



$$x = r\cos\omega t; \quad z = r\cdot\sin\omega t$$

$$\operatorname{tg}\varphi = \frac{z}{a+x} = \underline{\underline{\frac{r\sin\omega t}{a+r\cos\omega t}}}$$

Bewegung des Gleitsteines

$$\omega\cdot r\cdot\cos\alpha = -\dot{\xi} \qquad \alpha = \varphi+\psi$$

$$\psi = \frac{\pi}{2}-\omega t$$

$$\omega r\cdot\cos(\varphi+\psi) = -\dot{\xi}$$

$$\omega r\cdot\cos\left(\frac{\pi}{2}-(\omega t-\varphi)\right) = -\dot{\xi}$$

$$\omega\cdot r[\sin\omega t\cos\varphi - \cos\omega t\sin\varphi] = -\dot{\xi}$$

für $\sin\varphi$ u. $\cos\varphi$ wird $\operatorname{tg}\varphi$ eingeführt:

$$\dot{\xi} = -\frac{\omega a r\sin\omega t}{\sqrt{a^2+r^2+2ar\cos\omega t}}$$

$$\xi = \int\dot{\xi}\cdot dt; \qquad z = a^2+r^2+2ar\cos\omega t$$

$$\frac{dz}{dt} = -2ar\omega\sin\omega t$$

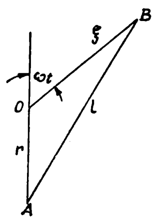
$$\xi = \int\frac{dz}{2\sqrt{z}}$$

$$\xi = \underline{\underline{\sqrt{a^2+r^2+2ar\cos\omega t}}}$$

Lösung 426

1) $x^2+(y+r)^2 = l^2$ (Kreis um A)

2)



Cosinussatz:

$$l^2 = r^2+\xi^2+2r\xi\cos\omega t$$

$$\xi = -r\cos\omega t+\sqrt{(l^2-r^2)+r^2\cos^2\omega t}$$

$$\xi = l\left(\sqrt{1-\lambda^2\sin^2\omega t}-\lambda\cos\omega t\right) \text{ exakt.}$$

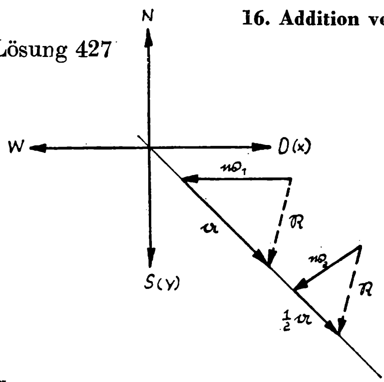
Näherung nach binomischem Lehrsatz:

$$\sqrt{1+z} = 1+\frac{z}{2} \quad \text{für kleine } z$$

$$\xi = \underline{\underline{l\left(1-\frac{\lambda^2}{2}\sin^2\omega t-\lambda\cos\omega t\right)}}$$

16. Addition von Punktgeschwindigkeiten

Lösung 427



Gesucht wird Richtung und Größe von $\mathfrak{R}$

$$-w_{x_1}\mathfrak{i}+a\frac{\sqrt{2}}{2}(\mathfrak{i}+\mathfrak{j}) = \mathfrak{R}$$

$$-w_{x_2}\mathfrak{i}+w_{x_2}\mathfrak{j}+a\frac{\sqrt{2}}{4}(\mathfrak{i}+\mathfrak{j}) = \mathfrak{R}$$

$$\mathfrak{i}\left(-w_{x_1}+\frac{a\sqrt{2}}{2}\right) = \mathfrak{i}\left(-w_{x_2}+\frac{a\sqrt{2}}{4}\right)$$

$$\mathfrak{j}\cdot\frac{a\sqrt{2}}{2} = \left(w_{y_2}+\frac{a\sqrt{2}}{4}\right)\cdot\mathfrak{j}$$

$$w_{y_2} = \frac{a\sqrt{2}}{4}$$

$w_{x_1} - w_{x_2} = \frac{a\sqrt{2}}{4}$ Aus dem Strahlensatz folgt:

$\frac{w_{x_1}}{w_{x_2}} = \frac{a}{a/2}$; $w_{x_1} = 2\,w_{x_2}$

$w_{x_1} = \frac{a\sqrt{2}}{2}$; $w_{x_2} = \frac{a\sqrt{2}}{4}$

Somit: $\mathfrak{R} = a\frac{\sqrt{2}}{2}\,\mathfrak{j}$

Größe: $|\mathfrak{R}| = a\frac{\sqrt{2}}{2}$ Knoten

Richtung: vom Norden

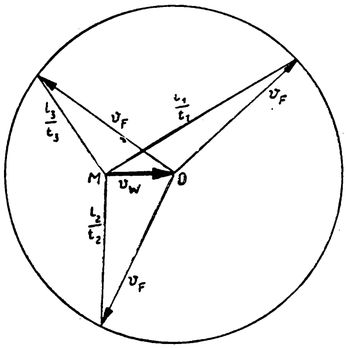
Lösung 428

$$l = (v + V)\,t_1;\quad l = (v - V)\,t_2$$

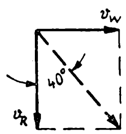
Eigengeschw.: $v = \frac{l}{2}\left(\frac{1}{t_1} + \frac{1}{t_2}\right)$

Windgeschw.: $V = \frac{l}{2}\left(\frac{1}{t_1} - \frac{1}{t_2}\right)$

Lösung 429



Lösung 430

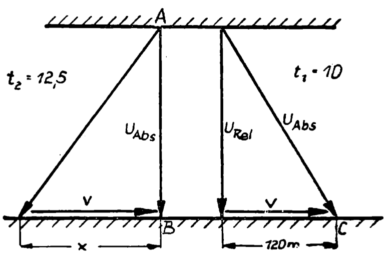


$v_w = 72$ km/h $\triangleq 20$ m/sek

$\operatorname{tg} 40° = \frac{v_w}{v_R}$

$v_R = v_w \cdot \operatorname{ctg} 40° = 23{,}8$ m/sek

Lösung 431



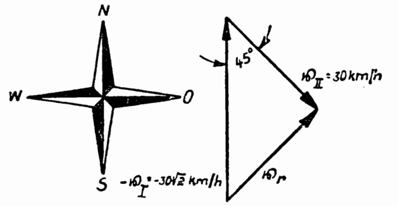
$v = \frac{120}{t_1} = \frac{120}{10} = 12$ m/min

$x = v \cdot t_2 = 150$ m

$u_{\text{rel.}} = \frac{\sqrt{150^2 + l^2}}{t_2} = \frac{l}{t_1}$

$l = 200$ m; $u_{\text{rel.}} = 20$ m/min

Lösung 432



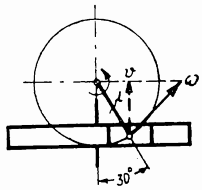
Relativgeschw. = Absolutgeschw. — Bezugsgeschw.

$\mathfrak{v}_r = \mathfrak{v}_{II} - \mathfrak{v}_I$

$|\mathfrak{v}_r| = 30\ \text{km/h};$

Richtung: Nord-Ost.

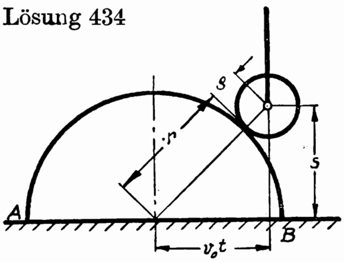
Lösung 433



$$\omega = \frac{\pi \cdot n}{30}; \qquad \omega \cdot l \cdot \sin 30° = v$$

$$v = \frac{\pi \cdot 90 \cdot 0{,}2 \cdot 0{,}5}{30} = \underline{\underline{0{,}942\ \text{m/sek}}}$$

Lösung 434

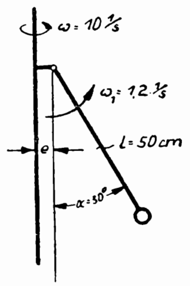


$$s^2 = (r + \varrho)^2 - v_0^2 t^2$$

$$2 s \dot{s} = -2 v_0^2 t$$

$$-\dot{s} = v = \underline{\underline{\frac{v_0^2 t}{\sqrt{(r+\varrho)^2 - v_0^2 t^2}}}}$$

Lösung 435

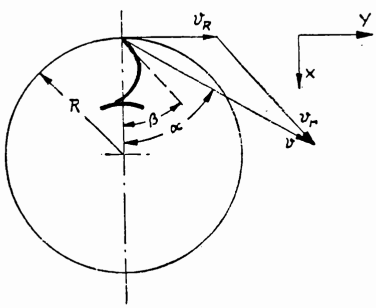


$$v = \sqrt{(\omega_1 \cdot l)^2 + \omega^2 (e + l \sin 30°)^2}$$

$$v = \sqrt{1{,}44 \cdot 0{,}25 + 100\,(0{,}05 + 0{,}25)^2}$$

$$v = 3{,}06\ \text{m/sek} \triangleq \underline{\underline{306\ \text{cm/sek}}}$$

Lösung 436



$$n = 30\ \text{U/min}; \qquad \omega = \frac{\pi \cdot n}{30} = \pi\ 1/\text{sek}$$

$$v_R = R \cdot \omega = 2\pi\ \text{m/sek}$$

$$\mathfrak{v}_r = \mathfrak{v} - \mathfrak{v}_R$$

$$v_{rx} = v_x = v \cos 60° = 7{,}5\ \text{m/sek}$$

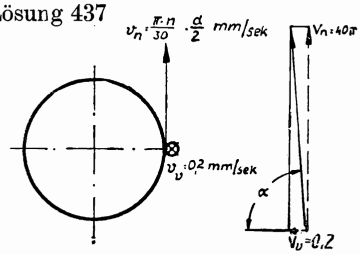
$$v_{ry} = v_y - v_{Ry} = \left(7{,}5\sqrt{3} - 2\pi\right) = 6{,}71\ \text{m/sek}$$

$$v_r = \sqrt{(7{,}5)^2 + (6{,}71)^2} = \underline{\underline{10{,}06\ \text{m/sek}}}$$

$$\operatorname{tg}(v_r, R) = \frac{v_{ry}}{v_{rx}} = 0{,}895;$$

$$\sphericalangle (v_r, R) = \underline{\underline{41° 50'}} = \beta$$

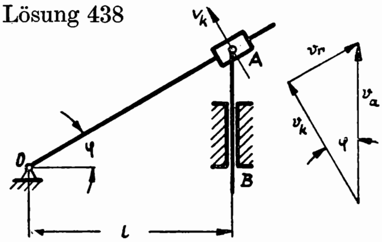
Lösung 437



$$v_r = \sqrt{\left(\frac{\pi \cdot 30 \cdot 80}{30 \cdot 2}\right)^2 + (0,2)^2} = \underline{\underline{125,7 \text{ mm/sek}}}$$

$$\operatorname{tg}\alpha = \frac{v_u}{v_v} = \frac{40\pi}{0,2} = \underline{\underline{628}}$$

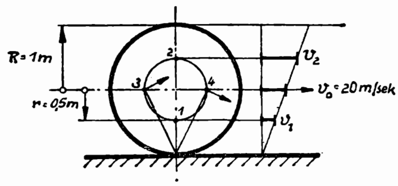
Lösung 438



$$v_k = \frac{l}{\cos\varphi} \cdot \omega$$

$$v_r = v_k \cdot \operatorname{tg}\varphi = \underline{\underline{l \cdot \omega \cdot \frac{\operatorname{tg}\varphi}{\cos\varphi}}}$$

Lösung 439



Da die Stange AB parallel geführt wird, kann M auch in A liegen.

$$\frac{v_2}{1,5} = \frac{v_0}{1} = \frac{v_1}{0,5}$$

$$v_2 = 1,5\, v_0 = \underline{\underline{30 \text{ m/sek}}}$$

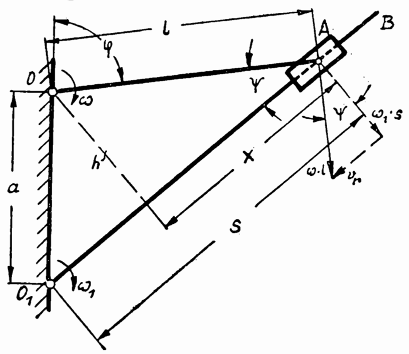
$$v_1 = 0,5\, v_0 = \underline{\underline{10 \text{ m/sek}}}$$

$$v_3 = v_4 = v_0 \cdot \sqrt{1^2 + 0,5^2} = \underline{\underline{22,36 \text{ m/sek}}}$$

Lösung 440

$v_A = d \cdot \omega = \frac{d \cdot v}{r}$ senkrecht zur Verbindungslinie der beiden Radzentren

Lösung 441



1. $\omega \cdot l \cdot \cos\psi = \omega_1 \cdot s$

$$\omega l \cdot \sin\psi = v_r$$

$$\sin\psi = \frac{h}{l}; \quad \cos\psi = \frac{x}{l}$$

$$a^2 = h^2 + (s - x)^2; \quad l^2 = h^2 + x^2$$

$$x = \frac{l^2 + s^2 - a^2}{2s}$$

$$h^2 = a^2 - \left(s + \frac{a^2 - l^2 - s^2}{2s}\right)^2$$

$$\omega_1 = \omega \frac{l}{s} \cdot \frac{x}{l}; \quad v_r = \omega \cdot l \cdot \frac{h}{l}$$

$$\underline{\underline{\omega_1 = \frac{\omega}{2}\left(1 + \frac{l^2 - a^2}{s^2}\right)}}$$

$$v_r = \frac{\omega}{2s}\sqrt{(l+s+a)(l+s-a)(a+l-s)(a+s-l)}$$

2. Bestimmung von $v_{r\max}$: $\frac{dv_r}{ds} = 0$, somit auch: $\frac{d\left(s + \frac{a^2 - l^2 - s^2}{2s}\right)}{ds} = 0$

$$s^2 = \overset{(+)}{-}(a^2 - l^2); \quad v_{r\max} = \omega \cdot h_{\max} = \omega \cdot a$$

$\omega_{1\max}$: Ein exaktes Maximum ist nicht vorhanden, da $\omega_1 = f(s)$ eine Hyperbel darstellt. Die Extremfälle werden deshalb aus der Konstruktion ermittelt.

$$s_{\max} = a + l \rightarrow \omega_{1\min}$$

$$s_{\min} = l - a \rightarrow \omega_{1\max}$$

$$\omega_{1\max} = \frac{\omega}{2}\left(1 + \frac{(l-a)(l+a)}{(l-a)^2}\right) = \omega \cdot \frac{l}{l-a}$$

$$\omega_{1\min} = \omega \cdot \frac{l}{l+a}$$

3. $\omega = \omega_1$ für:

$$1 = \frac{1}{2}\left(1 + \frac{l^2 - a^2}{s^2}\right); \quad s^2 = l^2 - a^2; \quad \text{d. h.: } O_1B \text{ steht senkrecht auf } O_1O$$

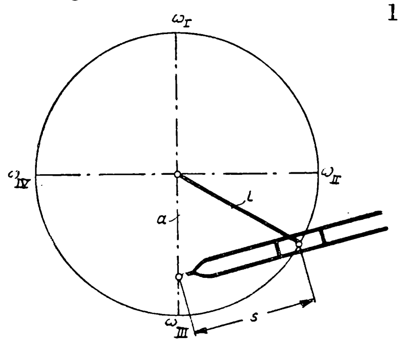
Lösung 442

$$\omega_E = \frac{R}{R_1} \cdot \omega_D = 2 \text{ 1/sek}; \quad \omega_{B_{o,u}} = \frac{O_1A}{BA} \cdot \omega_E$$

$$\omega_{B_o} = \omega_{\mathrm{I}} = \frac{300}{1000} \cdot 2 = 0{,}6 \text{ 1/sek}; \quad \omega_{Bu} = \omega_{\mathrm{III}} = \frac{300}{400} \cdot 2 = 1{,}5 \text{ 1/sek}$$

$$\omega_{Br,l} = \omega_{\mathrm{II}} = \omega_{\mathrm{IV}} = 0$$

Lösung 443



1. Vertikale Kurbellage:

$$s_a = l - a; \quad s_b = l + a$$

$$\omega_{1a} = \frac{\omega}{2}\left(1 + \frac{(l-a)(l+a)}{(l-a)^2}\right) = \omega \cdot \frac{l}{l-a}$$

$$\omega_{1a} = \frac{\pi \cdot n \cdot l}{30\,(l-a)} = \omega_{\mathrm{III}} = 4\pi \text{ 1/sek}$$

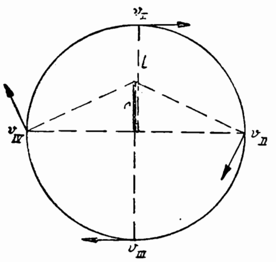
$$\omega_{1b} = \frac{\omega}{2}\left(1 + \frac{(l-a)(l+a)}{(l+a)^2}\right) = \frac{\omega l}{l+a}$$

$$\omega_{1b} = \frac{\pi \cdot l \cdot n}{30\,(l+a)} = \omega_{\mathrm{I}} = \frac{4}{7}\pi \text{ 1/sek}$$

2. Horizontale Kurbellage: $s^2 = l^2 + a^2$

$$\omega_1 = \frac{\omega}{2}\left(1 + \frac{l^2 - a^2}{l^2 + a^2}\right) = \omega \cdot \frac{l^2}{l^2 + a^2} = \frac{\pi n l^2}{30\,(l^2 + a^2)} = \omega_{\mathrm{II}} = \omega_{\mathrm{IV}} = 0{,}64\pi \text{ 1/sek}$$

Lösung 444



$$n = 1200\ \mathrm{U/min}$$

$$\omega = \frac{\pi n}{30} = 40\pi\ 1/\mathrm{sek}$$

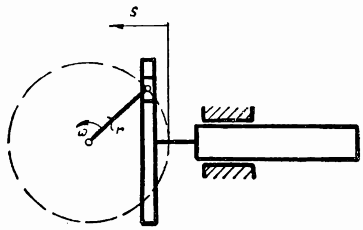
$$v_{\mathrm{I}} = \omega(l - r) = \underline{\underline{20{,}11\ \mathrm{m/sek}}}$$

$$v_{\mathrm{II}} = v_{\mathrm{IV}} = \omega\sqrt{l^2 + r^2} = \underline{\underline{33{,}51\ \mathrm{m/sek}}}$$

$$v_{\mathrm{III}} = \omega\,(l + r) = \underline{\underline{40{,}21\ \mathrm{m/sek}}}$$

17. Addition der Punktbeschleunigungen beim Übertragen vorwärtsschreitender Bewegung

Lösung 445



$$s = r - r\cos\omega t,$$

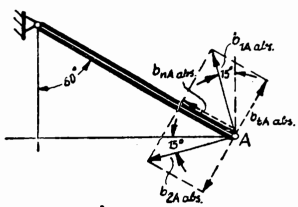
$$\dot{s} = r\,\omega\sin\omega t;$$

$$\ddot{s} = r\,\omega^2\cos\omega t;$$

$$\omega = \frac{\pi\cdot n}{30} = 4\pi\ 1/\mathrm{sek}; \qquad r = 40\ \mathrm{cm}$$

$$\ddot{s} = b = \underline{\underline{6320\cdot\cos 4\pi t\ \mathrm{cm/sek^2}}}$$

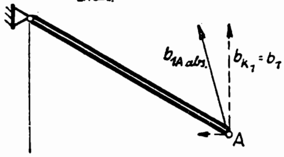
Lösung 446

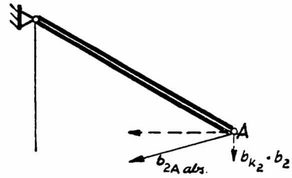


$$b_{n\,A\,\mathrm{abs}} = \omega^2\cdot r = 0{,}5\ \mathrm{m/sek^2}$$

$$b_{t\,A\,\mathrm{abs}} = \varepsilon\cdot r = \pm 0{,}5\ \mathrm{m/sek^2}$$

$$b_{A\,\mathrm{abs}} = 0{,}707\ \mathrm{m/sek^2}$$

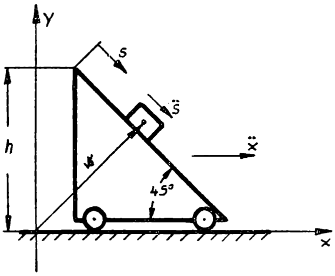




$$b_{k_1} = b_1 = b_{1\,A\,\mathrm{abs}}\cdot\cos 15° = \underline{\underline{0{,}683\ \mathrm{m/sek^2}}}$$

$$b_{k_2} = b_2 = b_{2\,A\,\mathrm{abs}}\cdot\sin 15° = \underline{\underline{0{,}183\ \mathrm{m/sek^2}}}$$

Lösung 447



$\mathfrak{r}$ = Ortsvektor des Körpers P

$$\ddot{\mathfrak{r}} = \mathfrak{i}\left(\ddot{x} + \ddot{s}\frac{\sqrt{2}}{2}\right) - \mathfrak{j}\ddot{s}\frac{\sqrt{2}}{2}; \quad \ddot{x} = \text{const}, \quad \ddot{s} = \text{const}$$

$$\dot{\mathfrak{r}} = \mathfrak{i}\left(\ddot{x} + \ddot{s}\frac{\sqrt{2}}{2}\right)t - \mathfrak{j}\ddot{s}\frac{\sqrt{2}}{2}\cdot t + \dot{\mathfrak{r}}_0$$

für $t = 0$; $\dot{\mathfrak{r}} = 0$: $\dot{\mathfrak{r}}_0 = 0$

$$\mathfrak{r} = \mathfrak{i}\left(\ddot{x} + \ddot{s}\frac{\sqrt{2}}{2}\right)\frac{t^2}{2} - \mathfrak{j}\ddot{s}\frac{\sqrt{2}}{4}t^2 + \mathfrak{r}_0$$

für $t = 0$; $r_x = x = 0$; $\mathfrak{r}_0 = h \cdot \mathfrak{j}$

$$r_y = y = h$$

Somit durch Einsetzen der gegebenen Werte:

$$\mathfrak{r} = \mathfrak{i}\left(1 + \frac{\sqrt{2}\sqrt{2}}{2}\right)\frac{t^2}{2} - \mathfrak{j}\left(\frac{\sqrt{2}\sqrt{2}}{4}t^2 - h\right)$$

$$\underline{\underline{\mathfrak{r} = \mathfrak{i}t^2 - \mathfrak{j}\left(\frac{t^2}{2} - h\right)}}$$

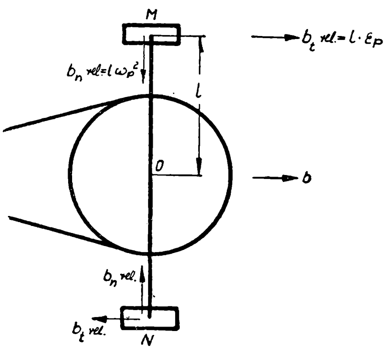
In Komponentendarstellung: $x = t^2$; $y = h - \frac{t^2}{2}$; $\underline{\underline{y = h - \frac{x}{2}}}$

Die absolute Geschwindigkeit beträgt: $\dot{\mathfrak{r}} = 2t\mathfrak{i} - t\mathfrak{j}$

$$|\dot{\mathfrak{r}}| = v = \sqrt{4t^2 + t^2} = \underline{\underline{\sqrt{5}\,t \text{ dm/sek}}}$$

Absolute Beschleunigung: $\ddot{\mathfrak{r}} = 2\mathfrak{i} - \mathfrak{j}$; $|\ddot{\mathfrak{r}}| = b = \sqrt{4+1} = \underline{\underline{\sqrt{5} \text{ dm/sek}^2}}$

Lösung 448



$$s = 0{,}1\,t^2;$$

$$\dot{s} = v = 0{,}2\,t; \quad v_{(t=10)} = 2 \text{ m/sek}$$

$$\ddot{s} = b = 0{,}2 \text{ m/sek}^2$$

$$\varphi_p = \frac{s}{R}\cdot\frac{Z_1}{Z_2}; \quad \omega_p = \frac{v}{R}\cdot\frac{Z_1}{Z_2} = 2{,}14 \text{ 1/sek}$$

$$\varepsilon_p = \frac{b}{R}\cdot\frac{Z_1}{Z_2} = 0{,}214 \text{ 1/sek}^2$$

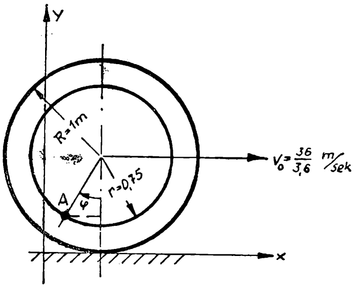
$$b_{n\,\text{rel}} = l\cdot\omega_p^2; \quad b_{t\,\text{rel}} = l\cdot\varepsilon_p$$

$$b_{M,N} = \sqrt{l^2\cdot\omega_p^4 + (b \pm l\cdot\varepsilon_p)^2}$$

$$b_{M,N} = \sqrt{(0{,}824)^2 + (0{,}2 \pm 0{,}039)^2}$$

$$b_M = \underline{\underline{0{,}860 \text{ m/sek}^2}}; \quad b_N = \underline{\underline{0{,}841 \text{ m/sek}^2}}$$

Lösung 449



Da AB parallel geführt wird, kann jeder Punkt auf AB betrachtet werden.

$$x = \varphi \cdot R - r\sin\varphi = R\left(\varphi - \frac{r}{R}\sin\varphi\right)$$

$$y = R - r\cos\varphi = R\left(1 - \frac{r}{R}\cos\varphi\right)$$

$$\omega_0 = \frac{v_0}{R}; \quad \omega_0 \cdot t = \varphi$$

$$x = R\left(\omega_0 t - \frac{r}{R}\sin\omega_0 t\right)$$

$$y = R\left(1 - \frac{r}{R}\cos\omega_0 t\right)$$

$$\dot{x} = R\left(\omega_0 - \frac{r}{R}\,\omega_0\cos\omega_0 t\right)$$

$$\dot{y} = r\,\omega_0\sin\omega_0 t$$

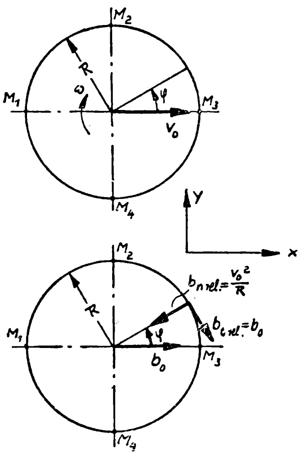
$$\ddot{x} = r\,\omega_0^2\sin\omega_0 t$$

$$\ddot{y} = r\,\omega_0^2\cos\omega_0 t$$

$$b = \sqrt{\ddot{x}^2 + \ddot{y}^2} = r\,\omega_0^2$$

$$b = 0{,}75 \cdot 100 = \underline{\underline{75\ \mathrm{m/sek}^2}}$$

Lösung 450



$$v_x = v_0 + R \cdot \omega\sin\varphi; \qquad \omega = \frac{v_0}{R}$$

$$v_y = -R\,\omega\cos\varphi;$$

$$v(\varphi) = \sqrt{v_x^2 + v_y^2} = v_0\sqrt{2 + 2\sin\varphi}$$

$$v_1 = v(\pi) = v_0\sqrt{2}; \qquad v_2 = v\left(\frac{\pi}{2}\right) = 2\,v_0$$

$$v_3 = v(0) = v_0\sqrt{2}; \qquad v_4 = v\left(\frac{3\pi}{2}\right) = 0$$

$$b_{x_1} = b_0 + \frac{v_0^2}{R}; \quad b_{y_1} = b_0; \quad b_1 = \sqrt{b_0^2 + \left(b_0 + \frac{v_0^2}{R}\right)^2}$$

$$b_{x_2} = 2\,b_0; \quad b_{y_2} = 0; \quad b_2 = 2\,b_0$$

$$b_{x_3} = b_0 - \frac{v_0^2}{R}; \quad b_{y_3} = -b_0; \quad b_3 = \sqrt{b_0^2 + \left(b_0 - \frac{v_0^2}{R}\right)^2}$$

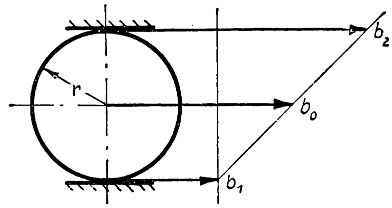
$$b_{x_4} = 0; \quad b_{y_4} = 0; \quad b_4 = 0$$

Lösung 451

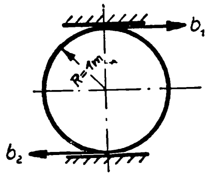
$$\frac{b_2 - b_1}{2r} = \frac{b_0 - b_1}{r}$$

$$b_0 = \frac{b_1 + b_2}{2} = \underline{\underline{2\ \mathrm{m/sek}^2}}$$

$$\varepsilon = \frac{b_0 - b_1}{r} = \underline{\underline{1\ \ 1/\mathrm{sek}^2}}$$



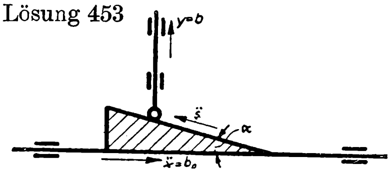
Lösung 452



$$\varepsilon = \frac{b_1 + b_2}{2R} = \underline{\underline{1{,}5\ 1/\text{sek}^2}}$$

$$b_0 = \frac{b_2 - b_1}{2} = \underline{\underline{0{,}5\ \text{m/sek}^2}}$$

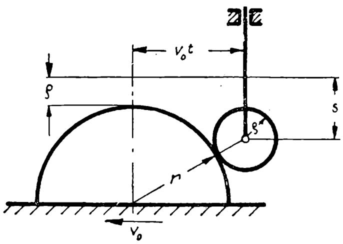
Lösung 453



Relativbeschleunigung: $\ddot{s} = \dfrac{\ddot{x}}{\cos\alpha}$; $\ddot{x} = b_0$

$$\ddot{y} = b = \ddot{s}\cdot\sin\alpha; \quad \underline{\underline{b = b_0\cdot\operatorname{tg}\alpha}}$$

Lösung 454



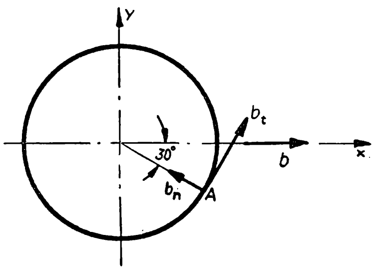
$$s = (r+\varrho) - \sqrt{(r+\varrho)^2 - v_0^2t^2}$$

$$\dot{s} = v = \frac{v_0^2\,t}{\sqrt{(r+\varrho)^2 - v_0^2t^2}}$$

$$\ddot{s} = b = \frac{v_0^2}{\sqrt{(r+\varrho)^2 - v_0^2t^2}} + \frac{v_0^2t\cdot v_0^2t}{\sqrt{(r+\varrho)^2 - v_0^2t^2}^{\,3}}$$

$$b = \underline{\underline{\frac{v_0^2\,(r+\varrho)^2}{\sqrt{(r+\varrho)^2 - v_0^2t^2}^{\,3}}}}$$

Lösung 455



$$\varphi = t^2$$

$$\dot{\varphi} = \omega = 2t; \quad \omega_{(t=1)} = 2\ 1/\text{sek}$$

$$\ddot{\varphi} = \varepsilon = 2\ 1/\text{sek}^2$$

$$b_t = \varepsilon\cdot r = 40\ \text{cm/sek}^2$$

$$b_n = \omega^2 r = 80\ \text{cm/sek}^2$$

$$b_x = b + b_t\cos 60^\circ - b_n\cos 30^\circ = 49{,}2 + 20 - 69{,}2 = 0$$

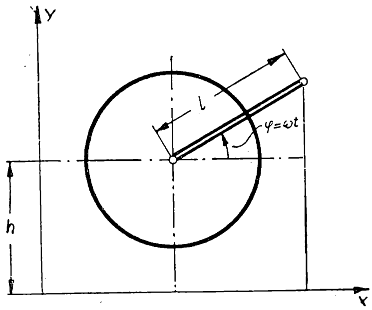
$$b_y = b_t\sin 60^\circ + b_n\sin 30^\circ = 34{,}6 + 40 = 74{,}6$$

$b = \underline{\underline{74{,}6\ \text{cm/sek}^2}}$ senkrecht nach oben gerichtet

Lösung 456

$$b = b_0 - \omega^2\cdot R; \quad b = 0{:}\ \omega = \sqrt{\frac{b_0}{R}} = \sqrt{\frac{49{,}2}{20}} = \underline{\underline{1{,}57\ 1/\text{sek}}}$$

Lösung 457



Gleichungen der Bewegungsbahn:

$x = a \sin \omega t + l \cos \omega t$

$y = h + l \sin \omega t$

Beschleunigungen:

$\ddot{x} = -a\omega^2 \sin \omega t - l\omega^2 \cos \omega$

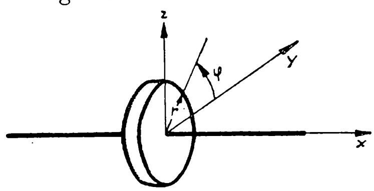
$\ddot{y} = -l\omega^2 \sin \omega t$

für $\omega t = \frac{\pi}{2}$ wird: $\ddot{x} = -a\omega^2$

$\ddot{y} = -l\omega^2$

$b = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \underline{\underline{\omega^2 \sqrt{a^2 + l^2}}}$

Lösung 458



Zylinderkoordinaten:

$b_x = b_0 \quad = 2\,\text{m/sek}^2$

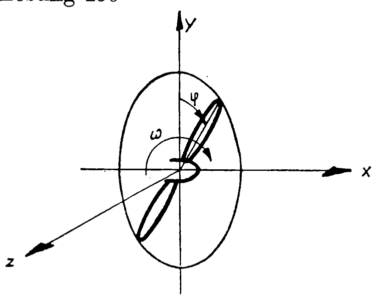
$b_\varphi = R \cdot \varepsilon \quad = 1\,\text{m/sek}^2$

$b_r = R \cdot \omega^2 = 4\,\text{m/sek}^2$

$b = \sqrt{b_x^2 + b_r^2 + b_\varphi^2} = \sqrt{21}$

$b = \underline{\underline{4{,}58\,\text{m/sek}^2}}$

Lösung 459



Bewegungsgleichungen:

$\ddot{x} = b_0; \quad x = b_0 \frac{t^2}{2}; \quad b_0 = 4\,\text{m/sek}^2$

$x = \underline{\underline{2t^2\,\text{m}}}$

$y = r \cos \varphi; \quad \varphi = \omega t; \qquad \omega = \frac{\pi \cdot n}{30}$

$z = r \sin \varphi; \quad r = 0{,}9\,\text{m}; \quad n = 1800\,\text{U/min}$

$y = \underline{\underline{0{,}9 \cos 60\pi t\,\text{m}}}$

$z = \underline{\underline{0{,}9 \sin 60\pi t\,\text{m}}}$

Geschwindigkeit: $\dot{x} = 4t;$ $\qquad \dot{s} = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} = v$

$\dot{y} = -0{,}9 \cdot 60\pi \cdot \sin 60\pi t;$

$\dot{z} = 0{,}9 \cdot 60\pi \cdot \cos 60\pi t; \qquad v = \underline{\underline{\sqrt{16t^2 + 2916\pi^2}\,\text{m/sek}}}$

Beschleunigung: $\ddot{x} = 4;$ $\qquad \ddot{s} = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2} = b$

$\ddot{y} = -0{,}9\,(60\pi)^2 \cos 60\pi t;$

$\ddot{z} = -0{,}9\,(60\pi)^2 \sin 60\pi t; \qquad b = \underline{\underline{31\,945\,\text{m/sek}^2}}$

18. Addition der Punktbeschleunigungen bei radial veränderlicher Drehbewegung um eine starre Achse

Lösung 460

$n = 180\ \text{U/min}; \qquad \omega = \frac{\pi n}{30} = 6\pi\ 1/\text{sek}$

$x = 10 + 5 \sin 8\pi t;$

$\dot{x} = 40\pi \cos 8\pi t; \qquad b_c = 2\dot{x} \cdot \omega$

$\ddot{x} = -320\pi^2 \sin 8\pi t; \qquad b_{c\max}$ entspricht $\dot{x}_{\max}$; $\dot{x}_{\max}$ entspr. $8\pi t = 0$

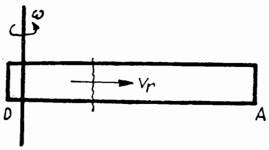
somit $\ddot{x} = 0$

$b_x = \ddot{x} - x\omega^2;$ für $8\pi t = 0$ wird $x = 10;$ $b_x = -360\pi^2\ \text{cm/sek}^2$

$b_{c\max} = 2\dot{x}_{\max} \cdot \omega = \pm 480\pi^2\ \text{cm/sek}^2; \qquad b_a = \sqrt{b_x^2 + b_c^2} = \underline{\underline{600\pi^2\ \text{cm/sek}^2}}$

In der äußersten Lage der Gewichte ist $\dot{x} = 0$, also $b_c = 0$

Lösung 461



$$b_c = 2\omega v_r = \frac{2\pi n}{30} v_r.$$

$$b_c = \underline{\underline{24\ \text{m/sek}^2}}$$

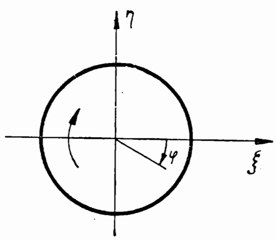
Lösung 462

$\varphi = \sin \pi t; \quad \omega_{\text{rel.}} = \dot{\varphi} = \pi \cos \pi t; \quad \omega_{\text{rel.}\,(t=2\frac{1}{6})} = \pi \cos \frac{1}{6}\pi = \frac{\pi\sqrt{3}}{2}$

$\varepsilon_{\text{rel.}} = \ddot{\varphi} = -\pi^2 \sin \pi t; \quad \varepsilon_{\text{rel.}\,(t=2\frac{1}{6})} = -\pi^2 \sin \frac{1}{6}\pi = -\frac{\pi^2}{2}$

$b_t = R \cdot \varepsilon_{\text{rel.}} = \underline{\underline{-4{,}93\ \text{m/sek}^2}}; \quad b_n = R(\omega + \omega_{\text{rel.}})^2 = \underline{\underline{13{,}84\ \text{m/sek}^2}}$

Lösung 463



$\ddot{\varphi} = \varepsilon = \text{const}$

$\dot{\varphi} = \varepsilon t + c; \qquad t = 0; \quad \dot{\varphi} = 0: \quad c = 0$

$\xi = \sin \pi t \qquad t = 1\frac{2}{3} = \frac{5}{3}\ \text{sek}:$

$\dot{\xi} = \pi \cos \pi t$

$\ddot{\xi} = -\pi^2 \sin \pi t$

$b_\xi = \ddot{\xi} - \xi \cdot \dot{\varphi}^2 = -\pi^2 \sin \pi t - \sin \pi t \cdot \varepsilon^2 t^2$

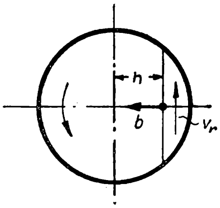
$= \pi^2 \cdot 0{,}866 + \frac{0{,}866 \cdot 25}{9}$

$b_\xi = \underline{\underline{10{,}95\ \text{dm/sek}^2}}$

$b_\eta = -b_\varphi = -2\dot{\xi}\dot{\varphi} - \xi\ddot{\varphi} = -\frac{2\pi}{2} \cdot \frac{5}{3} + 0{,}866$

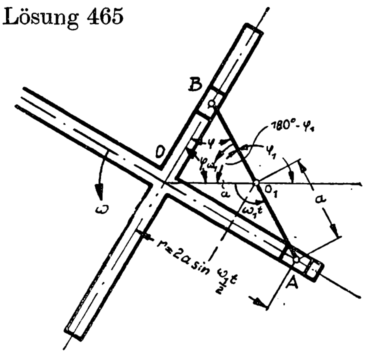
$b_\eta = \underline{\underline{-4{,}37\ \text{dm/sek}^2}}$

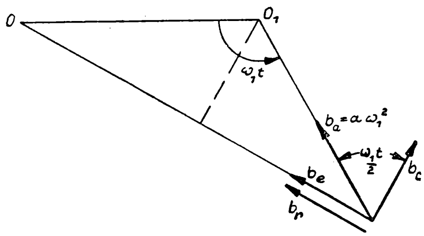
Lösung 464



$$v = v_0 + v_r = \underline{\underline{h\omega + v_r}}$$
$$b = b_n + b_c = \underline{\underline{\omega^2 h + 2\omega v_r}}$$

Lösung 465





Winkelsumme im Dreieck $= 180°$

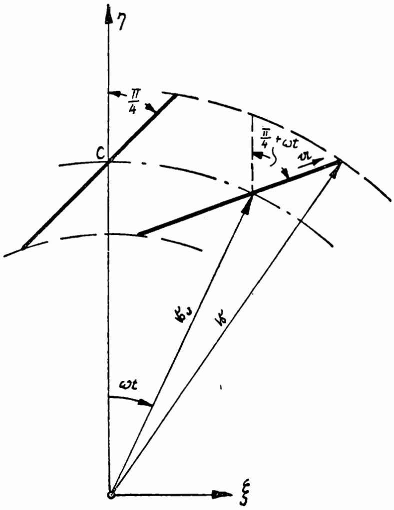
$$2\varphi + 180 - \varphi_1 = 180$$
$$2\varphi - \varphi_1 = 0; \quad \varphi = \frac{\varphi_1}{2}$$
$$\underline{\underline{\omega = \frac{\omega_1}{2}}}$$
$$v_e = r \cdot \omega = \underline{\underline{a\omega_1 \sin\frac{\omega_1 t}{2}}}$$
$$b_e = r \cdot \omega^2 = \underline{\underline{\frac{a\omega_1^2}{2} \sin\frac{\omega_1 t}{2}}}$$
$$r = 2a \sin\frac{\omega_1 t}{2}$$
$$v_r = \dot{r} = \underline{\underline{a\omega_1 \cos\frac{\omega_1 t}{2}}}$$
$$|b_r| = \ddot{r} = \underline{\underline{\frac{a\omega_1^2}{2} \sin\frac{\omega_1 t}{2}}}$$
$$\mathfrak{b}_a = \mathfrak{b}_c + \mathfrak{b}_e + \mathfrak{b}_r$$
$$\mathfrak{b}_c = \mathfrak{b}_a - \mathfrak{b}_e - \mathfrak{b}_r$$
$$b_c = b_a \cdot \cos\frac{\omega_1 t}{2} = \underline{\underline{a\omega_1^2 \cos\frac{\omega_1 t}{2}}}$$

Lösung 466

$$v_{\mathrm{bez}} = \omega \cdot r = 2 \text{ m/sek}; \quad v = v_r - v_{\mathrm{bez}} = 2 \text{ m/sek}; \quad b_n = \frac{v^2}{r} = \underline{\underline{1 \text{ m/sek}^2}}$$
$$b_n = 0 \quad \text{für} \quad v = 0, \quad \text{somit: } v_r = v_b = \underline{\underline{2 \text{ m/sek}}}$$

Lösung 467

$$\mathfrak{r} = \mathfrak{r}_0 + \mathfrak{a}$$
$$\mathfrak{r}_0 = r(\cos\omega t\,\mathfrak{j} + \sin\omega t\,\mathfrak{i})$$
$$\mathfrak{a} = v_r \cdot t\left[\cos\left(\frac{\pi}{4} + \omega t\right)\mathfrak{j} + \sin\left(\frac{\pi}{4} + \omega t\right)\mathfrak{i}\right]$$
$$\mathfrak{r} = \mathfrak{i}\left[v_r t \sin\left(\frac{\pi}{4} + \omega t\right) + v \sin\omega t\right] + \mathfrak{j}\left[v_r t \cos\left(\frac{\pi}{4} + \omega t\right) + r\cos\omega t\right]$$



$$\dot{\mathfrak{r}} = \mathfrak{i}\left[v_r \sin\left(\frac{\pi}{4} + \omega t\right) + v_r t\omega \cos\left(\frac{\pi}{4} + \omega t\right) + r\omega\cos\omega t\right] + \mathfrak{j}\left[v_r \cos\left(\frac{\pi}{4} + \omega t\right) - v_r t\omega \sin\left(\frac{\pi}{4} + \omega t\right) - r\omega\sin\omega t\right]$$

$$\ddot{\mathfrak{r}} = \mathfrak{i}\left[2v_r\omega\cos\left(\frac{\pi}{4} + \omega t\right) - v_r t\omega^2 \sin\left(\frac{\pi}{4} + \omega t\right) - r\omega^2\sin\omega t\right] + \mathfrak{j}\left[-2v_r\omega\sin\left(\frac{\pi}{4} + \omega t\right) - v_r t\omega^2\cos\left(\frac{\pi}{4} + \omega t\right) - r\omega^2\cos\omega t\right]$$

Für Punkt C ist $\mathfrak{a} = 0$, also $t = 0$.

Die Komponenten der Vektoren ergeben sich somit zu:

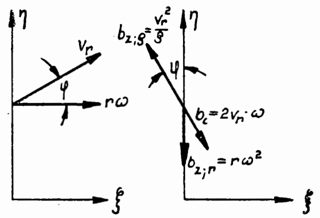
$$v_\xi = v_r \sin\frac{\pi}{4} + r\omega = \underline{\underline{7{,}7 \text{ m/sek}}}$$

$$v_\eta = v_r \cos\frac{\pi}{4} = \underline{\underline{1{,}414 \text{ m/sek}}}$$

$$b_\xi = 2v_r\omega\cos\frac{\pi}{4} = \underline{\underline{35{,}54 \text{ m/sek}^2}}$$

$$b_\eta = -2v_r\omega\sin\frac{\pi}{2} - r\omega^2 = \underline{\underline{-\,114{,}5 \text{ m/sek}^2}}$$

Lösung 468



$$v_\xi = v_r \cdot \cos\varphi + r\cdot\omega$$

$$v_\eta = v_r \cdot \sin\varphi$$

$$b_\xi = \left(2v_r\omega - \frac{v_r^2}{\varrho}\right)\sin\varphi$$

$$b_\eta = -\left[r\omega^2 + \left(2v_r\omega - \frac{v_r^2}{\varrho}\right)\cos\varphi\right]$$

Lösung 469

Nach Aufgabe 425 ist: $\operatorname{tg}\varphi = \dfrac{r\sin\omega t}{a + r\cos\omega t}$; $\varphi = \operatorname{arc\,ctg}\left(\dfrac{a}{r\sin\omega t} + \operatorname{ctg}\omega t\right)$

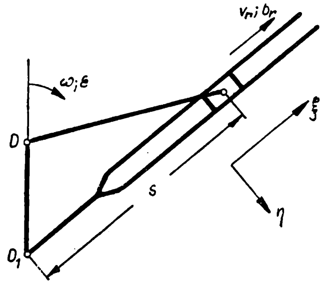
$$\dot{\varphi} = -\frac{1}{1 + \left(\dfrac{a + r\cos\omega t}{r\sin\omega t}\right)^2}\left(-\frac{\omega a\cos\omega t}{r\sin^2\omega t} - \frac{\omega}{\sin^2\omega t}\right)$$

$$\dot{\varphi} = \frac{r\cdot\omega\,(a\cos\omega t + r)}{r^2\sin\omega^2 t + (a + r\cos\omega t)^2} = \frac{r\omega\,(a\cos\omega t + r)}{r^2 + a^2 + 2ar\cos\omega t}$$

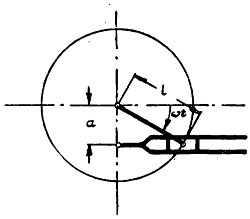
$$\varepsilon_1 = \ddot{\varphi} = \underline{\underline{\frac{(r^2 - a^2)\,ar\omega^2\sin\omega t}{(r^2 + a^2 + 2ar\cos\omega t)^2}}}$$

Lösung 470

$$b_\xi = b_r - b_n = b_r - s\,\omega^2$$
$$b_\eta = b_t + b_c = s \cdot \varepsilon + 2 v_r \omega$$



Lösung 471



Nach Aufgabe 469 gilt:

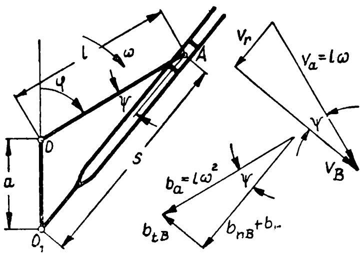
$$\varepsilon = \ddot{\varphi} = \frac{(l^2 - a^2)\, a l \omega^2 \sin \omega t}{(r^2 + a^2 + 2ar \cos \omega t)^2}$$

für $\omega t = 0; \pi \,:\, \ddot{\varphi} = 0$

für $\omega t = \frac{\pi}{2} \quad : \ddot{\varphi} = +\,1{,}21\ \mathrm{1/sek^2}$

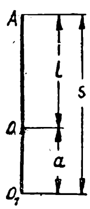
für $\omega t = \frac{3\pi}{2} \quad : \ddot{\varphi} = -\,1{,}21\ \mathrm{1/sek}$

Lösung 472



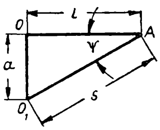
$$v_B = v_a \cdot \cos \psi = l \cdot \omega \cdot \cos \psi$$
$$b_{nB} + b_r = b_a \cdot \cos \psi = l \omega^2 \cos \psi$$
$$b_{nB} = \frac{v_B^2}{s} = \frac{l^2 \omega^2 \cos^2 \psi}{s}$$
$$b_r = l \omega^2 \cos \psi - b_{nB}$$
$$\underline{\underline{b_r = l \omega^2 \cos \psi - \frac{l^2}{s}\, \omega^2 \cos^2 \psi}}$$

1. $\varphi = 0$: $\quad s = a + l; \quad \psi = 0$



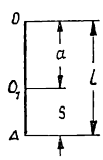
$$b_r = \omega^2 \left(l - \frac{l^2}{a + l}\right)$$
$$b_r = \omega^2 \cdot \frac{a \cdot l}{a + l} = 3^2 \cdot \frac{40 \cdot 30}{40 + 30}$$
$$b_r = \underline{\underline{154{,}3\ \mathrm{cm/sek^2}}}$$

2. $\varphi = 90°$ und $\varphi = 270°$: $\quad s = \sqrt{a^2 + l^2} = \frac{5}{4}\, l$



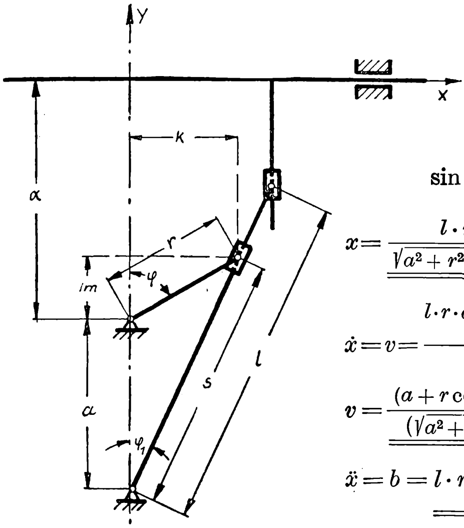
$$\cos \psi = \frac{4}{5}$$
$$b_r = \omega^2 \cdot l \cdot \left[\frac{4}{5} - \frac{4}{5} \cdot \left(\frac{4}{5}\right)^2\right]$$
$$b_r = \frac{36}{125} \cdot \omega^2 \cdot l$$
$$b_r = \underline{\underline{103{,}7\ \mathrm{cm/sek^2}}}$$

3. $\varphi = 180°$: $\quad s = l - a; \quad \psi = 0$



$$b_r = \omega^2 \left(l - \frac{l^2}{l - a}\right)$$
$$b_r = -\,\omega^2 \frac{a l}{l - a}$$
$$b_r = \underline{\underline{-\,1080\ \mathrm{cm/sek^2}}}$$

Lösung 473



$$x = l\sin\varphi_1; \quad s\cdot\sin\varphi_1 = r\sin\varphi$$

$$s^2 = (a+m)^2 + k^2; \quad m = r\cos\varphi$$

$$k = r\sin\varphi$$

$$s^2 = a^2 + r^2 + 2ar\cos\varphi$$

$$\sin\varphi_1 = \frac{r\sin\varphi}{\sqrt{a^2+r^2+2ar\cos\varphi}}; \quad \varphi = \omega t$$

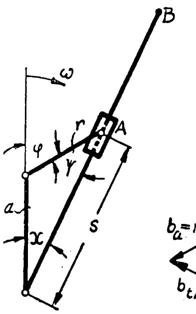
$$\underline{\underline{x = \frac{l\cdot r\cdot\sin\omega t}{\sqrt{a^2+r^2+2ar\cos\omega t}}}} = \frac{l\cdot r\cdot\sin\omega t}{\sqrt{f(t)}}$$

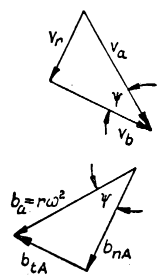
$$\dot{x} = v = \frac{l\cdot r\cdot\omega\cos\omega t\sqrt{f(t)} + l\cdot r\cdot\sin\omega t\cdot\dfrac{1}{2\sqrt{f(t)}}\cdot 2ar\sin\omega t}{\sqrt{f(t)}^{\,2}}$$

$$\underline{\underline{v = \frac{(a+r\cos\omega t)\,(r+a\cos\omega t)}{\left(\sqrt{a^2+r^2+2ar\cos\omega t}\right)^3}\,l\cdot r\cdot\omega}}$$

$$\underline{\underline{\ddot{x} = b = l\cdot r\,\omega^2\cdot\frac{a\,(r^2-a^2)\,(a+r\cos\omega t) - r^2\,(a\cos\omega t + r)^2}{\left(\sqrt{a^2+r^2+2ar\cos t}\right)^5}}}$$

Lösung 474





$$v_b = v_a\cdot\cos\psi = r\cdot\omega\cdot\cos\psi$$

$$b_{nA} = \frac{v_b^2}{s} = \frac{r^2\omega^2\cos^2\psi}{s}$$

$$b_{tA} = b_a\sin\psi = r\,\omega^2\sin\psi$$

$$b_{xA} = -b_{tA}\cos\chi - b_{nA}\sin\chi$$

$$b_{xB} = b_{xA}\cdot\frac{l}{s}$$

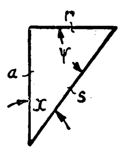
$$b_{xB} = -\,\omega^2\left[\frac{r\cdot l}{s}\sin\psi\cos\chi + \frac{r^2 l}{s^2}\cos^2\psi\sin\chi\right]$$

1. $\varphi = 0$ und $\varphi = 180°$: $\sin\psi = \sin\chi = 0$

$$\underline{\underline{b_{XB} = 0}}$$

2. $\varphi = 90°$ und $\varphi = 270°$: $s = \sqrt{r^2 + a^2}$

$$s = \sqrt{1000}\ \text{cm}$$



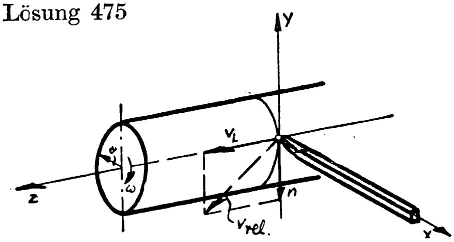
$$\sin\psi = \cos\chi = \frac{a}{s}$$

$$\cos\psi = \sin\chi = \frac{r}{s}$$

$$b_{xB} = -\omega^2 l\left[\frac{r}{s}\cdot\frac{a^2}{s^2} + \frac{r^2}{s^2}\cdot\frac{r^3}{s^3}\right]$$

$$b_{xB} = \underline{\underline{-276{,}2\ \text{cm/sek}^2}}$$

Lösung 475



Der Schnittstahl beschreibt relativ zum Werkstück eine Schraubenlinie.
Schraubengleichung:

$$\mathfrak{r} = a\cos\omega t\,\mathfrak{i} + a\sin\omega t\cdot\mathfrak{j} + \frac{v_l}{\omega}\cdot\omega t\,\mathfrak{k}$$

$$\dot{\mathfrak{r}} = -a\omega\sin\omega t\,\mathfrak{i} + a\omega\cos\omega t\cdot\mathfrak{j} + v_l\mathfrak{k}$$

$$|\dot{\mathfrak{r}}| = v_{\text{rel.}} = \sqrt{v_l^2 + (a\omega)^2} = \underline{\underline{125{,}7\ \text{mm/sek}}}$$

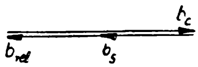
$$\ddot{\mathfrak{r}} = -a\omega^2\cos\omega t\,\mathfrak{i} - a\omega^2\sin\omega t\,\mathfrak{j}$$

$$|\ddot{\mathfrak{r}}| = b_{\text{rel.}} = b_e = a\omega^2 = 40\cdot\frac{\pi^2\cdot 900}{900} = \underline{\underline{394{,}8\ \text{mm/sek}^2}}$$

$$b_c = 2v_{\text{rel.}}\cdot\omega\cdot\sin\beta; \qquad \sin\beta = \frac{u}{v_{\text{rel.}}}$$

$$b_c = 2\cdot\omega\cdot u = 80\pi^2 = \underline{\underline{789{,}5\ \text{mm/sek}^2}}$$

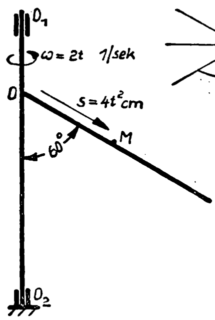
$\mathfrak{b}_{\text{rel.}} = \mathfrak{b}_a - \mathfrak{b}_s - \mathfrak{b}_c$; Da alle drei bekannten Vektoren auf der x-Achse liegen, gilt:

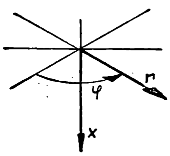


$$b_a = b_{\text{rel.}} + b_s + b_c$$

$$b_s = a\cdot\omega^2; \quad b_a = -394{,}8 - 394{,}8 + 789{,}5 = 0$$

Lösung 476





$$s = 4t^2$$

$$\dot{s} = v = 8t; \quad v_{(t=1)} = 8\ \text{cm/sek}$$

$$\ddot{s} = b = 8\ \text{cm/sek}^2$$

$$b_x = b\cos 60^\circ = \frac{b}{2} = 4\ \text{cm/sek}^2$$

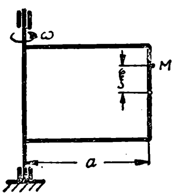
$$b_r = (b - s\omega^2)\sin 60^\circ = -4\sqrt{3}\ \text{cm/sek}^2$$

$$b_\varphi = 2v\omega\sin 60^\circ = 20\sqrt{3}\ \text{cm/sek}^2$$

$$b_M = \sqrt{b_x^2 + b_r^2 + b_\varphi^2} = \sqrt{1264}$$

$$b_M = \underline{\underline{35{,}56\ \text{cm/sek}^2}}$$

Lösung 477



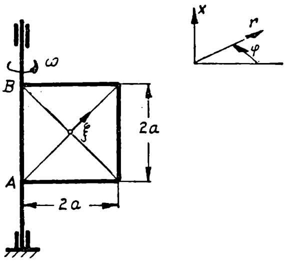
$$\mathfrak{b}_r = \mathfrak{b}_a - \mathfrak{b}_s - \mathfrak{b}_c; \quad b_a = \sqrt{b_r^2 + b_s^2 + b_c^2}$$

$$b_r = \ddot{\xi} = -a\frac{\pi^2}{4}\sin\frac{\pi}{2}t; \quad b_{r(t=1)} = -a\frac{\pi^2}{4}$$

$$b_s = a\cdot\omega^2 = a\frac{\pi^2}{4}; \quad b_c = 0$$

$$\underline{\underline{b_a = \frac{a\pi^2}{4}\sqrt{2}\ \text{cm/sek}^2}}$$

Lösung 478



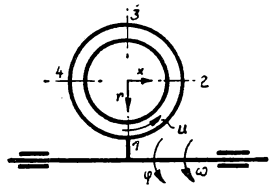
$$b_x = b_\xi \cdot \frac{\sqrt{2}}{2};$$

$$b_r = -\omega^2\left(a + \xi\frac{\sqrt{2}}{2}\right) + b_\xi\frac{\sqrt{2}}{2}$$

$$b_\varphi = 2 v_\xi \cdot \frac{\sqrt{2}}{2} \cdot \omega$$

	$t = 1$ sek	$t = 2$ sek
$\xi = a\cos\frac{\pi}{2}t$	0	$-a$
$v_\xi = \dot{\xi} = -a\frac{\pi}{2}\sin\frac{\pi}{2}t$	$-a\frac{\pi}{2}$	0
$b_\xi = \ddot{\xi} = -a\frac{\pi^2}{4}\cos\frac{\pi}{2}t$	0	$a\frac{\pi^2}{4}$
b_x	0	$a\frac{\pi^2\sqrt{2}}{8} = 0{,}177\, a\pi^2$
b_r	$-2\pi^2 a$	$-2\pi^2 a\left(1-\frac{\sqrt{2}}{2}\right) + a\frac{\pi^2\sqrt{2}}{8} = -0{,}404\, a\pi^2$
b_φ	$-\pi^2 a$	0
$b = \sqrt{b_x^2 + b_r^2 + b_\varphi^2}$	$\underline{\underline{a\pi^2\sqrt{5}\ \mathrm{cm/sek^2}}}$	$\underline{\underline{0{,}44\, a\pi^2\ \mathrm{cm/sek^2}}}$

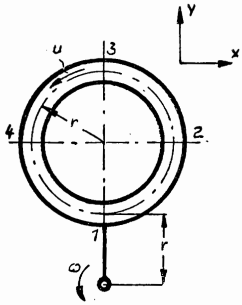
Lösung 479



	Punkt 1	2	3	4
b_x	0	$-\frac{u^2}{r}$	0	$+\frac{u^2}{r}$
b_r	$r\omega^2 - \frac{u^2}{r}$	$2r\omega^2$	$3r\omega^2 + \frac{u^2}{r}$	$2r\omega^2$
b_φ	0	$2u\omega$	0	$-2u\omega$

$$b = \sqrt{b_x^2 + b_r^2 + b_\varphi^2};\quad r\omega^2 - \frac{u^2}{r};\quad \sqrt{4r^2\omega^4 + \frac{u^4}{r^2} + 4\omega^2 u^2};\quad 3r\omega^2 + \frac{u^2}{r};\quad \sqrt{4r^2\omega^4 + \frac{u^4}{r^2} + 4\omega^2 u^2}$$

Lösung 480

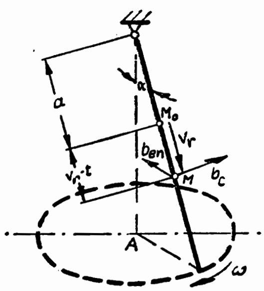


$$\mathfrak{b}_a = \mathfrak{b}_b + \mathfrak{b}_r; \quad b_{ax} = b_{bx} + b_{rx}$$
$$b_{ay} = b_{by} + b_{ry}$$

1)	1	2	3	4
b_{bx}	0	$-r\omega^2$	0	$r\omega^2$
b_{by}	$-r\omega^2$	$-2r\omega^2$	$-3r\omega^2$	$-2r\omega^2$
b_{rx}	0	$-2u\omega-\frac{u^2}{r}$	0	$2u\omega+\frac{u^2}{r}$
b_{ry}	$+\frac{u^2}{r}+2u\omega$	0	$-\frac{u^2}{r}-2u\omega$	0
b_{ax}	0	$-r\omega^2-2u\omega-\frac{u^2}{r}$	0	$r\omega^2+2u\omega+\frac{u^2}{r}$
b_{ay}	$-r\omega^2+\frac{u^2}{r}+2u\omega$	$-2r\omega^2$	$-3r\omega^2+\frac{u^2}{r}+2u\omega$	$-2r\omega^2$
	$\underline{\underline{r\omega^2-\frac{u^2}{r}-2u\omega}}$		$\underline{\underline{3r\omega^2+\frac{u^2}{r}+2u\omega}}$	
$b_a=\sqrt{b_{ax}^2+b_{ay}^2}$		$\underline{\underline{\sqrt{\left(r\omega^2+2u\omega+\frac{u^2}{r}\right)^2+4r^2\omega^4}}}$		$\underline{\underline{\sqrt{\left(r\omega^2+2u\omega+\frac{u^2}{r}\right)^2+4r^2\omega^2}}}$

Für 2) muß $+u$ durch $(-u)$ ersetzt werden.

Lösung 481

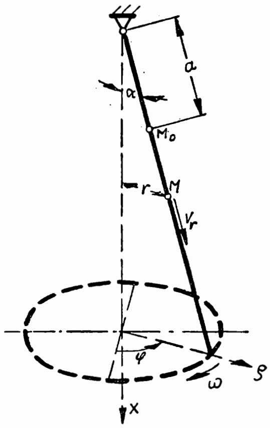


Die Beschleunigung liegt in einer Ebene, auf der die Drehachse senkrecht steht. Die Resultierende wird aus folgenden Komponenten gebildet:

$$b_{en} = \underline{\underline{\omega^2(a+v_r t)\sin\alpha}}$$

$$b_c = \underline{\underline{2 v_r \omega \sin\alpha}}$$

Lösung 482



Für $t = 1$ sek gilt:

$$v_r = b_r \cdot t + v_{r_0} = 10 \text{ cm/sek}$$

$$OM = s = a + \frac{b_r}{2} t^2 = 20 \text{ cm}$$

$$r = s \cdot \sin\alpha = 10 \text{ cm}$$

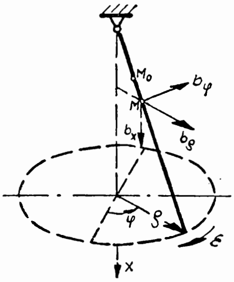
$$b_x = b_r \cdot \cos\alpha = 5\sqrt{3} \text{ cm/sek}^2$$

$$b_\varrho = b_r \cdot \sin\alpha - \omega^2 \cdot r = -5 \text{ cm/sek}^2$$

$$b_\varphi = 2 v_r \omega \sin\alpha = 10 \text{ cm/sek}^2$$

$$b = \sqrt{b_x^2 + b_\varrho^2 + b_\varphi^2} = \underline{\underline{14{,}14 \text{ cm/sek}^2}}$$

Lösung 483



$$\omega = \omega_0 + \varepsilon t; \quad \omega_0 = 0$$

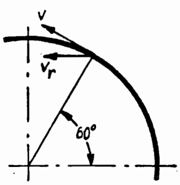
$$b_\varrho = -(a + v_r \cdot t)\, \omega^2 \cdot \sin\alpha$$

$$b_\varphi = 2\omega v_r \cdot \sin\alpha - (a + v_r \cdot t) \cdot \varepsilon$$

$$b_x = 0$$

$$b = \sqrt{b_\varrho^2 + b_\varphi^2} = \sqrt{12^2 + 9^2} = \underline{\underline{15 \text{ cm/sek}^2}}$$

Lösung 484

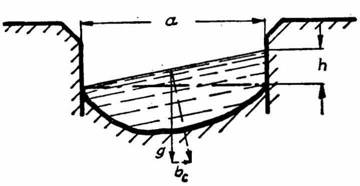


$$5 \text{ km/h} \mathrel{\hat=} \frac{5}{3{,}6} \text{ m/sek} = v$$

$$v_r = v \cdot \sin 60° = \frac{5}{7{,}2} \cdot \sqrt{3} \text{ m/sek}$$

$$\omega = 2\pi \cdot \frac{1}{24 \cdot 3600} \text{ 1/sek}$$

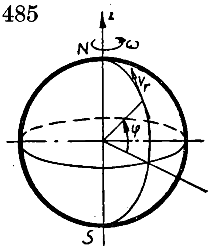
$$b_c = 2 v_r \cdot \omega = \underline{\underline{0{,}0175 \text{ cm/sek}^2}} \text{ nach Osten}$$



$$\frac{h}{a} = \frac{b_c}{g}; \quad h = a \cdot \frac{b_c}{g} = \frac{1750}{981}$$

$h = \underline{\underline{1{,}782 \text{ cm}}}$ Differenz des Wasserstandes zwischen rechtem und linkem Ufer.

Lösung 485



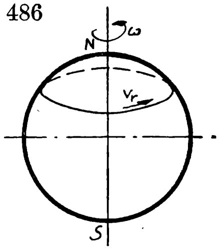
$$b_c = 2 v_r \cdot \omega \cdot \sin\varphi$$

$$\omega = 2\pi \cdot \frac{1}{24 \cdot 3600}\ 1/\text{sek}$$

$$v_r = \frac{90}{3{,}6}\ \text{m/sek} \triangleq \frac{90 \cdot 100}{3{,}6}\ \text{cm/sek}$$

$$b_c = \underline{\underline{0{,}266\ \text{cm/sek}^2}}$$

Lösung 486



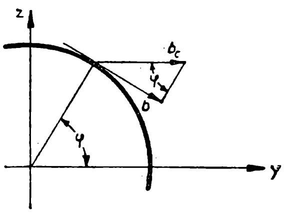
$$b_c = 2\,\omega v_r$$

$$\omega = 2\,\pi \frac{1}{24 \cdot 3600}\ 1/\text{sek}$$

$$b_c = 2 \cdot \frac{2\pi}{24 \cdot 3600} \cdot 20 = 0{,}00291\ \text{m/sek}^2$$

$$b_c = \underline{\underline{0{,}291\ \text{cm/sek}^2}}$$

Lösung 487

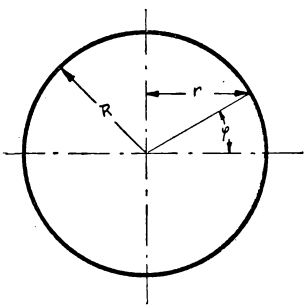


$$b_c = 2\,\omega \cdot v_r$$

$$b_{BC} = b_c \cdot \sin\varphi = 2 \cdot \frac{2\pi}{24 \cdot 3600} \cdot \frac{4}{3{,}6} \cdot \frac{\sqrt{3}}{2}\ \text{m/sek}^2$$

$$b_{BC} = \underline{\underline{1396 \cdot 10^{-5}\ \text{cm/sek}^2}}$$

Lösung 488



$$v_r = 4\ \text{km/h} \triangleq \frac{10}{9}\ \text{m/sek}$$

$$r = R \cdot \cos\varphi = 32 \cdot 10^5\ \text{m}$$

$$b_e = b_{en} = r\,\omega^2 = 32 \cdot 10^5 \cdot \left(\frac{2\pi}{24 \cdot 3600}\right)^2$$

$$= 16{,}92 \cdot 10^{-3}\ \text{m/sek}^2$$

$$b_e = \underline{\underline{1692 \cdot 10^{-3}\ \text{cm/sek}^2}}$$

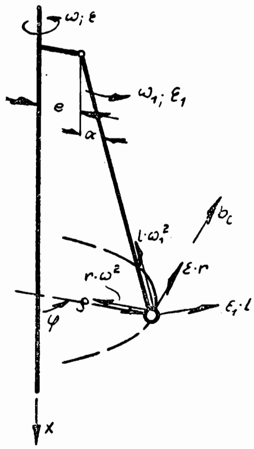
$$b_c = 2 \cdot \omega \cdot v_r = 16{,}16 \cdot 10^{-5}\ \text{m/sek}^2$$

$$b_c = \underline{\underline{1616 \cdot 10^{-5}\ \text{cm/sek}^2}}$$

$$b_r = b_{n\,\text{rel.}} = \frac{v_r^2}{r} = \frac{100}{81 \cdot 32 \cdot 10^5} = 3{,}86 \cdot 10^{-7}\ \text{m/sek}^2$$

$$b_{n\,\text{rel.}} = \underline{\underline{386 \cdot 10^{-7}\ \text{cm/sek}^2}}$$

Lösung 489



$$r = (e + l \sin\alpha)$$

$$b_c = 2 v_r \cdot \omega = 2\omega_1 \cdot l \cdot \frac{\sqrt{2}}{2} \cdot \omega$$

$$b_\varrho = \varepsilon_1 l \cdot \cos\alpha - l\omega_1^2 \sin\alpha - (e + l\sin\alpha)\,\omega^2$$

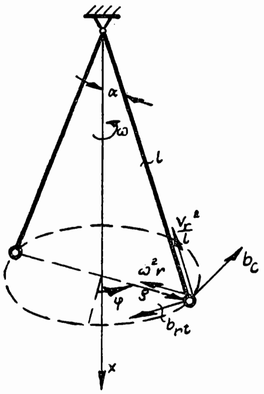
$$b_\varphi = \varepsilon\,(e + l\sin\alpha) + b_c$$

$$b_x = -(\varepsilon_1 \cdot l \cdot \sin\alpha + l\omega_1^2 \cos\alpha)$$

$$b = \sqrt{b_\varrho^2 + b_\varphi^2 + b_x^2} = \sqrt{175^2 + 215{,}9^2 + 101{,}6^2}$$

$$b = \underline{\underline{293{,}7 \text{ cm/sek}^2}}$$

Lösung 490



$$\frac{v_r^2}{l} = 200 \text{ cm/sek}^2$$

$$\omega^2 \cdot r = \omega^2 \cdot l \cdot \sin\alpha = 25\pi^2 \text{ cm/sek}^2$$

$$b_\varphi = b_c = 2\omega \cdot v_r \cdot \cos\alpha = 544{,}15 \text{ cm/sek}^2$$

$$b_\varrho = -\left(\omega^2 l \sin\alpha + b_{rt} \cdot \cos 30^\circ + \frac{v_r^2}{l} \sin 30^\circ\right)$$

$$b_\varrho = -355{,}40 \text{ cm/sek}^2$$

$$b_x = -\frac{v_r^2}{l} \cdot \cos 30^\circ + b_{rt} \sin 30^\circ$$

$$b_x = -168{,}21 \text{ cm/sek}^2$$

$$b = \sqrt{b_\varphi^2 + b_\varrho^2 + b_x^2} = \underline{\underline{671{,}3 \text{ cm/sek}^2}}$$

Lösung 491

$$\varphi = \varphi_0 \sin\omega t; \quad \dot\varphi = \varphi_0 \cdot \omega \cos\omega t$$

$$\ddot\varphi = -\varphi_0 \cdot \omega^2 \cdot \sin\omega t$$

$$\alpha = \omega t$$

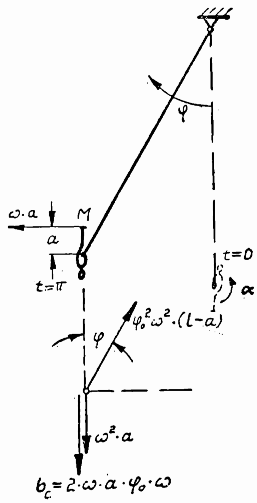
$$t = \frac{\pi}{\omega}: \quad \alpha = \pi; \quad \ddot\varphi = 0; \quad \dot\varphi = -\varphi_0 \cdot \omega$$

Da die Schwingungsgleichung nur für kleine Ausschläge gilt, kann gesetzt werden:

$$\varphi = \operatorname{tg}\varphi = \sin\varphi$$

Somit:

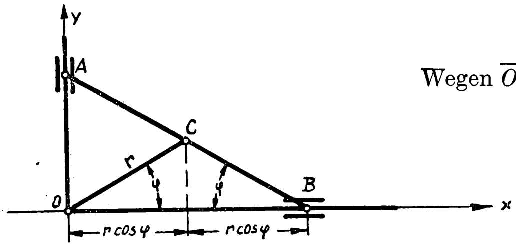
$$b_M = \underline{\underline{\omega^2\,[\varphi_0\,(l - a) - a\,(2\varphi_0 + 1)]}}$$



VI. Ebene Bewegung starrer Körper

19. Bewegungsgleichung einer ebenen Figur und ihrer Punkte

Lösung 492

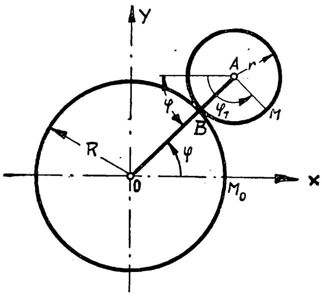


Wegen $\overline{OC} = \overline{CB}$ ist $\sphericalangle COB = \sphericalangle CBO = \varphi$

$$\varphi = \omega_0 t$$

$$x_0 = 2r\cos\varphi = \underline{\underline{2r\cos\omega_0 t}}; \quad \underline{\underline{y_0 = 0}}$$

Lösung 493



$\overline{OA} = R + r$

Für gleichförmig beschleunigte Bewegung gilt: $\varphi = \frac{\varepsilon_0 t^2}{2}$

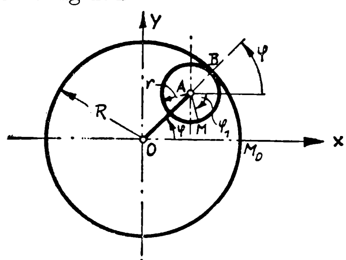
$$\underline{\underline{x_0 = (R + r)\cos\frac{\varepsilon_0 t^2}{2}}}$$

$$\underline{\underline{y_0 = (R + r)\sin\frac{\varepsilon_0 t^2}{2}}}$$

$$\overset{\frown}{M_0 B} = \overset{\frown}{MB}; \quad R\cdot\varphi = r(\varphi_1 - \varphi)$$

$$\varphi_1 = \left(\frac{R}{r} + 1\right)\varphi = \underline{\underline{\left(\frac{R}{r} + 1\right)\cdot\frac{\varepsilon_0 t^2}{2}}}$$

Lösung 494



$$\overline{OB} = R - r; \quad \varphi = \omega_0 t$$

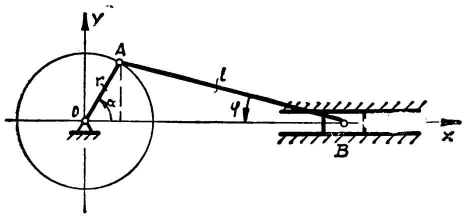
$$x_0 = \underline{\underline{(R - r)\cos\omega_0 t}}$$

$$y_0 = \underline{\underline{(R - r)\sin\omega_0 t}}$$

$$\overset{\frown}{M_0 B} = \overset{\frown}{MB}; \quad R\varphi = r(\varphi - \varphi_1)$$

$$\underline{\underline{\varphi_1 = \psi = -\left(\frac{R}{r} - 1\right)\cdot\omega_0 t}}$$

Lösung 495



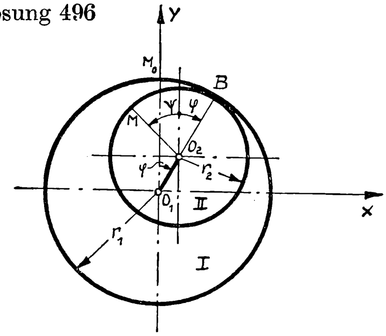
$$\alpha = \omega_0 \cdot t$$

$$x = \underline{\underline{r\cos\omega_0 t}}; \quad y = \underline{\underline{r\sin\omega_0 t}}$$

$$y = r\sin\omega_0 t = l\sin(-\varphi)$$

$$\underline{\underline{\varphi = -\arcsin\left(\frac{r}{l}\sin\omega_0 t\right)}}$$

Lösung 496



$$\varphi = \omega t = \frac{\pi \cdot 270}{30} t = 9\pi t$$

Nach Aufgabe 494 ist:

$$\psi = -\left(\frac{r_1}{r_2} - 1\right)\varphi = -\frac{2}{3}\varphi = -6\pi t$$

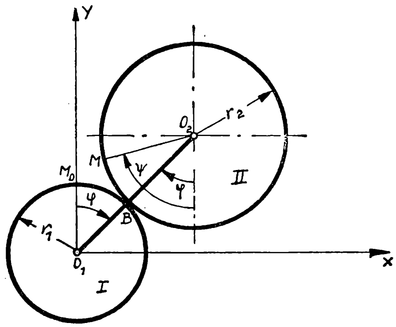
$$x_M = \overline{O_1O_2}\sin\varphi + r_2 \sin\psi$$

$$\underline{\underline{x_M = 8\sin 9\pi t - 12\sin 6\pi t}}$$

$$y_M = \overline{O_1O_2}\cos\varphi + r_2\cos\psi$$

$$\underline{\underline{y_M = 8\cos 9\pi t + 12\cos 6\pi t}}$$

Lösung 497



$$\varphi = \frac{\pi n t}{30} = 8\pi t$$

Nach Aufgabe 493 ist:

$$\psi = \left(\frac{r_1}{r_2} + 1\right)\varphi = 14\pi t$$

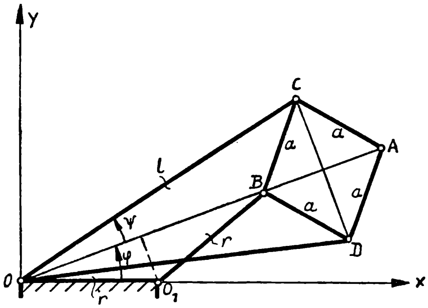
$$x_M = \overline{O_1O_2}\sin\varphi - r_2\sin\psi$$

$$\underline{\underline{x_M = 28\sin 8\pi t - 16\sin 14\pi t}}$$

$$y_M = \overline{O_1O_2}\cos\varphi - r_2\cos\psi$$

$$\underline{\underline{y_M = 28\cos 8\pi t - 16\cos 14\pi t}}$$

Lösung 498



$$x = \overline{OA}\cos\varphi$$

$$\overline{OA} = 2r\cos\varphi + 2\,(l\cos\psi - 2r\cos\varphi)$$

Kosinussatz für das Dreieck BOC:

$$a^2 = l^2 + 4r^2\cos^2\varphi - 4lr\cos\varphi\cos\psi$$

$$\cos\psi = \frac{4r^2\cos^2\varphi + l^2 - a^2}{4rl\cos\varphi}$$

$$\overline{OA} = 2r\cos\varphi + 2\left(-r\cos\varphi + \frac{l^2 - a^2}{4r\cos\varphi}\right)$$

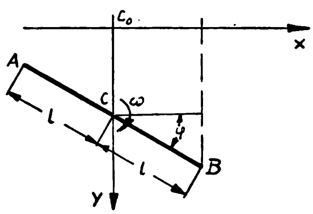
$$\underline{\underline{x = \overline{AO}\cos\varphi = \frac{l^2 - a^2}{2r}}}$$

Lösung 499

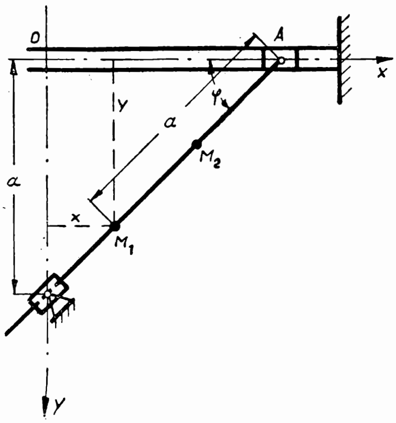
$$y_B = \frac{gt^2}{2} + l\sin\varphi; \quad x_B = l\cos\varphi$$

$$\varphi = \omega t$$

$$\underline{\underline{x_B = l\cos\omega t}}; \quad \underline{\underline{y_B = \frac{gt^2}{2} + l\sin\omega t}}$$



Lösung 500



1) $x = a \operatorname{ctg} \varphi - a \cos \varphi$

$$x = a \cos \varphi \left(\frac{1}{\sin \varphi} - 1 \right)$$

$$y = a \sin \varphi$$

$$\cos \varphi = \sqrt{1 - \sin^2 \varphi} = \sqrt{1 - \frac{y^2}{a^2}}$$

$$\underline{\underline{x^2 y^2 = (a - y)^2 (a^2 - y^2)}}$$

2) $x = a \operatorname{ctg} \varphi - \frac{a}{2} \cos \varphi$

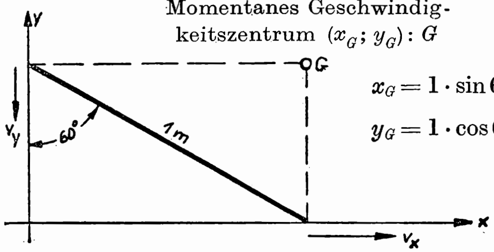
$$x = a \cos \varphi \left(\frac{1}{\sin \varphi} - \frac{1}{2} \right); \quad y = \frac{a}{2} \sin \varphi$$

$$\underline{\underline{4 x^2 y^2 = (a - y)^2 (a^2 - 4 y^2)}}$$

20. Geschwindigkeiten von Körperpunkten bei ebener Bewegung Momentanes Geschwindigkeitszentrum

Lösung 501

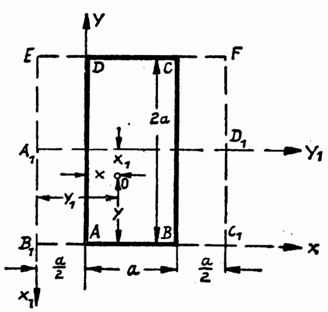
Momentanes Geschwindigkeitszentrum $(x_G; y_G)$: G



$$x_G = 1 \cdot \sin 60° = \underline{\underline{0{,}866 \text{ m}}}$$

$$y_G = 1 \cdot \cos 60° = \underline{\underline{0{,}5 \text{ m}}}$$

Lösung 502



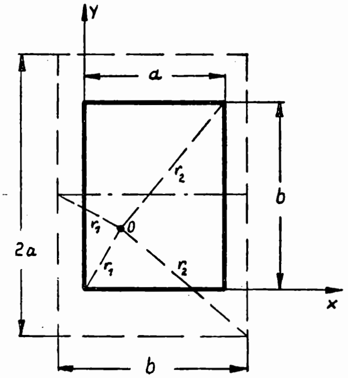
$$y = a - x_1; \quad x_1 = x; \quad y_1 = y$$

$$y_1 = \frac{a}{2} + x;$$

$$x = \frac{a}{4} = \frac{56}{4} = \underline{\underline{14 \text{ cm}}}$$

$$y = 28 + 14 = \underline{\underline{42 \text{ cm}}}$$

Lösung 503



$$r_2^2 = (a-x)^2 + (b-y)^2$$

$$r_2^2 = \left(y + a - \frac{b}{2}\right)^2 + \left(\frac{b+a}{2} - x\right)^2$$

$$x(b-a) + \frac{b^2}{2} - y\,(2a+b) + \frac{ab}{2} - \frac{a^2}{4} = 0 \qquad (1)$$

$$r_1^2 = \left(\frac{b-a}{2} + x\right)^2 + \left(\frac{b}{2} - y\right)^2$$

$$r_1^2 = y^2 + x^2$$

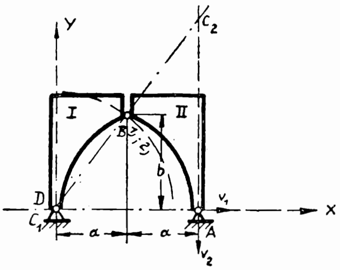
$$x(b-a) + \frac{b^2}{2} - by - \frac{ab}{2} + \frac{a^2}{4} = 0 \qquad (2)$$

Gleichung (1) — Gleichung (2):

$$2ay - ab + \frac{a^2}{2} = 0; \quad \underline{\underline{y = \frac{b}{2} - \frac{a}{4}}}$$

$$\underline{\underline{x = \frac{a}{4}}}$$

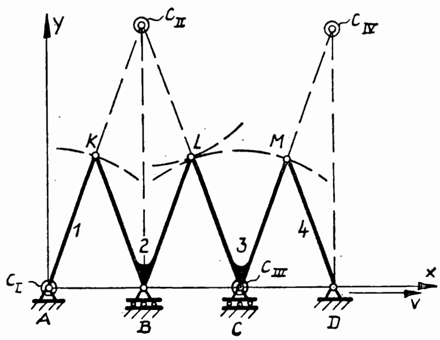
Lösung 504



Das momentane Geschwindigkeitszentrum ist der Schnittpunkt der Bahnnormalen in A und B bzw. D und B.

1. $x_{C_{\mathrm{I}}} = 0$; $y_{C_{\mathrm{I}}} = 0$; $x_{C_{\mathrm{II}}} = 2a$; $y_{C_{\mathrm{II}}} = 2b$
2. $x_{C_{\mathrm{I}}} = 0$; $y_{C_{\mathrm{I}}} = 0$; $x_{C_{\mathrm{II}}} = 0$; $y_{C_{\mathrm{II}}} = 0$

Lösung 505



Teil I: Bewegung nur um C_{I} möglich.

Teil II: B kann sich horizontal verschieben, Bewegung von K ist durch AK bestimmt.

Teil III: C kann sich horizontal verschieben, Bewegung von L durch C_{II} festgelegt.

Teil IV: Bewegung von M durch C_{III} festgelegt, D kann sich horizontal verschieben.

Die Schnittpunkte der Bahnnormalen ergeben die einzelnen Geschwindigkeitspole.

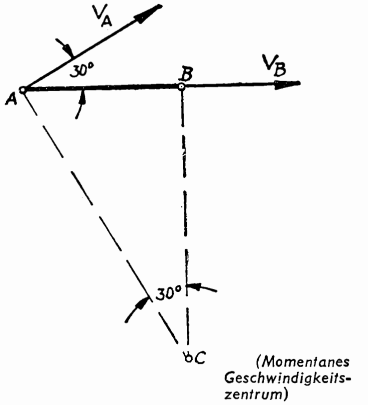
$$x_{C_{\mathrm{I}}} = 0; \quad y_{C_{\mathrm{I}}} = 0$$

$$x_{C_{\mathrm{II}}} = 2a; \quad y_{C_{\mathrm{II}}} = 2b$$

$$x_{C_{\mathrm{III}}} = 4a; \quad y_{C_{\mathrm{III}}} = 0$$

$$x_{C_{\mathrm{IV}}} = 6a; \quad y_{C_{\mathrm{IV}}} = 2b$$

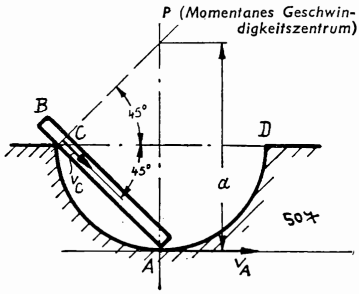
Lösung 506



$$\frac{v_A}{a} = \frac{v_B}{a \cos 30^\circ}; \quad \overline{AC} = a$$

$$v_B = v_A \cdot \cos 30^\circ = \underline{\underline{156 \text{ cm/sek}}}$$

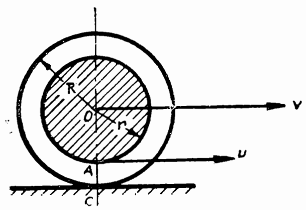
Lösung 507



$$\frac{v_C}{\frac{a\sqrt{2}}{2}} = \frac{v_A}{a}; \quad v_C = v_A \frac{\sqrt{2}}{2}$$

$$v_C = \underline{\underline{2{,}83 \text{ m/sek}}}$$

Lösung 508



Das momentane Geschwindigkeitszentrum lieg
in C, somit:

$$\frac{v}{u} = \frac{R}{R-r}; \qquad \underline{\underline{v = u \cdot \frac{R}{R-r}}}$$

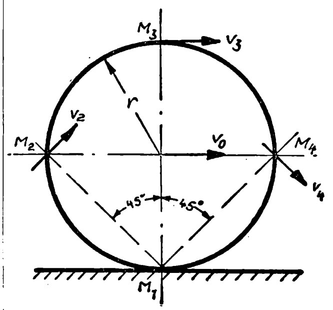
Lösung 509

Die Umfangsgeschwindigkeit des Rades C ist gleich der Geschwindigkeit des Fahrrades

$$v = u = \frac{d \cdot \pi \cdot n}{100} \text{ m/sek}; \quad d = 70 \text{ cm}; \quad n = 1 \cdot \frac{26}{9} \text{ U/sek}$$

$$v = \frac{70 \cdot \pi \cdot 26}{100 \cdot 9} = 6{,}35 \text{ m/sek}; \quad v = 6{,}35 \cdot 3{,}6 = \underline{\underline{22{,}87 \text{ km/h}}}$$

Lösung 510



Das momentane Geschwindigkeitszentrum liegt in M_1

$$\frac{v_0}{r} = \frac{v_2 \cos 45^\circ}{r} = \frac{v_3}{2r} = \frac{v_4 \cdot \cos 45^\circ}{r}$$

$$v_1 = \underline{\underline{0}}$$

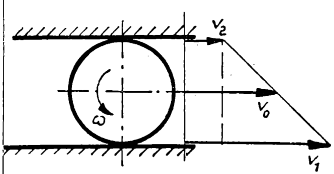
$$v_2 = \frac{v_0}{\cos 45^\circ} = \underline{\underline{14{,}14 \text{ m/sek}}}$$

$$v_3 = 2 v_0 = \underline{\underline{20 \text{ m/sek}}}$$

$$v_4 = \frac{v_0}{\cos 45^\circ} = \underline{\underline{14{,}14 \text{ m/sek}}}$$

$$\omega = \frac{v_0}{r} = \underline{\underline{20 \text{ 1/sek}}}$$

Lösung 511

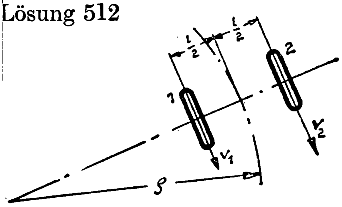


$$\frac{v_0 - v_1}{r} = \frac{v_2 - v_1}{2r}$$

$$v_0 = \frac{v_1 + v_2}{2} = \underline{\underline{4 \text{ m/sek}}}$$

$$\omega = \frac{v_1 - v_2}{2r} = \underline{\underline{4 \text{ 1/sek}}}$$

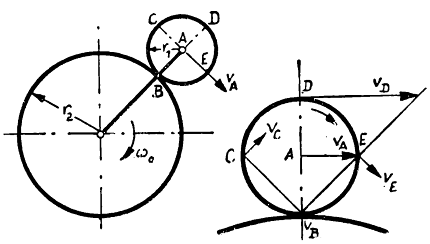
Lösung 512



$$\frac{v_1}{\rho - \frac{l}{2}} = \frac{v_2}{\rho + \frac{l}{2}}$$

$$\rho = \frac{v_2 + v_1}{v_2 - v_1} \cdot \frac{l}{2} = \frac{24 + 18}{24 - 18} \cdot 1 = \underline{\underline{7 \text{ m}}}$$

Lösung 513



$$v_A = \omega_0 (r_1 + r_2) = 2{,}5 \cdot 20 = \underline{\underline{50 \text{ cm/sek}}}$$

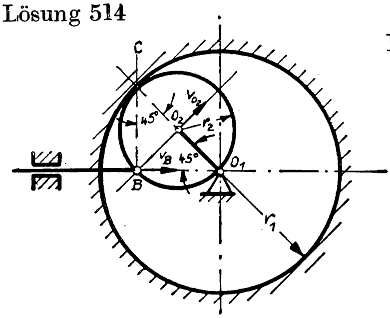
vergl. Aufgabe 510:

$$v_D = 2 v_A = \underline{\underline{100 \text{ cm/sek}}}$$

$$v_B = \underline{\underline{0}}$$

$$v_C = v_E = v_A \sqrt{2} = \underline{\underline{70{,}7 \text{ cm/sek}}}$$

Lösung 514



Das momentane Geschwindigkeitszentrum liegt in C

$$\frac{v_{O_2}}{\sqrt{2}} = \frac{v_B \cdot \cos 45^\circ}{\sqrt{2}}; \qquad v_B = \frac{v_{O_2}}{\cos 45^\circ}$$

$$v_{O_2} = \overline{O_1 O_2}\, \omega = (r_1 - r_2)\frac{\pi n}{30}$$

$$v_B = \frac{(r_1 - r_2)\,\pi n}{30 \cos 45^\circ} = \underline{\underline{239{,}87 \text{ cm/sek}}}$$

Lösung 515

$$x_C = x_A - m \cdot \cos\varphi; \qquad \cos\varphi = \frac{x_A}{l}; \qquad x_A = a \sin\omega t;$$

$$y_C = y_B - n \cdot \sin\varphi; \qquad \sin\varphi = \frac{y_B}{l}; \qquad y_B = \sqrt{l^2 - x_A^2}$$

$$x_C = a \sin\omega t\left(1 - \frac{m}{l}\right); \qquad \dot{x} = a\,\omega \cos\omega t\left(1 - \frac{m}{l}\right);$$

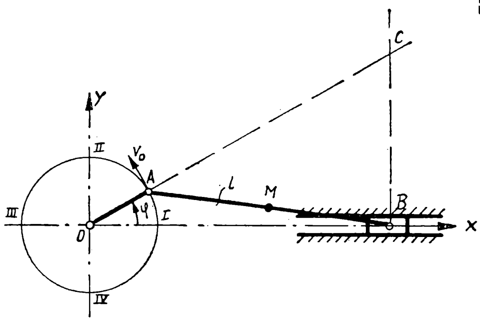
$$y_C = \sqrt{l^2 - a^2 \sin^2\omega t}\left(1 - \frac{n}{l}\right); \qquad \dot{y} = \left(\frac{n}{l} - 1\right)\frac{a^2 \omega \sin\omega t \cos\omega t}{\sqrt{l^2 - a^2\sin^2\omega t}}$$

$$v = \sqrt{\dot{x}_C^2 + \dot{y}_C^2}: \qquad \underline{\underline{v = \frac{a\omega}{l} \cdot \cos\omega t \sqrt{n^2 - m^2 + \frac{m^2 l^2}{l^2 - a^2 \sin^2\omega t}}}}$$

Lösung 516

$$v_0 = r\,\omega_0 = r\,\frac{\pi n}{30} = \underline{\underline{754 \text{ cm/sek}}}$$

C = Momentanes Geschwindigkeitszentrum



Stellung I: C liegt in B

$$\omega = -\frac{v_0}{l} = -\frac{\pi n \cdot r}{30 \cdot l} = \underline{\underline{-\frac{6}{5}\pi \text{ 1/sek}}}$$

$$\frac{v_0}{l} = \frac{2 v_M}{l}; \qquad v_M = \underline{\underline{377 \text{ cm/sek}}}$$

Stellung II: C liegt im Unendlichen

$$\omega = 0; \qquad v_M = v_0 = \underline{\underline{754 \text{ cm/sek}}}$$

Stellung III: C liegt in B

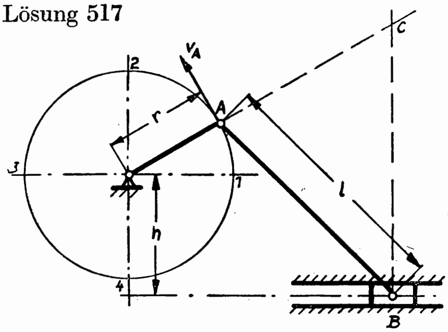
$$\omega = \frac{v_0}{l} = \underline{\underline{\frac{6}{5}\pi \text{ 1/sek}}}$$

$$v_M = \frac{v_0}{2} = \underline{\underline{377 \text{ cm/sek}}}$$

Stellung IV: C liegt im Unendlichen

$$\underline{\underline{\omega = 0}}; \qquad v_M = v_0 = \underline{\underline{754 \text{ cm/sek}}}$$

Lösung 517



$v_A = r \cdot \omega = 40 \cdot 1{,}5 = 60$ cm/sek

Stellung 1 und 3:

$$\frac{v_A}{\sqrt{l^2 - h^2}} = \frac{v_B}{h}$$

$$v_{B_{1;3}} = \frac{v_A \cdot h}{\sqrt{l^2 - h^2}} = \frac{60 \cdot 20}{\sqrt{39600}} = 6{,}03 \text{ cm/sek}$$

Stellung 2 und 4:

C liegt im Unendlichen

$v_{B_{2,4}} = v_A = \underline{\underline{60 \text{ cm/sek}}}$

Lösung 518

Das momentane Geschwindigkeitszentrum für $\varphi = 45°$ liegt in E

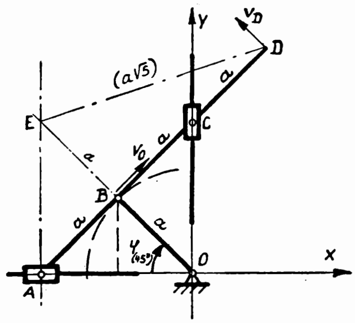
$a = 12$ cm: $\quad v_0 = \omega \cdot a = 24$ cm/sek

$\frac{v_D}{a\sqrt{5}} = \frac{v_0}{a}; \quad v_D = v_0\sqrt{5} = \underline{\underline{53{,}66 \text{ cm/sek}}}$

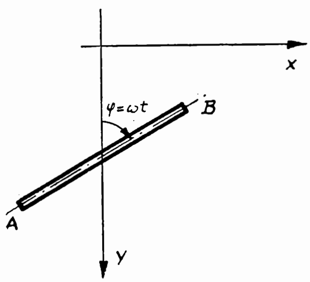
$x = 3a\cos\varphi - 2a\cos\varphi = a\cos\varphi$

$y = 3a\sin\varphi; \quad \cos^2\varphi + \sin^2\varphi = 1$

$\left(\frac{x}{12}\right)^2 + \left(\frac{y}{36}\right)^2 = 1$ (Ellipse)



Lösung 519



$x_A = -l\sin\omega t; \qquad \dot{x}_A = -l\omega\cos\omega t$

$y_A = \frac{1}{2}gt^2 + l\cos\omega t; \qquad \dot{y}_A = gt - l\omega\sin\omega t$

$x_B = l\sin\omega t; \qquad \dot{x}_B = l\omega\cos\omega t$

$y_B = \frac{1}{2}gt^2 - l\cos\omega t; \qquad \dot{y}_B = gt + l\omega\sin\omega t$

$\omega t = \varphi = \frac{\pi}{4}; \quad t = \frac{\pi}{4 \cdot 2{,}75} = 0{,}286$ sek; $\quad l = 33$ cm

$v_A = \sqrt{\dot{x}_A^2 + \dot{y}_A^2} = \sqrt{64{,}4^2 + 215{,}9^2} = \underline{\underline{225{,}3 \text{ cm/sek}}}$

$v_B = \sqrt{\dot{x}_B^2 + \dot{y}_B^2} = \sqrt{64{,}4^2 + 344{,}7^2} = \underline{\underline{350{,}6 \text{ cm/sek}}}$

Lösung 520

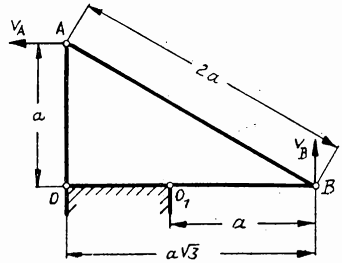
Das momentane Geschwindigkeitszentrum liegt in O

$v_A = \omega \cdot a; \quad v_B = v_A \cdot \sqrt{3} = \omega \cdot a\sqrt{3} = a \cdot \omega_{O_1 B}$

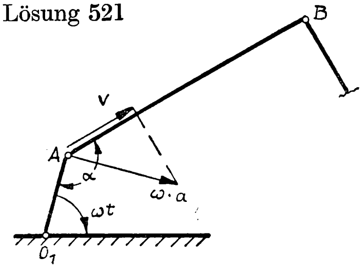
$\omega_{O_1 B} = \omega \cdot \sqrt{3} = \underline{\underline{5{,}2 \text{ 1/sek}}}$

Da A sowohl ein Punkt des Gliedes DA als auch der Stange AB ist und das momentane Geschwindigkeitszentrum in O liegt, ist

$$\omega_{AB} = \omega = \underline{\underline{3 \text{ 1/sek}}}$$



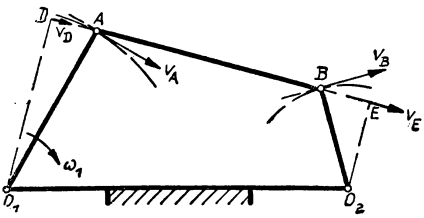
Lösung 521



$$v_M = a \cdot \omega \cdot \cos\left(\alpha - \frac{\pi}{2}\right)$$

$$\underline{\underline{v_M = a \cdot \omega \cdot \sin\alpha}}$$

Lösung 522



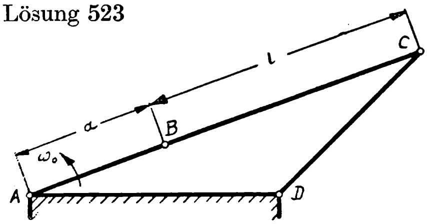
$$\omega_1 = \frac{v_A}{\overline{O_1A}}; \quad \omega_2 = \frac{v_B}{\overline{O_2B}}$$

$$\frac{v_A}{v_D} = \frac{\overline{O_1A}}{\overline{O_1D}}; \quad \frac{v_B}{v_E} = \frac{\overline{O_2B}}{\overline{O_2E}}; \quad v_E = v_D$$

$$\omega_1 = \frac{v_D}{\overline{O_1D}}; \quad \omega_2 = \frac{v_E}{\overline{O_2E}};$$

$$\underline{\underline{\omega_2 = \frac{\overline{O_1D}}{\overline{O_2E}} \cdot \omega_1}}$$

Lösung 523



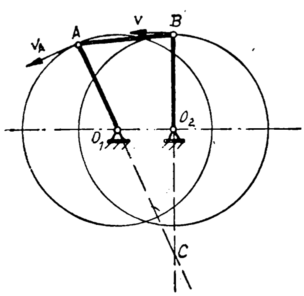
$$v = \omega_0 \cdot a = \omega_{BC} \cdot l$$

Punkt C bleibt in diesem Augenblick in Ruhe

$$\omega_{BC} = \omega_0 \cdot \frac{a}{l} = 6\pi \cdot \frac{1}{3} = \underline{\underline{2\pi \text{ 1/sek}}}$$

$$\underline{\underline{\omega_{CD} = 0}}$$

Lösung 524



Momentanes Geschwindigkeitszentrum von $\overline{AB}$ ist C

Fall 1: C liegt in O_2

$$\frac{v_1}{\overline{O_2B}} = \frac{v_A}{\overline{O_2O_1A}}; \quad v_A = 10 \cdot \frac{60 \cdot \pi}{30} = 62{,}8 \text{ cm/sek}$$

$$v_1 = \frac{10}{14} \cdot 62{,}8 = \underline{\underline{44{,}9 \text{ cm/sek}}}$$

Fall 2: $\overline{CB} = \overline{CA}$

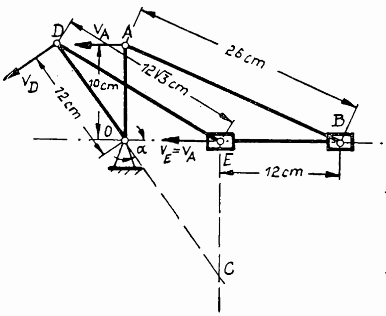
$$v_2 = v_A = \underline{\underline{62{,}8 \text{ cm/sek}}}$$

Fall 3: C liegt in O_1

$$\frac{v_A}{\overline{O_1A}} = \frac{v_3}{\overline{O_1O_2B}}; \quad v_3 = \frac{14}{10} \cdot 62{,}8 = \underline{\underline{88 \text{ cm/sek}}}$$

Lösung 525 Lösung vgl. Aufgabenstellung

Lösung 526 Das momentane Geschwindigkeitszentrum liegt in C



$$\overline{OE} = \sqrt{26^2 - 10^2} - 12 = 12 \text{ cm}$$

$$\cos\alpha = -\frac{\overline{OD}^2 + \overline{OE}^2 - \overline{DE}^2}{2\,\overline{OD}\,\overline{OE}} = -\frac{12^2 + 12^2 - (12\sqrt{3})^2}{2 \cdot 12 \cdot 12} = \frac{1}{2}$$

$$\sin\alpha = \frac{\sqrt{3}}{2}$$

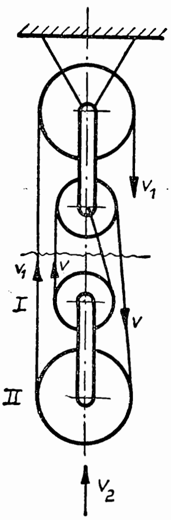
$$\overline{OC} = \frac{OE}{\cos\alpha} = 24 \text{ cm}; \quad \overline{EC} = \overline{OC}\sin\alpha = 12\sqrt{3} \text{ cm}$$

$$\frac{v_D}{\overline{DOC}} = \frac{v_A}{\overline{EC}}; \quad v_D = \frac{\overline{DOC}}{\overline{EC}}\,\overline{OA}\cdot\omega_0 = \frac{360}{\sqrt{3}} \text{ cm/sek}$$

$$\omega_{OD} = \frac{v_D}{\overline{OD}} = \underline{\underline{10\sqrt{3}\ \ 1/\text{sek}}}$$

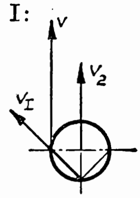
$$\omega_{DE} = \frac{v_D}{\overline{DOC}} = \underline{\underline{\frac{10}{3}\sqrt{3}\ \ 1/\text{sek}}}$$

Lösung 527



$$s_2 = \frac{s_1}{4}; \quad \dot{s}_2 = \frac{\dot{s}_1}{4} = v_2 = \frac{12}{4} = \underline{\underline{3 \text{ cm/sek}}}$$

I:



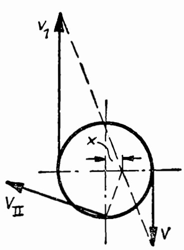
$$v = 2v_2 = 6 \text{ cm/sek}$$

$$\frac{v_{\mathrm{I}}}{r_{\mathrm{I}}\sqrt{2}} = \frac{v_2}{r_{\mathrm{I}}}$$

$$v_{\mathrm{I}} = v_2\sqrt{2} = \underline{\underline{4{,}24 \text{ cm/sek}}}$$

$$\omega_{\mathrm{I}} = \frac{v_2}{r} = \underline{\underline{0{,}5\ \ 1/\text{sek}}}$$

II:



$$\frac{v_1}{r_{\mathrm{II}} + x} = \frac{v}{r_{\mathrm{II}} - x}$$

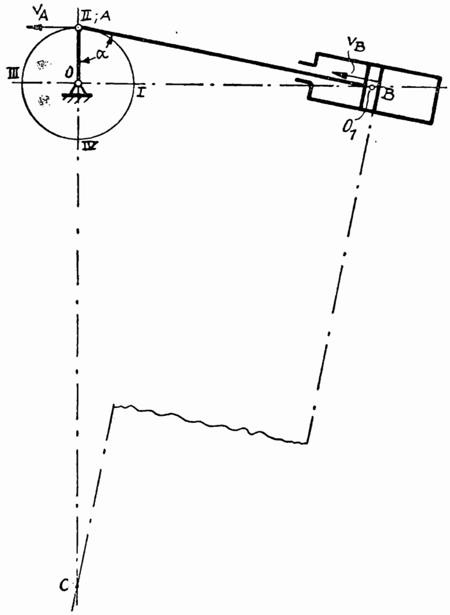
$$x = \frac{r_{\mathrm{II}}}{3}$$

$$\frac{v_{\mathrm{II}}}{\sqrt{r_{\mathrm{II}}^2 + x^2}} = \frac{v_2}{r_{\mathrm{II}}}$$

$$v_{\mathrm{II}} = \underline{\underline{9{,}49 \text{ cm/sek}}}$$

$$\omega_{\mathrm{II}} = \frac{v_2}{x} = \underline{\underline{1\ \ 1/\text{sek}}}$$

Lösung 528



Stellung I und III: Das momentane Geschwindigkeitszentrum liegt in O_1

$$v_A = \overline{OA}\cdot\omega = 12\cdot 5 = 60 \text{ cm/sek}$$

$$\frac{v_A}{\overline{IO_1}} = \frac{v_I}{\overline{O_1B}}; \qquad \overline{IO_1} = 48 \text{ cm} \qquad \overline{O_1B} = 12 \text{ cm}$$

$$v_I = \frac{12}{48}\cdot 60 = \underline{\underline{15 \text{ cm/sek}}}$$

$$\overline{IIIO_1} = 72 \text{ cm}$$

$$\frac{v_A}{\overline{IIIO_1}} = \frac{v_{III}}{\overline{O_1B}}; \qquad v_{III} = \frac{12}{72}\cdot 60 = \underline{\underline{10 \text{ cm/sek}}}$$

Stellung II und IV: Das momentane Geschwindigkeitszentrum liegt in C

$$\frac{\overline{OA}}{\overline{OO_1}} = \frac{\overline{OO_1}}{\overline{OC}}; \qquad \overline{OC} = \frac{\overline{OO_1}^2}{\overline{OA}} = \frac{60^2}{12} = 300 \text{ cm}$$

$$\overline{CB} = \sqrt{\overline{AB}^2 + \overline{IIC}^2 - 2\,\overline{AB}\,\overline{IIC}\cos\alpha}$$

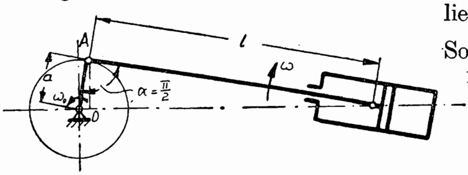
$$\alpha = 78{,}69^\circ$$

$$\overline{CB} = \underline{303{,}9 \text{ cm}}$$

$$\frac{v_A}{\overline{IIC}} = \frac{v_{II}}{\overline{BC}}; \qquad v_{II} = \frac{303{,}9}{312}\cdot 60 = \underline{\underline{58{,}5 \text{ cm/sek}}}$$

$$\underline{\underline{v_{IV} = v_{II}}}$$

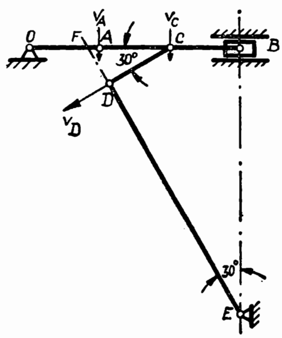
Lösung 529



Das momentane Geschwindigkeitszentrum liegt im Unendlichen.

Somit $\omega = 0$; $v = a\cdot\omega_0 = \underline{\underline{225 \text{ cm/sek}}}$

Lösung 530



$$\frac{v_A}{\overline{AB}} = \frac{v_C}{\overline{CB}}; \qquad \frac{\overline{AB}}{\overline{CB}} = 2; \qquad v_C = \frac{v_A}{2} = \frac{\omega_0\cdot\overline{OA}}{2}$$

$$v_C = \frac{8\cdot 25}{2} = 100 \text{ cm/sek}$$

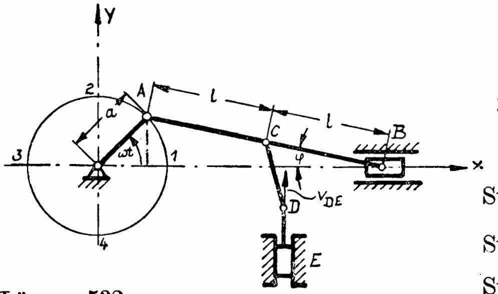
Das momentane Geschwindigkeitszentrum für $\overline{CD}$ liegt in F

$$\frac{v_C}{\overline{FC}} = \frac{v_D}{\overline{FD}}; \qquad \frac{\overline{FD}}{\overline{FC}} = \sin 30^\circ = \frac{1}{2}$$

$$v_D = \frac{v_C}{2} = \overline{ED}\cdot\omega_{ED}$$

$$\omega_{ED} = \frac{100}{2\cdot 100} = \underline{\underline{0{,}5 \text{ 1/sek}}}$$

Lösung 531



$$y_C = y_{DE} = a \sin \omega t - l \sin \varphi$$

$$a \sin \omega t = 2 l \sin \varphi$$

$$\sin \varphi = \frac{a}{2l} \sin \omega t$$

$$\dot{y}_C = v_{DE} = a\omega \cos \omega t - \frac{a\omega}{2} \cos \omega t$$

$$v_{DE} = \frac{a\omega}{2} \cos \omega t$$

Stellung 1: $\omega t = 0$; $v_1 = \frac{a\omega}{2} = \underline{\underline{400\,\text{cm/sek}}}$

Stellung 2: $\omega t = \frac{\pi}{2}$; $v_2 = \underline{\underline{0}}$

Stellung 3: $\omega t = \pi$; $v_3 = \underline{\underline{-400\,\text{cm/sek}}}$

Stellung 4: $\omega t = \frac{3\pi}{2}$; $v_4 = \underline{\underline{0}}$

Lösung 532

Das momentane Geschwindigkeitszentrum von $\overline{AB}$ liegt in G

$$\overline{GA} = \frac{\sqrt{40^2 - 5^2}}{\cos 30^\circ} = 45{,}8\ \text{cm}$$

$$\overline{GB} = \sqrt{40^2 - 5^2} \cdot \operatorname{tg} 30^\circ + 5 = 27{,}9\ \text{cm}$$

$$v_B = \frac{\overline{GB}}{\overline{GA}} \cdot v_A$$

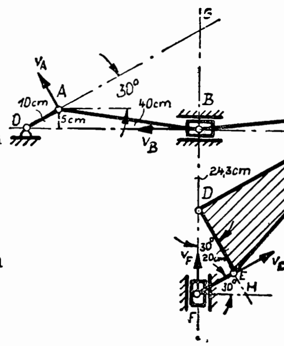
Das momentane Geschwindigkeitszentrum von BDE liegt in D

$$v_E = \frac{\overline{DE}}{\overline{DB}} \cdot v_B$$

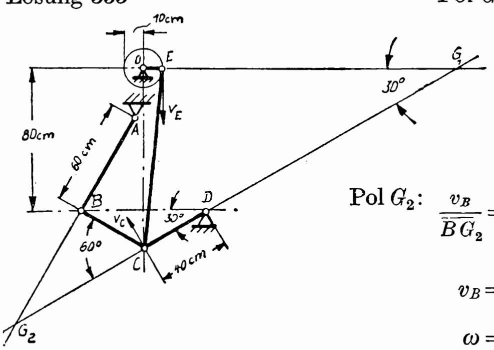
Das momentane Geschwindigkeitszentrum von EF liegt in H

$$v_F = \frac{\overline{FH}}{\overline{EH}} v_E = \frac{v_E}{\sin 30^\circ} = \frac{\overline{GB}\,\overline{DE}}{\overline{GA}\,\overline{DB} \sin 30^\circ} v_A$$

$$v_F = \frac{27{,}9 \cdot 20 \cdot 40 \cdot 2}{45{,}8 \cdot 24{,}3} = \underline{\underline{39{,}94\ \text{cm/sek}}}$$



Lösung 533



$$v_E = \frac{\overline{OE} \cdot 2 \cdot \pi \cdot n}{60} = \frac{20 \cdot \pi \cdot 100}{60}$$

Pol G_1: $\frac{v_E}{\overline{EG_1}} = \frac{v_C}{\overline{CG_1}}$;

$$\overline{CG_1} = \frac{80 + 40 \cdot \sin 30^\circ}{\sin 30^\circ} = 200\ \text{cm}$$

$$\overline{EG_1} = \overline{CG_1} \cos 30^\circ - \overline{OE} = 173{,}2 - 10 = 163{,}2\ \text{cm}$$

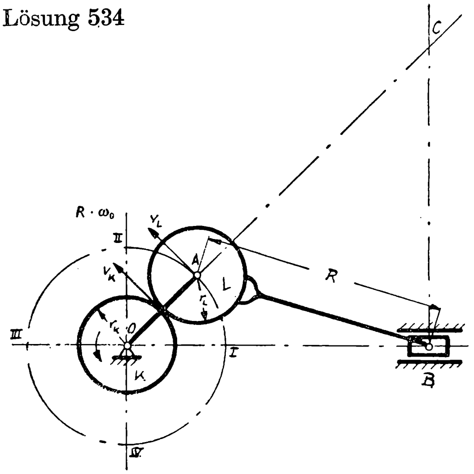
Pol G_2: $\frac{v_B}{\overline{BG_2}} = \frac{v_C}{\overline{CG_2}}$; $\overline{CG_2} = \frac{\overline{BC}}{\cos 60^\circ} = 80\ \text{cm}$

$$\overline{BG_2} = \overline{CG_2} \sin 60^\circ = 69{,}4\ \text{cm}$$

$$v_B = \frac{v_E \cdot \overline{CG_1} \cdot \overline{BG_2}}{\overline{EG_1} \cdot \overline{CG_2}} = \omega \cdot \overline{BA}$$

$$\omega = \underline{\underline{1{,}852\ 1/\text{sek}}}$$

Lösung 534



$$v_K = \frac{\pi n}{30} \cdot r_K = 20\pi \text{ cm/sek}$$

Für Stellung I und III liegt das momentane Geschwindigkeitszentrum C in B

I. $\frac{v_K}{R + r_L} = \frac{r_L}{R}$; $\quad \omega_1 = \frac{v_A}{r_K + r_L}$

$$\omega_1 = \frac{R}{(R + r_L)(r_K + r_L)} \cdot v_K = \underline{\underline{\frac{10}{11}\pi \text{1/sek}}}$$

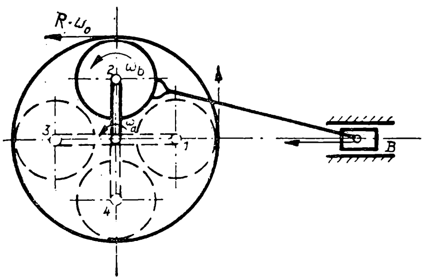
III. $\frac{v_K}{R - r_L} = \frac{v_A}{R}$; $\omega_2 = \frac{R}{(R - r_L)(r_K + r_L)} v_K$

$$\omega_2 = \underline{\underline{\frac{10}{9}\pi \text{ 1/sek}}}$$

Für Stellung II und IV liegt das momentane Geschwindigkeitszentrum im Unendlichen: $v_K = v_A$

$$\omega_2 = \omega_4 = \frac{v_K}{r_K + r_L} = \underline{\underline{\pi \text{ 1/sek}}}$$

Lösung 535



Stellung 2 und 4: $R \cdot \omega_0 = (R - r)\,\omega_a + r\,\omega_b$. Das momentane Geschwindigkeitszentrum liegt im Unendlichen, also $\omega_b = 0$

$$\omega_{a_{2,4}} = \omega_0 \frac{R}{R - r} = 10\frac{25}{15} = \underline{\underline{16{,}67 \text{ 1/sek}}}$$

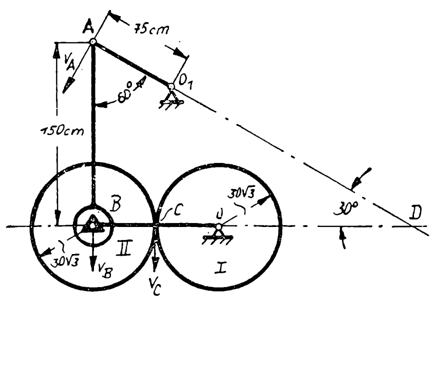
Stellung 1 und 3: Das momentane Geschwindigkeitszentrum liegt in B

1. $\frac{\omega_0 \cdot R}{\overline{AB} - r} = \omega_{a_1} \cdot \frac{(R - r)}{\overline{AB}}$

$$\omega_{a_1} = \omega_0 \cdot \frac{25}{140} \cdot \frac{150}{15} = \underline{\underline{17{,}81 \text{ 1/sek}}}$$

3. $\frac{\omega_0 \cdot R}{\overline{AB} + r} = \omega_{a_3} \frac{(R - r)}{\overline{AB}}$; $\omega_{a_3} = \underline{\underline{15{,}62 \text{ 1/sek}}}$

Lösung 536



Das momentane Geschwindigkeitszentrum von $\overline{AB}$ liegt in C

$$v_B = \frac{\overline{DB}}{\overline{DA}} \cdot v_A; \quad v_C = \frac{\overline{DB} - \overline{BC}}{\overline{DA}} \cdot v_A$$

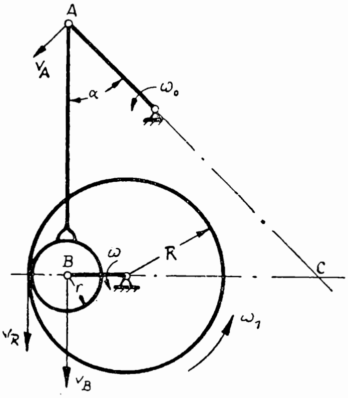
$$v_A = \overline{O_1 A} \cdot \omega_0 = 75 \cdot 6 = 450 \text{ cm/sek}$$

$$\overline{DA} = \frac{\overline{AB}}{\sin 30^\circ} = 300 \text{ cm}; \quad \overline{DB} = \frac{\overline{AB}}{\operatorname{tg} 30^\circ} = 259{,}5 \text{ cm}$$

$$\omega_{0B} = \frac{v_B}{\overline{OB}} = \frac{259{,}5}{300 \cdot 60\sqrt{3}} \cdot 450 = \underline{\underline{3{,}75 \text{ 1/sek}}}$$

$$\omega_1 = \frac{v_C}{\overline{OC}} = \frac{259{,}5 - 30\sqrt{3}}{300 \cdot 30\sqrt{3}} \cdot 450 = \underline{\underline{6 \text{ 1/sek}}}$$

Lösung 537



Das momentane Geschwindigkeitszentrum von $\overline{AB}$ liegt in C

$$\frac{v_B}{\overline{AB}\,\mathrm{tg}\,\alpha}=\frac{v_A\cos\alpha}{\overline{AB}}; \qquad v_B=\omega\cdot\overline{OB}$$

$$v_A=\omega_0\cdot\overline{AO_1}$$

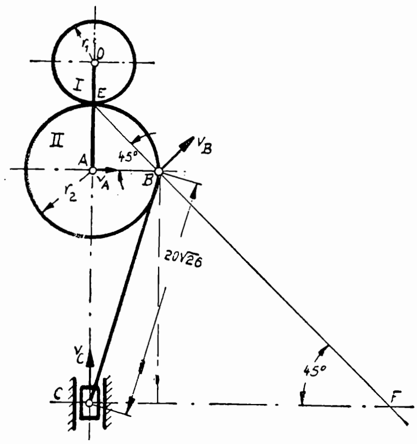
$$v_R=\omega_1\cdot R$$

$$\omega_0=\frac{\omega\cdot\overline{OB}\cdot\overline{AB}}{\overline{AB}\,\mathrm{tg}\,\alpha\cdot\cos\alpha\cdot\overline{AO_1}}=\frac{8\cdot 15\cdot 2}{\sqrt{2}\cdot 30\cdot\sqrt{2}}=\underline{\underline{4\ 1/\mathrm{sek}}}$$

$$\frac{v_R}{\overline{AB}\,\mathrm{tg}\,\alpha+r}=\frac{v_A\cdot\cos\alpha}{\overline{AB}}$$

$$\omega_1=\frac{\omega_0\cdot\overline{AO_1}\cdot\cos\alpha\,(\overline{AB}\,\mathrm{tg}\,\alpha+r)}{R\,\overline{AB}}=\underline{\underline{5{,}12\ 1/\mathrm{sek}}}$$

Lösung 538



$$v_A=\omega_0\,(r_1+r_2)=0{,}5\cdot 30=15\ \mathrm{cm/sek}$$

Das momentane Geschwindigkeitszentrum für Rad II liegt in E

$$v_B=\frac{v_A}{\cos 45^\circ}=\underline{\underline{21{,}2\ \mathrm{cm/sek}}}$$

Das momentane Geschwindigkeitszentrum für $\overline{BC}$ liegt in F

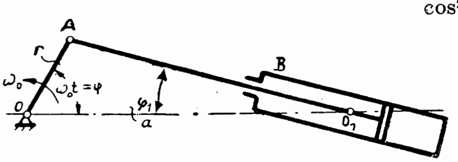
$$\overline{BF}=\frac{\sqrt{20^2\cdot 26-20^2}}{\cos 45^\circ}=\underline{\underline{141{,}4\ \mathrm{cm}}}$$

$$\overline{CF}=\sqrt{20^2\cdot 26-20^2}+20=\underline{\underline{120\ \mathrm{cm}}}$$

$$\omega_{BC}=\frac{v_B}{\overline{BF}}=\frac{21{,}2}{141{,}4}=\underline{\underline{0{,}15\ 1/\mathrm{sek}}}$$

$$v_C=\frac{\overline{CF}}{\overline{BF}}\,v_B=\frac{120}{141{,}4}\cdot 21{,}2=\underline{\underline{18\ \mathrm{cm/sek}}}$$

Lösung 539



$$\mathrm{tg}\,\varphi_1=\frac{r\sin\varphi}{a-r\cos\varphi}; \qquad \varphi=\omega_0 t$$

$$\frac{1}{\cos^2\varphi_1}\cdot\dot{\varphi}_1=\frac{r\dot{\varphi}\cos\varphi\,(a-r\cos\varphi)-r^2\sin^2\varphi\dot{\varphi}}{(a-r\cos\varphi)^2}$$

$$\dot{\varphi}_1=\dot{\varphi}\,\frac{(ar\cos\varphi-r^2)\cos^2\varphi_1}{(a-r\cos\varphi)^2}; \qquad \dot{\varphi}=\omega_0$$

$$\cos^2\varphi_1=\frac{(a-r\cos\varphi)^2}{(r\sin\varphi)^2+(a-r\cos\varphi)^2}$$

$$\underline{\underline{\dot{\varphi}_1=\omega_1=\omega_0 r\,\frac{(a\cos\varphi-r)}{r^2+a^2-2ar\cos\varphi}}}$$

$$\underline{\underline{\omega_{1\max}=\frac{\omega_0\cdot r}{a-r}}}; \qquad \text{für } \varphi=0$$

$$\omega_1=0 \quad \text{für } \underline{\underline{\varphi=\arccos\frac{r}{a}}}$$

$$\underline{\underline{\omega_{1\min}=-\frac{\omega_0 r}{a+r}}}; \qquad \text{für } \varphi=\pi$$

Lösung 540

Das Ergebnis der vorigen Aufgabe lautete: $\omega_1 = \dfrac{\omega_0 r (a\cos\varphi - r)}{a^2 + r^2 - 2ar\cos\varphi}$; $\varphi = \omega_0 t$

$$\varepsilon_1 = \dot{\omega}_1 = \omega_0 r \frac{(-a\omega_0 \sin\varphi)(a^2 + r^2 - 2ar\cos\varphi) - 2ar\omega_0 \sin\varphi (a\cos\varphi - r)}{(a^2 + r^2 - 2ar\cos\varphi)^2}$$

$$\underline{\underline{\varepsilon_1 = \frac{\omega_0^2 ra \sin\varphi (r^2 - a^2)}{(a^2 + r^2 - 2ar\cos\varphi)^2}}};$$

Für $\omega_1 = \text{konst.}$ wird $\varepsilon_1 = 0$

Somit $r^2 - a^2 = 0$

$$\underline{\underline{r = a}}$$

Lösung 541

$x_z = r\cos\omega t + (l - z)\cos\alpha;$ $\sin\alpha = \dfrac{r}{l}\sin\omega t;$ $\varphi = \omega t$

$y_z = z\sin\alpha;$

Reihenentwicklung:

$$\sqrt{1 - \left(\frac{r}{l}\sin\varphi\right)^2} = 1 - \frac{1}{2}\left(\frac{r}{l}\right)^2 \sin^2\varphi$$

$$x_z = r\cos\omega t + (l - z)\left[1 - \frac{1}{2}\left(\frac{r}{l}\right)^2 \sin^2\omega t\right]$$

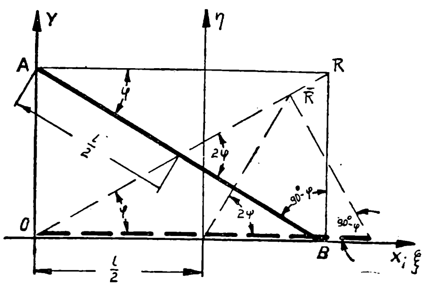
$$v_x = \dot{x}_z = -\omega\left[r\sin\omega t + \frac{(l-z)r^2}{2l^2}\sin 2\omega t\right]; \qquad v_y = \dot{y}_z = \frac{rz\omega}{l}\cos\omega t$$

$$b_x = \ddot{x}_z = -\omega^2\left[r\cos\omega t + \frac{(l-z)r^2}{l^2}\cos 2\omega t\right]; \qquad b_y = \ddot{y}_z = -\frac{rz\omega^2}{l}\sin\omega t$$

21. Rast- und Gangpolbahnen

Die unbewegliche Polbahn oder Rastpolbahn ist die Verbindungslinie der Schnittpunkte der momentanen Bahnnormalen.
Die bewegliche Polbahn oder Gangpolbahn ist die Verbindungslinie aller Punkte, die im Laufe der Bewegung zu Drehpolen werden, bezogen auf das sich bewegende Glied.
Die Gangpolbahn wird durch Zurückdrehen in die Anfangslage gefunden.
Bei der Bewegung rollt die Gangpolbahn auf der Rastpolbahn ab.

Lösung 542



Rastpolbahn: $y = l\sin\varphi$
$x = l\cos\varphi$

$\underline{\underline{x^2 + y^2 = l^2}} = 1\,\text{m}^2$ [Kreis vom Radius 1 m um O]

Gangpolbahn: $\bar{y} = l\cos\varphi\sin\varphi$
$\bar{x} = l\cos\varphi\cos\varphi$

$\eta = \bar{y} = \dfrac{l}{2}\sin 2\varphi;$ $\underline{\underline{\eta^2 + \xi^2 = \dfrac{l^2}{4}}}$

$\xi = \bar{x} - \dfrac{l}{2} = \dfrac{l}{2}\cos 2\varphi$: [Kreis vom Radius $\dfrac{1}{2}$ m um den Mittelpunkt von AB]

Lösung 543

Vgl. Abbildung zur Lösung von Aufgabe 527.

Scheibe B: Rastpolbahn: Senkrechte Gerade im Abstand x vom Scheibenmittelpunkt.

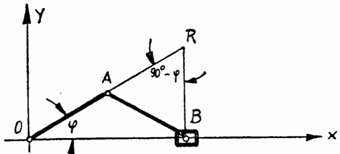
Gangpolbahn: Kreis vom Radius $x - \frac{r_B}{3}$ um den Scheibenmittelpunkt.

Scheibe A: Rastpolbahn: Senkrechte Gerade im Abstand r_A rechts vom Scheibenmittelpunkt.

Gangpolbahn: Kreis vom Radius r_A um den Scheibenmittelpunkt.

Lösung 544

Rastpolbahn:

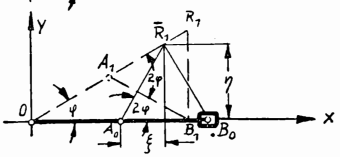


$x = \overline{OA}\cos\varphi + \overline{AB}\cos\varphi$

$x = 2r\cos\varphi$ [Kreis vom Radius $2r$

$y = 2r\sin\varphi$ um O]

Gangpolbahn:



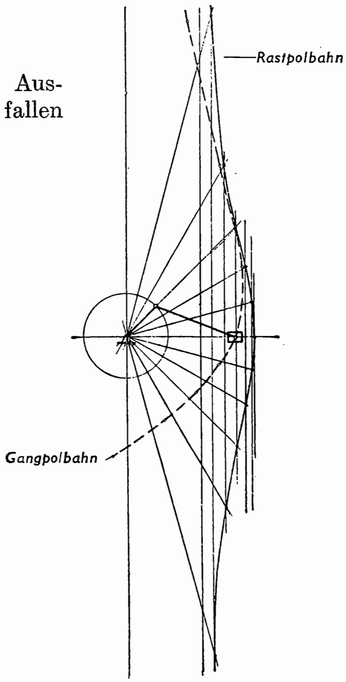
$\xi = AB\cos 2\varphi = r\cos 2\varphi$

$\eta = AB\sin 2\varphi = r\sin 2\varphi$

[Kreis vom Radius r um A]

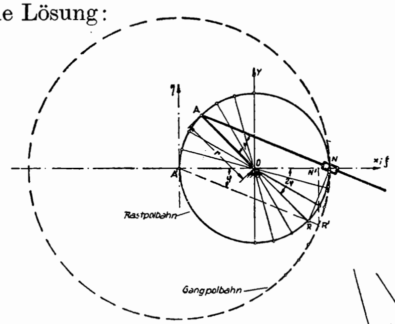
Lösung 545

Die gezeichnete Kurbelstellung gilt als Ausgangsstellung, d. h., in dieser Stellung fallen Gangpol und Rastpol zusammen.



Lösung 546

Graphische Lösung:



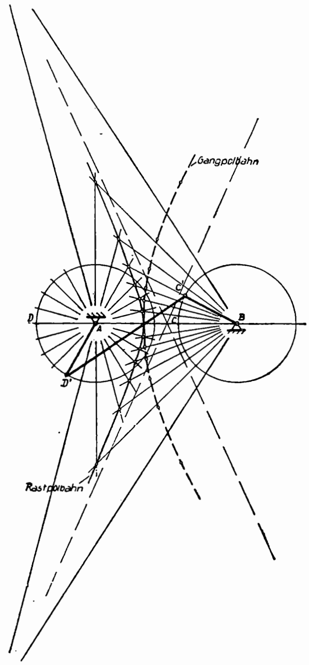
Analytische Lösung:

$y = r \cdot \sin 2\varphi;$ $x = r \cdot \cos 2\varphi;$ Kreis vom Radius r um O: Rastpolbahn

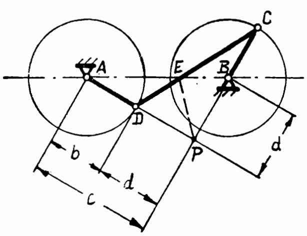
$\eta = 2r \cdot \sin \varphi;$ $\xi = 2r \cdot \cos \varphi;$ Kreis vom Radius $2r$ um A': Gangpolbahn

Lösung 547

Graphische Lösung
(Ein Hyperbelzweig)



Analytische Lösung:



Rastpolbahn: Da die Dreiecke BEP und EPD deckungsgleich sind, gilt: $\overline{EB} = \overline{DE}$ also:

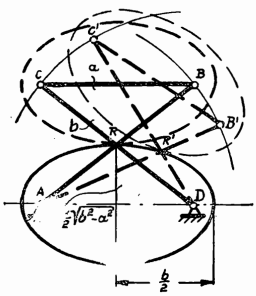
$$c - d = b$$

Dies ist die Bildungsgleichung einer Hyperbel mit den Brennpunkten A und B.

Gangpolbahn: Da $AB = CD$ ist, ergibt sich die Gangpolbahn analog als Hyperbel mit den Brennpunkten in C und D (vgl. graphische Lösung).

Lösung 548

Analytische Lösung:



Rastpolbahn: Da die Dreiecke $AR'D$ und $R'B'C'$ deckungsgleich sind, ist auch $AR' = R'C'$ und $R'D = R'B'$, somit also:

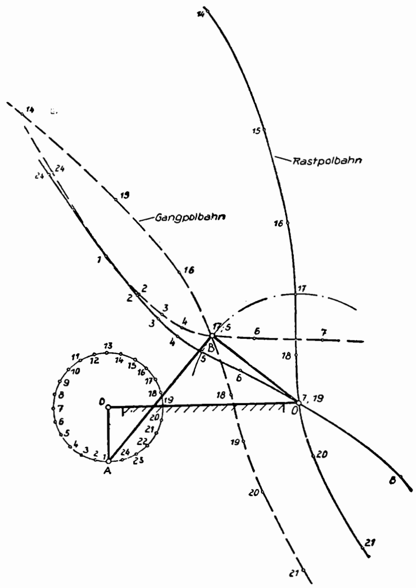
$$AR' + R'B' = b = AR' + R'D$$

Dies ist das Bildungsgesetz einer Ellipse mit den Brennpunkten in A und D. Halbachsen:

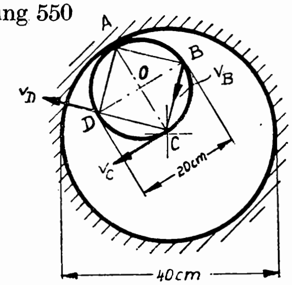
$$\frac{1}{2}\sqrt{b^2 - a^2}\,;\ \frac{b}{2}$$

Gangpolbahn: Aus Symmetriegründen ist die Gangpolbahn die gleiche Ellipse mit den Brennpunkten in C und B

Lösung 549



Lösung 550



Die Rastpolbahn ist ein Kreis um C vom Radius CA

Die Gangpolbahn ist ein Kreis um O vom Radius OC

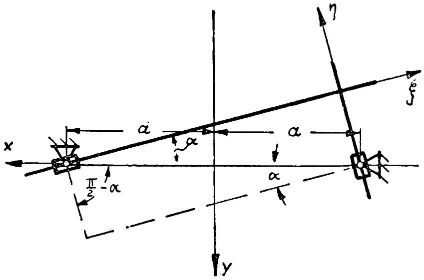
$v_A = 0$, da A auf der Rastpolbahn liegt

$$v_0 = 2 \cdot \pi \cdot 10 \quad = \quad 62{,}8 \text{ cm/sek}$$

$$v_C = 2 \cdot \pi \cdot 20 \quad = 125{,}66 \text{ cm/sek}$$

$$v_B = v_D = 2\pi \cdot 10 \cdot \sqrt{2} = \quad 88{,}84 \text{ cm/sek}$$

Lösung 551



Rastpolbahn:

$$y_C = (a + x_C)\,\mathrm{tg}\,\alpha = (a - x_C)\,\mathrm{ctg}\,\alpha$$

$$\frac{a - x_C}{a + x_C} = \mathrm{tg}^2\alpha = \frac{y_C^2}{(a + x_C)^2}$$

$$(a - x_C)(a + x_C) = y_C^2$$

$$\underline{\underline{x_C^2 + y_C^2 = a^2}}$$

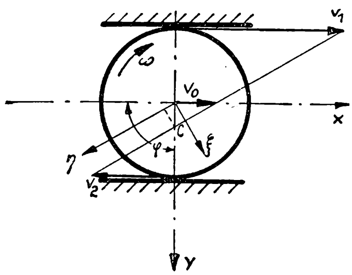
Gangpolbahn:

$$\xi_C = 2a\cos\alpha$$

$$\eta_C = 2a\sin\alpha$$

$$\underline{\underline{\xi_C^2 + \eta_C^2 = 4a^2}}$$

Lösung 552



Rastpolbahn:

$$\frac{v_1}{a - y_C} = \frac{v_2}{a + y_C}$$

$$(a + y_C)\,v_1 = (a - y_C)\,v_2$$

$$\underline{\underline{y_C = a\,\frac{v_1 - v_2}{v_1 + v_2}}}$$

Gangpolbahn:

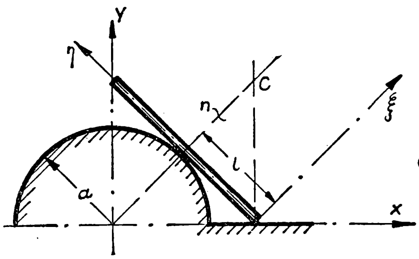
$$\xi = y_C \cdot \sin\varphi$$

$$\eta = y_C \cdot \cos\varphi$$

$$\underline{\underline{\xi_C^2 + \eta_C^2 = a^2\left(\frac{v_1 - v_2}{v_1 + v_2}\right)^2}}$$

$$\underline{\underline{\omega = \frac{v_1 + v_2}{2a}}}; \qquad \underline{\underline{v_0 = \frac{v_1 - v_2}{2}}}$$

Lösung 553



Rastpolbahn:

$$a^2 + l^2 = x_C^2; \qquad x_C^2 + y_C^2 = (a + n)^2$$

$$\frac{n^2 + l^2 = y_C^2}{a^2 - n^2 = x_C^2 - y_C^2};$$

$$2x_C^2 = a^2 - n^2 + a^2 + 2an + n^2; \quad n = \frac{x_C^2 - a^2}{a}$$

$$y_C^2 + x_C^2 = \left(\frac{a^2 + x_C^2 - a^2}{a}\right)^2;$$

$$\underline{\underline{x_C^2(x_C^2 - a^2) - a^2 y_C^2 = 0}}$$

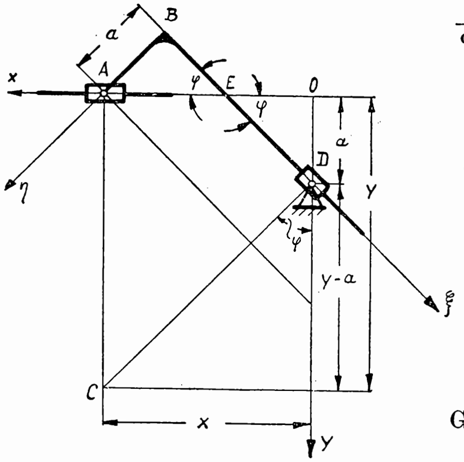
Gangpolbahn: $\xi_C^2 + \eta_C^2 = y_C^2$; $x_C^2 + y_C^2 = (a + \xi_C)^2$

$$\frac{a^2 + \eta_C^2 = x_C^2}{\xi_C^2 + a^2 + 2\eta_C^2 = x_C^2 + y_C^2};$$

$$\xi_C^2 + a^2 + 2\eta_C^2 = a^2 + 2a\xi_C + \xi_C^2$$

$$\underline{\underline{\eta_C^2 = a\,\xi_C}}$$

Lösung 554



Rastpolbahn: $$\operatorname{tg}\varphi = \frac{x_C}{y_C - a} = \frac{a\left(1 + \dfrac{1}{\cos\varphi}\right)}{x_C}$$

$$\frac{1}{\cos\varphi} = \sqrt{1 + \operatorname{tg}^2\varphi} = \sqrt{1 + \left(\frac{x_C}{y_C - a}\right)^2}$$

$$\frac{x_C}{y_C - a} - \frac{a}{x_C} = \frac{a}{x_C}\sqrt{1 + \left(\frac{x_C}{y_C - a}\right)^2}$$

$$\left(\frac{x_C}{y_C - a}\right)^2 + \left(\frac{a}{x_C}\right)^2 - \frac{2\,a\,x_C}{(y_C - a)\,x_C} =$$

$$\left(\frac{a}{x_C}\right)^2\left[1 + \left(\frac{x_C}{y_C - a}\right)^2\right]$$

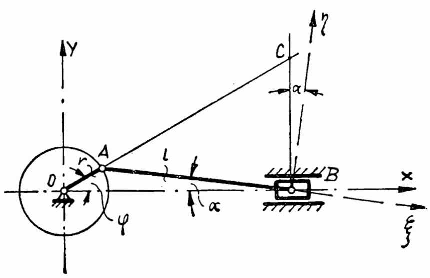
$$\frac{x_C^2}{y_C - a}\left[1 - \left(\frac{a}{x_C}\right)^2\right] = 2a$$

$$\underline{\underline{x_C^2 = a\,(2\,y_C - a)}}$$

Gangpolbahn: Da die Dreiecke ABE und DEO deckungsgleich sind, gilt:

$$\underline{\underline{\xi_C^2 = a\,(2\,\eta_C - a)}}$$

Lösung 555



Rastpolbahn: $x_C = r\cos\varphi + l\cos\alpha$; $\cos\alpha = 1$

$$x_C = r\cos\varphi + l$$

$$y_C = x_C \operatorname{tg}\varphi; \qquad \cos\varphi = \frac{x_C - l}{r}$$

$$y_C^2 = x_C^2 \cdot \frac{1 - \cos^2\varphi}{\cos^2\varphi}$$

$$= x_C^2\,\frac{r^2 - (x_C - l)^2}{(x_C - l)^2}$$

$$\underline{\underline{(x_C - l)^2\,(x_C^2 + y_C^2) = r^2\,x_C^2}}$$

Gangpolbahn: $\alpha = \dfrac{r\sin\varphi}{l}$; $\qquad \alpha = -\dfrac{\xi_C}{\eta_C}$

$(r \ll l)$:

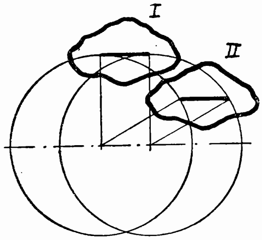
$$y_C = \eta_C = l\operatorname{tg}\varphi; \quad \eta_C^2(1 - \sin^2\varphi) = l^2\sin^2\varphi$$

$$\eta_C^2\left(1 - \frac{l^2\,\xi_C^2}{r^2\,\eta_C^2}\right) = \frac{l^4\,\xi_C^2}{r^2\,\eta_C^2}$$

$$\underline{\underline{r^2\,\eta_C^4 = l^2\,\xi_C^2\,(l^2 + \eta_C^2)}}$$

22. Beschleunigung von Körperpunkten bei ebener Bewegung
Momentanes Beschleunigungszentrum

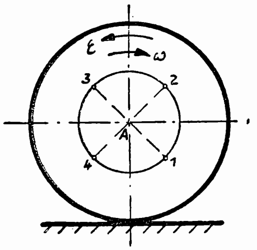
Lösung 556



Bei reiner Translation auf gerader Strecke legen alle Punkte der bewegten Ebene gleiche Wege zurück, somit sind auch Geschwindigkeit und Beschleunigung aller Punkte gleich.

Bei translatorischer Kreisbewegung beschreiben alle Punkte der bewegten Ebene Kreise gleicher Durchmesser. Die Zentripetalbeschleunigungen haben also gleiche Größe und Richtung.

Lösung 557

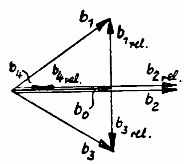


$$\omega = \frac{v_0}{R} = 2\,\frac{1}{\text{sek}}; \quad \varepsilon = \frac{b_0}{R} = \frac{2}{0{,}5} = 4 \ \text{1/sek}^2$$

$$b_{1_{\text{rel}}} = b_{2_{\text{rel}}} = b_{3_{\text{rel}}} = b_{4_{\text{rel}}} = \sqrt{\omega^4 + \varepsilon^2} \cdot r$$

$$= \sqrt{2}\ \text{m/sek}^2$$

Aus dem Beschleunigungsplan ergibt sich:

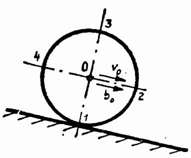


$$b_1 = b_3 = \sqrt{(\sqrt{2})^2 + b_0^2} = \underline{\underline{2{,}449\ \text{m/sek}^2}}$$

$$b_4 = b_0 - \sqrt{2} = \underline{\underline{0{,}586\ \text{m/sek}^2}}$$

$$b_2 = b_0 + \sqrt{2} = \underline{\underline{3{,}414\ \text{m/sek}^2}}$$

Lösung 558

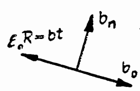


Für die einzelnen Punkte gilt allgemein:

$$b_n = \frac{v_0^2}{R} = 2\ \text{m/sek}^2$$

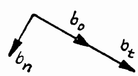
$$b_t = \varepsilon_0 \cdot R = b_0 = 3\ \text{m/sek}^2$$

1.



$$b_1 = \sqrt{(b_0 - b_t)^2 + b_n^2} = \underline{\underline{2\ \text{m/sek}^2}}$$

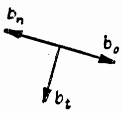
3.



$$b_3 = \sqrt{(b_0 + b_t)^2 + b_n^2}$$

$$b_3 = \sqrt{40} = 6{,}32\ \text{m/sek}^2$$

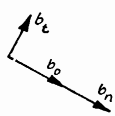
2.



$$b_2 = \sqrt{(b_0 - b_n)^2 + b_t^2}$$

$$b_2 = \sqrt{10} = \underline{\underline{3{,}16\ \text{m/sek}^2}}$$

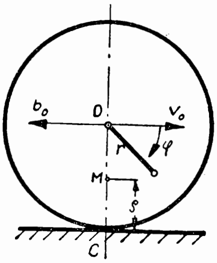
4.



$$b_4 = \sqrt{(b_0 + b_n)^2 + b_t^2}$$

$$= \sqrt{34} = \underline{\underline{5{,}83\ \text{m/sek}^2}}$$

Lösung 559



$$x = x_0 + r\cos\varphi; \quad x_0 = v_0 t - \frac{1}{2} b_0 t^2$$
$$y = R - r\sin\varphi; \quad \dot{x}_0 = v_0 - b_0 t$$
$$\ddot{x}_0 = -b_0$$
$$\dot{\varphi} = \frac{v_0}{R}; \quad \ddot{\varphi} = -\frac{b_0}{R}$$
$$\dot{x} = \dot{x}_0 - r\dot{\varphi}\sin\varphi$$
$$\dot{y} = -r\dot{\varphi}\cos\varphi$$
$$\ddot{x} = \ddot{x}_0 - r\ddot{\varphi}\sin\varphi - r\dot{\varphi}^2\cos\varphi$$
$$\ddot{y} = -r\ddot{\varphi}\cos\varphi + r\dot{\varphi}^2\sin\varphi$$

Momentanes Beschleunigungszentrum:

$$\ddot{x} = 0; \quad \ddot{y} = 0; \quad \varphi = \theta; \quad r = r_b$$
$$\ddot{y} = 0: \quad \operatorname{tg}\theta = \frac{\ddot{\theta}}{\dot{\theta}^2} = -\frac{b_0 R^2}{R \cdot v_0^2} = -1$$
$$\underline{\underline{\theta = -\frac{\pi}{4}}}$$

$$\ddot{x} = 0: \quad 0 = -b_0 - r_b \cdot \frac{b_0}{R} \cdot \frac{\sqrt{2}}{2} - r_b \cdot \frac{v_0^2}{R^2} \cdot \frac{\sqrt{2}}{2}; \quad |r_b| = \underline{\underline{0{,}354\ \mathrm{m}}}$$

Beschleunigung des Punktes C, der das momentane Geschwindigkeitszentrum bildet:

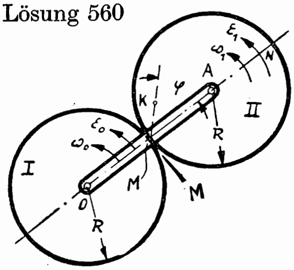
$$r = R; \quad \varphi = \frac{\pi}{2}; \quad \ddot{x}_C = \ddot{x}_0 - R \cdot \ddot{\varphi};$$
$$\ddot{y}_C = R \cdot \dot{\varphi}^2$$
$$b_C = \sqrt{\ddot{x}_C^2 + \ddot{y}_C^2} = \sqrt{\frac{v_0^4}{R^2}}; \quad b_C = \frac{v_0^2}{R} = \underline{\underline{0{,}5\ \mathrm{m/sek^2}}}$$

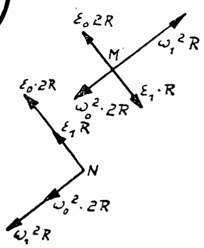
Beschleunigung des Punktes M: $\quad r = \frac{R}{2}; \quad \varphi = \frac{\pi}{2}$

$$\ddot{x}_M = \ddot{x}_0 + \frac{R}{2} \cdot \frac{b_0}{R} = -\frac{b_0}{2}; \quad b_M = \sqrt{\ddot{x}_M^2 + \ddot{y}_M^2} = \frac{1}{4}\sqrt{2} = \underline{\underline{0{,}354\ \mathrm{m/sek^2}}}$$
$$\ddot{y}_M = \frac{R}{2} \cdot \frac{v_0^2}{R^2} = \frac{v_0^2}{2R};$$

Der Krümmungsradius der Bewegungsbahn des Punktes M ist: $\varrho = \underline{\underline{0{,}25\ \mathrm{m}}}$

Lösung 560





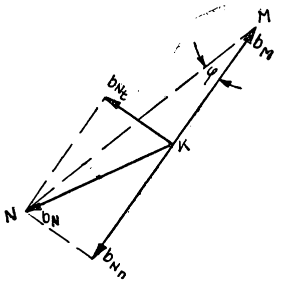
$$\begin{aligned} \varepsilon_1 &= 2\varepsilon_0 = 16\ 1/\mathrm{sek^2} \\ \omega_1 &= 2\omega_0 = 4\ 1/\mathrm{sek} \end{aligned}; \quad b = \sqrt{b_t^2 + b_n^2}$$

Das momentane Geschwindigkeitszentrum des Rades II liegt in M:

$$b_M = 2\,\omega_0^2 \cdot R = \underline{\underline{96\ \mathrm{cm/sek^2}}}$$

$$b_N = R\sqrt{(4\,\varepsilon_0)^2 + (6\,\omega_0^2)^2}$$
$$b_N = \underline{\underline{480\ \mathrm{cm/sek^2}}}$$

Mit Hilfe dieser zwei bekannten Beschleunigungen kann der Beschleunigungsplan von K aus gezeichnet werden. Die Lage des momentanen Beschleunigungszentrums K wird aus der Ähnlichkeit der Dreiecke im Lageplan und Beschleunigungsplan ermittelt.



$$\sphericalangle NMK = \sphericalangle AMK = \varphi$$

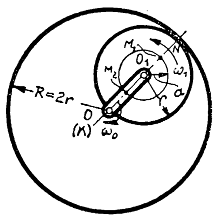
$$\operatorname{tg} \varphi = \frac{b_{N_t}}{b_M + b_{N_n}} = \frac{R \cdot 4\,\varepsilon_0}{R\,(6\,\omega_0^2 + 2\,\omega_0^2)} = 1$$

$$\underline{\underline{\varphi = 45^\circ}}$$

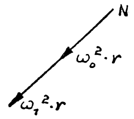
$$(MN)_B = R \cdot 4\,\varepsilon_0 \sqrt{2}\,; \quad (MN)_L = 2R$$

$$(MK)_L = \frac{2\,\omega_0^2 \cdot R}{R \cdot 4\,\varepsilon_0 \cdot \sqrt{2}} \cdot 2R = \underline{\underline{4{,}24 \text{ cm}}}$$

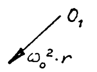
Lösung 561



$$\omega_1 = \omega_0$$

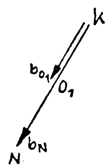


$$b_N = 2\,\omega_0^2 \cdot r$$



$$b_{O_1} = \omega_0^2 \cdot r$$

Beschleunigungsplan:



$$\frac{(O_1N)_L}{(O_1K)_L} = \frac{(O_1N)_B}{(O_1K)_B}$$

$$(O_1K)_L = r \cdot \frac{\omega_0^2 r}{2\,\omega_0^2 r - \omega_0^2 r}$$

$$(O_1K)_L = r$$

Das momentane Geschwindigkeitszentrum C liegt in N

$$\underline{\underline{b_C = b_N = 2\,\omega_0^2 \cdot r}}$$

Das momentane Beschleunigungszentrum liegt in O

$$\underline{\underline{v_K = 2\,r\,\omega_0}}$$

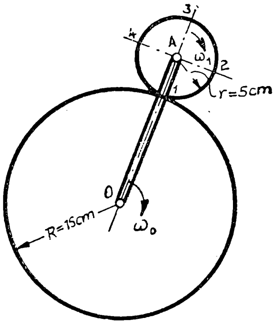
Lösung 562

$\omega_1 = \omega_0$ (vgl. Aufgabe 561)

$$v_{M_{1,2}} = (r \mp a)\,\omega_0; \quad b_{M_{1,2}} = \omega_0^2\,(r \pm a); \quad b_{M_{1,2}} = \frac{v_{M_{1,2}}^2}{\varrho_{1,2}}$$

$$\underline{\underline{\varrho_1 = \frac{(r-a)^2}{r+a}}}; \quad \underline{\underline{\varrho_2 = \frac{(r+a)^2}{r-a}}}$$

Lösung 563



$$v_A = \omega_0 \cdot (R + r) = \omega_1 \cdot r; \qquad \omega_1 = \omega_0 \frac{(R+r)}{r}$$

$$\omega_1 = 4\,\omega_0$$

$$b_1 = \omega_1^2 \cdot r - \omega_0^2 (R + r)$$

$$b_1 = \underline{\underline{540 \text{ cm/sek}^2}}$$

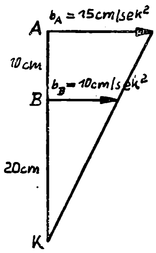
$$b_2 = b_4 = \sqrt{(\omega_1^2 \cdot r)^2 + (\omega_0^2 (R + r))^2}$$

$$b_2 = b_4 = \underline{\underline{742 \text{ cm/sek}^2}}$$

$$b_3 = \omega_1^2 \cdot r + \omega_0^2 (R + r)$$

$$b_3 = \underline{\underline{900 \text{ cm/sek}^2}}$$

Lösung 564

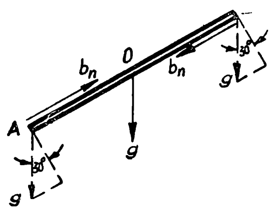


$\omega = 0$, da keine Zentripetalbeschleunigung auftritt.

Das momentane Beschleunigungszentrum befindet sich auf der Geraden AB im Abstand $AK = 30$ cm

$$\varepsilon = \frac{b_A}{AK} = \frac{15}{30} = \underline{\underline{0{,}5 \text{ 1/sek}^2}}$$

Lösung 565



$$b_{n_{A,B}} = \omega^2 \cdot \frac{l}{2} = 100 \cdot \frac{39{,}24}{2}$$

$$b_{n_{A,B}} = 1962 \text{ cm/sek}^2$$

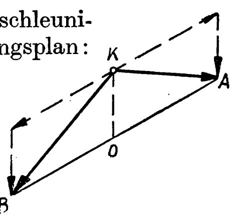
$$b_A = \sqrt{(b_n - g \sin 30°)^2 + (g \cos 30°)^2}$$

$$b_A = \underline{\underline{17 \text{ m/sek}^2}}$$

$$b_B = \sqrt{(b_n + g \sin 30°)^2 + (g \cos 30°)^2}$$

$$b_B = \underline{\underline{25{,}96 \text{ m/sek}^2}}$$

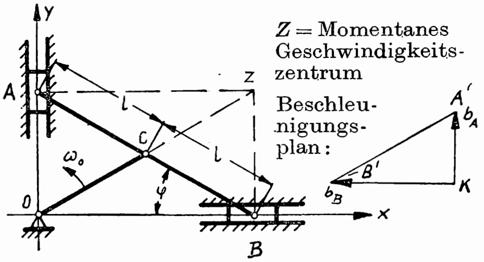
Beschleunigungsplan:



$$\frac{(\overline{AB})_L}{2 b_n} = \frac{(\overline{OK})_L}{g}$$

$$(\overline{OK})_L = (\overline{AB})_L \cdot \frac{g}{2 b_n} = \underline{\underline{0{,}0981 \text{ m}}}$$

Lösung 566



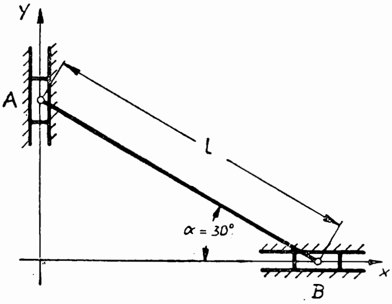
$b_Z = \omega_0^2 \cdot 2l = \underline{\underline{80 \text{ cm/sek}^2}}$

$b_A = \omega_0^2 \cdot 2l \sin 30^\circ = \underline{\underline{40 \text{ cm/sek}^2}}$

$b_B = \omega_0^2 \cdot 2l \cos 30^\circ = \underline{\underline{69{,}3 \text{ cm/sek}^2}}$

Das Dreieck $A'KB'$ ist ähnlich dem Dreieck ABO. Das momentane Beschleunigungszentrum K liegt also in O.

Lösung 567



$\dot{x} = v_{B_x} = -20 \text{ cm/sek}$

$\ddot{x} = b_{B_x} = -10 \text{ cm/sek}^2$

$x^2 + y^2 = l^2$

$2x\dot{x} + 2y\dot{y} = 0$

$x = l\cos\alpha; \quad y = l\sin\alpha$

$\dot{x}\cos\alpha + \dot{y}\sin\alpha = 0; \quad \dot{y} = -\dot{x}\operatorname{ctg}\alpha$

$\dot{y} = v_{A\,y} = \underline{\underline{34.64 \text{ cm/sek}}}$

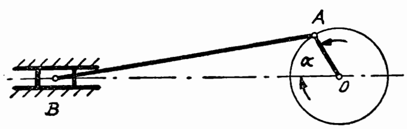
$\dot{x}^2 + x\ddot{x} + \dot{y}^2 + y\ddot{y} = 0$

$\dot{x}^2(1 + \operatorname{ctg}^2\alpha) + \ddot{x}\,l\cos\alpha + l\sin\alpha \cdot \ddot{y} = 0$

$$\ddot{y} = -\frac{\ddot{x}\,l\cos\alpha + \dot{x}^2(1 + \operatorname{ctg}^2\alpha)}{l\sin\alpha}$$

$b_{A\,y} = \ddot{y} = \underline{\underline{-142{,}68 \text{ cm/sek}^2}}$

Lösung 568



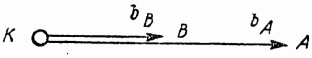
Allgemein gilt vektoriell

$$\mathfrak{b}_B = \mathfrak{b}_A + \mathfrak{b}_{A\,B}$$
$$\mathfrak{b}_{A\,B} = \mathfrak{b}_{A\,Bn} + \mathfrak{b}_{A\,Bt}$$

Der momentane Beschleunigungspol für $\alpha = 0$ und $\alpha = \pi$ liegt auf BO, da $b_{A\,Bt} = 0$. Also gilt:

1. $\underline{\alpha = 0:}$

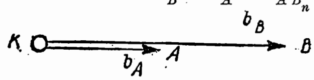
$$b_B = b_A + b_{AB_n} = \overline{AO}\,\omega_0^2 + \overline{AB}\left(\omega_0 \cdot \frac{\overline{AO}}{\overline{BA}}\right)^2 = \underline{\underline{108 \text{ m/sek}^2}}$$



$$\frac{b_B}{b_A} = \frac{\overline{BK}}{\overline{BK} - \overline{AB}}; \quad \overline{BK} = \frac{b_B}{b_B - b_A} \cdot \overline{AB} = \underline{\underline{12 \text{ m}}}$$

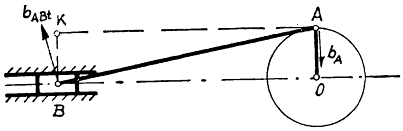
3. $\underline{\alpha = \pi:}$

$$b_B = b_A - b_{A\,B_n} = \underline{\underline{72 \text{ m/sek}^2}}$$

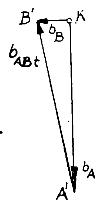


$$\overline{BK} = -\frac{b_B}{b_B - b_A} \cdot \overline{AB} = \underline{\underline{8 \text{ cm}}}$$

2. $\alpha = \frac{\pi}{2}$:



Die Winkelgeschwindigkeit ω_{AB} ist in dieser Stellung: $\omega_{AB} = 0$, da das momentane Geschwindigkeitszentrum im Unendlichen liegt.



Somit auch $b_{AB_n} = 0$.

Aus der Ähnlichkeit der Dreiecke ergibt sich:

$$\frac{b_B}{b_A} = \frac{40}{\sqrt{200^2 - 40^2}}$$

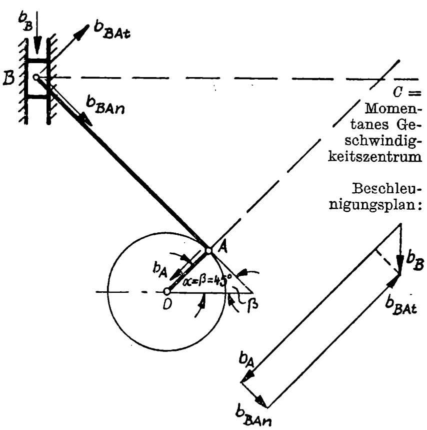
$$b_B = b_A \cdot \frac{4}{\sqrt{384}} = \underline{\underline{18{,}37\ \text{m/sek}^2}}$$

Die Lage von K folgt aus dem gleichen Dreieck zu

$$(\overline{AK})_B = (\overline{BO})_L = \underline{\underline{196\ \text{cm}}}$$

$$(\overline{BK})_B = (\overline{AO})_L = \underline{\underline{40\ \text{cm}}}.$$

Lösung 569



Allgemein gilt vektoriell

$$\mathfrak{b}_B = \mathfrak{b}_A + \mathfrak{b}_{AB}$$

$$\omega_0 \cdot \overline{OA} = \omega \cdot \overline{AC}; \qquad \overline{AC} = \overline{AB}$$

$$\omega = \omega_0 \cdot \frac{\overline{OA}}{\overline{AB}} = 10\,\frac{20}{100} = \underline{\underline{2\ 1/\text{sek}}}$$

$$b_A = \overline{OA} \cdot \omega_0^2 = 2000\ \text{cm/sek}^2$$

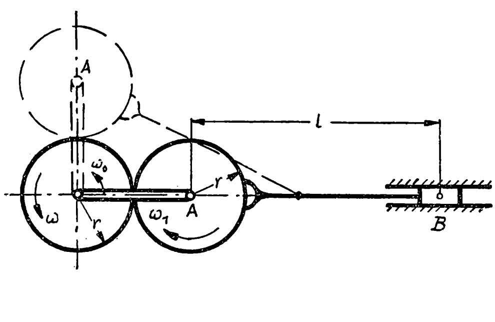
$$b_{BA_n} = \overline{AB} \cdot \omega^2 = \ 400\ \text{cm/sek}^2$$

Aus dem Beschleunigungsplan ergibt sich:

$$b_B = b_{BA_n} \cdot \sqrt{2} = \underline{\underline{565{,}6\ \text{cm/sek}^2}}$$

$$\varepsilon = \frac{b_{BA_t}}{\overline{AB}} = \frac{b_A - b_{BA_n}}{\overline{AB}} = \underline{\underline{16\ 1/\text{sek}^2}}$$

Lösung 570



Allgemein

$$\omega_0 \cdot 2r + \omega_1 \cdot r = \omega \cdot r$$

Horizontale Kurbelstellung:
Das momentane Geschwindigkeitszentrum von AB liegt in B, also

$$\omega_0 \cdot 2r = \omega_1 \cdot l$$

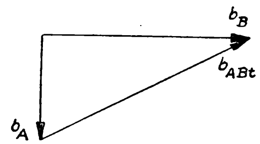
$$\omega_1 = \omega_0 \cdot \frac{2r}{l}$$

$$\underline{\underline{\omega = \omega_0\, 2\left(1 + \frac{r}{l}\right)}}$$

Da b_A und b_B im Beschleunigungsplan auf einer Geraden liegen, ist

$$b_{AB_t} = 0 \qquad \text{also} \qquad \underline{\underline{\varepsilon = 0}}$$

Vertikale Kurbelstellung:



Das momentane Geschwindigkeitszentrum liegt im Unendlichen, also $\omega_1 = 0$

$$\underline{\underline{\omega = 2\omega}}$$

$$b_A = 2r \cdot \omega_0^2; \quad b_{ABn} = l \cdot \omega_1^2 = 0$$

$$\mathfrak{b}_B = \mathfrak{b}_A + \mathfrak{b}_{ABt}$$

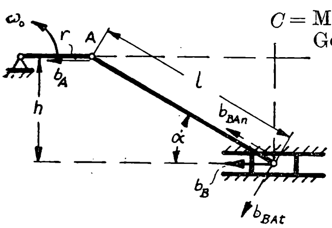
$$\frac{b_{ABt}}{b_A} = \frac{l}{\sqrt{l^2 - 4r^2}}; \quad \varepsilon = \frac{b_{ABt}}{l}$$

$$\underline{\underline{\varepsilon = \frac{2r\omega_0^2}{\sqrt{l^2 - 4r^2}}}}$$

(Verzögerung, da b_{ABt} entgegen ω_1 gerichtet ist.)

Lösung 571

1. Horizontale, rechte Lage:



C = Momentanes Geschwindigkeitszentrum

$$\omega \sqrt{l^2 - h^2} = \omega_0 \cdot r$$

$$\underline{\underline{\omega = \frac{r\omega_0}{\sqrt{l^2 - h^2}}}}$$

$$v_B = h \cdot \omega = \underline{\underline{\frac{h \cdot r\omega_0}{\sqrt{l^2 - h^2}}}}$$

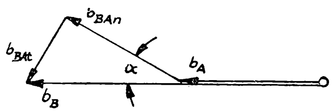
$$b_A = \omega_0^2 \cdot r; \quad b_{BAn} = \omega^2 \cdot l$$

$$b_{BAt} = b_{BAn} \cdot \operatorname{tg}\alpha; \quad \operatorname{tg}\alpha = \frac{h}{\sqrt{l^2 - h^2}}$$

$$b_{BAt} = \frac{\omega_0^2 \cdot r^2 \cdot h \cdot l}{(l^2 - h^2)^{3/2}}$$

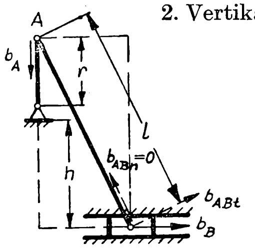
$$\varepsilon = \frac{b_{BAt}}{l} = \underline{\underline{\frac{\omega_0^2 \cdot r^2 h}{(l^2 - h^2)^{3/2}}}}$$

Beschleunigungsplan:



$$b_B = b_A + \frac{b_{BAn}}{\cos\alpha}; \quad \cos\alpha = \frac{\sqrt{l^2 - h^2}}{l}; \quad \underline{\underline{b_B = \omega_0^2 \cdot r \left[1 + \frac{r \cdot l^2}{(l^2 - h^2)^{3/2}}\right]}}$$

2. Vertikale Lage



Das momentane Geschwindigkeitszentrum liegt im Unendlichen, also:

$$\underline{\underline{\omega = 0}}; \quad b_{ABn} = 0$$

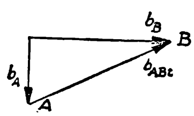
$$\underline{\underline{v_B = r \cdot \omega_0}}$$

$$\frac{b_B}{b_A} = \frac{r + h}{\sqrt{l^2 - (r+h)^2}}; \quad \underline{\underline{b_B = \frac{\omega_0^2 \cdot r(r+h)}{\sqrt{l^2 - (r+h)^2}}}}$$

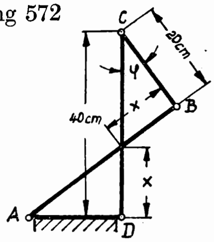
$$\frac{b_{ABt}}{b_A} = \frac{l}{\sqrt{l^2 - (r+h)^2}}; \quad \varepsilon = \frac{b_{ABt}}{l}$$

$$\underline{\underline{\varepsilon = \frac{r\omega_0^2}{\sqrt{l^2 - (r+h)^2}}}}$$

Beschleunigungsplan:



Lösung 572



$$x^2 + (20)^2 = (40 - x)^2$$

$$x = 15\ \text{cm}$$

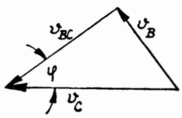
$$v_B = 40 \cdot \omega_0; \quad v_C = v_B \cdot \frac{(40 - x)}{x} = \frac{200}{3}\,\omega_0$$

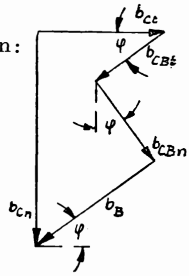
$$v_{CB} = \frac{v_B \cdot 20}{x} = \frac{160}{3}\,\omega_0$$

$$\omega_{CB} = \frac{v_{CB}}{\overline{BC}} = \underline{\underline{\frac{8}{3}\,\omega_0}}$$

Beschleunigungsplan:

Geschwindigkeitsplan:





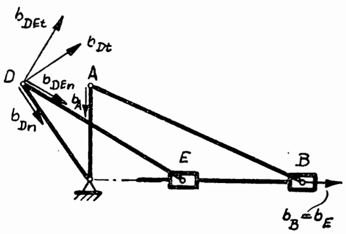
$$b_B = 40 \cdot \omega_0^2; \quad b_{C_n} = \frac{v_C^2}{40} = \frac{1000}{9}\,\omega_0^2$$

$$b_{CB_n} = 20 \cdot \omega_{CB}^2 = \frac{1280}{9} \cdot \omega_0^2$$

$$b_{CB_t} = \frac{5}{3}\,b_{C_n} - b_B - \frac{4}{3}\,b_{CB_n} = -\frac{400}{9}\,\omega_0^2$$

$$\varepsilon_{CB} = \frac{b_{CB_t}}{\overline{CB}} = \underline{\underline{-\frac{20}{9}\,\omega_0^2}}$$

Lösung 573



Vgl. Aufg. 526

Das momentane Geschwindigkeitszentrum von AB liegt im Unendlichen, somit:

$$\omega_{AB} = 0; \quad b_{AB_n} = 0$$

$$\frac{b_B}{b_A} = \frac{10}{\sqrt{26^2 - 10^2}}; \quad b_B = \frac{10}{\sqrt{26^2 - 10^2}} \cdot 10 \cdot 12^2 = 600\ \text{cm/sek}^2$$

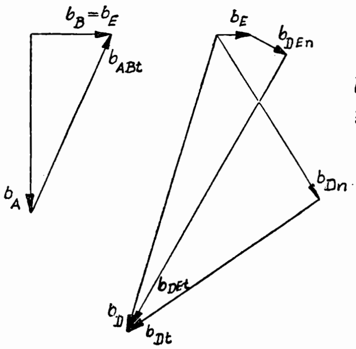
$$b_B = b_E$$

$$b_{D_n} = 12 \cdot \omega_{0D}^2 = 12 \cdot 100 \cdot 3 = 3600\ \text{cm/sek}^2$$

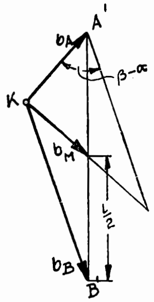
$$b_{DE_n} = 12 \cdot \sqrt{3} \cdot \omega_{DE}^2 = 693\ \text{cm/sek}^2$$

b_D ergibt sich aus dem gezeichneten Beschleunigungsplan zu:

$$\underline{\underline{b_D = 5240\ \text{cm/sek}^2}}$$



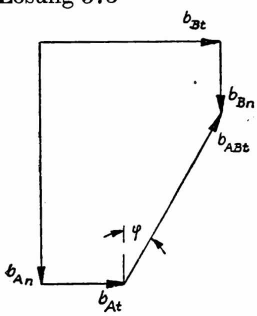
Lösung 574



$$b_M = \frac{1}{2}\sqrt{b_A^2 + b_B^2 - 2\,b_A\,b_B\cos(\beta - \alpha)}$$

$$b_M = \underline{\underline{8{,}66\ \mathrm{cm/sek^2}}}$$

Lösung 575



Es gilt die Vektorgleichung:

$$\mathfrak{b}_{Bn} + \mathfrak{b}_{Bt} = \mathfrak{b}_{An} + \mathfrak{b}_{At} + \mathfrak{b}_{ABn} + \mathfrak{b}_{BAt}$$

$$b_{An} = \frac{v_A^2}{r}; \qquad v_A = \omega_0 \cdot r = 200\ \mathrm{cm/sek}$$

Da das momentane Geschwindigkeitszentrum im Unendlichen liegt, ist:

$$v_B = v_A = \underline{\underline{200\ \mathrm{cm/sek}}}$$

$$b_{At} = \varepsilon_0 \cdot r = 100\ \mathrm{cm/sek^2}$$

$$b_{An} = \frac{v_A^2}{r} = 2000\ \mathrm{cm/sek^2}$$

$$b_{Bn} = \frac{v_B^2}{R} = \underline{\underline{400\ \mathrm{cm/sek^2}}}$$

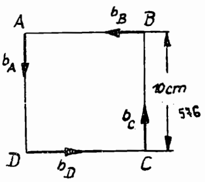
$$b_{BAn} = 0 \quad (\omega_{BA} = 0)$$

$$b_{Bt} = b_{At} + \operatorname{tg}\varphi\,(b_{An} - b_{Bn}) = \underline{\underline{370{,}45\ \mathrm{cm/sek^2}}}$$

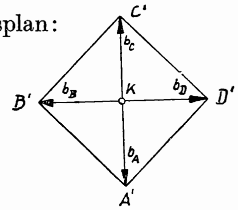
$$\operatorname{tg}\varphi = \frac{r}{\sqrt{l^2 - r^2}}$$

Lösung 576

Lageplan:



Beschleunigungsplan:



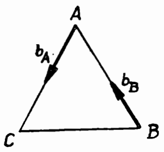
Aus der Ähnlichkeit der Quadrate $ABCD$ und $A'B'C'D'$ ergibt sich

$$b_C = b_D = 10\ \mathrm{cm/sek^2}$$

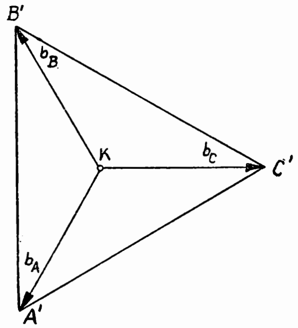
K = Momentanes Beschleunigungszentrum.

Lösung 577

Lageplan:



Beschleunigungsplan:



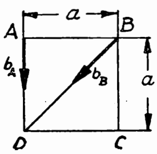
Aus der Ähnlichkeit der Dreiecke ABC und $A'B'C'$ ergibt sich:

$$b_C = 16 \text{ cm/sek}^2$$

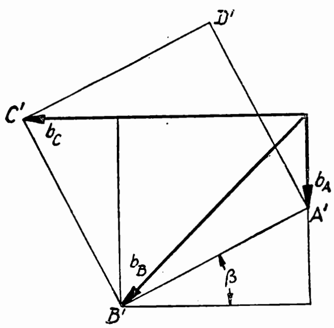
Die Richtung von b_C ist aus dem Beschleunigungsplan zu erkennen.

Lösung 578

Lageplan:



Beschleunigungsplan:



Aus dem Beschleunigungsplan ergeben sich Richtung und Größe von b_C.

$b_C = \underline{\underline{6 \text{ cm/sek}^2}}$; Richtung: Auf $\overline{DC}$ nach D.

$$\varepsilon = \frac{b_A}{a} = \underline{\underline{1 \, 1/\text{sek}^2}}$$

$\operatorname{tg}\beta = \frac{\varepsilon}{\omega^2}$; aus der Lage und Größe von b_A und b_B folgt:

$$\operatorname{tg}\beta = \frac{1}{2}$$

$$\omega = \sqrt{\frac{\varepsilon}{\operatorname{tg}\beta}} = \underline{\underline{\sqrt{2} \; 1/\text{sek}}}$$

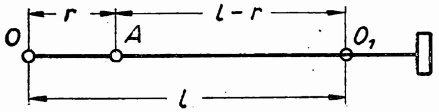
Lösung 579

1. Fall:

$$b_A = \omega_0^2 \cdot r = 300 \text{ cm/sek}^2$$

$$b_{AB} = \omega_{AB}^2 \cdot l$$

$$\omega_0 \cdot r = \omega_{AB} \cdot (l - r);$$



$$\omega_{AB} = \frac{\omega_0 \cdot r}{(l - r)}$$

$$b_{AB} = \frac{\omega_0^2 \cdot r^2 \cdot l}{(l - r)^2} = \frac{(5 \cdot 12)^2 \cdot 60}{(60 - 12)^2} = 93{,}75 \text{ cm/sek}^2$$

$$b_B = b_A - b_{AB} = \underline{\underline{206{,}25 \text{ cm/sek}^2}}$$

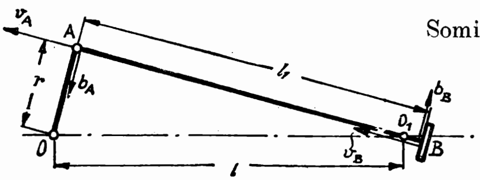
$$\varrho = \frac{v_B^2}{b_B} = \frac{15^2}{206{,}25} = \underline{\underline{1{,}09 \text{ cm}}}$$

2. Fall: Der Winkel zwischen Kurbel und Kurbelstange beträgt 90°

Das momentane Geschwindigkeitszentrum liegt im Unendlichen:

$$v_A = v_B;$$

$$b_{AB_n} = b_{B_n} = 0$$



Somit: $$\frac{b_B}{b_A} = \frac{l_1 - \sqrt{l^2 - r^2}}{\sqrt{l^2 - r^2}} = 0{,}0208$$

$$b_A = \omega_0^2 \cdot r = 300 \text{ cm/sek}^2;$$

$$b_B = \underline{\underline{6{,}24 \text{ cm/sek}^2}}$$

$$b_B = \frac{v_B^2}{\varrho};$$

$$\varrho = \frac{v_B^2}{b_B} = \frac{(5 \cdot 12)^2}{6{,}24} = \underline{\underline{576 \text{ cm}}}$$

23. Addition ebener Körperbewegungen

Lösung 580

$$\omega_2 = \omega_3 + \omega_{23}$$

$$\underline{\underline{\omega_{23} = \pm \frac{r_1 \omega_3}{r_2}}}$$

(+) gilt für äußeren Eingriff
(—) gilt für inneren Eingriff

$$\omega_2 = \pm \frac{r_1 \omega_3}{r_2} + \omega_3; \qquad \omega_2 = \omega_3 \underline{\underline{\frac{r_2 \pm r_1}{r_2}}}$$

Lösung 581

Nach vorhergehender Aufgabe (580) ist mit $r_1 = r_2$:

$$\underline{\underline{\omega_{23} = \omega_0}}; \quad \underline{\underline{\omega_2 = 2\,\omega_0}}$$

Lösung 582

$$\omega_{24} = \frac{r_3}{r_2}\,\omega_4; \quad \omega_1 = \frac{r_2}{r_1}\,\omega_{24} + \omega_4 = \left(\frac{r_3}{r_1} + 1\right)\omega_4$$

oder $\frac{r_3}{r_1} + 1 = 12$

$$\underline{\underline{r_1 = \frac{1}{11}\,r_3}}$$

Lösung 583

Die Drehzahlen bezogen auf die Kurbel sind:

$$n_3^* = \frac{n_2^* \cdot z_2}{z_3}; \qquad n_0^* = \frac{n_1^* z_1}{z_0}; \qquad n_1^* = n_2^*$$

$$n_3^* = +\frac{n_0^* z_0 z_2}{z_1 \cdot z_3}$$

Absolut gilt: $n_0^* = -n_0$; $n_3 = +n_3^* + n_0 = \underline{\underline{n_0\left(1 - \frac{z_0 z_2}{z_1 - z_3}\right) = -60\ \text{U/min}}}$

Lösung 584

Nach Aufgabe 582 gilt mit $\omega_3 = 0$:

$$\omega_1' = \omega_0\left(1 + \frac{r_3}{r_1}\right)$$

Mit $\omega_0 = 0$:

$$\omega_1'' = \frac{r_3}{r_1}\,|\omega_3|$$

somit:

$$\omega_1 = \omega_1' + \omega_1'' = \underline{\underline{\omega\left(1 + \frac{r_3}{r_1}\right) + \frac{r_3}{r_1}\,|\omega_3|}}$$

Lösung 585

$$n_{\mathrm{I}} = n_{\mathrm{II}} - \frac{z_2}{z_1}\,n_{2\,\mathrm{II}}; \qquad n_{2\,\mathrm{II}} = -\frac{z_3}{z_2}\,n_{\mathrm{II}}$$

$$n_{\mathrm{I}} = n_{\mathrm{II}} + \frac{z_2 \cdot z_3}{z_1 \cdot z_2} \cdot n_{\mathrm{II}}; \quad n_{\mathrm{II}} = \frac{n_{\mathrm{I}}}{1 + \frac{z_3}{z_1}} = \frac{4500}{1 + \frac{70}{20}} = \underline{\underline{1000\ \text{U/min}}}$$

$$n_2 = n_{2\,\mathrm{II}} + n_{\mathrm{II}} = -\frac{70}{25} \cdot 1000 + 1000$$

$$= \underline{\underline{-1800\ \text{U/min}}}$$

Lösung 586

$$n_{2\,\mathrm{I}} = n_{3\,\mathrm{I}} = -\frac{z_1}{z_2} \cdot n_{\mathrm{I}}; \quad n_{\mathrm{II}} = n_{\mathrm{I}} - \frac{z_3}{z_4} \cdot n_{3\,\mathrm{I}}$$

$$n_{\mathrm{II}} = n_{\mathrm{I}} + \frac{z_1 \cdot z_3}{z_2 \cdot z_4}\,n_{\mathrm{I}} = \underline{\underline{3000\ \text{U/min}}}$$

Lösung 587

$$n_{2\mathrm{I}} = n_{3\mathrm{I}} = \frac{r_1}{r_2} \cdot n_{\mathrm{I}}; \quad n_{\mathrm{II}} = n_{\mathrm{I}} + \frac{r_3}{r_4} \cdot n_{3\mathrm{I}}$$

$$n_{\mathrm{II}} = n_{\mathrm{I}} + \frac{r_1 \cdot r_3}{r_2 \cdot r_4} n_{\mathrm{I}} = 1800 + \frac{40 \cdot 30}{20 \cdot 90} \cdot 1800 = \underline{\underline{3000\ \mathrm{U/min}}}$$

Lösung 588

Für $\omega_1 = 0$: $\omega_{2\mathrm{I}} = \omega_{3\mathrm{I}} = \frac{z_1}{z_2} \cdot \omega_{\mathrm{I}}$; $\omega'_{\mathrm{II}} = -\frac{z_3}{z_4} \omega_{2\mathrm{I}} + \omega_{\mathrm{I}}$

Für $\omega_{\mathrm{I}} = 0$: $\omega''_{\mathrm{II}} = \frac{z_1 \cdot z_3}{z_3 \cdot z_4} \cdot \omega_1$

Demnach:

$$\omega_{\mathrm{II}} = \omega'_{\mathrm{II}} + \omega''_{\mathrm{II}} = \omega_{\mathrm{I}} + \frac{z_1 \cdot z_3}{z_2 \cdot z_4} (\omega_1 - \omega_{\mathrm{I}}) = \underline{\underline{280\ 1/\mathrm{sek}}}$$

Lösung 589

Drehzahl des Rades 2 infolge n_{I}: $n'_2 = n_{\mathrm{I}} \left(\frac{z_1}{z_2} - 1 \right)$

Drehzahl des Rades 2 infolge n_{I}: $n''_2 = n_1 \cdot \frac{z_1}{z_2}$

Resultierende Drehzahl des Rades 2:

$$n_2 = n'_2 + n''_2 = \frac{n_{\mathrm{I}}(z_1 - z_2) + n_1 z_1}{z_2}; \quad n_2 = n_3$$

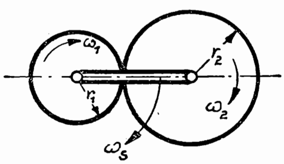
Entsprechend gilt:

$$n_3 \cdot z_3 = n_{\mathrm{I}} (z_4 - z_3) + n_{\mathrm{II}} z_4$$

$$n_{\mathrm{II}} = \frac{n_3 z_3 - n_{\mathrm{I}} (z_4 - z_3)}{z_4}$$

$$n_{\mathrm{II}} = \frac{n_{\mathrm{I}} (z_1 z_3 - z_2 z_4) + n_1 z_1 z_3}{z_2 z_4} = \frac{6000 + 3360}{16} = \underline{\underline{585\ \mathrm{U/min}}}$$

Lösung 590



Allgemeine Formel für Planetengetriebe (vgl. LEWENSON, Kinematik und Dynamik der Getriebe):

$$i_{12} = \frac{\omega_1 - \omega_{\mathrm{Steg}}}{\omega_2 - \omega_{\mathrm{Steg}}} = \pm \frac{z_2}{z_1}$$

(+) gilt für Innenverzahnung
(—) gilt für Außenverzahnung

In der Aufgabe gilt:

$\omega \mathrel{\widehat{=}} n \qquad n_2 = n_3 \qquad n_1 = 0$

$$\frac{-n_{\mathrm{I}}}{n_2 - n_{\mathrm{I}}} = \frac{z_2}{z_1}; \qquad n_2 = n_{\mathrm{I}} \left(1 - \frac{z_1}{z_2}\right)$$

$$\frac{n_3 - n_{\mathrm{I}}}{n_4 - n_{\mathrm{I}}} = \frac{z_4}{z_3}; \qquad n_4 = n_{\mathrm{I}} + \frac{z_3}{z_4} (n_2 - n_{\mathrm{I}})$$

$$n_4 = n_{\mathrm{I}} \left(1 - \frac{z_1 z_3}{z_2 z_4}\right) = 1200 \left(1 - \frac{30 \cdot 70}{80 \cdot 20}\right) = \underline{\underline{-375\ \mathrm{U/min}}}$$

Lösung 591

$$\omega_a = \omega_1; \quad \omega_B = \omega_b$$
$$\omega_2 = \omega_3; \quad \omega_4 = 0$$

$$\frac{\omega_1 - \omega_B}{\omega_2 - \omega_B} = -\frac{z_2}{z_1}; \qquad \omega_a - \omega_B = -\frac{z_2}{z_1}(\omega_2 - \omega_B)$$

$$\frac{\omega_3 - \omega_B}{\omega_4 - \omega_B} = \frac{z_4}{z_3}; \qquad \omega_2 - \omega_B = -\omega_B \cdot \frac{z_4}{z_3}; \qquad \omega_a - \omega_B = \omega_B \cdot \frac{z_2}{z_1} \cdot \frac{z_4}{z_3}$$

$$\frac{\omega_a}{\omega_b} = 1 + \frac{z_2 z_4}{z_1 z_3} = 1 + \frac{28 \cdot 54}{12 \cdot 14} = \underline{\underline{10}}$$

Lösung 592

$$\frac{n_4 - n_R}{n_3 - n_R} = -\frac{r_3}{r_4}; \qquad n_4 - n_R = -\frac{r_3}{r_4}(n_3 - n_R)$$

$$n_3 = n_2$$

$$\frac{n_2 - n_R}{n_1 - n_R} = -\frac{r_1}{r_2}; \qquad n_2 - n_R = -\frac{r_1}{r_2}(n_1 - n_R)$$

$$n_1 = n_{\mathrm{I}}$$

$$\frac{n_R}{n_0} = -\frac{r_0}{R}$$

$$n_{\mathrm{II}} = \underline{\underline{\left(n_{\mathrm{I}} + n_0 \frac{r_0}{R}\right) \frac{r_1 r_3}{r_2 r_4} - n_0 \frac{r_0}{R}}}$$

Lösung 593

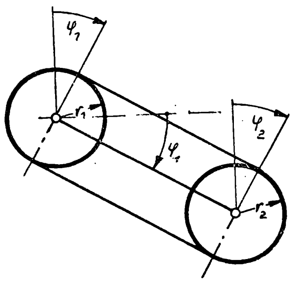
$$\frac{\omega_a - \omega_s}{\omega_b - \omega_s} = -\frac{r_b}{r_a}; \quad \begin{matrix} r_b = r_a \\ \omega_a = 0 \end{matrix}; \qquad -\omega_s = -(\omega_b - \omega_s)$$

$$\frac{\omega_b - \omega_s}{\omega_c - \omega_s} = -\frac{r_c}{r_b}; \quad r_c = r_b; \quad \omega_b - \omega_s = -(\omega_c - \omega_s)$$

$$-\omega_s = \omega_c - \omega_s$$

$$\underline{\underline{\omega_c = 0}}$$

Lösung 594



Bei der Bewegung wickeln sich die gleichen Längen der Kette auf und ab.

$$\varphi_1 \cdot r_1 = \varphi_2 \cdot r_2$$
$$r_1 = r_2; \quad \varphi_1 = \varphi_2$$

Da sich dabei das Relativsystem gegen das Absolutsystem ebenfalls um φ_1 gedreht hat, führt das Kettenrad nur eine Kreisverschiebung aus, also:

$$\underline{\underline{\omega = 0}}$$
$$\underline{\underline{\varepsilon = 0}}$$

Alle Punkte des Rades 2 führen demnach eine Kreisverschiebung vom Radius l aus. Es gilt somit:

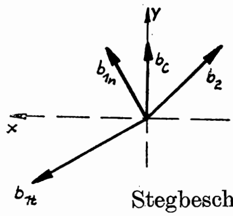
$$v_M = v_A = \underline{\underline{l \cdot \omega_0}} \qquad b_M = b_A = \underline{\underline{l \cdot \omega_0^2}}$$

Lösung 595

$$\frac{\omega_2-\omega_s}{\omega_0-\omega_s}=-\frac{r_0}{r_2}; \qquad \frac{\omega_3-\omega_s}{\omega_2-\omega_s}=-\frac{r_2}{r_3}$$

$$\frac{\omega_3-\omega_s}{\omega_0-\omega_s}=\frac{r_0}{r_3}; \qquad \omega_3=\omega_s+(\omega_0-\omega_s)\frac{r_0}{r_3}$$

$$r_0=r_3: \underline{\underline{\omega_3=\omega_0}}$$



$$v=\sqrt{(3R\omega_0)^2+(R\omega_0)^2}=\underline{\underline{R\omega_0\sqrt{10}}}$$

Stegbeschleunigung b_1; b_{1t} = Stegtangentialbeschleunigung
b_{1n} = Stegnormalbeschleunigung
Beschleunigung des Rades gegenüber dem Steg: b_2
Coriolisbeschleunigung: b_c

$$b_{1ty}=-3R\varepsilon_0; \quad b_{1tx}=R\varepsilon_0; \quad b_{1ny}=R\omega_0^2; \quad b_{1nx}=3R\omega_0^2$$

$$b_{2x}=R(2\omega_0)^2; \quad b_{2x}=-2R\varepsilon_0; \quad b_{cy}=-2\omega_0\cdot R\cdot 2\omega_0; \quad b_{cx}=0$$

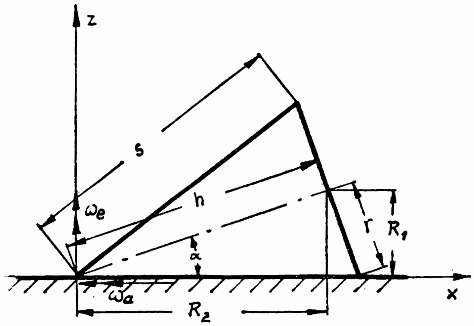
$$b_y=R(\omega_0^2-3\varepsilon_0); \qquad b_x=R(3\omega_0^2-\varepsilon_0)$$

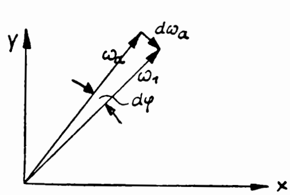
$$b=\sqrt{b_x^2+b_y^2}=\underline{\underline{R\sqrt{10(\omega_0^4+\varepsilon_0^2)-12\omega_0^2\varepsilon_0}}}$$

VII. Drehung des starren Körpers um einen festen Punkt

24. Drehung des starren Körpers um einen festen Punkt

Lösung 596





$$\frac{R_1}{h}=\frac{r}{s}$$

$$R_1=\frac{h\cdot r}{s}=\frac{4\cdot 3}{\sqrt{4^2+3^2}}=2{,}4\text{ cm}$$

$$\omega_a=\frac{v_C}{R_1}=\frac{48}{2{,}4}=\underline{\underline{20\ 1/\text{sek}}}$$

$$R_2=h\cos\alpha=\frac{h^2}{s}=3{,}2\text{ cm}$$

Umlaufzeit: $T=\dfrac{2\pi R_2}{v_C}=\dfrac{2\pi}{15}$

$$\omega_e=\frac{2\pi}{T}=15\ 1/\text{sek}$$

$$x_1=|\omega_a|\cdot\cos\omega_e t=\underline{\underline{20\cos 15t}}$$

$$y_1=|\omega_a|\cdot\sin\omega_e t=\underline{\underline{20\sin 15t}}$$

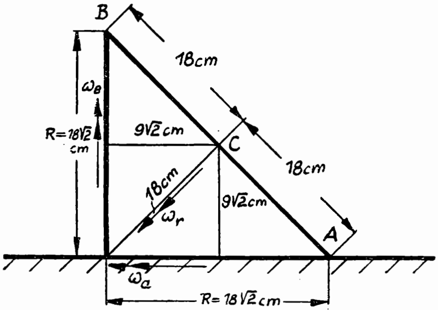
$$\underline{\underline{z_1=0}}$$

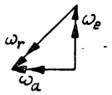
$$d\omega_a=d\varphi\cdot\omega_a$$

$$\frac{d\omega}{dt}=\omega_a\cdot\frac{d\varphi}{dt}=\omega_a\cdot\omega_e$$

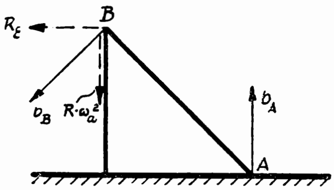
$$\varepsilon=\omega_a\cdot\omega_e=\underline{\underline{300\ 1/\text{sek}^2}}$$

Lösung 597





$$\vec{\omega}_\iota = \vec{\omega}_e + \vec{\omega}_r$$



$$T = 1\,\text{sek}; \quad \omega_e = \frac{2\pi}{T} = 2\pi\ 1/\text{sek}$$

$$v_C = 9\sqrt{2}\,\omega_e = 18\sqrt{2}\,\pi\ \text{cm/sek}$$

$$\omega_a = \frac{v_C}{9\sqrt{2}} = 2\pi\ 1/\text{sek}$$

$$\omega_r = \sqrt{\omega_a^2 + \omega_e^2} = 2\sqrt{2}\,\pi\ 1/\text{sek}$$

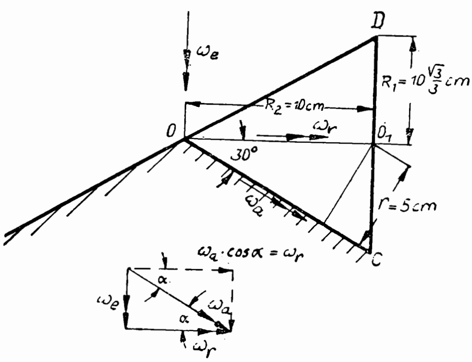
$$v_B = 18\sqrt{2}\,\omega_a = \underline{\underline{36\sqrt{2}\,\pi\ \text{cm/sek}}}$$

$$\varepsilon = \omega_a \cdot \omega_e = 4\pi^2 = \underline{\underline{39{,}5\ 1/\text{sek}^2}}$$

$b_A = R \cdot \varepsilon = \underline{\underline{1000\ \text{cm/sek}^2}}$, da ω_A durch A geht

$$b_B = \sqrt{(\omega_a^2 \cdot R)^2 + (R\cdot\varepsilon)^2} = \underline{\underline{1000\sqrt{2}\,\frac{\text{cm}}{\text{sek}^2}}}$$

Lösung 598



$$T_z = \frac{1}{2}\,\text{sek}$$

$$\omega_e = \frac{2\pi}{T_z} = \underline{\underline{4\pi \cdot 1/\text{sek}}}$$

$$v_{01} = 10\,\omega_e = 40\,\pi\ \text{cm/sek}$$

$$\omega_a = \frac{v_{01}}{r} = \frac{40\,\pi}{5} = \underline{\underline{8\,\pi\ \text{cm/sek}}}$$

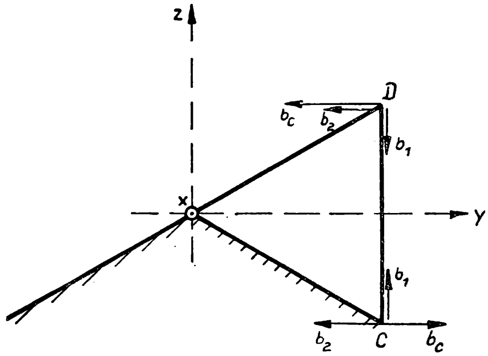
$$\omega_r = \sqrt{\omega_a^2 - \omega_e^2} = \pi\sqrt{8^2 - 4^2}$$

$$= \underline{\underline{6{,}92\,\pi\ 1/\text{sek}}}$$

$$\varepsilon_a = \omega_e \cdot \omega_a \cos\alpha = \omega_e \cdot \omega_r$$

$$\varepsilon_a = \underline{\underline{27{,}68\,\pi^2\ 1/\text{sek}^2}}$$

Lösung 599



$\underline{\underline{v_C = 0}}$, da der Vektor ω_a durch C geht.

$v_D = \overline{CD} \cdot \omega_a = 10 \cdot 8\pi = \underline{\underline{80\pi \text{ cm/sek}}}$

$b_1 = R_1 \omega_r^2 = \frac{10}{3}\sqrt{3} \cdot 48\pi^2 = 160\sqrt{3}\pi^2$

$b_2 = R_2 \omega_e^2 = 10 \cdot 16\pi^2 = 160\pi^2$

$b_C = 2 R_1 \omega_r \omega_e = 320\pi^2$

Punkt C:

$b_y = b_c - b_2 = 159{,}5\pi^2$

$b_z = b_1 = 277\pi^2$

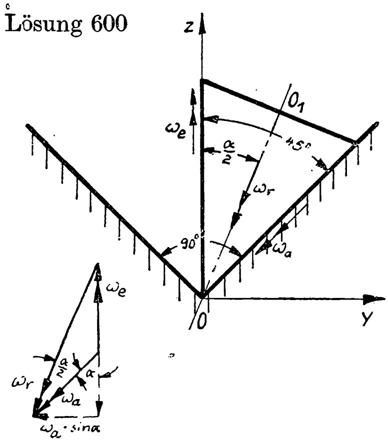
$b_C = \sqrt{b_y^2 + b_z^2} = \underline{\underline{320\pi^2 \text{ cm/sek}^2}}$

Punkt D:

$b_y = -b_2 - b_c = \underline{\underline{-480\pi^2 \text{ cm/sek}^2}}$

$b_z = -b_1 = \underline{\underline{-160\sqrt{3}\pi^2 \text{ cm/sek}^2}}$

Lösung 600



$T_z = 0{,}5 \text{ sek}$

$\omega_e = \frac{2\pi}{T_z} = \frac{2\pi}{0{,}5} = \underline{\underline{4\pi \text{ 1/sek}}}$

$\omega_a = \omega_e = 4\pi$ 1/sek (vgl. Geschwindigkeitsplan)

$\omega_r = 2\,\omega_a \cos\frac{\alpha}{2} = 2 \cdot 4\pi \cdot 0{,}924 = \underline{\underline{7{,}39\pi \text{ 1/sek}}}$

$\varepsilon_a = \omega_e \cdot \omega_a \sin\alpha; \quad \omega_e = \omega_a$

$\varepsilon_a = \omega_a^2 \cdot \sin\alpha = 16\pi^2 \cdot \frac{\sqrt{2}}{2} = \underline{\underline{11{,}3\pi^2 \text{ 1/sek}^2}}$

Lösung 601

$$v_0 = \omega_a \cdot x = \omega_a \cdot 100 \sin\frac{\alpha}{2} = 4\pi \cdot 100 \cdot 0{,}383 = \underline{\underline{153{,}2\pi \text{ cm/sek}}}$$

$$v_1 = \omega_a \cdot 2x = \underline{\underline{306{,}4\pi \text{ cm/sek}}}; \quad v_2 = \omega_a \cdot 0 = \underline{\underline{0}}$$

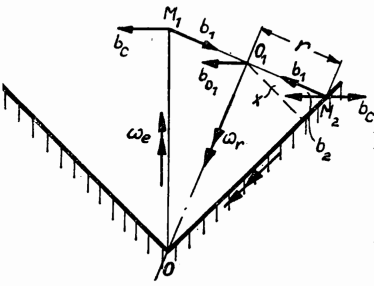
Punkt M_2: $\quad b_{M_2 y} = b_c - b_2 - b_{1y}$

$$b_{M_2 z} = b_{1z}$$

$$b_1 = \omega_r^2 \cdot r; \quad b_{1y} = \omega_r^2 \cdot r \cos\frac{\alpha}{2} = \omega_r^2 \cdot 100 \cdot \operatorname{tg}\frac{\alpha}{2} \cdot \cos\frac{\alpha}{2}$$

$$b_{1y} = \omega_r^2 \cdot 100 \cdot \sin\frac{\alpha}{2} = 2090\pi^2$$

$$b_{1z} = \omega_r^2 \cdot r \sin\frac{\alpha}{2} = 865\pi^2$$



$$b_2 = 2x \cdot \omega_e^2 = 2 \cdot 100 \cdot \sin\frac{\alpha}{2} \cdot \omega_e^2 = 1224\pi^2$$

$$b_c = 2v_r \cdot \omega_e = 2r\,\omega_r\,\omega_e = 2446\pi^2$$

$$b_{M_2} = \sqrt{b_{M_2y}^2 + b_{M_2z}^2} = \underline{\underline{1225\pi^2 \text{ cm/sek}^2}}$$

Punkt M_1: $\quad b_{M_1y} = b_{1y} - b_c = (2090 - 2446)\,\pi^2$

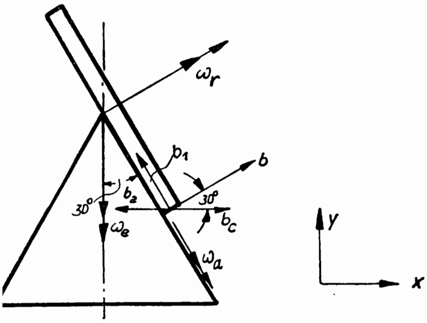
$$b_{M_1y} = \underline{\underline{-362\pi^2 \text{ cm/sek}^2}}$$

$$b_{M_1z} = -b_{1z} = \underline{\underline{-865\pi^2 \text{ cm/sek}^2}}$$

Punkt O_1: $\quad b_1 = 0; \quad b_c = 0$

$$b_{O_1} = x \cdot \omega_e^2 = \underline{\underline{612{,}8\pi^2 \text{ cm/sek}^2}}$$

ösung 602



$$b_1 = R \cdot \omega_r^2; \quad b_{1y} = b_1 \cdot \cos 30^\circ = \frac{\sqrt{3}}{2}\, b_1$$

$$b_y = b \cdot \sin 30^\circ = 48 \cdot 0{,}5 = 24$$

$$b_y = b_{1y}; \quad \omega_r^2 = \frac{2\,b_y}{R\sqrt{3}} = \frac{2 \cdot 24}{4\sqrt{3}\,\sqrt{3}} = 4$$

$$\omega_r = \underline{\underline{2 \text{ 1/sek}}}$$

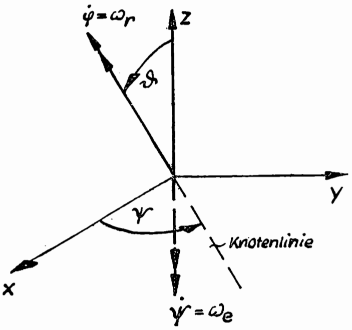
ösung 603

$$\mathfrak{v} = [\vec{\omega} \times \mathfrak{r}]; \qquad \vec{\omega} = \sqrt{3}\,\mathfrak{i} + \sqrt{5}\,\mathfrak{j} + \sqrt{7}\,\mathfrak{k}$$

$$\mathfrak{r} = \sqrt{12}\,\mathfrak{i} + \sqrt{20}\,\mathfrak{j} + \sqrt{28}\,\mathfrak{k}$$

$$\mathfrak{v} = \begin{vmatrix} \mathfrak{i} & \mathfrak{j} & \mathfrak{k} \\ \sqrt{3} & \sqrt{5} & \sqrt{7} \\ \sqrt{12} & \sqrt{20} & \sqrt{28} \end{vmatrix} = 2 \begin{vmatrix} \mathfrak{i} & \mathfrak{j} & \mathfrak{k} \\ \sqrt{3} & \sqrt{5} & \sqrt{7} \\ \sqrt{3} & \sqrt{5} & \sqrt{7} \end{vmatrix} = 0$$

ösung 604



Nach Sommerfeld (Vorlesungen über theoretische Physik, Mechanik) werden die Eulerschen Winkel wie folgt definiert:

ϑ ist der Winkel zwischen Vertikale und Figurenachse; $\dot{\vartheta}$ ist eine Drehung um die zu beiden senkrechte Knotenlinie.

ψ ist der Winkel, den die Knotenlinie mit einer festen Richtung in der Horizontalebene, z. B. der x-Achse, bildet; $\dot{\psi}$ ist eine Drehung um die Vertikale.

φ ist der Winkel, den die Knotenlinie mit einer festen Richtung in der Äquatorebene des Kreisels einschließt; $\dot{\varphi}$ ist eine Drehung um die Figurenachse.

$$\varphi = 4t;\quad \psi = \frac{\pi}{2} - 2t;\quad \vartheta = \frac{\pi}{3};\quad \omega_e = \dot\psi = -2;\quad \omega_r = 4$$

$$\vec\omega_r = \omega_r\left\{\sin\vartheta\cos\left(\frac{\pi}{2}-\psi\right)\mathfrak{i} - \sin\vartheta\sin\left(\frac{\pi}{2}-\psi\right)\mathfrak{j} + \cos\vartheta\,\mathfrak{k}\right\}$$

$$\vec\omega_r = 4\sin\vartheta\sin\psi\,\mathfrak{i} - 4\sin\vartheta\cos\psi\,\mathfrak{j} + 4\cos\vartheta\,\mathfrak{k}$$

$$\vec\omega_e = -2\mathfrak{k};\quad \vec\omega_a = \vec\omega_r + \vec\omega_e$$

$$\vec\omega_a = \omega_x\mathfrak{i} + \omega_y\mathfrak{j} + \omega_z\mathfrak{k} = 4\sin\vartheta\sin\psi\,\mathfrak{i} - 4\sin\vartheta\cos\psi$$

$$\left.\begin{aligned} x &= \omega_x = 2\sqrt{3}\cos 2t \\ y &= \omega_y = -2\sqrt{3}\sin 2t \\ z &= \omega_z = 0 \end{aligned}\right\|$$

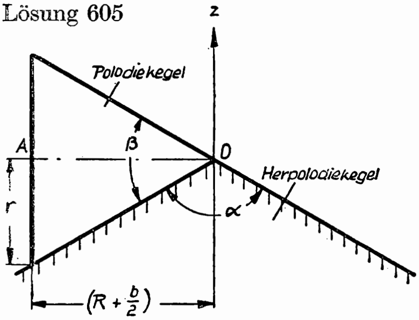
$$|\omega_a| = \sqrt{\omega_x^2+\omega_y^2+\omega_z^2} = \underline{\underline{2\sqrt{3}\ 1/\text{sek}}}$$

$$\vec\varepsilon = [\vec\omega_e \times \vec\omega_r] = \begin{vmatrix} \mathfrak{i} & \mathfrak{j} & \mathfrak{k} \\ 0 & 0 & -2 \\ 4\sin\vartheta\sin\psi & -4\sin\vartheta\cos\psi & 4\cos\vartheta \end{vmatrix}$$

$$= -8\sin\vartheta(\sin\psi\,\mathfrak{j} + \cos\psi\,\mathfrak{i})$$

$$\varepsilon = \underline{\underline{8\sin\vartheta = 4\sqrt{3}\ 1/\text{sek}^2}}$$

Lösung 605



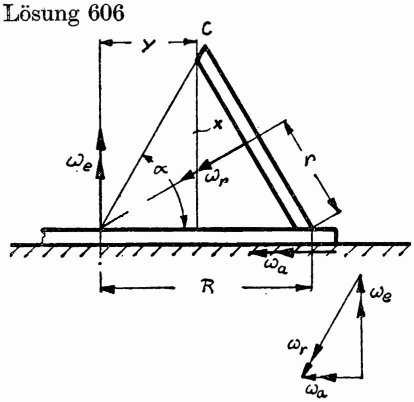
Die Bewegung eines Körpers um einen feste Punkt kann so dargestellt werden, daß e körperfester Drehkegel (Polodiekegel) a einem raumfesten Drehkegel (Herpolodi kegel) ohne zu gleiten abrollt.

$$\operatorname{tg}\frac{\beta}{2} = \frac{r}{R+\frac{b}{2}} = \frac{0{,}25}{5+0{,}4} = 0{,}0463$$

$$\beta = 2\operatorname{arc\,tg} 0{,}0463 = \underline{\underline{5^\circ\,18'}}$$

$$\alpha = 180 - \beta = \underline{\underline{174^\circ\,42'}}$$

Lösung 606



$$T_e = 12\text{ sek.}\quad \omega_e = \frac{2\pi}{12} = \frac{\pi}{6}\ 1/\text{sek}$$

$$x = r\cos\alpha;\quad \sin\alpha = \frac{r}{R} = \frac{1}{2}$$

$$y = R - r\sin\alpha = r(2-\sin\alpha) = \frac{3}{2}r$$

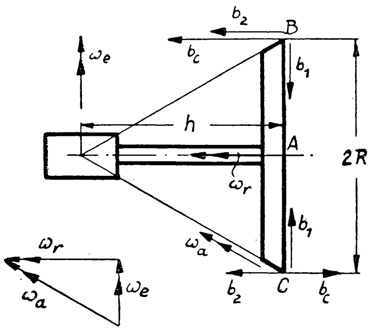
$$v_c = \omega_e\cdot y = \frac{3}{2}r\cdot\omega_e$$

$$\omega_a = \frac{v_c}{x} = \frac{3r\omega_e}{2r\cos\alpha} = \underline{\underline{0{,}907\ 1/\text{sek}}}$$

$$\omega_r^2 = \omega_a^2 + \omega_e^2 = \frac{9\omega_e^2}{4\cos^2\alpha} + \omega_e^2 = 4\omega_e^2$$

$$\omega_r = 2\omega_e = \frac{\pi}{3} = \underline{\underline{1{,}047\ 1/\text{sek}}}$$

Lösung 607



$$h = R;$$

$$\omega_e = \frac{v_A}{h} = \frac{20}{10\sqrt{2}} = \sqrt{2}\ 1/\text{sek}$$

$$\omega_r = \frac{v_A}{R} = \omega_e = \sqrt{2}\ 1/\text{sek}$$

$$\omega_a = \sqrt{\omega_e^2 + \omega_r^2} = 2\ 1/\text{sek}$$

$$v_c = \omega_a \cdot 0 = \underline{\underline{0}}$$

$$v_B = \omega_a \cdot R \cdot \sqrt{2} = 2 \cdot 10\sqrt{2}\ \sqrt{2} = \underline{\underline{40\ \text{cm/sek}}}$$

$$b_1 = R \cdot \omega_r^2 = 20\sqrt{2}; \quad b_2 = h \cdot \omega_e^2 = 20\sqrt{2}$$

$$b_c = 2 v_r \cdot \omega_e = 40\sqrt{2}$$

Punkt C: $\quad b_{Cx} = b_c - b_2 = 20\sqrt{2}$, $\quad b_{Cy} = b_1 = 20\sqrt{2}$ $\qquad b_C = \underline{\underline{40\ \text{cm/sek}^2}}$

Punkt B: $\quad b_{Bx} = b_c + b_2 = 60\sqrt{2}$, $\quad b_{By} = b_1 = 20\sqrt{2}$ $\qquad b_B = \underline{\underline{40\sqrt{5}\ \text{cm/sek}^2}}$

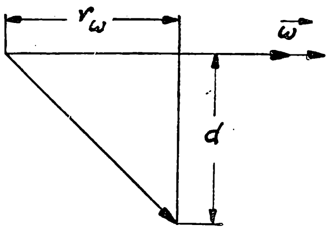
Lösung 608

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1; \qquad \cos\beta = \sqrt{1 - \cos^2\alpha - \cos^2\gamma} = \frac{3}{7}$$

$$\mathfrak{r} = 0\mathfrak{i} + 2\mathfrak{j} + 0\mathfrak{k} = 2\mathfrak{j}$$

$$\vec{\omega} = \omega(\cos\alpha\,\mathfrak{i} + \cos\beta\,\mathfrak{j} + \cos\gamma\,\mathfrak{k}) = 2\mathfrak{i} + 3\mathfrak{j} + 6\mathfrak{k}$$

$$\mathfrak{v} = [\vec{\omega} \times \mathfrak{r}] = \begin{vmatrix} \mathfrak{i} & \mathfrak{j} & \mathfrak{k} \\ 2 & 3 & 6 \\ 0 & 2 & 0 \end{vmatrix} = -2 \begin{vmatrix} \mathfrak{i} & \mathfrak{k} \\ 2 & 6 \end{vmatrix} = -12\mathfrak{i} + 4\mathfrak{k}$$



$$v_x = \underline{\underline{-12\ \text{m/sek}}}; \qquad v_y = \underline{\underline{0}}; \qquad v_z = \underline{\underline{4\ \text{m/sek}}}$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \underline{\underline{12{,}65\ \text{m/sek}}}$$

$$d = \sqrt{r^2 - r_\omega^2}; \qquad r_\omega = \frac{(\mathfrak{r} \cdot \vec{\omega})}{\omega}$$

$$(\mathfrak{r} \cdot \vec{\omega}) = \omega(0\cos\alpha + 2\cos\beta + 0\cos\gamma) = 6$$

$$r_\omega = \frac{6}{7}; \quad r = 2; \quad d = \sqrt{4 - \frac{36}{49}} = \underline{\underline{1{,}82\ \text{m}}}$$

Lösung 609

$$\vec{\omega} = A\mathfrak{i} + B\mathfrak{j} + C\mathfrak{k}; \qquad \mathfrak{r}_1 = 2\mathfrak{k}; \qquad \mathfrak{r}_2 = \mathfrak{j} + 2\mathfrak{k}$$

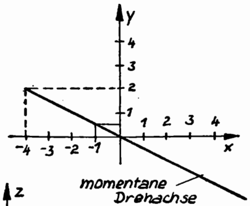
$$\mathfrak{v}_1 = [\vec{\omega} \times \mathfrak{r}_1] = \begin{vmatrix} \mathfrak{i} & \mathfrak{j} & \mathfrak{k} \\ A & B & C \\ 0 & 0 & 2 \end{vmatrix} = 2B\mathfrak{i} - 2A\mathfrak{j}; \qquad \mathfrak{v}_1 = \mathfrak{i} + 2\mathfrak{j}$$

$$A = -1; \quad B = \frac{1}{2}$$

$$\mathfrak{v}_2 = \left[\vec{\omega} \times \mathfrak{r}_2\right] = \begin{vmatrix} \mathfrak{i} & \mathfrak{j} & \mathfrak{k} \\ A & B & C \\ 0 & 1 & 2 \end{vmatrix} = 2(B\mathfrak{i} - A\mathfrak{j}) - (C\mathfrak{i} - A\mathfrak{k})$$

$$\mathfrak{v}_2 = (1 - C)\,\mathfrak{i} + 2\mathfrak{j} - \mathfrak{k} = v_2(\cos\alpha\,\mathfrak{i} + \cos\beta\,\mathfrak{j} + \cos\gamma\,\mathfrak{k})$$

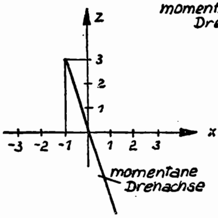
$$v_2 = \sqrt{(1-C)^2 + 2^2 + 1^2}; \qquad \cos\gamma = -\frac{1}{\sqrt{C^2 - 2C + 6}} = -\frac{1}{3}; \qquad C = 3$$



$$\vec{\omega} = -\mathfrak{i} + \frac{1}{2}\mathfrak{j} + 3\mathfrak{k};$$

$$\omega = \sqrt{(-1)^2 + \left(\frac{1}{2}\right)^2 + 3^2} = \underline{\underline{3{,}2\ 1/\text{sek}}}$$

$$\underline{\underline{x + 2y = 0}}$$



$$\underline{\underline{3x + z = 0}}$$

Lösung 610

Nach Aufgabe 604 gilt:

$$\omega_r = n$$

$$\omega_e = an$$

$$\vec{\omega}_e = an\mathfrak{k}$$

$$\vec{\omega}_r = n\sin\vartheta\sin\psi\,\mathfrak{i} - n\sin\vartheta\cos\psi\,\mathfrak{j} + n\cos\vartheta\,\mathfrak{k}$$

$$\vec{\omega}_a = \vec{\omega}_e + \vec{\omega}_r$$

$$\vec{\omega}_a = n\sin\vartheta\sin\psi\,\mathfrak{i} - n\sin\vartheta\cos\psi\,\mathfrak{j} + n(a + \cos\vartheta)\,\mathfrak{k}$$

$$\vec{\omega}_a = \frac{n\sqrt{3}}{2}\cos ant\,\mathfrak{i} + n\frac{\sqrt{3}}{2}\sin ant\,\mathfrak{j} + n\left(a + \frac{1}{2}\right)\mathfrak{k}$$

$$\omega_x = \underline{\underline{\frac{n\sqrt{3}}{2}\cos ant}}$$

$$\omega_y = \underline{\underline{\frac{n\sqrt{3}}{2}\sin ant}}$$

$$\omega_z = \underline{\underline{n\left(a + \frac{1}{2}\right)}}$$

$$\vec{\varepsilon} = \left[\vec{\omega}_e \times \vec{\omega}_r\right] = \frac{n}{2}\begin{vmatrix} \mathfrak{i} & \mathfrak{j} & \mathfrak{k} \\ 0 & 0 & an \\ \sqrt{3}\cos ant & \sqrt{3}\sin ant & 1 \end{vmatrix}$$

$$\vec{\varepsilon} = -\frac{\sqrt{3}\,an^2}{2}\begin{vmatrix} \mathfrak{i} & \mathfrak{j} \\ \cos ant & \sin ant \end{vmatrix} = -\frac{\sqrt{3}\,an^2}{2}(\mathfrak{i}\sin ant - \mathfrak{j}\cos ant)$$

$$\varepsilon_x = -\frac{\sqrt{3}\,an^2}{2}\sin ant$$

$$\varepsilon_y = \frac{\sqrt{3}\,an^2}{2}\cos ant$$

$$\varepsilon_z = 0$$

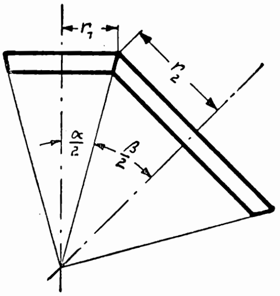
Oxy ist Herpolodiekegel, wenn $\vec{\omega}_a$ in der xy-Ebene rotiert:

$$\omega_{az} = 0; \qquad a + \frac{1}{2} = 0$$

$$a = -\frac{1}{2}$$

25. Addition von Drehbewegungen fester Körper um sich schneidende Achsen

Lösung 611

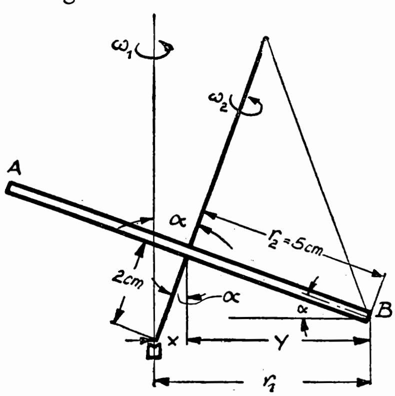


$$\omega_1 r_1 = \omega_2 r_2; \quad \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2} = \frac{\sin\frac{\alpha}{2}}{\sin\frac{\beta}{2}}$$

$$\sin\frac{\alpha}{2} = 0{,}2538 \qquad \sin\frac{\beta}{2} = 0{,}5$$

$$\omega_2 = 5{,}16\ 1/\text{min}$$

Lösung 612



Rotation um die Vertikale:

$$x = 2\sin\alpha = 2\cdot 0{,}342 = 0{,}684$$

$$y = 5\cos\alpha = 5\cdot 0{,}940 = 4{,}700$$

$$r_1 = x + y = 5{,}384\ \text{m}$$

$$\omega_1 = \frac{2\pi}{T_1} = \frac{2\pi}{6} = \frac{\pi}{3}\ 1/\text{sek}$$

$$v_1 = r_1\omega_1 = \frac{\pi}{3}\cdot 5{,}384 = 5{,}63\ \text{m/sek}$$

Rotation um die Karussellachse:

$$\omega_2 = \frac{2\pi}{T_2} = \frac{2\pi}{10} = \frac{\pi}{5}\ 1/\text{sek}$$

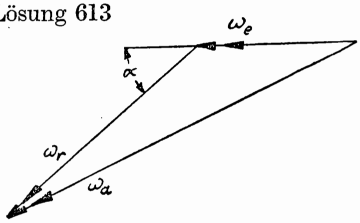
$$r_2 = 5\ \text{m}$$

$$v_2 = r_2\omega_2 = 5\cdot\frac{\pi}{5} = \pi\ \text{m/sek}$$

Beide Geschwindigkeiten addiert:

$$v = v_1 + v_2 = 8{,}77\ \text{m/sek}$$

Lösung 613



$$\omega_e = \omega_0$$

$$\omega_r = \frac{R}{r}\,\omega_e = \frac{R}{r}\,\omega_0$$

$$\omega_a^2 = \omega_e^2 + \omega_r^2 + 2\,\omega_e\,\omega_r \cos\alpha$$

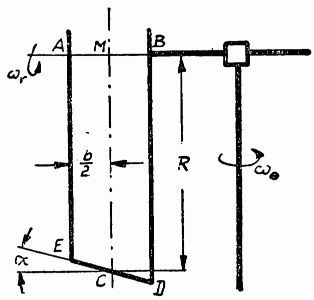
$$\omega_a^2 = \omega_0^2 + \frac{R^2}{r^2}\,\omega_0^2 + 2\,\frac{R}{r}\,\omega_0^2 \cos\alpha$$

$$\underline{\underline{\omega_a = \frac{\omega_0}{r}\sqrt{r^2 + R^2 + 2\,R\,r\cos\alpha}}}$$

$$\varepsilon = \omega_e \cdot \omega_r \cdot \sin(180 - \alpha)$$

$$\varepsilon = \omega_e \cdot \omega_r \sin\alpha = \underline{\underline{\omega_0^2\,\frac{R}{r}\sin\alpha}}$$

Lösung 614



$$\omega_e = 1\ 1/\text{sek}$$

$$v_M = r\,\omega_e = 1 \cdot 60 = 60\ \text{cm/sek}$$

$$\omega_r = \frac{v_M}{R} = \frac{60}{100} = 0{,}6\ 1/\text{sek}$$

$$\frac{b}{2}\operatorname{tg}\alpha = 25 \cdot 0{,}2 = 5\ \text{cm}$$

$$v_A = \omega_e\left(\frac{b}{2} + r\right) = 1\,(25 + 60) = 85\ \text{cm/sek}$$

$$v_E^A = \left(R - \frac{b}{2}\operatorname{tg}\alpha\right)\omega_r = (100 - 5) \cdot 0{,}6$$

$$= 57\ \text{cm/sek}$$

$$v_E = v_A - v_E^A = 85 - 57 = \underline{\underline{28\ \text{cm/sek}}}$$

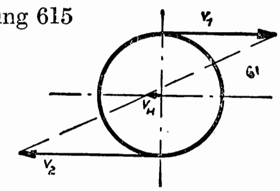
$$v_B = \left(r - \frac{b}{2}\right)\omega_e = (60 - 25) \cdot 1 = 35\ \text{cm/sek}$$

$$v_D^B = \left(R + \frac{b}{2}\operatorname{tg}\alpha\right)\omega_r = (100 + 5)\,0{,}6$$

$$= 63\ \text{cm/sek}$$

$$v_D = v_D^B - v_B = 63 - 35 = \underline{\underline{28\ \text{cm/sek}}}$$

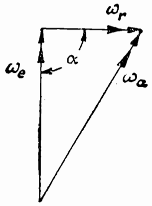
Lösung 615



$$v_H = \frac{v_1 - v_2}{2} = \frac{3 - 2}{2} = \underline{\underline{0{,}5\ \text{m/sek}}}$$

$$\omega_r = \frac{v_1 + v_2}{2\,r} = \frac{7}{2 \cdot 0{,}05} = \underline{\underline{70\ 1/\text{sek}}}$$

Lösung 616



$$\omega_r = 70\ 1/\text{sek}$$

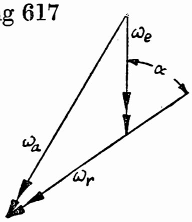
$$\omega_e = \frac{v_H}{\overline{HI}} = \frac{0{,}5 \cdot 14}{1} = \underline{\underline{7\ 1/\text{sek}}}$$

$$\omega_a = \sqrt{\omega_e^2 + \omega_r^2} = \underline{\underline{\sqrt{4949}\ 1/\text{sek}}}$$

$$\vec{\varepsilon} = \left[\overrightarrow{\omega_e} \times \overrightarrow{\omega_r}\right]; \qquad \varepsilon = \omega_e \cdot \omega_r \cdot \sin\alpha$$

$$\varepsilon = \omega_e \cdot \omega_r = \underline{\underline{490\ 1/\text{sek}^2}}$$

Lösung 617



$$T_e = \frac{60}{n}\ \text{sek/U}; \qquad \omega_e = \frac{2\pi}{T_e} = \frac{2\pi n}{60} = \frac{\pi n}{30}\ 1/\text{sek}$$

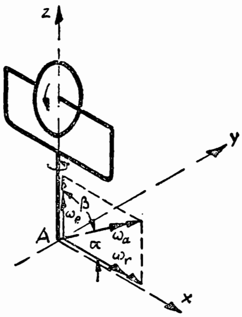
$$\omega_r = \omega_1 = 1/\text{sek}$$

$$\omega_a^2 = \omega_e^2 + \omega_r^2 - 2\,\omega_e\,\omega_r \cos(180 - \alpha)$$

$$\omega = \omega_a = \underline{\underline{\sqrt{\omega_1^2 + \left(\frac{\pi n}{30}\right)^2 + 2\,\omega_1 \frac{\pi n}{30} \cos\alpha}}}$$

$$\varepsilon = \omega_r\,\omega_e \sin(180 - \alpha) = \omega_r\,\omega_e \sin\alpha = \underline{\underline{\omega_1 \frac{\pi n}{30} \sin\alpha}}$$

Lösung 618



$$\omega_e = \omega_2 = 3\ 1/\text{sek}; \qquad \omega_r = \omega_1 = 5\ 1/\text{sek}$$

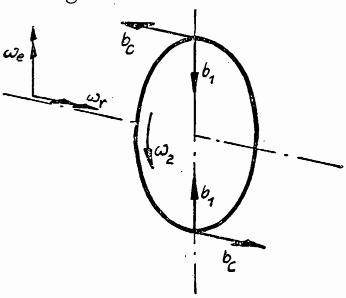
$$\omega_a = \sqrt{\omega_e^2 + \omega_r^2} = \sqrt{5^2 + 3^2} = \sqrt{34} = \underline{\underline{5{,}82\ 1/\text{sek}}}$$

$$\operatorname{tg}\alpha = \frac{\omega_e}{\omega_r} = \frac{3}{5} = 0{,}6; \qquad \underline{\underline{\alpha = 30°\,41'}}$$

$$\varepsilon = \omega_e\,\omega_r = \underline{\underline{15\ 1/\text{sek}^2}}$$

ε steht senkrecht auf ω_e und ω_r, liegt also in Richtung der y-Achse.

Lösung 619



$$\mathfrak{v}_A = \mathfrak{v}_{Ar} + \mathfrak{v}_{Ae}; \qquad \mathfrak{v}_{Ae} = 0; \qquad v_A = \underline{\underline{R\,\omega_r}}$$

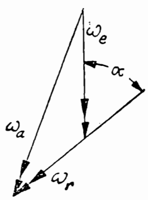
$$\mathfrak{v}_B = \mathfrak{v}_{Br} + \mathfrak{v}_{Be}; \qquad \mathfrak{v}_{Be} = 0; \qquad v_B = \underline{\underline{R\,\omega_r}}$$

$$b_1 = R\,\omega_r^2$$

$$b_c = 2\,v\,\omega_e = 2\,R\,\omega_r\,\omega_e$$

$$b_A = b_B = \sqrt{b_1^2 + b_c^2} = \underline{\underline{R\,\omega_r \sqrt{4\,\omega_e^2 + \omega_r^2}}}$$

Lösung 620



$$\omega_e = \omega_r = \frac{2\pi}{30} = \frac{\pi}{15}\ 1/\text{sek}$$

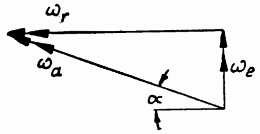
$$\omega_a = 2\,\omega_e \cdot \cos\frac{\alpha}{2}$$

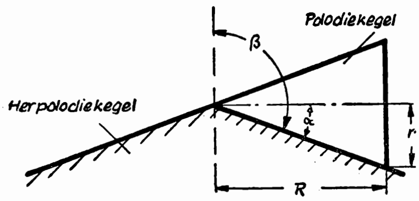
$$\omega_a = \omega_e \sqrt{2(1 + \cos\alpha)} = \frac{\pi}{15} \sqrt{2(1 + 0{,}707)} = \underline{\underline{0{,}387\ 1/\text{sek}}}$$

$$\varepsilon = \omega_e \cdot \omega_r \cdot \sin(180 - \alpha) = \omega_e^2 \sin\alpha = \frac{\pi^2}{15^2} \cdot \frac{\sqrt{2}}{2}$$

$$= \underline{\underline{0{,}031\ 1/\text{sek}^2}}$$

Lösung 621





$$\omega_e = \Omega; \quad v_A = R\,\omega_e = r\,\omega_r$$

$$\omega_r = \frac{R}{r}\,\omega_e = \frac{R}{r}\,\Omega$$

$$\omega = \omega_a = \sqrt{\omega_e^2 + \omega_r^2} = \Omega\sqrt{1 + \left(\frac{R}{r}\right)^2}$$

$$= \underline{\underline{\frac{\Omega}{r}\sqrt{r^2 + R^2}}}$$

$$\operatorname{tg}\alpha = \frac{\omega_e}{\omega_r} = \frac{\Omega}{\frac{R}{r}\,\Omega} = \frac{r}{R} \mathrel{\hat{=}} \text{Richtung } OC$$

Der Vektor der absoluten Winkelgeschwindigkeit liegt also auf der Geraden OC.

$$\alpha = \operatorname{arc\,tg}\frac{r}{R}$$

$$\beta = \pi - \operatorname{arc\,tg}\frac{R}{r}$$

Lösung 622

$$\frac{\omega_1 - \omega_4}{\omega_3 - \omega_4} = -\frac{r_3}{r_1} \qquad \frac{\omega_1 - \omega_4}{\omega_2 - \omega_4} = -\frac{r_2}{r_1} = -1$$

$$\frac{\omega_3 - \omega_4}{\omega_2 - \omega_4} = +\frac{r_2}{r_3}$$

$$\omega_4 = \frac{\omega_2 + \omega_1}{2} = \frac{5+3}{2} = \underline{\underline{4\ 1/\text{sek}}}$$

$$v_A = R\,\omega_4 = 7 \cdot 4 = \underline{\underline{28\ \text{cm/sek}}}; \quad v_{\text{Umfang}} = R\,\omega_1 = 7 \cdot 5 = 35\ \text{cm/sek}$$

$$\Delta v = v_U - v_A = 35 - 28 = 7\ \text{cm/sek}; \quad \omega_{34} = \frac{\Delta v}{r} = \frac{7}{2} = \underline{\underline{3{,}5\ 1/\text{sek}}}$$

Lösung 623

$$\frac{\omega_1 - \omega_4}{\omega_3 - \omega_4} = -\frac{r_3}{r_1} \qquad \frac{\omega_1 - \omega_4}{-\omega_2 - \omega_4} = -\frac{r_2}{r_1}$$

$$\frac{\omega_3 - \omega_4}{-\omega_2 - \omega_4} = \frac{r_2}{r_3}$$

$$\omega_4 = \frac{\omega_1 - \omega_2}{2} = \frac{7-3}{2} = \underline{\underline{2\ 1/\text{sek}}}$$

$$v_A = R\,\omega_4 = 5 \cdot 2 = \underline{\underline{10\ \text{cm/sek}}}; \quad v_{\text{Umfang}} = R\,\omega_1 = 5 \cdot 7 = 35\ \text{cm/sek}$$

$$\Delta v = v_U - v_A = 35 - 10 = 25\ \text{cm/sek}; \quad \omega_{34} = \frac{\Delta v}{r} = \frac{25}{2{,}5} = \underline{\underline{10\ 1/\text{sek}}}$$

Lösung 624

$$v_m = \frac{36}{3{,}6} = 10\ \text{m/sek}$$

$$v_i = \frac{\varrho - \frac{l}{2}}{\varrho}\,v_m = \frac{5-1}{5} \cdot 10 = 8\ \text{m/sek}; \qquad \omega_i = \frac{v_i}{R} = \frac{8}{0{,}5} = 16\ 1/\text{sek}$$

$$v_a = \frac{\varrho + \frac{l}{2}}{\varrho}\,v_m = \frac{5+1}{5} \cdot 10 = 12\ \text{m/sek}; \qquad \omega_a = \frac{v_a}{R} = \frac{12}{0{,}5} = 24\ 1/\text{sek}$$

$$\omega_A = \omega_a = \underline{\underline{24\ 1/\text{sek}}}; \qquad \omega_B = \omega_i = \underline{\underline{16\ 1/\text{sek}}}$$

$$\omega_D = \frac{\omega_A + \omega_B}{2} = \frac{24+16}{2} = 20\ 1/\text{sek}$$

$$v_F = \omega_D \cdot 2r$$

$$v_A = \omega_A \cdot 2r$$

$$\Delta v = 2r(\omega_A - \omega_D) = 2r \cdot 4\ \text{cm/sek}; \qquad \omega_r = \frac{\Delta v}{r_F} = \frac{2r\cdot 4}{r} = \underline{\underline{8\ 1/\text{sek}}}$$

Lösung 625

$$n_1 = n_1' \qquad n_1' = n_4 \frac{z_4}{z_1'} = n_4 \cdot \frac{x}{m}$$

$$n_2 = n_2'$$

$$n_5 = n_4 \qquad n_2' = n_5 \frac{z_5}{z_2'} = n_4 \cdot \frac{y}{n}$$

$$\omega = \frac{\omega_1 + \omega_2}{2} = \frac{\omega_4}{2}\left(\frac{x}{m} + \frac{y}{n}\right)$$

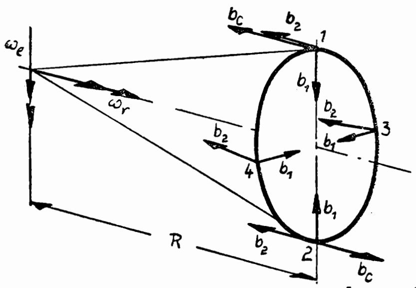
$$\underline{\underline{\frac{\omega}{\omega_0} = \frac{1}{2}\left(\frac{x}{m} + \frac{y}{n}\right)}}$$

Lösung 626

I′ und II′ werden durch das eingesetzte Zwischenrad gegenläufig bewegt.

$$\omega = \frac{\omega_1 - \omega_2}{2} = \frac{\omega_0}{2}\left(\frac{x}{m} - \frac{y}{m}\right); \qquad \underline{\underline{\frac{\omega}{\omega_0} = \frac{1}{2}\left(\frac{x}{m} - \frac{y}{m}\right)}}$$

Lösung 627



$$\omega_e = \frac{\omega_1 + \omega_2}{2} = 5 1/\text{sek}$$

$$v_A = \omega_e \cdot R = 5 \cdot 6 = 30\ \text{cm/sek}$$

$$v_U = \omega_1 \cdot R = 6 \cdot 6 = 36\ \text{cm/sek}$$

$$\Delta v = v_U - v_A = 6\ \text{cm/sek}$$

$$\omega_r = \frac{\Delta v}{r} = \frac{6}{3} = 2\ 1/\text{sek}$$

$$b_1 = \omega_r^2 \cdot r = 4 \cdot 3 = 12\ \text{cm/sek}^2$$

$$b_{2_{1,2}} = \omega_e^2 \cdot R = 25 \cdot 6 = 150\ \text{cm/sek}^2$$

$$b_{c_{1,2}} = 2 v_r \omega_e = 2r\omega_r\omega_e = 2\cdot 3\cdot 2\cdot 5 = 60\ \text{cm/sek}^2$$

$$b_{c_{3,4}} = 0; \qquad b_{2_{3,4}} = \omega_e^2 r\sqrt{5} = 25 \cdot 3\sqrt{5} = 167{,}7\ \text{cm/sek}^2$$

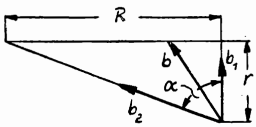
Punkt 2. $b_x = b_2 - b_c = 150 - 60 = 90\ \text{cm/sek}^2$ $\qquad b_2 = \sqrt{90^2 + 12^2} = \underline{\underline{90{,}8\ \text{cm/sek}^2}}$

$b_y = b_1 = 12\ \text{cm/sek}^2$

Punkt 1. $b_x = b_2 + b_c = 150 + 60 = 210\ \text{cm/sek}^2$ $\qquad b_1 = \sqrt{210^2 + 12^2} = \underline{\underline{210{,}4\ \text{cm/sek}^2}}$

$b_y = b_1 = 12\ \text{cm/sek}^2$

Punkt 3,4. $\operatorname{tg}\alpha = \frac{R}{r} = 2$ $\qquad \cos\alpha = \frac{1}{\sqrt{1+\operatorname{tg}^2\alpha}} = \frac{1}{\sqrt{1+4}} = \frac{1}{\sqrt{5}}$



$$b^2 = b_1^2 + b_2^2 + 2b_1 b_2 \cos\alpha$$

$$b^2 = 12^2 + (25\cdot 3\sqrt{5})^2 + 2\cdot 12\cdot 25\cdot 3\sqrt{5}\,\frac{1}{\sqrt{5}} = 30069$$

$$b_{3,4} = \underline{\underline{173{,}4\ \text{cm/sek}^2}}$$

Lösung 628

$$\frac{\omega_2-\omega_a}{\omega_1-\omega_a}=-\frac{r_2}{r_1} \qquad \frac{\omega_1-\omega_a}{\omega_3-\omega_a}=-\frac{r_3}{r_1}$$

$$\frac{\omega_2-\omega_a}{\omega_3-\omega_a}=+\frac{r_3}{r_2}$$

$$r_3=r_1:\quad \omega_1-\omega_a=-\omega_3+\omega_a;\quad \omega_3=\omega_b;\quad \omega_1=\omega_4;$$

$$\underline{\omega_b=2\,\omega_a-\omega_4}$$

1. $\omega_4=0$: $\omega_b=\underline{\underline{2\,\omega_a}}$ 4. $\omega_4=2\,\omega_a$: $\omega_b=\underline{\underline{0}}$

2. $\omega_4=+\omega_4^*$: $\omega_b=\underline{\underline{2\,\omega_a-\omega_4^*}}$ 5. $\omega_4=-\omega_4^*$: $\omega_b=\underline{\underline{2\,\omega_a+\omega_4^*}}$

3. $\omega_4=\omega_a$: $\omega_b=\underline{\underline{\omega_a}}$

Lösung 629

$\omega_4=2\,\omega_a=\underline{\underline{120\ 1/\text{min}}}$ gleichsinnig.

Lösung 630

$$\frac{\omega_a-\omega_4}{\omega_2-\omega_4}=-\frac{r_2}{r_1} \qquad \frac{\omega_a-\omega_4}{\omega_3-\omega_4}=-\frac{r_3}{r_1}$$

$$\frac{\omega_2-\omega_4}{\omega_3-\omega_4}=+\frac{r_3}{r_2}$$

$$\omega_3=\omega_b;\quad r_3=r_1;\quad \omega_a-\omega_4=-\omega_b+\omega_4$$

$$\underline{\omega_b=2\,\omega_4-\omega_a}$$

1. $\omega_4=\omega_a$: $\omega_b=\underline{\underline{\omega_a}}$ 3. $\omega_4=0$: $\omega_b=\underline{\underline{-\omega_a}}$

2. $\omega_4=-\omega_a$: $\omega_b=\underline{\underline{-3\,\omega_a}}$

Lösung 631

$$\frac{\omega_a-\omega_b}{\omega_2-\omega_b}=-\frac{r_2}{r_1} \qquad \frac{\omega_a-\omega_b}{\omega_3-\omega_b}=-\frac{r_3}{r_1}$$

$$\frac{\omega_2-\omega_b}{\omega_3-\omega_b}=+\frac{r_3}{r_2}$$

$$r_3=r_1;\quad \omega_3=0:\quad \omega_a-\omega_b=\omega_b;\quad 2\,\omega_b=\omega_a;\quad \underline{\underline{\frac{\omega_b}{\omega_a}=0.5}}$$

Lösung 632

$$\frac{\omega_1-\omega_3}{\omega_4-\omega_3}=-\frac{r_1}{R_1} \qquad \frac{\omega_1-\omega_3}{\omega_2-\omega_3}=-\frac{r_1}{r_2}\cdot\frac{R_2}{R_1}$$

$$\frac{\omega_4-\omega_3}{\omega_2-\omega_3}=+\frac{R_2}{r_2}$$

$$\omega_1-\omega_3=-\frac{r_1}{r_2}\cdot\frac{R_2}{R_1}(\omega_2-\omega_3);\quad \omega_3=\frac{\omega_1+\omega_2\cdot\frac{r_1}{r_2}\cdot\frac{R_2}{R_1}}{1+\frac{r_1}{r_2}\cdot\frac{R_2}{R_1}}$$

$$\omega_3 = \frac{4{,}5 + 9 \cdot \frac{5}{2} \cdot \frac{5}{10}}{1 + \frac{5}{2} \cdot \frac{5}{10}} = \frac{45 + 18}{9} = \underline{\underline{7\ 1/\text{sek}}}$$

$$v_4^* = R_1 \cdot \omega_3 = 10 \cdot 7 = 70\ \text{cm/sek}; \quad v_1 = R_1 \cdot \omega_1 = 10 \cdot 4{,}5 = 45\ \text{cm/sek}$$

$$\Delta v = 25\ \text{cm/sek}; \quad \omega_{34} = \frac{\Delta v}{r_1} = \frac{25}{5} = \underline{\underline{5\ 1/\text{sek}}}.$$

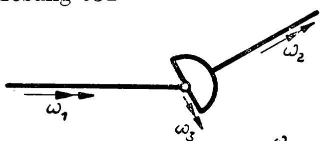
Lösung 633

$$\omega_3 = \frac{\omega_1 - \omega_2 \cdot \frac{r_1}{r_2} \frac{R_2}{R_1}}{1 + \frac{r_1}{r_2} \frac{R_2}{R_1}} = \frac{45 - 18}{9} = \underline{\underline{3\ 1/\text{sek}}}$$

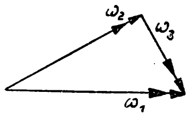
$$v_4^* = R_1 \omega_3 = 10 \cdot 3 = 30\ \text{cm/sek}; \quad v_1 = R_1 \omega_1 = 10 \cdot 4{,}5 = 45\ \text{cm/sek}$$

$$\Delta v = 75\ \text{cm/sek}; \qquad \omega_{34} = \frac{\Delta v}{r_1} = \frac{75}{5} = \underline{\underline{15\ 1/\text{sek}}}$$

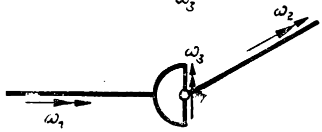
Lösung 634



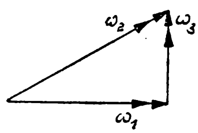
$$\vec{\omega}_2 + \vec{\omega}_3 = \vec{\omega}_1$$



$$\underline{\underline{\frac{\omega_1}{\omega_2} = \frac{1}{\cos\alpha} = 2}}$$

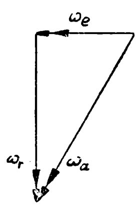


$$\vec{\omega}_1 + \vec{\omega}_3 = \vec{\omega}_2$$



$$\underline{\underline{\frac{\omega_1}{\omega_2} = \cos\alpha = \frac{1}{2}}}$$

Lösung 635



$$\omega_r = \frac{z_1}{z_2} \cdot \frac{z_3}{z_4}\, \omega_I = \frac{80}{43} \cdot 4{,}3 = 8{,}0\ \ 1/\text{sek}$$

$$\omega_e = \omega = 4{,}3\ \ 1/\text{sek}$$

$$\omega_a = \sqrt{\omega_r^2 + \omega_e^2} = \underline{\underline{9{,}08\ 1/\text{sek}}}$$

$$\varepsilon_a = \omega_r \cdot \omega_e = \underline{\underline{34{,}4\ 1/\text{sek}^2}}$$

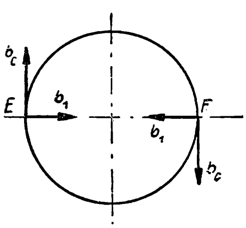
$$v_E = v_F = r \cdot \omega_r = 5 \cdot 8 = \underline{\underline{40\ \text{cm/sek}}}$$

$$b_1 = r \cdot \omega_r^2 = 320\ \text{cm/sek}^2$$

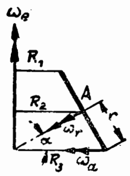
$$b_2 = 0 \cdot \omega_e^2 = 0$$

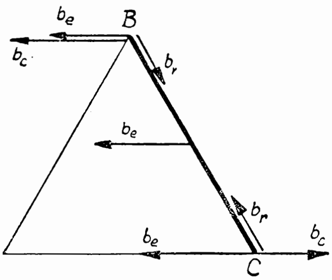
$$b_c = 2 v_r \cdot \omega_e = 2 r\, \omega_r\, \omega_e = 344\ \text{cm/sek}^2$$

$$b_E = b_F = \sqrt{b_1^2 + b_c^2} = \underline{\underline{468\ \text{cm/sek}^2}}$$



Lösung 636





$$\omega_e = \omega_0 = 0{,}1 \text{ 1/sek}$$

$$v_A = \omega_c \cdot R_2 = \omega_e \cdot r \cdot \left(\frac{1}{\sin\alpha} - \sin\alpha\right)$$

$$v_A = \omega_e \cdot r \cdot \frac{\cos^2\alpha}{\sin\alpha}; \quad \cos\alpha = \frac{84}{85}$$

$$\sin\alpha = \frac{13}{85}$$

$$v_A = 0{,}1 \cdot 25 \cdot \frac{84^2 \cdot 85}{85^2 \cdot 13} = \underline{\underline{15{,}96 \text{ cm/sek}}}$$

$$\omega_a = \frac{v_A}{r\cos\alpha} = \frac{\cos\alpha}{\sin\alpha} \cdot \omega_e = \underline{\underline{0{,}646 \text{ 1/sek}}}$$

$$\omega_r = \frac{v_A}{r} = 0{,}638 \text{ 1/sek}$$

$$\varepsilon = \omega_e \cdot \omega_r \cdot \sin\alpha = \omega_e \cdot \omega_a = \underline{\underline{0{,}0646 \text{ 1/sek}^2}}$$

$$v_B = \omega_a \cdot 2 \cdot r \cdot \cos\alpha = 2\,v_A = \underline{\underline{31{,}92 \text{ cm/sek}}}$$

$$v_c = \underline{\underline{0}}$$

Punkt A: $b_r = 0$

$$b_e = R_2 \cdot \omega_e^2 = \underline{\underline{1{,}596 \text{ cm/sek}^2}} = b_A$$

$$b_c = 0$$

Punkt B: $b_r = \omega_r^2 \cdot r = 10{,}176 \text{ cm/sek}^2$

$$b_c = 2\,\omega_r \cdot r \cdot \omega_e = 3{,}190 \text{ cm/sek}^2$$

$$b_e = \omega_e^2 \cdot R_1 = \omega_e^2 \left(\frac{r}{\sin\alpha} - 2\,r\sin\alpha\right) = 1{,}558 \text{ cm/sek}^2$$

$$b_{Bx} = b_e + b_c - b_r \cdot \sin\alpha = 3{,}192 \text{ cm/sek}^2$$

$$b_{By} = b_r \cos\alpha = 10{,}056 \text{ cm/sek}^2$$

$$b_B = \sqrt{b_{Bx}^2 + b_{By}^2} = \underline{\underline{10{,}50 \text{ cm/sek}^2}}$$

Punkt C: $b_r = \omega_r^2 \cdot r = 10{,}176 \text{ cm/sek}^2$

$$b_c = 2\,\omega_r \cdot r \cdot \omega_e = 3{,}190 \text{ cm/sek}^2$$

$$b_e = \omega_e^2 \cdot R_3 = \omega_e^2 \cdot \frac{r}{\sin\alpha} = 1{,}635 \text{ cm/sek}^3$$

$$b_{cx} = b_c - b_c + b_r \sin\alpha = 0$$

$$b_{cy} = b_r \cdot \cos\alpha = \underline{\underline{10{,}056 \text{ cm/sek}^2}} = b_o$$

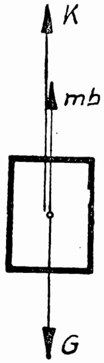
Dritter Teil

Dynamik

VIII. Dynamik des materiellen Punktes

26. Bestimmung der Kräfte aus der gegebenen Bewegung

Lösung 637

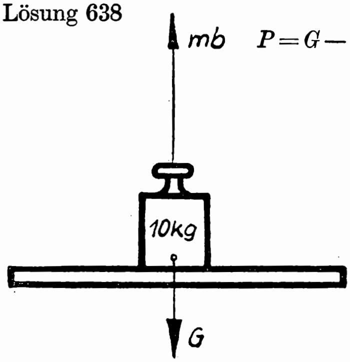


$$s = \frac{b t^2}{2}; \quad b = \frac{2s}{t^2} = \frac{2 \cdot 35}{10^2} = 0{,}7 \text{ m/sek}^2$$

$$K = G - mb = m(g - b) = \frac{280}{9{,}81}(9{,}81 - 0{,}7) = \underline{\underline{260 \text{ kg}}}$$

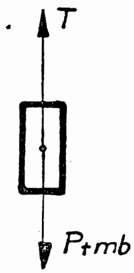
$K =$ Seilkraft

Lösung 638



$$P = G - mb = m(g - b) = \frac{10}{9{,}81}(9{,}81 - 4) = \underline{\underline{5{,}92 \text{ kg}}}$$

Lösung 639



$$T = P + m b$$

$$b = \frac{T - P}{m} = \frac{1{,}2}{3} \cdot 9{,}81 = \underline{\underline{3{,}92\ \text{m/sek}^2}}$$

Lösung 640

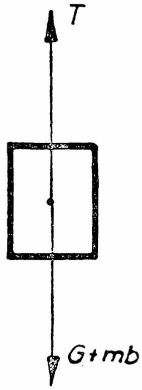
$$v = b t + v_0; \quad b = \frac{v - v_0}{t}$$

$$b_1 = \frac{5}{2} = 2{,}5\ \text{m/sek}^2; \quad b_2 = \frac{5-5}{6} = 0; \quad b_3 = \frac{0-5}{2} = -2{,}5\ \text{m/sek}^2$$

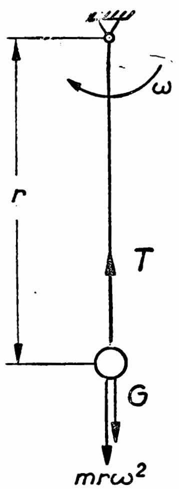
$$T_1 = m(g + b_1) = \frac{480}{9{,}81}(9{,}81 + 2{,}5) = \underline{\underline{602{,}4\ \text{kg}}}$$

$$T_2 = m g = \underline{\underline{480\ \text{kg}}}$$

$$T_3 = m(g + b_3) = \frac{480}{9{,}81}(9{,}81 - 2{,}5) = \underline{\underline{357{,}6\ \text{kg}}}$$



Lösung 641

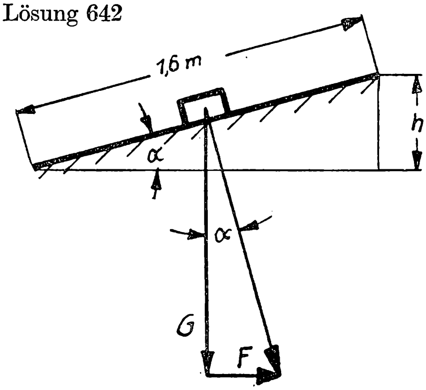


An der tiefsten Stelle der Bewegungsbahn gilt:

$$T = G + m r \omega^2$$

$$\omega = \sqrt{\frac{T - g}{m \cdot r}} = \underline{\underline{4{,}44\ 1/\text{sek}}}$$

Lösung 642



$$\operatorname{tg}\alpha = \frac{F}{G} = \frac{G r \omega^2}{g G} = \frac{v^2}{r g} = \frac{10^2}{400 \cdot 9{,}81}$$
$$= 0{,}0255$$

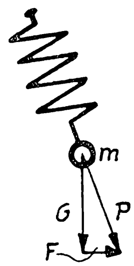
Da der Winkel α klein ist, kann gesetzt werden:

$$\operatorname{tg}\alpha = \alpha,$$

also:

$h = 0{,}0255 \cdot 169 \cong \underline{\underline{4{,}1\ \text{cm}}}$

Lösung 643

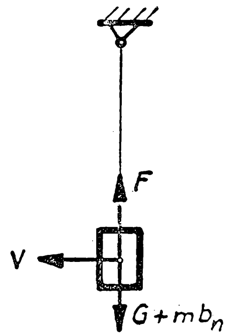


$$F = \frac{m v^2}{r} = \sqrt{P^2 - G^2}$$

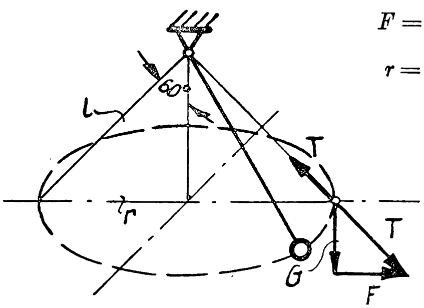
$$r = \frac{m v^2}{\sqrt{P^2 - G^2}} = \frac{5 \cdot \left(\frac{72}{3{,}6}\right)^2}{\sqrt{5{,}1^2 - 5^2}} = \underline{\underline{202\ \text{m}}}$$

Lösung 644

$$F = G + m b_n = m\left(g + \frac{v^2}{r}\right) = \frac{2}{9{,}81}\left(9{,}81 + \frac{5^2}{1}\right) = \underline{\underline{7{,}1\ \text{kg}}}$$



Lösung 645



$$F = m \frac{v^2}{r}$$

$$r = l \sin 60^\circ = l\, \frac{1}{2} \sqrt{3}\,; \qquad F = G \operatorname{tg} 60^\circ = G \sqrt{3}$$

$$G \operatorname{tg} 60^\circ = \frac{m v^2}{l \sin 60^\circ}$$

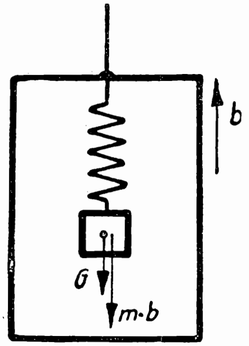
$$v = \sqrt{g l \cdot \sqrt{3}\, \frac{\sqrt{3}}{2}}\,; \qquad v = \sqrt{9{,}81 \cdot 30 \cdot \frac{3}{2}}$$
$$= \underline{\underline{210\ \text{cm/sek}}}$$

$$T = \frac{G}{\cos 60^\circ} = \frac{1}{0{,}5} = \underline{\underline{2\ \text{kg}}}$$

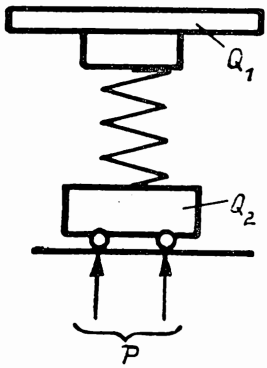
Lösung 646

$$P = G - F = m\left(g - \frac{v^2}{r}\right)$$
$$= \frac{1000}{9{,}81}\left(9{,}81 - \frac{10^2}{50}\right) = \underline{\underline{796\ \text{kg}}}$$

Lösung 647



$$P = G + m \cdot b$$
$$b = \frac{P - G}{m} = \frac{0{,}1}{5}\, 9{,}81 = \underline{\underline{0{,}196\ \text{m/sek}^2}}$$



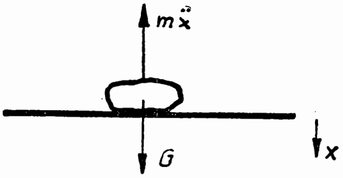
Lösung 648

$$N = (Q_1 + Q_2) + m_1 b_1$$
$$x = 2 \sin 10\,t \qquad \ddot{x} = -200 \sin 10\,t$$
$$b_{\max} = \pm 200\ \text{cm/sek}^2 \triangleq \pm 2\ \text{m/sek}^2$$
$$N_1 = N_{\max} = Q_2 + m_1 (g + b) = \underline{\underline{13{,}04\ \text{t}}}$$
$$N_2 = N_{\min} = Q_2 + m_1 (g - b) = \underline{\underline{8{,}96\ \text{t}}}$$

Lösung 649

$$x = r\left(\cos \omega t + \frac{r}{4l} \cos 2\,\omega t\right); \qquad \dot{x} = r\,\omega\left(-\sin \omega t - \frac{r}{4l} \cdot 2 \sin 2\,\omega t\right)$$
$$\ddot{x} = r\,\omega^2\left(-\cos \omega t - \frac{r}{l} \cos 2\,\omega t\right); \qquad \ddot{x}_{\max} = (\ddot{x})_{\omega t = 0} = -r\,\omega^2\left(1 + \frac{r}{l}\right)$$
$$\underline{\underline{P = \frac{Q}{g}\, r\,\omega^2\left(1 + \frac{r}{l}\right)}}$$

Lösung 650



$$x = \alpha \sin \omega t; \qquad \ddot{x} = -\omega^2 \cdot x \qquad \ddot{x} \cdot m = G$$
$$\ddot{x}_{\max} = \omega^2 x_{\max}; \qquad x_{\max} = \alpha$$
$$\omega^2 = \frac{g}{x_{\max}} = \frac{981}{5}; \qquad \underline{\underline{\omega = 14\ 1/\text{sek}}}$$

Lösung 651

$$s = 10 \sin \frac{\pi}{2} t; \qquad \ddot{s} = -10 \frac{\pi^2}{4} \sin \frac{\pi}{2} t$$

$$P = m\ddot{s} = -m\, 10 \frac{\pi^2}{4} \cdot \sin \frac{\pi}{2} t$$

$$= -\frac{G}{g} \frac{\pi^2}{4} s = \underline{\underline{-5{,}03 \cdot s\,\mathrm{g}}}$$

$$P_{\max} = 5{,}03 \cdot s_{\max} = \underline{\underline{50{,}3\ \mathrm{g}}}$$

Lösung 652

$$x = 3 \cos 2\pi t\,(\mathrm{cm}); \qquad y = 4 \sin \pi t\,(\mathrm{cm})$$

$$\ddot{x} = -4\pi^2 x; \qquad \ddot{y} = -\pi^2 y$$

$$X = m\ddot{x} = -\frac{2 \cdot 4\pi^2}{981} x; \qquad Y = m\ddot{y} = -\frac{2}{981} \cdot \pi^2 y$$

$$X = \underline{\underline{-0{,}08\, x\,\mathrm{g}}}; \qquad Y = \underline{\underline{-0{,}02\, y\,\mathrm{g}}}$$

Lösung 653

$$x = 490t - 245(1 - e^{-2t}); \qquad \dot{x} = 490 - 2 \cdot 245 e^{-2t}$$

$$\ddot{x} = 4 \cdot 245 e^{-2t}$$

$$m\ddot{x} = G - P_L$$

$$P_L = m(g - \ddot{x}) = 2m(490 - 490 e^{-2t}) = \underline{\underline{2mv}}$$

Lösung 654

1. $b = \frac{v - v_0}{t} = \frac{0{,}5}{0{,}5} = 1\ \mathrm{m/sek^2}$

$$P_1 = \frac{Q_1 + Q_2}{g} \cdot b + (Q_1 + Q_2)\,\mu_1 = \underline{\underline{242\ \mathrm{kg}}}$$

2. $P_2 = (Q_1 + Q_2)\,\mu_2 = \underline{\underline{70\ \mathrm{kg}}}$

Lösung 655

Für konstante Geschwindigkeit gilt:

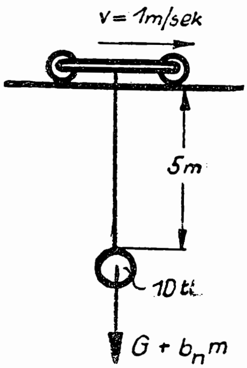
$$S_1 = Q \sin\alpha - Q \cos\alpha\,\mu = 700\,(0{,}258 - 0{,}966 \cdot 0{,}015) = \underline{\underline{171{,}5\,\mathrm{kg}}}$$

Für den Bremsweg gilt:

$$b = \frac{v}{t} = \frac{1{,}6}{4} = \underline{\underline{0{,}4\ \mathrm{m/sek^2}}}$$

$$S_2 = Q\left(\sin\alpha - \cos\alpha\,\mu + \frac{b}{g}\right) = \underline{\underline{200{,}1\ \mathrm{kg}}}$$

Lösung 656



$$S = m\left(g + \frac{v^2}{r}\right) = \frac{10\,000}{9{,}81}\left(9{,}81 + \frac{1}{5}\right) = 10\,200 \text{ kg}$$

$$S = \underline{\underline{10{,}2 \text{ t}}}$$

Lösung 657

$$\operatorname{tg}\alpha = \frac{F}{G} = \frac{m\frac{v^2}{r}}{mg} = \frac{v^2}{gr}; \qquad \alpha = \underline{\underline{18^\circ\,47'}}$$

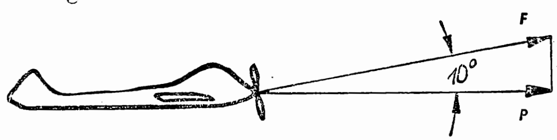
$$N = \frac{G}{\cos\alpha} = \underline{\underline{1{,}585 \text{ t}}}$$

Lösung 658

$$b = \frac{v}{t} = \frac{54}{3{,}6 \cdot 60} = 0{,}25 \text{ m/sek}^2$$

$$Z = mb + N\mu = \frac{200}{9{,}81} \cdot 0{,}25 + 200 \cdot 0{,}005 = \underline{\underline{6{,}1 \text{ t}}}$$

Lösung 659



$$P = mb + kv^2$$

$$k = 0{,}05$$

$$P = \frac{2000}{9{,}81} \cdot 5 + 0{,}05 \cdot 200^2$$

$$= 3020 \text{ kg}$$

$$F = \frac{P}{\cos 10^\circ} = \underline{\underline{3080 \text{ kg}}}$$

Lösung 660

$$v = bt + v_0; \quad t = \frac{v - v_0}{b}; \quad s = \frac{bt^2}{2} + v_0 t = \frac{v^2 - v_0^2}{2b}$$

$$b = \frac{v^2 - v_0^2}{2s}; \quad \begin{matrix} v = 0 \\ s = 10 \text{ m} \end{matrix}; \quad b = -\frac{v_0^2}{2s} = -\frac{6^2}{20} = -\underline{\underline{1{,}8 \text{ m/sek}^2}}$$

$$P = mb = \frac{6}{9{,}81} \cdot 1{,}8 = 1{,}1 \text{ t}$$

verteilt auf 2 Seile ergibt die Seilkraft: $\underline{\underline{550 \text{ kg}}}$

Lösung 661

$$\omega T = 2\pi; \quad \omega = \frac{2\pi}{TP}$$

$$x_A = 1 \cdot \sin \omega t = \sin \frac{2\pi}{TP} t = \sin \frac{2\pi}{0{,}25} t = \underline{\underline{\sin 8\pi t}}$$

$$\ddot{x} = -64\pi^2 \sin 8\pi t$$

$$R_{\max} = P_A + P_B + m_A \cdot b_{A\max}; \quad b_{A\max \atop \min} = \pm 64\pi^2; \quad R_{\max} = \underline{\underline{72{,}8\ \mathrm{kg}}}$$

$$R_{\min} = P_A + P_B + m_A b_{A\min} = \underline{\underline{47{,}2\ \mathrm{kg}}}$$

Lösung 662

$$\ddot{x} + \frac{c}{m} x = 0$$

$$x = A \sin \sqrt{\frac{c}{m}}\, t$$

Zeit einer Schwingung: $t_0 = \frac{2{,}1}{6} = 0{,}35$ sek

$$\sqrt{\frac{c}{m}}\, t_0 = 2\pi; \quad c = \left(\frac{2 \cdot \pi}{0{,}35}\right)^2 \cdot \frac{5}{981} = \underline{\underline{1{,}65\ \mathrm{kg/cm}}}$$

Lösung 663

$$v = \frac{1000}{3{,}6} = 278\ \mathrm{m/sek}$$

$$P = m\left(g + \frac{v^2}{r}\right) = \frac{80}{9{,}81}\left(9{,}81 + \frac{278^2}{600}\right) = \underline{\underline{1130\ \mathrm{kg}}}$$

Lösung 664

$$G_{\text{Erde}} = m g_{\text{Erde}}$$

$$G_{\text{Mond}} = m g_{\text{Mond}} = \frac{G_{\text{Erde}}}{g_{\text{Erde}}} g_{\text{Mond}} = \frac{1}{9{,}81} \cdot 1{,}7 = \underline{\underline{0{,}1735\ \mathrm{kg}}}$$

$$G_{\text{Sonne}} = \frac{G_{\text{Erde}}}{g_{\text{Erde}}} g_{\text{Sonne}} = \frac{1}{9{,}81} \cdot 270 = \underline{\underline{27{,}55\ \mathrm{kg}}}$$

Lösung 665

Umfangsgeschwindigkeit des Schmiertopfes:

$$u_K = v \frac{r \cdot 2}{D}$$

$$b_n = \frac{u_K^2}{r} = \frac{v^2 r \cdot 4}{D^2}$$

Ausfluß wenn $b_n = g$

$$v^2 = \frac{g\left(\frac{D}{2}\right)^2}{r}$$

$$v = \sqrt{\frac{9{,}81 \cdot 0{,}95^2}{0{,}325}} = 5{,}22\ \mathrm{m/sek}$$

$$v \geqq \underline{\underline{18{,}8\ \mathrm{km/h}}}$$

Lösung 666

Krümmung der Brückendurchbiegung:

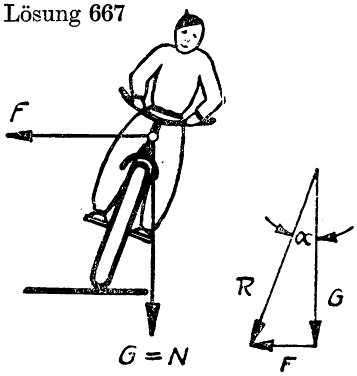
$$y'' = \frac{1}{\varrho} = -\frac{M}{EJ} = \frac{Ql}{EJ \cdot 4} \qquad \varrho = \text{Krümmungsradius}$$

$$h = \frac{Ql^3}{48EJ}; \quad EJ = \frac{Ql^3}{48h}; \qquad \frac{1}{\varrho} = \frac{Ql48h}{Ql^3 4} = 12\frac{h}{l^2}$$

Zusätzliche Belastung:

$$C = \frac{mv^2}{\varrho} = \frac{Qv^2 12h}{gl^2} = \underline{\underline{0{,}88\ \text{t}}}$$

Lösung 667



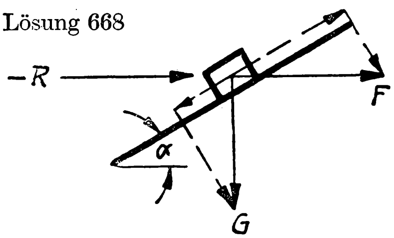
$$F = m\frac{v^2}{r}$$

$$\operatorname{tg}\alpha = \frac{F}{G} = \frac{v^2}{rg} = 0{,}255$$

$$\alpha = \underline{\underline{14^\circ\ 20'}}$$

$$N\mu = F; \qquad \mu = \frac{F}{N} = \underline{\underline{0{,}255}}$$

Lösung 668



$$F = \frac{mv^2}{R}; \quad \text{Gleichgewicht } \mu N = \sum P_T$$

$$\mu\left(G\cos\alpha + \frac{G}{g}\frac{v^2}{R}\sin\alpha\right) = \pm G\sin\alpha \mp \frac{G}{g}\frac{v^2}{R}\cos\alpha$$

$$\frac{v^2}{gR}(\mu\sin\alpha \pm \cos\alpha) = \pm\sin\alpha - \mu\cos\alpha$$

$$v^2 = gR\frac{\pm\operatorname{tg}\alpha - \mu}{\mu\operatorname{tg}\alpha \pm 1}$$

$$\underline{\underline{v_{\min} = \sqrt{gR\frac{\operatorname{tg}\alpha - \mu}{1 + \mu\operatorname{tg}\alpha}}}}$$

$$\underline{\underline{v_{\max} = \sqrt{gR\frac{\operatorname{tg}\alpha + \mu}{1 - \mu\operatorname{tg}\alpha}}}}$$

Lösung 669

$$K = m\frac{v^2}{r}; \quad K = \text{Federkraft}$$

$$v = \omega r = \frac{\pi n}{30}\cdot r = 4\pi r; \qquad K = m\,16\pi^2 r$$

$$K_{\max} = m\cdot 16\pi^2 r_{\max}; \qquad r_{\max} = 147{,}5 + 2{,}5 = 150\ \text{cm}$$

$$= \frac{1{,}5}{9{,}81}\cdot 16\pi^2\cdot 150 = \underline{\underline{36{,}2\ \text{kg}}}$$

Annahme: Die Feder ist bei $n = 0$ ohne Vorspannung

$$c = \frac{K_{\max}}{e} = \frac{36{,}2}{2{,}5} = \underline{\underline{14{,}5\ \text{kg/cm}}}$$

Lösung 670

$$P = cf = mr\omega^2 \qquad n = 120; \quad \omega = 4\pi$$

$$f = (r-5)$$

$$c(r-5) = mr\omega^2; \qquad r = \frac{5}{1 - \frac{m\omega^2}{c}} = \underline{\underline{6{,}58\ \text{cm}}}$$

Lösung 671

$$F_1 = mr_1\omega_1^2 = m \cdot 0{,}85 \cdot (50\pi)^2$$

$$F_2 = mr_2\omega_2^2 = m \cdot 1{,}3 \cdot (55\pi)^2$$

$$c = \frac{F_2 - F_1}{f} = \frac{m\pi^2(1{,}3 \cdot 55^2 - 0{,}85 \cdot 50^2)}{0{,}45} = \underline{\underline{9{,}08\ \text{kg/cm}}}$$

Lösung 672

$$x = v_0 t; \qquad y = +\frac{b}{a}\sqrt{a^2 - x^2}$$

$$\frac{dy}{dt} = \frac{dy}{dx}\,\frac{dx}{dt} \qquad \frac{dx}{dt} = v_0; \quad \frac{dy}{dx} = -\frac{b}{a}\,\frac{x}{\sqrt{a^2-x^2}}$$

$$\frac{d^2y}{dt^2} = \frac{dy}{dx}\,\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 \frac{d^2y}{dx^2} \qquad \frac{d^2x}{dt^2} = 0; \quad \frac{d^2y}{dx^2} = -\frac{b}{a}\,\frac{(a^2-x^2)+x^2}{(a^2-x^2)^{3/2}} = -\frac{b^4}{a^2y^3}$$

$$F_y = m\frac{d^2y}{dt^2} = \underline{\underline{-\frac{v_0^2 b^4}{a^2y^3} \cdot m}}$$

Lösung 673

$$x = a\cos kt \qquad \ddot{x} = -k^2 x$$

$$y = b\sin kt \qquad \ddot{y} = -k^2 y$$

$$b = \sqrt{\ddot{x}^2 + \ddot{y}^2} = k^2\sqrt{x^2+y^2} = k^2 \cdot r$$

$$\underline{\underline{F = mb = mk^2 r}}$$

27. Bewegungsgleichungen der Punktdynamik

a) Geradlinige Bewegung

Lösung 674

$$s = \frac{gt^2}{2}; \qquad \text{Fallzeit} \quad t_F = \sqrt{\frac{2s}{g}}$$

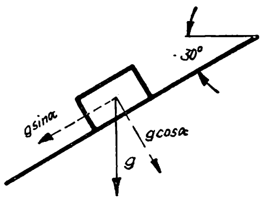
$$\text{Schallzeit} \quad t_s = \frac{s}{v_s}$$

$$t_{\text{ges}} = t_F + t_s = \sqrt{\frac{2s}{G}} + \frac{s}{v_s}$$

Auflösen nach s: $\frac{2s}{g} = \left(t_{\text{ges}} - \frac{s}{v_s}\right)^2$; $\quad s^2 - \left(2t_{\text{ges}} + \frac{2v_s}{g}\right)v_s s + t_{\text{ges}}^2 v_s^2 = 0$

$$\underline{\underline{s = 175\ \text{m}}}$$

Lösung 675



$$s = \frac{bt^2}{2} + v_0 t \qquad b = g \sin\alpha$$

$$s = \frac{gt^2 \sin\alpha}{2} + v_0 t$$

$$t^2 + \frac{2v_0}{g\sin\alpha} t - \frac{2s}{g\sin\alpha} = 0$$

$$t_{1,(2)} = \frac{v_0}{g\sin\alpha} \underset{-}{(+)} \sqrt{\left(\frac{v_0}{g\sin\alpha}\right)^2 + \frac{2s}{g\sin\alpha}}$$

$$\underline{\underline{t = 1{,}61 \text{ sek}}}$$

Lösung 676

$$P = m \cdot b; \qquad b = \frac{v}{t}; \qquad t = \frac{2l}{v}; \qquad t = \frac{4}{570} = \underline{\underline{0{,}007 \text{ sek}}}; \qquad b = \frac{v^2}{2l}$$

$$P = \frac{G}{g} \cdot \frac{v^2}{2l} = \frac{6}{9{,}81} \cdot \frac{570^2}{2 \cdot 2} = 49{,}7 \cdot 10^3 \text{ kg} \mathrel{\hat{=}} \underline{\underline{49{,}7 \text{ t}}}$$

Lösung 677

$$\dot{x} = bt + v_0 \qquad t = 5 \text{ sek}; \quad \dot{x} = 0: \qquad v_0 = -5b \text{ m/sek}$$

$$x = b\left(\frac{t^2}{2} - 5t\right) \qquad b = \frac{x}{\frac{t^2}{2} - 5t} = \frac{24{,}5}{12{,}5 - 5 \cdot 5}$$

$$G\mu = mb$$

$$\underline{\underline{\mu = \frac{b}{g} = 0{,}2}}$$

Lösung 678

$$P = m \cdot b$$

$$300 = \frac{1000}{9{,}81} \cdot b; \qquad b = 0{,}3\,g; \qquad t = \frac{v_0}{b} = \frac{36}{3{,}6 \cdot 0{,}3 \cdot g} = \underline{\underline{3{,}4 \text{ sek}}}$$

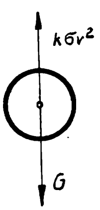
$$s = v_0 t - \frac{bt^2}{2} = \frac{36}{3{,}6} \cdot 3{,}4 - \frac{0{,}3 \cdot g \cdot 3{,}4^2}{2} = \underline{\underline{16{,}9 \text{ m}}}$$

Lösung 679

$$v = bt + v_0 \qquad v = 0: \quad b = -\frac{v_0}{t}$$

$$s = \frac{bt^2}{2} + v_0 t = \frac{v_0 t}{2} \qquad \underline{\underline{t = \frac{2s}{v_0} = 0{,}2 \text{ sek}}}$$

Lösung 680



$$b = \frac{P}{m} = \frac{G - k\sigma v^2}{m}$$

$$b = 0 \quad \text{wenn} \quad v = v_{\max}$$

$$v_{\max} = \sqrt{\frac{G}{k\sigma}} = \underline{\underline{144 \text{ m/sek}}}$$

Lösung 681

$$v_{1\max} = \sqrt{\frac{G_1}{k\sigma}} \qquad \text{da} \quad k_1 = k_2 \qquad \frac{v_{1\max}}{v_{2\max}} = \sqrt{\frac{G_1}{G_2}} = \underline{\underline{\sqrt{\frac{\gamma_1}{\gamma_2}}}}$$

$$v_{2\max} = \sqrt{\frac{G_2}{k\sigma}} \qquad \sigma_1 = \sigma_2$$

Lösung 682

$$b = \frac{P}{m} = \frac{-(G\sin\alpha + G\mu\cos\alpha)}{\frac{G}{g}} = -g(\sin\alpha + \mu\cos\alpha)$$

$$v = bt + v_0; \qquad \text{Endbedingung:} \quad v = 0: \quad t = -\frac{v_0}{b}$$

$$s = \frac{bt^2}{2} + v_0 t = -\frac{v_0^2}{2b} = \frac{v_0^2}{2g(\sin\alpha + \mu\cos\alpha)} = \underline{\underline{19{,}55\,\text{m}}}$$

$$T = \frac{v_0}{g(\sin\alpha + \mu\cos\alpha)} = \underline{\underline{2{,}61\,\text{sek}}}$$

Lösung 683

$$G\sin\alpha = G\mu\cos\alpha + a v_{\max}^2; \qquad v_{\max} = \sqrt{\frac{G(\sin\alpha - \mu\cos\alpha)}{a}}$$

$$v_{1\max} = 3{,}6\sqrt{\frac{90\,\frac{\sqrt{2}}{2}\,(1-0{,}1)}{0{,}0635}} = \underline{\underline{108\,\text{km/h}}}$$

$$v_{2\max} = 3{,}6\sqrt{\frac{90\,\frac{\sqrt{2}}{2}\,(1-0{,}05)}{0{,}0635}} = \underline{\underline{111\,\text{km/h}}}$$

Lösung 684

$$T = W_{\max}; \qquad T_0\left(1 - \frac{v_{\max}}{v_s}\right) = a v_{\max}^2$$

$$v_{\max}^2 + \frac{T_0}{a v_s} v_{\max} - \frac{T_0}{a} = 0$$

$$v_{\max} = -\frac{T_0}{2 a v_s} \pm \sqrt{\left(\frac{T_0}{2 a v_s}\right)^2 + \frac{T_0}{a}} = -15{,}2 + \sqrt{15{,}2^2 + 100}$$

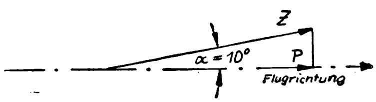
$$= 20\,\text{m/sek} \mathrel{\hat{=}} \underline{\underline{72\,\text{km/h}}}$$

Lösung 685

$$P = Z \cdot \cos\alpha = 3030\,\text{kg}$$

$$W = a v^2; \quad \text{für} \quad v = 1\,\text{m/sek} \quad \text{ist} \quad W = 0{,}05\,\text{kg}$$

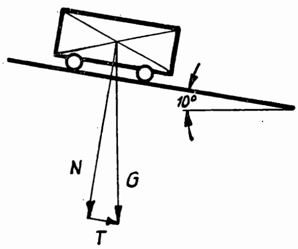
$$\text{also} \quad a = \frac{W}{v^2} = \frac{0{,}05}{1} = 0{,}05\,\frac{\text{kg}\,\text{sek}^2}{\text{m}^2}$$



$$P = W_{\max}; \quad P = a v_{\max}^2$$

$$v_{\max} = \sqrt{\frac{P}{a}} = \sqrt{\frac{3030}{0{,}05}} = \underline{\underline{246\,\text{m/sek}}}$$

Lösung 686



1. Bestimmung des Reibungswiderstandes:
   Bei $\alpha = 10°$ bewegt sich der Wagen mit konstanter Geschwindigkeit, also:

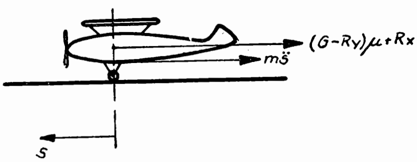
$$G\mu\cos\alpha = G\sin\alpha$$
$$\mu = \operatorname{tg}\alpha = 0{,}1765$$

2. Beschleunigte Bewegung:

$$G\mu\cos\beta + mb = G\sin\beta$$
$$b = (\sin\beta - \mu\cos\beta)\,g = (\sin\beta - \operatorname{tg}\alpha\cos\beta)\,g$$
$$= \frac{\sin(\beta-\alpha)}{\cos\alpha}\,g = \underline{\underline{0{,}87\ \text{m/sek}^2}}$$
$$v = bt = \frac{\sin(\beta-\alpha)}{\cos\alpha}\,g\cdot t = \underline{\underline{1{,}74\ \text{m/sek}}}$$
$$s = \frac{bt^2}{2} = \frac{\sin(\beta-\alpha)}{\cos\alpha}\,g\,\frac{t^2}{2} = \underline{\underline{17{,}4\ \text{m}}}$$

Lösung 687

$$R_x = k_x\cdot v^2;\qquad k_x = 0{,}09\ \text{kg}\,\frac{\text{sek}^2}{\text{m}^2}$$
$$R_y = k_y\cdot v^2;\qquad k_y = 0{,}7\ \text{kg}\,\frac{\text{sek}^2}{\text{m}^2}$$
$$m\ddot{s} + k_x\dot{s}^2 + \mu\,(mg - k_y\cdot\dot{s}^2) = 0$$
$$\underline{\ddot{s} + \frac{k_x - \mu k_y}{m}\cdot\dot{s}^2 + \mu\cdot g = 0}$$



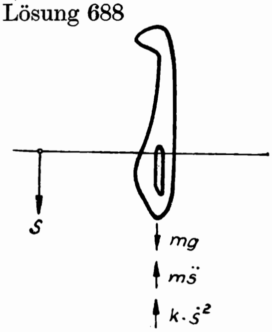
$$\ddot{s} = \frac{d\dot{s}}{ds}\cdot\dot{s};\qquad \frac{k_x-\mu k_y}{m} = A;\qquad \mu\cdot g = B$$
$$\dot{s}\,\frac{d\dot{s}}{ds} + A\dot{s}^2 + B = 0;$$
$$-\frac{\dot{s}\,d\dot{s}}{B + A\dot{s}^2} = ds;\qquad s = 0 : \dot{s} = v_0$$
$$-\frac{1}{2A}\ln\left(\frac{B + A\dot{s}^2}{B + Av_0^2}\right) = s$$

$$s_{(\dot{s}=0)} = \frac{1}{2A}\cdot\ln\left(1 + \frac{A}{B}\,v_0^2\right) = \frac{1}{34\cdot 9{,}81}\cdot 10^6\ln\left(1 + \frac{34}{16}\cdot 10^{-4}\cdot 18{,}5^2\right) = \underline{\underline{216\ \text{m}}}$$

$$\dot{v} + Av^2 + B = 0;\qquad \frac{dv}{Av^2 + B} = -\,dt;\qquad \frac{1}{\sqrt{AB}}\operatorname{arc\,tg}\sqrt{\frac{A}{B}}\,v = -t + T;\qquad t = 0 : v = v_0$$

$$T = \frac{1}{\sqrt{AB}}\operatorname{arc\,tg}\sqrt{\frac{A}{B}}\,v_0 = \frac{10^4}{g\sqrt{136}}\operatorname{arc\,tg}\left(\sqrt{\frac{34}{16}}\cdot 10^{-2}\cdot 18{,}5\right) = \underline{\underline{22{,}5\ \text{sek}}}$$

Lösung 688



$$m\ddot{s}+k\dot{s}^2-mg=0;\quad \ddot{s}=\frac{d\dot{s}}{ds}\cdot\frac{ds}{dt}$$

$$mv\cdot\frac{dv}{ds}+kv^2-mg=0$$

$$\frac{mv\,dv}{mg-kv^2}=ds;\quad -\frac{m}{2k}\ln C\,(mg-kv^2)=s$$

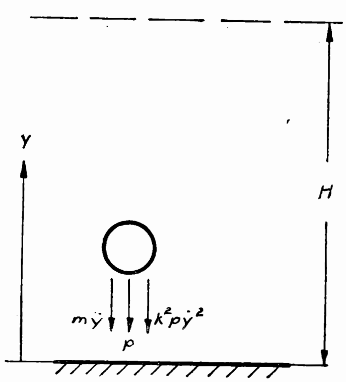
$$s=0;\quad v=0:\quad C=\frac{1}{mg}$$

$$\underline{1-\frac{k}{mg}\cdot v^2=e^{-\frac{2k}{m}\cdot s}}$$

$$s\to\infty;\quad v\to v_{\max}:\quad \underline{v_{\max}^2=\frac{mg}{k}};\quad 1-\frac{v^2}{v_{\max}^2}=e^{-\frac{2k}{m}\cdot s}$$

$$\underline{\underline{v=v_{\max}\sqrt{1-e^{-\frac{2gs}{v_{\max}^2}}}}}$$

Lösung 689



$$m\ddot{y}+k^2p\dot{y}^2+p=0$$

$$\dot{v}+\frac{k^2p}{m}v^2+\frac{p}{m}=0;\quad p=m\cdot g;\quad k^2\cdot g=a$$

$$\frac{dv}{dt}+av^2+g=0;\qquad \frac{dv}{av^2+g}=-dt$$

$$-t=\frac{1}{\sqrt{ag}}\operatorname{arc\,tg}\left(\sqrt{\frac{a}{g}}\cdot v\right)+C;\quad t=0;\quad v=v_0$$

$$T=\frac{1}{\sqrt{ag}}\operatorname{arc\,tg}\sqrt{\frac{a}{g}}\,v_0$$

$$\underline{\underline{T=\frac{1}{gk}\operatorname{arc\,tg}(kv_0)}}$$

$$\frac{dv}{ds}\cdot v+av^2+g=0;\quad \frac{v\,dv}{av^2+g}=-ds$$

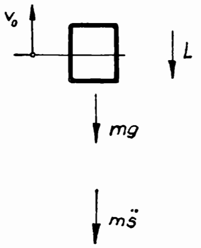
$$-s=\frac{1}{2a}\ln\frac{(av^2+g)}{C};\quad s=0;\quad v=v_0$$

$$C=av_0^2+g$$

$$s=H;\quad v=0$$

$$H=\frac{1}{2a}\ln\frac{av_0^2+g}{g};\quad \underline{\underline{H=\frac{\ln(k^2v_0^2+1)}{2k^2g}}}$$

Lösung 690



$$m\ddot{s} + k\dot{s} + mg = 0$$

$$\frac{m\,d\dot{s}}{mg + k \cdot \dot{s}} = -dt$$

$$\frac{m}{k} \ln C\,(mg + k\dot{s}) = -t$$

$$t = 0; \qquad v = v_0; \qquad C = \frac{1}{mg + kv_0}$$

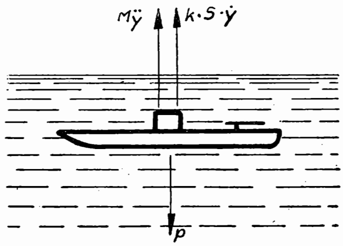
$$-t = \frac{m}{k} \ln\left(\frac{mg + k\dot{s}}{mg + kv_0}\right)$$

$$t = t_s \quad \text{für} \quad v = 0:$$

$$t_s = \frac{m}{k} \ln\left(\frac{mg + kv_0}{mg}\right)$$

$$t_s = 5{,}1 \cdot \ln 1{,}41; \qquad t_s = \underline{\underline{1{,}7\,\text{sek}}}$$

Lösung 691



$$M \cdot \ddot{y} + k \cdot S \cdot \dot{y} - p = 0$$

$$\dot{v} + \frac{k \cdot S \cdot v}{M} - \frac{p}{M} = 0$$

$$\frac{M\,dv}{p - kSv} = dt; \qquad -\frac{M}{kS} \ln C\,(p - kSv) = t$$

$$t = 0; \qquad v = 0: \qquad C = \frac{1}{p}$$

$$\underline{\underline{v = \frac{p}{kS}\left(1 - e^{-\frac{kS}{M} \cdot t}\right)}}$$

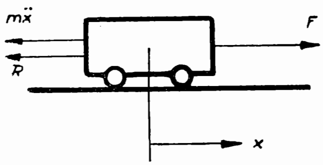
Lösung 692

$$\dot{s} = v = \frac{p}{k \cdot S}\left(1 - e^{-\frac{kS}{M} \cdot t}\right); \qquad s = \frac{p}{k \cdot S}\left(t + \frac{M}{kS} \cdot e^{-\frac{kS}{M}t} + C\right)$$

$$t = 0; \qquad s = 0: \qquad C = -\frac{M}{k \cdot S}$$

$$t = T; \qquad s = z; \qquad \underline{\underline{z = \frac{p}{kS}\left[T - \frac{M}{kS}\left(1 - e^{-\frac{kS}{M} \cdot T}\right)\right]}}$$

Lösung 693



$$R = Q\,(2{,}5 + 0{,}05v)\,\text{kg}$$

$$Q \text{ in t}; \quad v \text{ in m/sek}$$

$$m = \frac{Q\,[\text{t}] \cdot 1000}{g} \; \frac{\text{kg}\,\text{sek}^2}{\text{m}}$$

$$\dot{v} + \frac{g \cdot 2{,}5}{1000} + \frac{g \cdot 0{,}05}{1000} \cdot v - \frac{F \cdot g}{Q \cdot 1000} = 0$$

$$\dot{v} + 24{,}5 \cdot 10^{-3} + 0{,}49 \cdot 10^{-3} v - 49 \cdot 10^{-3} = 0$$

$$\dot{v} - 24{,}5 \cdot 10^{-3} + 0{,}49 \cdot 10^{-3} v = 0$$

$$\frac{dv \cdot 10^3}{24{,}5 - 0{,}49v} = dt; \qquad -\frac{10^3}{0{,}49}\ln C(24{,}5 - 0{,}49v) = t \qquad t = 0; \qquad v = 0$$

$$C = \frac{1}{24{,}5}$$

$$t = T; \qquad v = 12\ \text{km/h} \mathrel{\hat{=}} 3{,}33\ \text{m/sek}:$$

$$T = \frac{10^3}{0{,}49}\ln\left(\frac{24{,}5}{22{,}9}\right) = \underline{\underline{141\ \text{sek}}}$$

$$\dot{v} = \frac{dv}{ds}\cdot v; \qquad \frac{dv}{ds}\cdot v - 24{,}5\cdot 10^{-3} + 0{,}49\cdot 10^{-3}v = 0$$

$$\frac{v\cdot dv\cdot 10^3}{24{,}5 - 0{,}49\cdot v} = ds;$$

$$-\frac{10^3}{0{,}49^2}[24{,}5\ln(24{,}5 - 0{,}49\cdot v) - 24{,}5 + 0{,}49v + C] = s$$

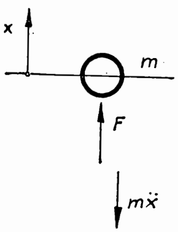
$$s = 0; \qquad v = 0: \qquad C = 24{,}5 - 24{,}5\ln 24{,}5$$

$$s_{\left(v = 12\frac{\text{km}}{\text{h}}\right)} = \frac{10^3}{0{,}49^2}\left[24{,}5\ln\frac{24{,}5}{24{,}5 - 0{,}49v} - 0{,}49v\right] = \underline{\underline{245\ \text{m}}}$$

Bewegung mit konst. Geschwindigkeit $v = 3{,}33$ m/sek:

$$Q(2{,}5 + 0{,}05\cdot 3{,}33) = N; \qquad N = \underline{\underline{106{,}6\ \text{kg}}}$$

Lösung 694



$$E = A\cdot\sin(kt); \qquad F = e\cdot E$$

$$m\ddot{x} - e\cdot A\cdot\sin(kt) = 0; \qquad \dot{x} = -\frac{eA}{km}\cos(kt) + C_1$$

$$t = 0; \quad \dot{x} = 0: \quad C_1 = \frac{eA}{km}$$

$$x = \frac{eA}{km}\left(t - \frac{1}{k}\sin(kt) + C_2\right)$$

$$t = 0; \quad x = 0: \quad C_2 = 0$$

$$\underline{\underline{x = \frac{eA}{km}\left(t - \frac{\sin(kt)}{k}\right)}}$$

Lösung 695

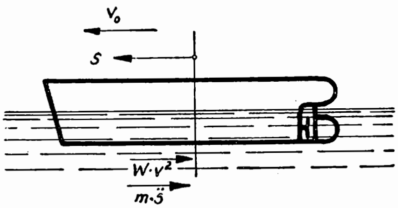
$$m\ddot{s} + a\dot{s}^2 - T = 0; \qquad \dot{v} + \frac{a}{m}v^2 - \frac{T}{m} = 0; \qquad \dot{v} = \frac{dv}{ds}\cdot v$$

$$\frac{v\cdot dv}{\frac{a}{m}v^2 - \frac{T}{m}} = -ds; \qquad \frac{m}{2a}\ln C\left(\frac{a}{m}v^2 - \frac{T}{m}\right) = -s$$

$$s = 0; \qquad v = v_0: \qquad C = \frac{m}{av_0^2 - T}; \qquad 2\frac{as}{m} = \ln\frac{av_0^2 - T}{av_1^2 - T}$$

$$\underline{\underline{T = \frac{a\left(v_0^2 - v_1^2 e^{\frac{2asg}{p}}\right)}{1 - e^{\frac{2asg}{p}}}\ \text{kg}}}$$

Lösung 696



$$v_0 = 16\,\text{m/sek}; \quad W = 30\,t\,\frac{\text{sek}^2}{\text{m}^2}$$

$$m\ddot{s} + W \cdot \dot{s}^2 = 0; \quad \frac{m}{W}\frac{d\dot{s}}{\dot{s}^2} = -dt$$

$$-\frac{m}{W \cdot \dot{s}} = -t + C_1; \quad t = 0; \quad \dot{s} = v_0:$$

$$C_1 = -\frac{m}{W \cdot v_0}$$

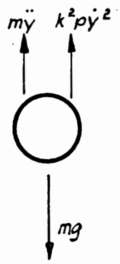
$$t = \frac{m}{W}\left(\frac{1}{\dot{s}} - \frac{1}{v_0}\right); \quad t_{(\dot{s} = 4\,\text{m/sek})} = \underline{\underline{6{,}38\,\text{sek}}}$$

$$\dot{s} = \frac{\frac{m}{W}}{\frac{m}{W v_0} + t}; \quad s = \frac{m}{W}\ln C\left(\frac{m}{W v_0} + t\right); \quad t = 0; \quad s = 0: \quad C = \frac{W v_0}{m}$$

$$s = \frac{m}{W}\ln\left(1 + \frac{t \cdot W \cdot v_0}{m}\right)$$

$$s_{(t = 6{,}38\,\text{sek})} = \underline{\underline{47{,}1\,\text{m}}}$$

Lösung 697



$$p = m \cdot g$$

$$m\dot{v} + k^2 p v^2 - mg = 0$$

$$\dot{v} + k^2 g v^2 - g = 0$$

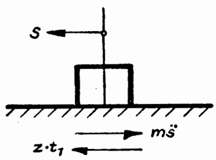
$$\frac{dv}{-k^2 g v^2 + g} = +dt; \quad t = 0; \quad v = 0$$

$$+t = \frac{1}{2kg}\ln\frac{kg + k^2 g v}{kg - k^2 g v}$$

$$e^{2kgt} = \frac{kg + k^2 g v}{kg - k^2 g v}; \quad \underline{\underline{v = \frac{1}{k}\,\frac{e^{kgt} - e^{-kgt}}{e^{kgt} + e^{-kgt}}}}$$

für $t \to \infty$ wird $\underline{\underline{v_{\max} = \frac{1}{k}}}$

Lösung 698



$$mg = 10000\,\text{kg}$$

$$W = \quad 200\,\text{kg}$$

$z =$ Zugkraftzuwachs $= 120$ kg/sek

$$m\ddot{s} - z \cdot t + R = 0$$

Mit $z \cdot t - R = z \cdot t_1$ ergibt sich: $t_1 = t - \frac{R}{z} = t - \frac{5}{3}$

$t =$ Zeit vom Beginn des Abschaltens von Widerständen.

$$m\ddot{s} - z \cdot t_1 = 0$$

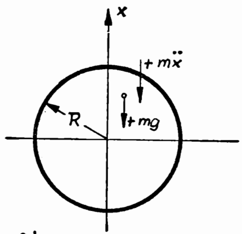
$$\dot{s} = \frac{z}{m}\frac{t_1^2}{2} + C_1 \qquad \left.\begin{matrix} t_1 = 0 \\ \dot{s} = 0 \end{matrix}\right\} C_1 = 0$$

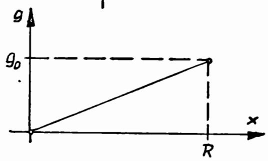
$$s = \frac{z}{m}\frac{t_1^3}{6} + C_2 \qquad \left.\begin{matrix} t_1 = 0 \\ s = 0 \end{matrix}\right\} C_2 = 0$$

$$s = \frac{z}{6m}\left(t - \frac{5}{3}\right)^3 = \frac{120 \cdot 9{,}81}{6 \cdot 10000}\left(t - \frac{5}{3}\right)^3$$

$$\underline{\underline{s = 0{,}01962\left(t - \frac{5}{3}\right)^3}} \quad [\text{m}]$$

Lösung 699





$$\underline{g = g_0 \cdot \frac{x}{R}}$$

$$\underline{m\ddot{x} + m g_0 \frac{x}{R} = 0}$$

Ansatz:

$$x = A \sin \alpha t + B \cos \alpha t$$
$$\dot{x} = A\alpha \cos \alpha t - B\alpha \sin \alpha t$$
$$\ddot{x} = -A\alpha^2 \sin \alpha t$$

Anfangsbedingungen:

$$t = 0 \qquad x = R \qquad \dot{x} = 0 \qquad \frac{R = B}{0 = A}; \qquad \alpha = \sqrt{\frac{g_0}{R}}$$

$$\underline{\underline{x = R \cos \sqrt{\frac{g_0}{R}}\, t}}$$

Geschwindigkeit im Erdmittelpunkt:

$$x = 0 = R \cos \sqrt{\frac{g_0}{R}}\, t; \qquad \cos \sqrt{\frac{g_0}{R}}\, t = 0$$

$$t = \frac{\pi}{2}\sqrt{\frac{R}{g_0}}; \qquad -\dot{x} = \sqrt{\frac{g_0}{R}} \cdot R \sin\left(\sqrt{\frac{g_0}{R}} \cdot \frac{\pi}{2}\sqrt{\frac{R}{g_0}}\right); \qquad \dot{x} = -\sqrt{g_0 \cdot R}$$

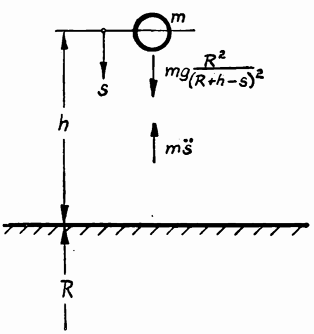
$$\dot{x} = -\sqrt{637 \cdot 10^6 \cdot 981} = 7{,}9 \cdot 10^5 \text{ cm/sek}$$

$$\underline{\underline{v = 7{,}9 \text{ km/sek}}}$$

Zeit bis zum Erreichen des Erdmittelpunktes:

$$\underline{\underline{T = \frac{\pi}{2}\sqrt{\frac{R}{g_0}} \mathrel{\hat{=}} 21{,}1 \text{ min}}}$$

Lösung 700



$$m\ddot{s} - mg \cdot \frac{R^2}{(R + h - s)^2} = 0$$

$$\ddot{s} = \frac{dv}{ds} \cdot v$$

$$v \cdot dv = \frac{g \cdot R^2 \cdot ds}{(R + h - s)^2}$$

$$\frac{v^2}{2} = \frac{g \cdot R^2}{R + h - s} + C_1; \qquad s = 0; \qquad v = 0$$

$$C_1 = -\frac{g R^2}{R + h}$$

$$\underline{\underline{v_{(s=h)} = \sqrt{\frac{2gRh}{R + h}}}}$$

$$\dot{s} = R\sqrt{\frac{2g}{R + h}}\sqrt{\frac{s}{R + h - s}}; \qquad T = \frac{1}{R}\sqrt{\frac{R + h}{2g}} \int\limits_{s=0}^{h} \sqrt{\frac{R + h - s}{s}}\, ds$$

$$\int_0^h \sqrt{\frac{R+h}{s}-1}\,ds = I; \qquad \frac{R+h}{s}-1=u^2; \qquad s=\frac{R+h}{u^2+1}$$

$$-\frac{R+h}{s^2}\,ds = 2u\,du; \qquad ds = -\frac{2u\,(R+h)}{(u^2+1)^2}\,du$$

Grenzen: Für $s=h$: $\quad u=\sqrt{\frac{R}{h}}$

Für $s=0$: $\quad u=\infty$

$$I=-2(R+h)\int_\infty^{\sqrt{\frac{R}{h}}} \frac{u^2\,du}{(u^2+1)^2}$$

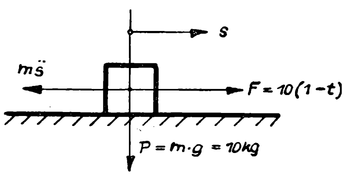
$$I=-2(R+h)\left[\int_\infty^{\sqrt{\frac{R}{h}}} \frac{du}{u^2+1} - \int_\infty^{\sqrt{\frac{R}{h}}} \frac{du}{(u^2+1)^2}\right]$$

$$I=-2(R+h)\left[-\operatorname{arc\,ctg} u - \frac{1}{2}\,\frac{u}{u^2+1} + \frac{1}{2}\operatorname{arc\,ctg} u\right]_\infty^{\sqrt{\frac{R}{h}}}$$

$$I=\sqrt{R\cdot h}+(R+h)\operatorname{arc\,ctg}\sqrt{\frac{R}{h}}=\sqrt{R\cdot h}+\frac{R+h}{2}\arccos\left(\frac{R-h}{R+h}\right)$$

Fallzeit: $$\underline{\underline{T=\frac{1}{R}\sqrt{\frac{R+h}{2g}}\left[\sqrt{R\cdot h}+\frac{R+h}{2}\arccos\left(\frac{R-h}{R+h}\right)\right]}}$$

Lösung 701



$$m\ddot{s}-10\,(1-t)=0$$

$$\ddot{s}=\frac{10}{m}\,(1-t)$$

$$\dot{s}=v=\frac{10}{m}\left(t-\frac{t^2}{2}\right)+v_0$$

Anfangsbedingungen: $\quad t=0; \quad v=v_0=20$ cm/sek

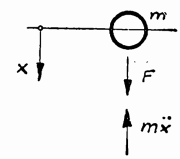
$$v=\frac{10\cdot g}{p}\left(t-\frac{t^2}{2}\right)+20\text{ cm/sek}$$

Stillstand: $\quad v=0: \quad t^2-2t-\frac{2\cdot 20\cdot p}{10g}=0; \quad t=\underline{\underline{2{,}02\text{ sek}}}$

Zurückgelegter Weg: $\quad s=\frac{10}{m}\left(\frac{t^2}{2}-\frac{t^3}{6}\right)+v_0t+s_0; \quad t=0; \quad s=0: s_0=0$

Mit $t=2{,}02$ sek ergibt sich: $\quad \underline{\underline{s=692\text{ cm}}}$

Lösung 702



$$F = F_0 \cos \omega t$$

$$m\ddot{x} - F_0 \cos \omega t = 0$$

$$\dot{x} = \frac{F_0}{m \cdot \omega} \sin \omega t + C_1; \quad \text{Anfangsbedingungen:} \quad t = 0; \quad \dot{x} = v_0$$

$$C_1 = v_0$$

$$x = -\frac{F_0}{m\omega^2} \cos \omega t + v_0 \cdot t + C_2; \quad t = 0; \quad x = 0: \quad C_2 = \frac{F_0}{m\omega^2}$$

$$\underline{\underline{x = \frac{F_0}{m\omega^2}(1 - \cos \omega t) + v_0 \cdot t}}$$

Lösung 703

$$m\dot{v} - F = 0; \quad m = \frac{G}{g} = 1; \quad \dot{v} = \frac{dv}{ds} \cdot v; \quad F = -\frac{2v^2}{3+s} \text{kg}$$

$$dv \cdot v = -\frac{2v^2}{3+s} \cdot ds; \quad \frac{dv}{2v} + \frac{ds}{3+s} = 0; \quad \frac{1}{2} \ln v + \ln(3+s) = \ln C_0$$

$$v = \frac{ds}{dt} = \left(\frac{C_0}{3+s}\right)^2; \quad (3+s)^2 ds = C_0^2 \cdot dt; \quad t = 0; \quad s_0 = 0:$$

$$s^3 + 9s^2 + 27s = 3C_0^2 \cdot t$$

$$(s+3)^3 = 3(C_0^2 t + 9)$$

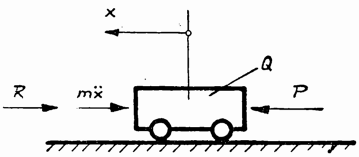
$$s = \sqrt[3]{3(C_0^2 t + 9)} - 3$$

Anfangsbedingung:

$$\left(\frac{ds}{dt}\right)_{t=0} = v_0 = \frac{C_0^2}{(\sqrt[3]{27})^2} = \frac{C_0^2}{9}; \quad C_0^2 = 9 \cdot v_0; \quad v_0 = 5 \text{ m/sek}$$

$$\underline{\underline{s = 3\left[\sqrt[3]{5t+1} - 1\right] \text{m}}}$$

Lösung 704



$$Q = 9216 \text{ kg}; \quad P = k \cdot S \cdot u^2 \text{kg}$$

$$R = Q/200 \text{ kg} \quad S = 6 \text{ m}^2$$

$$k = 0{,}12 \,[\text{kg sek}^2/\text{m}^4]; \quad w = 12 \text{ m/sek}$$

$$m\ddot{x} + R - P = 0; \quad u = w - \dot{x}$$

$$\dot{u} = -\ddot{x}$$

$$\ddot{x} + \frac{Q}{m \cdot 200} - \frac{k \cdot S \cdot u^2}{m} = 0$$

$$-\dot{u} + \frac{g}{200} - \frac{k \cdot S u^2}{m} = 0$$

$$-\int \frac{du}{\frac{g}{200}\left(1 - \frac{200kS}{mg} \cdot u^2\right)} = -t + C; \quad \alpha^2 = \frac{200kS}{mg}$$

$$\frac{200}{\alpha g}\left[\frac{1}{2}\int\frac{-d(\alpha u)}{(\alpha u+1)}+\frac{1}{2}\int\frac{d(\alpha u)}{(\alpha u-1)}\right]=-t+C$$

$$\frac{100}{\alpha g}\ln\frac{\alpha u-1}{\alpha u+1}=-t+C;\qquad t=0;\qquad \dot{x}=0;\qquad u=w:$$

$$C=\frac{100}{\alpha g}\ln\frac{\alpha w-1}{\alpha w+1}$$

$$\underline{-t=\frac{100}{\alpha g}\ln\frac{(\alpha u-1)(\alpha w+1)}{(\alpha u+1)(\alpha w-1)}};\qquad \alpha=\sqrt{\frac{200\,kS}{m\cdot g}}=\frac{1}{8}$$

1. Die maximale Geschwindigkeit wird erreicht für $t\to\infty$:

$$\text{somit:}\quad \alpha u-1=0;\quad \alpha(w-\dot{x}_\infty)-1=0$$

$$\dot{x}_\infty=w-\frac{1}{\alpha}=w-\sqrt{\frac{mg}{200\,k\cdot S}}$$

$$=12-\sqrt{\frac{9216}{200\cdot 0{,}12\cdot 6}}$$

$$v_{\max}=\dot{x}_\infty=\underline{\underline{4\,\frac{\text{m}}{\text{sek}}}}$$

2. Nach vorhergehendem beträgt die Zeit bis zum Erreichen der Höchstgeschwindigkeit

$$\underline{\underline{T=\infty}}$$

3. Zurückgelegter Weg für $\dot{x}=3$ m/sek:

$$\dot{u}=\frac{du}{d\xi}\cdot u;\qquad \dot{\xi}=u=w-\dot{x};\qquad \dot{u}=-\ddot{x};\qquad \xi=wt-x$$

$$-\frac{du}{d\xi}u+\frac{g}{200}-\frac{kSu^2}{m}=0;\qquad \frac{du\cdot u}{\dfrac{g}{200}-\dfrac{kSu^2}{m}}=d\xi;\qquad \frac{200}{g}\cdot\frac{du\cdot u}{\left(1-\dfrac{kS\cdot 200}{m\cdot g}u^2\right)}=d\xi$$

$$\alpha^2=\frac{k\cdot S\cdot 200}{mg};\qquad -\frac{m}{2k\cdot S}\ln C(1-\alpha^2u^2)=\xi=wt-x;\quad x=0;\quad t=0;\quad u=w:$$

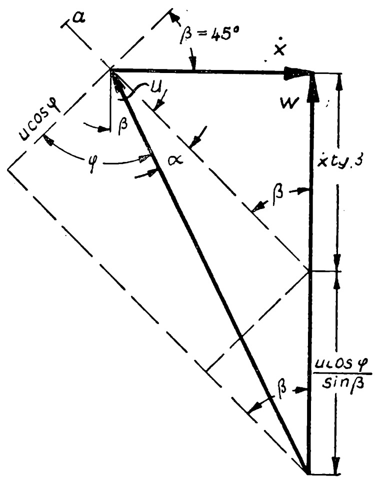
$$C=\frac{1}{1-\alpha^2w^2}$$

$$\underline{x=wt+\frac{m}{2ks}\ln\frac{1-\alpha^2u^2}{1-\alpha^2w^2}}$$

Für $x=x_1$; $\quad\dot{x}=3$ m/sek; $\quad u=9$ m/sek; $\quad -t=\dfrac{100}{\frac{1}{8}g}\ln\dfrac{1\cdot 20}{17\cdot 4}=99{,}8$ sek

$$x_1=99{,}8\cdot 12+\frac{9216}{9{,}81\cdot 0{,}12\cdot 6}\cdot\ln\frac{0{,}265}{1{,}25}=\underline{\underline{187\text{ m}}}$$

Lösung 705



$$w = \frac{u\cos\varphi}{\sin\beta} + \frac{\dot{x}}{\operatorname{tg}\beta}$$

$$u\cos\varphi = w\sin\beta - \dot{x}\cos\beta$$

$$P = k \cdot S \cdot u^2\cos^2\varphi$$

$$m\ddot{x} = P\cos\beta = k\cdot S\cdot\cos\beta\,(w\sin\beta - \dot{x}\cos\beta)^2$$

$$\underline{\underline{\ddot{x} = \varepsilon(w\sin\beta - \dot{x}\cos\beta)^2}}$$

1. Für $v_{\max}$ ist $\ddot{x} = 0$, also:

$$w\sin\beta = v\cos\beta$$

$$\text{mit } \beta = 45^\circ = \frac{\pi}{4}: \quad \dot{x}_{\max} = \underline{\underline{v_{\max} = w}}$$

2. Der Wimpel weht in Richtung von u

$$\operatorname{tg}(\beta - \alpha) = \frac{v_{\max}}{w}$$

$$\beta - \alpha = \frac{\pi}{4}$$

$$\text{mit } \beta = \frac{\pi}{4}: \quad \underline{\underline{\alpha = 0^\circ}}$$

3. Zurückgelegter Weg:

$$\frac{dv}{dx}\cdot v = \varepsilon(w\sin\beta - v\cos\beta)^2; \qquad \frac{v\,dv}{(w\sin\beta - v\cos\beta)^2} = \varepsilon\cdot dx$$

Substitution: $(w\sin\beta - v\cos\beta) = z$: $\quad -\frac{1}{\cos^2\beta}\left[\frac{(w\sin\beta - z)}{z^2}\,dz\right] = \varepsilon dx$

$$-\frac{1}{\cos^2\beta}\left(-\frac{w\sin\beta}{w\sin\beta - v\cos\beta} - \ln(w\sin\beta - v\cos\beta) + C\right) = \varepsilon\cdot x$$

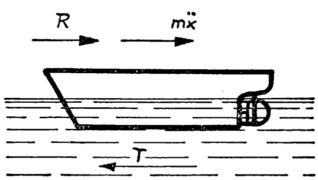
$v = 0$; $\quad x = 0$: $\quad C = 1 + \ln w\sin\beta$

$$x = -\frac{1}{\varepsilon\cos^2\beta}\left(-\frac{w\sin\beta}{w\sin\beta - v\cos\beta} + \ln\frac{w\sin\beta}{w\sin\beta - v\cos\beta} + 1\right)$$

Für $v = \frac{3}{2}w$ und $\beta = \frac{\pi}{4}$ ist $x = x_1$:

$$x_1 = -\frac{2\sqrt{2}\,m}{k\cdot S}(-3 + \ln 3 + 1) = \underline{\underline{90\text{ m}}}$$

Lösung 706



$$m\ddot{x} + \alpha\dot{x}^2 - T_0\left(1 - \frac{\dot{x}}{v_s}\right) = 0$$

$$m\cdot\frac{d\dot{x}}{T_0\left(1 - \frac{\dot{x}}{v_s}\right) - \alpha\dot{x}^2} = dt$$

$$-m\frac{d\dot{x}}{\alpha\dot{x}^2 + \frac{T_0\dot{x}}{v_s} - T_0} = dt$$

$$-\frac{m}{2\sqrt{T_0\left(\alpha+\frac{T_0}{4v_s^2}\right)}}\ln\left[\frac{\sqrt{T_0\left(\alpha+\frac{T_0}{4v_s^2}\right)}-\frac{T_0}{2v_s}-\alpha\dot{x}}{\sqrt{T_0\left(\alpha+\frac{T_0}{4v_s^2}\right)}+\frac{T_0}{2v_s}+\alpha\dot{x}}\right]=t+C;\qquad \sqrt{T_0\left(\alpha+\frac{T_0}{4v_s^2}\right)}=\beta$$

$$t=0;\qquad \dot{x}=v_0:\qquad C=-\frac{m}{2\beta}\ln\left[\frac{\beta-\frac{T_0}{2v_s}-\alpha v_0}{\beta+\frac{T_0}{2v_s}+\alpha v_0}\right]$$

$$t=-\frac{m}{2\beta}\ln\left[\frac{\left(\beta-\frac{T_0}{2v_s}-\alpha\dot{x}\right)\left(\beta+\frac{T_0}{2v_s}+\alpha v_0\right)}{\left(\beta+\frac{T_0}{2v_s}+\alpha\dot{x}\right)\left(\beta-\frac{T_0}{2v_s}-\alpha v_0\right)}\right]$$

$$\beta=4{,}2;\qquad \frac{T_0}{2v_s}=1{,}8;\qquad \frac{2\beta}{m}=0{,}055$$

$$\frac{(50+\dot{x})}{(20-\dot{x})}\cdot\frac{(20-v_0)}{(50+v_0)}=e^{0{,}055\,t}$$

$$\underline{\underline{\dot{x}=v=\frac{70v_0+20(v_0+50)(e^{0{,}055\,t}-1)}{70+(v_0+50)(e^{0{,}055\,t}-1)}}};\qquad v_0 \text{ in m/sek}$$

Lösung 707

Nach Aufgabe 706 gilt:

$$m\ddot{x}+\alpha\dot{x}^2-T_0\left(1-\frac{\dot{x}}{v_s}\right)=0$$

$$m\cdot\frac{dv}{dx}\cdot v+\alpha v^2-T_0\left(1-\frac{v}{v_s}\right)=0$$

$$\frac{m v\,dv}{\alpha v^2+\frac{T_0}{v_s}v-T_0}=-dx$$

$$x=\int\frac{m v\,dv}{T_0-\frac{T_0}{v_s}v-\alpha v^2}+x_0$$

$$x_0-x=\frac{m}{\alpha\cdot 2}\ln\left\{1-\left(\frac{\varphi+v}{\sqrt{\varepsilon}}\right)^2\right\}+\frac{m\varphi\sqrt{\varepsilon}}{\alpha\cdot\varepsilon\cdot 2}\cdot\ln\left\{\frac{1+\frac{\varphi+v}{\sqrt{\varepsilon}}}{1-\frac{\varphi+v}{\sqrt{\varepsilon}}}\right\}$$

$$\varphi=\frac{T_0}{2\alpha v_s}=15\,\frac{\text{m}}{\text{sek}};\qquad \varepsilon=\frac{T_0}{\alpha}\left(1+\frac{T_0}{4v_s^2\alpha}\right)=1225\,\frac{\text{m}^2}{\text{sek}^2}$$

$$x_0-x=637{,}5\cdot\ln\left\{1-\left(\frac{15+v}{35}\right)^2\right\}+273{,}9\ln\left\{\frac{50+v}{20-v}\right\}$$

für $t=0;\quad x=0;\quad v=v_0$:

$$x_0=637{,}5\ln\left\{1-\left(\frac{15+v_0}{35}\right)^2\right\}+273{,}9\ln\left(\frac{50+v_0}{20-v_0}\right)$$

Somit wird:

$$\underline{\underline{x=637{,}5\ln\frac{v_0^2+30v_0-1000}{v^2+30v-1000}+273{,}9\ln\frac{(v-20)(v+50)}{(v_0-20)(v_0+50)}}}$$

Lösung 708

Nach Aufgabe 706 gilt:

$$dx = \frac{20\,(v_0+50)\,(e^{0,055\,t}-1)+70\,v_0}{(v_0+50)\,(e^{0,055\,t}-1)+70}\,dt = 20\,dt - \frac{70\,(20-v_0)\,dt}{(v_0+50)\,e^{0,055\,t}+(20-v_0)}$$

$$C + x = 20\,\mathrm{t} - 70\,\frac{20-v_0}{v_0+50}\int\frac{dt}{e^{0,055\,t}+\dfrac{20-v_0}{v_0+50}}; \qquad \frac{20-v_0}{v_0+50} = \alpha$$

Substitution: $\qquad e^{0,055\,t} = y$

$$dt = \frac{1}{0,055\,y}\,dy$$

$$C + x = 20\,\mathrm{t} - \frac{70\,\alpha}{0,055\,\alpha}\ln\frac{e^{0,055\,t}}{e^{0,055\,t}+\dfrac{20-v_0}{v_0+50}}$$

$$t = 0; \quad x = 0: \quad C = -\frac{70}{0,055}\ln\frac{1}{1+\dfrac{20-v_0}{v_0+50}} = 199,3$$

$$\underline{\underline{s = x_{v_0=10} = 20\,\mathrm{t} - 127^2\ln\frac{6\,e^{0,055\,t}}{6\,e^{0,055\,t}+1} - 199,3}}$$

b) Krummlinige Bewegung

Lösung 709

Theoretische Schußbahn: $\quad x = v_0 t\cos\alpha$

$$y = v_0 t\sin\alpha - \frac{g t^2}{2}$$

Maximale Schußhöhe: $\quad \dfrac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{\dot y}{\dot x} = 0; \quad \dot y = 0; \quad 0 = v_0\sin\alpha - g t; \quad t = \frac{v_0\sin\alpha}{g}$$

$$y_{\max} = \frac{v_0^2\sin^2\alpha}{g} - \frac{v_0^2\sin^2\alpha}{2g} = \frac{v_0^2\sin^2\alpha}{2g}$$

1. $\alpha = 45°$: $\quad y_{\max} = 12,5\cdot 10^3\,\mathrm{m}; \quad y_{\max} - y_{\mathrm{tats.}} = 12,5 - 5 = \underline{\underline{7,5\,\mathrm{km}}}$

2. $\alpha = 75°$: $\quad y_{\max} = 23,2\cdot 10^3\,\mathrm{m}; \quad y_{\max} - y_{\mathrm{tats.}} = 23,2 - 11,2 = \underline{\underline{12\,\mathrm{km}}}$

Maximale Schußweite: $\quad y = 0$

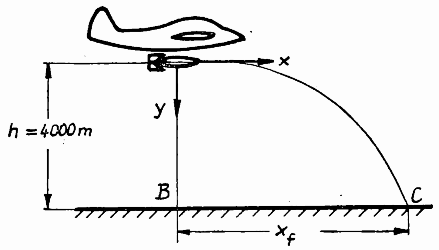
$$v_0\sin\alpha = \frac{g t}{2}; \quad t = \frac{2 v_0\sin\alpha}{g}$$

$$x_{\max} = \frac{2 v_0^2\sin\alpha\cos\alpha}{g} = \frac{v_0^2}{g}\sin 2\alpha$$

1. $\alpha = 45°$: $\quad x_{\max} = 50\cdot 10^3\,\mathrm{m}; \quad x_{\max} - x_{\mathrm{tats.}} = 50 - 13,4 = \underline{\underline{36,6\,\mathrm{km}}}$

2. $\alpha = 75°$: $\quad x_{\max} = 25\cdot 10^3\,\mathrm{m}; \quad x_{\max} - x_{\mathrm{tats.}} = 25 - 8,3 = \underline{\underline{16,7\,\mathrm{km}}}$

Lösung 710



$$\dot{x} = 500\ \mathrm{km/h}$$

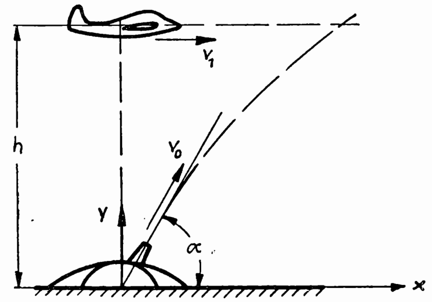
$$x = \frac{500}{3{,}6} \cdot t\ \mathrm{m}$$

$$y = \frac{gt^2}{2}; \quad t = \sqrt{\frac{2y}{g}}$$

$$y = h; \quad t = t_f: \quad t_f = \sqrt{\frac{2h}{g}} = 28{,}55\ \mathrm{sek}$$

$$x_f = \frac{500}{3{,}6} \cdot 28{,}55 = \underline{\underline{3960\ \mathrm{m}}}$$

Lösung 711



Flieger: $y_1 = h; \quad x_1 = v_1 \cdot t_1$

Geschoß: $x_0 = v_0 t_0 \cos\alpha$

$$y_0 = v_0 t_0 \sin\alpha - \frac{g t_0^2}{2}$$

Das Geschoß trifft den Flieger bei:

$$x_0 = x_1; \quad y_0 = y_1; \quad t_0 = t_1$$

Somit: $v_1 t_1 = v_0 t_0 \cos\alpha;$

$$\underline{\underline{\cos\alpha = \frac{v_1}{v_0}}}$$

Aus $y_0 = v_0 t_0 \sin\alpha - \frac{g t_0^2}{2}$ folgt: $t_0 = \frac{v_0 \sin\alpha}{g} \pm \sqrt{\frac{v_0^2 \sin^2\alpha - 2 y_0 g}{g^2}}$

Da t_0 reell ist, gilt: $v_0^2 \sin^2\alpha - 2 y_0 g > 0; \quad y_0 = y_1 = h; \quad \cos\alpha = \frac{v_1}{v_0}$

$$\underline{\underline{v_0^2 \geqq v_1^2 + 2gh}}$$

Lösung 712

$$x = v_0 \cos\alpha \cdot t$$

$$y = v_0 \sin\alpha \cdot t - \frac{gt^2}{2}; \qquad y = 0: \quad t = \frac{2 v_0}{g} \sin\alpha$$

$$x = L; \quad t = \frac{2 v_0}{g} \sin\alpha: \quad v_0 = \sqrt{\frac{L \cdot g}{2 \cos\alpha \sin\alpha}}$$

Die größte Schußweite wird erreicht bei $\alpha = 45°$

Somit: $v_0 = \sqrt{L \cdot g}$

$$\alpha = 30°; \quad x = l: \ t_l = \frac{2\sqrt{L \cdot g}}{g} \cdot \frac{1}{2} = \sqrt{\frac{L}{g}}; \quad l = \sqrt{L \cdot g} \cos\alpha \cdot \sqrt{\frac{L}{g}}$$

$$\underline{\underline{l = \frac{\sqrt{3}}{2} \cdot L}}$$

Schußbahnhöhe h: $\dot{y} = 0$: $t = \frac{v_0 \sin\alpha}{g}$

$$h = \frac{L \cdot g}{g} \sin^2\alpha - \frac{g L \cdot g}{g^2} \frac{\sin^2\alpha}{2} = \frac{L \sin^2\alpha}{2};$$

$$\alpha = 30°: \quad \underline{\underline{h = \frac{L}{8}}}$$

Lösung 713

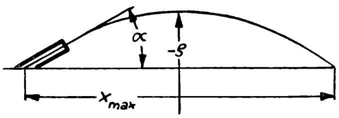
$$x = v_0 t \cos\alpha; \qquad \text{Schußweite bei } y = 0:$$

$$y = v_0 t \sin\alpha - \frac{g t^2}{2}; \quad l_\alpha = v_0^2 \frac{2}{g} \sin\alpha \cos\alpha$$

$$l_{\frac{\alpha}{2}} = v_0^2 \frac{2}{g} \sin\frac{\alpha}{2} \cos\frac{\alpha}{2} = v_0^2 \frac{\sin\alpha}{g}$$

$$\frac{l_\alpha}{l_{\frac{\alpha}{2}}} = \frac{v_0^2 \cdot 2\sin\alpha \cdot \cos\alpha \cdot g}{g \cdot v_0^2 \cdot \sin\alpha} = 2\cos\alpha; \quad l_{\frac{\alpha}{2}} = \frac{l_\alpha}{2\cos\alpha}$$

Lösung 714



$$x = v_0 t \cos\alpha; \quad y = v_0 t \sin\alpha - \frac{g t^2}{2}$$

$$+\varrho = \frac{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}{\dot{x}\ddot{y} - \dot{y}\ddot{x}}$$

$$+\varrho = \frac{[v_0^2(\cos^2\alpha + \sin^2\alpha) + g^2 t^2 - 2 v_0 g t \sin\alpha]^{\frac{3}{2}}}{-v_0 g \cos\alpha}$$

$$t = \frac{v_0}{g}\sin\alpha: \quad \varrho = -\varrho_0 = -16000\ \text{m}: \quad -\varrho_0 = \frac{[v_0^2(1 + \sin^2\alpha - 2\sin^2\alpha)]^{\frac{3}{2}}}{-v_0 g \cos\alpha}$$

$$+\varrho_0 = \frac{v_0^2 \cos^2\alpha}{g}; \qquad x_{\max} = \frac{2 v_0^2}{g} \cdot \cos\alpha \sin\alpha$$

$$x_{\max} = 2\varrho_0 \cdot \text{tg}\,\alpha = \underline{\underline{18480\ \text{m}}}$$

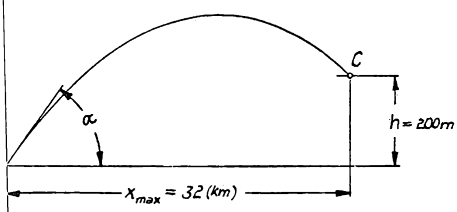
Lösung 715

Nach Aufgabe 709 gilt: Schußweite: $x = \frac{v_0^2}{g}\sin 2\alpha$

$$\sin 2\alpha = \frac{g \cdot x}{v_0^2}; \quad x = 32000\ \text{m}; \quad v_0 = 600\ \text{m/sek}: \quad \sin 2\alpha = 0{,}872$$

$$2\alpha_1 = 60^\circ\,36'; \quad 2\alpha_2 = 119^\circ\,24'; \quad \alpha_1 = \underline{\underline{30^\circ\,18'}}; \quad \alpha_2 = \underline{\underline{59^\circ\,42'}}$$

Lösung 716



$$x = v_0 \cos\alpha \cdot t;$$

$$y = v_0 \sin\alpha \cdot t - \frac{g t^2}{2};$$

$$t_c = \frac{x_{\max}}{v_0 \cos\alpha}$$

$$h = v_0 \sin\alpha \frac{x_{\max}}{v_0 \cos\alpha} - \frac{g x_{\max}^2}{2 v_0^2 \cos^2\alpha}$$

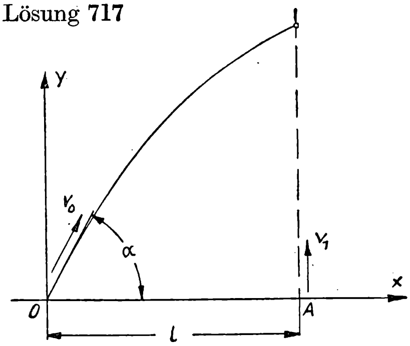
$$h = x_{\max} \cdot \text{tg}\,\alpha - \frac{g x_{\max}^2}{2 v_0^2}(1 + \text{tg}^2\alpha)$$

$$g^2\alpha - \frac{2 \cdot v_0^2}{g x_{\max}} \text{tg}\,\alpha = -\left(1 + \frac{2 h v_0^2}{g x_{\max}^2}\right); \quad \text{tg}\,\alpha_{2,1} = \frac{v_0^2}{g x_{\max}}\left[1 \pm \sqrt{1 - \left(\frac{g x_{\max}}{v_0^2}\right)^2 - \frac{2hg}{v_0^2}}\right]$$

$$\text{max} = 32000\ \text{m}; \quad h = 200\ \text{m}; \quad v_0 = 600\ \text{m/sek}: \quad \text{tg}\,\alpha_2 = 1{,}697; \quad \alpha_2 = \underline{\underline{59^\circ\,23'}}$$

$$\text{tg}\,\alpha_1 = 0{,}597; \quad \alpha_1 = \underline{\underline{30^\circ\,45'}}$$

Lösung 717



$x_0 = v_0 t_0 \cos\alpha$

$y_0 = v_0 t_0 \sin\alpha - \frac{g t_0^2}{2}$

$x_1 = l$

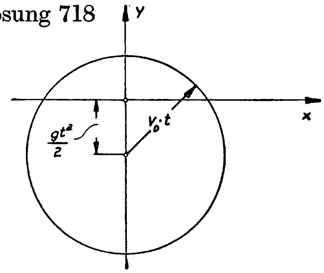
$y_1 = v_1 t - \frac{g t_1^2}{2}$

Treffbedingung: $y_0 = y_1$; $t_0 = t_1 = t$

$$v_0 t \sin\alpha - \frac{g t^2}{2} = v_1 t - \frac{g t^2}{2}$$

$$\underline{\underline{v_1 = v_0 \sin\alpha}}$$

Lösung 718



$x = v_0 t \cos\alpha$

$y = v_0 t \sin\alpha - \frac{g t^2}{2}$

Parameterdarstellung eines Kreises vom Radius

$$r = v_0 t$$

Kreismittelpunkt: $y_0 = -\frac{g t^2}{2}$

Lösung 719

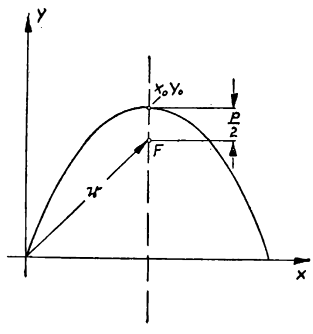
Allgemeine Parabelgleichung: $y + y_0 = \frac{(x - x_0)^2}{2p}$

$\frac{p}{2}$ = Abstand des Brennpunktes F vom Scheitel

x_0; y_0 = Scheitelkoordinaten

$$x_0 = \frac{v_0^2 \sin 2\alpha}{2g}; \quad y_0 = \frac{v_0^2 \cdot \sin^2\alpha}{2g}$$

Mit $x = 0$; $y = 0$ gilt: $p = \frac{v_0^2 \cos^2\alpha}{g}$



$$\mathfrak{r} = x_0 \mathfrak{i} + \left(y_0 - \frac{p}{2}\right)\mathfrak{j}$$

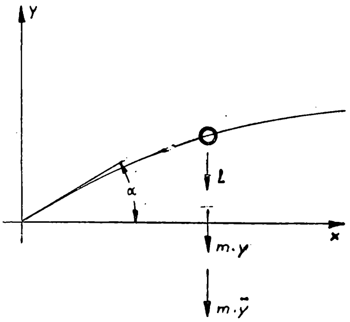
$$r^2 = x_0^2 + \left(y_0 - \frac{p}{2}\right)^2$$

$r = \frac{v_0^2}{2g}$; Der geometrische Ort aller Brennpunkte ist also ein Kreis vom Radius

$$\frac{v_0^2}{2g} \; (v_0 = \text{konst.})$$

$$\underline{\underline{x^2 + y^2 = \frac{v_0^4}{4g^2}}}$$

Lösung 720



$$m\ddot{y} + mg + \dot{y}kP = 0; \qquad \ddot{y} = \frac{d\dot{y}}{dy}\cdot\dot{y}$$

$$\frac{m\dot{y}\,d\dot{y}}{mg + \dot{y}kP} = -dy$$

$$\frac{m}{kP}\left[\frac{(\dot{y}kP + mg - mg)\,d\dot{y}}{mg + \dot{y}kP}\right] = -dy$$

$$\frac{m}{kP}\left[\dot{y} - \frac{mg}{kP}\ln(my + \dot{y}kP)\right] = -y + C$$

$$y = 0; \qquad \dot{y} = v_0\sin\alpha:$$

$$C = \frac{m}{kP}\left[v_0\sin\alpha - \frac{mg}{kP}\ln(mg + v_0kP\sin\alpha)\right]$$

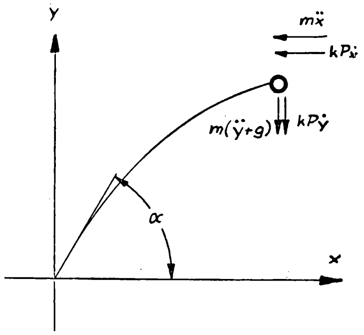
$$y = h; \qquad \dot{y} = 0$$

$$h = \frac{m}{kP}\left[v_0\sin\alpha - \frac{mg}{kP}\ln\frac{(mg + kPv_0\sin\alpha)}{mg}\right]$$

$$P = mg$$

$$\underline{\underline{h = \frac{v_0\sin\alpha}{gk} - \frac{1}{gk^2}\ln(1 + kv_0\sin\alpha)}}$$

Lösung 721



1. $m\ddot{x} + kP\dot{x} = 0$

$$\ddot{x} + gk\dot{x} = 0$$

Lösungsansatz: $x = C_1e^{\varepsilon t} + C_2$

$$\dot{x} = C_1\varepsilon e^{\varepsilon t}$$

$$\ddot{x} = C_1\varepsilon^2e^{\varepsilon t}$$

Nach Einsetzen in 1. ergibt sich

$$\varepsilon = -gk$$

Anfangsbedingungen:

$$t = 0; \quad x = 0; \quad \dot{x} = v_0\cos\alpha:$$

$$0 = C_1 + C_2$$

$$C_1 = -\frac{v_0\cos\alpha}{kg} = -C_2$$

$$\underline{\underline{x = \frac{v_0\cos\alpha}{gk}(1 - e^{-gkt})}}$$

2. $m(\ddot{y} + g) + kP\dot{y} = 0$

$$\ddot{y} + g + gk\dot{y} = 0$$

Lösungsansatz: $y = C_1e^{\varepsilon t} + C_2 + C_3t$

$$\dot{y} = C_1e^{\varepsilon t}\varepsilon + C_3$$

$$\ddot{y} = C_1\varepsilon^2e^{\varepsilon t}$$

In 2. eingesetzt, ergibt sich:

$$\varepsilon^2C_1e^{\varepsilon t} + g + gk\varepsilon C_1e^{\varepsilon} + C_3gk = 0$$

$$\varepsilon = -gk; \qquad C_3 = -\frac{1}{k}$$

Die Anfangsbedingungen liefern:

$$C_1 = -C_2 = -\frac{v_0\sin\alpha + \frac{1}{k}}{gk}$$

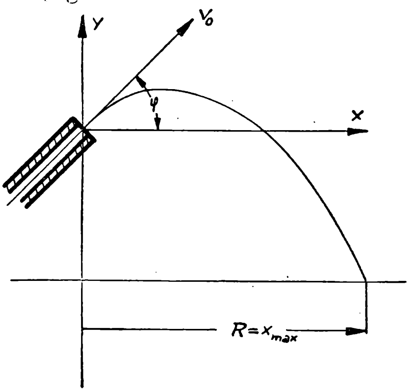
Somit:

$$\underline{\underline{y = \frac{v_0\sin\alpha + \frac{1}{k}}{gk}(1 - e^{-gk}) - \frac{t}{k}}}$$

Lösung 722 Nach Aufgabe 721 gilt:
$$x = \frac{v_0 \cos\alpha}{gk}\left(1 - e^{-gkt}\right)$$
$$\dot y = \frac{v_0 \sin\alpha + \frac{1}{k}}{gk} \cdot gk\, e^{-gkt} - \frac{1}{k}$$

Der Punkt erreicht seine höchste Lage bei $\dot y = 0$, also:
$$e^{-gkt} = \frac{1}{kv_0 \sin\alpha + 1}$$
$$\underline{\underline{s = \frac{v_0^2 \sin 2\alpha}{2g\,(kv_0 \sin\alpha + 1)}}}$$

Lösung 723

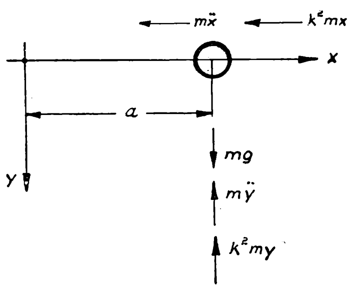


$$x = v_0 \cdot t \cos\varphi; \qquad y = v_0 t \sin\varphi - \frac{gt^2}{2}$$
$$v_0^2 = \frac{4g}{3\cos\varphi}; \qquad y = x \operatorname{tg}\varphi - \frac{3x^2}{8\cos\varphi}$$

Bedingung für $x_{\max}$: $\dfrac{dy}{d\varphi} = 0$
$$0 = x \cdot \frac{1}{\cos^2\varphi} - \frac{3x^2}{8} \cdot \frac{\sin\varphi}{\cos^2\varphi}$$
$$\sin\varphi = \frac{8}{3x}$$
$$y = -h; \qquad x = x_{\max} = R:$$
$$-h = R \cdot \frac{8}{3R\sqrt{1 - \frac{64}{9R^2}}} - \frac{3R^2}{8\sqrt{1 - \frac{64}{9R^2}}}$$

Nach R aufgelöst: $R^4 - \dfrac{64}{9} R^2 = \left(\dfrac{24}{9}\right)^2; \qquad R = \sqrt{8} = \underline{\underline{2{,}83 \text{ m}}}$

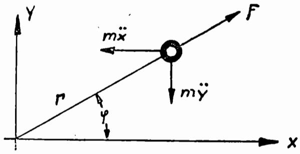
Lösung 724



Anfangsbedingungen:
$$t = 0: \qquad x = a; \quad y = 0$$
$$\dot x = 0; \quad \dot y = 0$$
$$\ddot x + k^2 x = 0; \qquad x = C_1 \cos kt + C_2 \sin kt$$
$$C_1 = a; \qquad C_2 = 0$$
$$\underline{\underline{x = a \cos kt}}$$
$$(\ddot y - g) + k^2 y = 0$$
$$\left(y - \frac{g}{k^2}\right)^{\cdot\cdot} + k^2\left(y - \frac{g}{k^2}\right) = 0$$
$$\left(y - \frac{g}{k^2}\right) = C_1^* \cos kt + C_2^* \sin kt$$
$$C_1^* = -\frac{g}{k^2}; \quad C_2^* = 0$$
$$\underline{\underline{y = \frac{g}{k^2}(1 - \cos kt)}}$$

Die Bewegung erfolgt demnach auf der Geraden: $\underline{\underline{y = \dfrac{g}{k^2}\left(1 - \dfrac{x}{a}\right)}}$

Lösung 725



$$F = mk^2 \cdot r$$
$$r\cos\varphi = x$$
$$r\sin\varphi = y$$

$$m\ddot{x} - F\cos\varphi = 0 \qquad m\ddot{y} - F\sin\varphi = 0$$
$$\ddot{x} - k^2 x = 0 \qquad \ddot{y} - k^2 y = 0$$

Ansatz: $x = C_1 e^{\varepsilon t} + C_2 e^{-\varepsilon t}$ $\qquad y = C_3 e^{\alpha t} + C_4 e^{-\alpha t}$

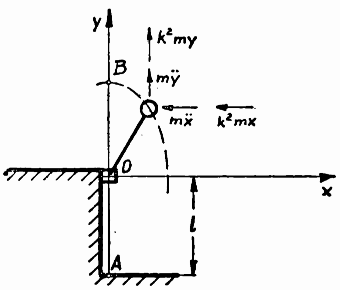
$$\ddot{x} = \varepsilon^2 C_1 e^{\varepsilon t} + \varepsilon^2 C_2 e^{-\varepsilon t} \qquad \ddot{y} = \alpha^2 C_3 e^{\alpha t} + C_4 \alpha^2 \cdot e^{-\alpha t}$$
$$\varepsilon = k \qquad \alpha = k$$

Anfangsbedingungen:

$$t = 0: \quad x = a \qquad t = 0: \quad y = 0$$
$$\dot{x} = 0 \qquad \dot{y} = v_0$$
$$a = C_1 + C_2 \qquad 0 = C_3 + C_4$$
$$0 = \varepsilon C_1 - \varepsilon C_2 \qquad v_0 = \alpha C_3 - \alpha C_4$$
$$C_1 = C_2 = \frac{a}{2} \qquad C_3 = -C_4 = \frac{v_0}{2k}$$
$$\underline{x = \frac{a}{2}(e^{kt} + e^{-kt})} \qquad \underline{y = \frac{v_0}{2k}(e^{kt} - e^{-kt})}$$

$$\frac{4x^2}{a^2} = e^{2kt} + e^{-2kt} + 2$$
$$\frac{4k^2y^2}{v_0^2} = e^{2kt} + e^{-2kt} - 2 \quad (-)$$
$$\frac{4x^2}{a^2} - \frac{4k^2y^2}{v_0^2} = 4; \qquad \underline{\underline{\left(\frac{x}{a}\right)^2 - \left(\frac{ky}{v_0}\right)^2 = 1 \text{ (Hyperbel)}}}$$

Lösung 726

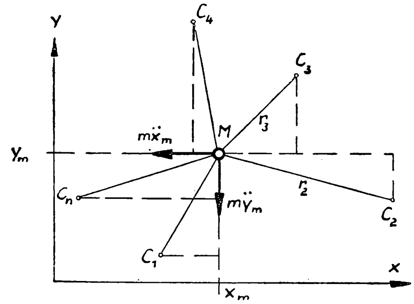


$$\ddot{x} + k^2 x = 0$$
$$x = C_1 \cos kt + C_2 \sin kt$$
$$t = 0: \quad x = 0; \quad \dot{x} = v_0: \quad C_1 = 0; \quad C_2 = \frac{v_0}{k}$$
$$\underline{\underline{x = \frac{v_0}{k}\sin kt}}$$
$$\ddot{y} + k^2 y = 0$$
$$y = C_1^* \cos kt + C_2^* \sin kt$$
$$t = 0: \quad y = l; \quad \dot{y} = 0: \quad C_1^* = l; \quad C_2^* = 0$$
$$\underline{\underline{y = l\cos kt}}$$

Bewegungsbahn:

$$\underline{\underline{\frac{v_0^2}{k^2}x^2 + \frac{y^2}{l^2} = 1}}$$

Lösung 727



$$F_i = k_i \cdot m \cdot r_i$$

$$\sum_{i=1}^{n} F_{xi} = \sum_{i=1}^{n} k_i m (x_i - x_m)$$

$$\sum_{i=1}^{n} F_{yi} = \sum_{i=1}^{n} k_i m (y_i - y_m)$$

$$m\ddot{x}_m - m \sum_{i=1}^{n} k_i (x_i - x_m) = 0$$

$$m\ddot{y}_m - m \sum_{i=1}^{n} k_i (y_i - y_m) = 0$$

Unter Berücksichtigung der Anfangsbedingungen erhält man:

$$x - a = (x_0 - a) \cos \sqrt{\sum k_i} \cdot t$$

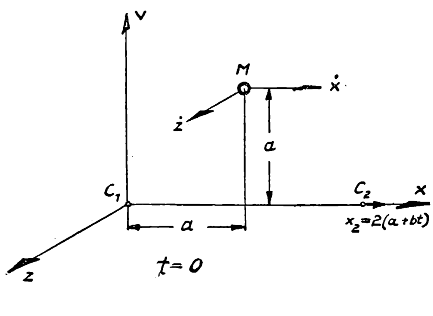
$$y - b = \frac{v_0}{\sqrt{\Sigma k_i}} \sin \sqrt{\sum k_i} \cdot t + (y_0 - b) \cos \sqrt{\sum k_i} \cdot t$$

$$a = \frac{\sum_{i=1}^{n} k_i x_i}{k}; \quad b = \frac{\sum_{i=1}^{n} k_i y_i}{k}; \quad k = \sum_{i=1}^{n} k_i$$

Durch Eliminieren von $\sin \sqrt{\sum k_i} \cdot t$ und $\cos \sqrt{\sum k_i} \cdot t$ erhält man:

$$\underline{\underline{\left(\frac{x-a}{x_0-a}\right)^2 + \left\{ y - b + \frac{x-a}{x_0-a} (b - y_0) \right\}^2 \frac{k}{v_0^2} = 1}}$$

Lösung 728



$$\ddot{x} + kx - k\{2(a+bt) - x\} = 0$$

$$\ddot{x} + 2kx = 2k(a+bt)$$

$$\ddot{y} + 2ky = 0$$

$$\ddot{z} + 2kz = 0$$

Ansatz:

$$x = C_1 \cos \sqrt{2k}t + C_2 \sin \sqrt{2k}t + a + bt$$

$$y = C_3 \sin \sqrt{2k}t + C_4 \cos \sqrt{2k}t$$

$$z = C_5 \sin \sqrt{2k}t + C_6 \cos \sqrt{2k}t$$

Anfangsbedingungen:

$$t = 0: \quad x = a: \quad C_1 = 0$$

$$y = a: \quad C_4 = a$$

$$z = 0: \quad C_6 = 0$$

$$\dot{x} = b: \quad C_2 = 0$$

$$\dot{y} = 0: \quad C_3 = 0$$

$$\dot{z} = b: \quad C_5 = \frac{b}{\sqrt{2k}}$$

$$x = a + bt$$

$$y = a \cos \sqrt{2k}t$$

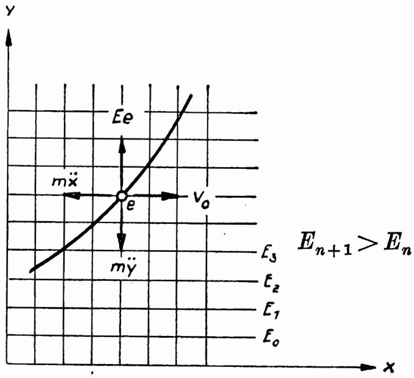
$$z = \frac{b}{\sqrt{2k}} \sin \sqrt{2k}t$$

Die Bewegungsbahn ist somit eine Schraubenlinie, die auf einem elliptischen Zylinder der Gleichung: $\frac{2k}{b^2} z^2 + \frac{y^2}{a^2} = 1$ verläuft.

Steigung der Schraube: $x = a + bt; \quad \sqrt{2k}\,t = 2\pi$

$$\underline{\underline{x_s = b \cdot t = \pi b \sqrt{\frac{2}{k}}}}$$

Lösung 729



$E_{n+1} > E_n$

$$m\ddot{x} = 0$$
$$\dot{x} = C_1$$
$$x = C_1 t + C_2$$
$$t = 0; \quad x = 0; \quad \dot{x} = v_0:$$
$$\underline{x = v_0 t}$$
$$m\ddot{y} - Ee = 0; \quad \ddot{y} = \frac{Ee}{m}$$
$$\dot{y} = \frac{Ee}{m} t + C_3$$
$$y = \frac{Ee}{2m} t^2 + C_3 t + C$$
$$t = 0; \quad y = 0; \quad \dot{y} = 0:$$
$$\underline{y = \frac{Eet^2}{2m}}$$
$$\underline{\underline{y = \frac{Ee}{2mv_0^2} x^2}}$$

Lösung 730

$$\mathfrak{F} = -e(\mathfrak{v} \times \mathfrak{H})$$

$$\mathfrak{F} = -e \begin{vmatrix} \mathfrak{i} & \mathfrak{j} & \mathfrak{k} \\ \dot{x} & \dot{y} & \dot{z} \\ H_x & H_y & H_z \end{vmatrix}; \quad \begin{matrix} H_x = 0 \\ H_z = 0 \end{matrix}$$

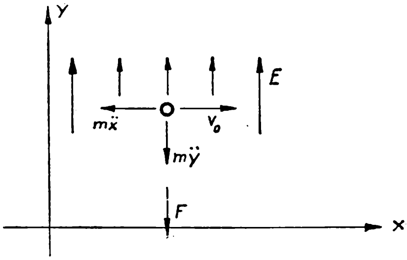
$$F_z = -eH\dot{x}; \quad F_x = eH\dot{z}$$
$$m\ddot{z} + eH\dot{x} = 0$$
$$m\ddot{x} - eH\dot{z} = 0$$

Ansatz: $x = C_1 \sin\omega t; \quad z = C_2 \cos\omega t$

$$-C_2\omega^2 + \frac{eH}{m}\omega C_1 = 0$$
$$-C_1\omega^2 + \frac{eH}{m}\omega C_2 = 0$$
$$C_1 = C_2 = C; \qquad \omega = \frac{eH}{m}$$
$$t = 0; \quad \dot{x} = v_0: \quad C = \frac{v_0}{\omega} = \frac{v_0 m}{eH}$$
$$\underline{\underline{x^2 + z^2 = r^2 = \left(\frac{v_0 m}{eH}\right)^2}}$$

Die Bewegungsbahn des Teilchens ist also ein Kreis vom Halbmesser r.

Lösung 731



$$m\ddot{x} = 0;$$

Mit den Anfangsbedingungen ergibt sich:

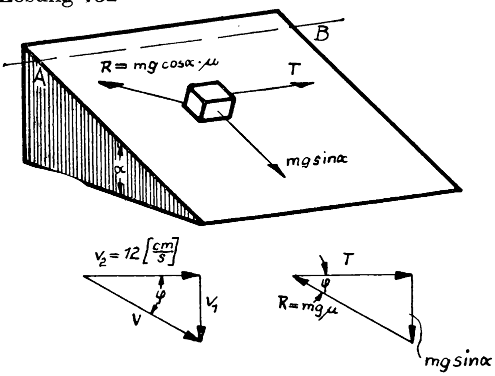
$$\underline{x = v_0 t}$$

$$m\ddot{y} + F = 0; \quad m\ddot{y} - eA\cos kt = 0$$

$$\underline{y = -\frac{eA}{mk^2}\cos kt + \frac{eA}{mk^2}}$$

$$\underline{\underline{y = \frac{eA}{mk^2}\left(1 - \cos\frac{kx}{v_0}\right)}}$$

Lösung 732



$$\operatorname{tg}\alpha = \frac{1}{30} \approx \sin\alpha; \quad \cos\alpha \cong 1$$

$$T = \sqrt{R^2 - (mg\sin\alpha)^2}$$

$$T = \sqrt{(300 \cdot 0{,}1)^2 - \left(\frac{300}{30}\right)^2} = \underline{\underline{28{,}3\ \mathrm{g}}}$$

$$v_1 = v_2 \cdot \operatorname{tg}\varphi = v_2 \cdot \frac{mg\sin\alpha}{T}$$

$$v_1 = 12 \cdot \frac{10}{28{,}3} = \underline{\underline{4{,}24\ \mathrm{cm/sek}}}$$

28. Impuls- und Flächensatz des Massenpunktes

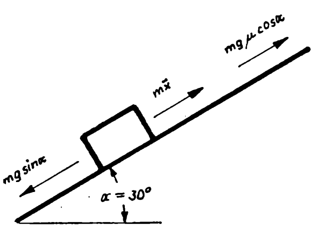
Lösung 733

Impulssatz: $m \cdot v = P \cdot t; \qquad P = 0{,}1 \cdot mg$

$$mv = 0{,}1 \cdot m \cdot g \cdot t; \qquad t = \frac{v}{0{,}1g} = \frac{72}{3{,}6 \cdot 0{,}1 \cdot 9{,}81} = \underline{\underline{20{,}4\ \mathrm{sek}}}$$

Bremsweg: $s = \frac{1}{2}bt^2; \qquad b = \frac{v}{t}; \qquad s = \frac{vt}{2} = \underline{\underline{204\ \mathrm{m}}}$

Lösung 734



$$m\ddot{x} - mg(\sin\alpha - \mu\cos\alpha) = 0$$

$$x = g(\sin\alpha - \mu\cos\alpha)\frac{t^2}{2}$$

$$x = L; \qquad t = T:$$

$$T = \sqrt{\frac{2L}{g(\sin\alpha - \mu\cos\alpha)}}$$

$$T = \sqrt{\frac{2 \cdot 39{,}2}{9{,}81(0{,}5 - 0{,}2 \cdot 0{,}866)}} = \underline{\underline{4{,}94\ \mathrm{sek}}}$$

Lösung 735

$$m(v_0 - v_1) = t(P - G(\mu \cos\alpha + \sin\alpha))$$

$$P = G\left\{(\mu\cos\alpha + \sin\alpha) - \frac{v_0 - v_1}{t\cdot g}\right\}$$

$$P = 400\left\{\left(\frac{0{,}005}{\sqrt{1+(0{,}006)^2}} + \frac{0{,}006}{\sqrt{1+(0{,}006)^2}} - \frac{15-12{,}5}{50\cdot 9{,}81}\right)\right\} = \underline{\underline{2{,}36\ \mathrm{t}}}$$

Lösung 736

Da kein einprägendes Moment vorhanden ist, ist der Drall konstant, also

$$mR^2\cdot\omega_1 = m\left(\frac{R}{2}\right)^2\cdot\omega_2$$

$$\omega_2 = \omega_1\cdot 4;\qquad n_2 = 4\cdot n_1 = \underline{\underline{480\ \mathrm{U/min}}}$$

Lösung 737

$$mv = Pt - \mu G t;\qquad G = \frac{Pt}{\dfrac{v}{g} + \mu t}$$

$$G = \frac{100{,}8\cdot 120}{\dfrac{57{,}6}{3{,}6\cdot 9{,}81} + 120\cdot 0{,}02} = \underline{\underline{3000\ \mathrm{t}}}$$

Lösung 738

$$m d\dot{x} = -mg\mu dt;\quad \dot{x} = -g\mu t + C$$

$$t = 0;\quad \dot{x} = v_0 : C = v_0$$

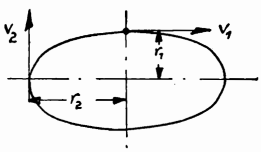
$$t = T;\quad \dot{x} = 0 : \mu = \frac{v_0}{gT} = \frac{20}{9{,}81\cdot 6} \geqq \underline{\underline{0{,}34}}$$

Lösung 739

$$mv = Pt;\quad \frac{Q}{g}v = p\cdot F\cdot t;\quad p = \frac{Q\cdot v}{g\cdot F\cdot t} = \frac{0{,}02\cdot 650\cdot 10^6}{9{,}81\cdot 150\cdot 9{,}5\cdot 10^{-4}\cdot 10^4}$$

$$p = \underline{\underline{931\ \mathrm{kg/cm^2}}}$$

Lösung 740



Der Drall bleibt konstant:

$$M r_2 v_2 = M r_1 v_1$$

$$v_2 = v_1\cdot\frac{r_1}{r_2} = \frac{v_1}{5} = \underline{\underline{6\ \mathrm{cm/sek}}}$$

Lösung 741

$$m\dot{x}_0 = m v_0 \cos\alpha$$

$$\underline{m\dot{x}_1 = m v_1}$$

$$m(\dot{x}_1 - \dot{x}_0) = -m(v_0\cos\alpha - v_1):$$

$$S_x = S_{x_1} - S_{x_0} = -\frac{100}{9{,}81}(250 - 200)$$

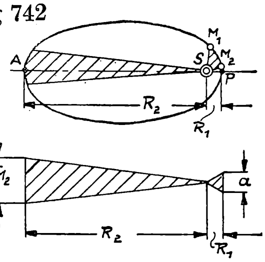
$$S_x = -\underline{\underline{510\ \mathrm{kgsek}}}$$

$$m\dot{y} = v_0\sin\alpha\cdot m - mgt;\quad S_{y_1} = 0$$

$$S_{y_0} = m v_0 \sin\alpha$$

$$S_y = S_{y_1} - S_{y_0} = -\underline{\underline{4410\ \mathrm{kgsek}}}$$

Lösung 742



Der Fahrstrahl überstreicht in gleichen Zeiten gleiche Flächen:

$$\frac{R_1 \cdot a}{2} = \frac{R_2 \cdot \overline{M_1 M_2}}{2}$$

$$\underline{\underline{\overline{M_1 M_2} = \frac{R_1}{R_2} \cdot a}}$$

Lösung 743

$$mv = S \cdot z - Rt; \quad v = \frac{S \cdot z - Rt}{m};$$

S = Stoßimpuls
z = Anzahl der Stöße
R = Fahrwiderstand
m = Gesamtmasse

$$v = \frac{(2 - 0{,}01 \cdot 80) \cdot 15 \cdot 9{,}81}{80} = \underline{\underline{2{,}2\ \text{m/sek}}}$$

Lösung 744

$$m(\Delta v) = P(\Delta t) = S;$$

Physikalisches Maßsystem:
$T = 4\,\text{sek}$
$m = 5\,\text{g}$

$$S = 2mv = 2 \cdot 5 \cdot 20 = \underline{\underline{200\ \text{dyn sek}}}$$

$$F_m \cdot \frac{T}{2} = S; \quad F_m = \frac{2S}{T} = \underline{\underline{100\ \text{dyn}}}$$

Lösung 745

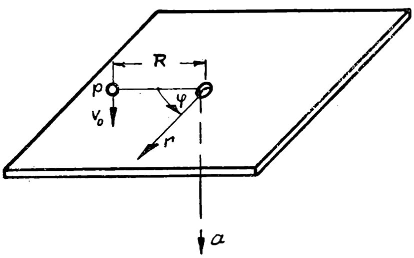
Die Schwingungszeit eines mathematischen Pendels ist:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Beide Pendel schwingen synchron, wenn $n T_1 = k T_2$ ist, wobei k/n einen ganzen rationalen Bruch darstellt.

Es gilt also: $\quad \frac{T_1}{T_2} = \frac{k}{n}; \quad \underline{\underline{\frac{k}{n} = \sqrt{\frac{l_1}{l_2}}}}$

Lösung 746



Polarkoordinaten: $\quad r = R - at$

$$\varphi = \frac{v_0 \cdot t}{R - at}$$

$$m \cdot b_r = T; \quad b_r = \ddot{r} + \dot{\varphi}^2 \cdot r$$

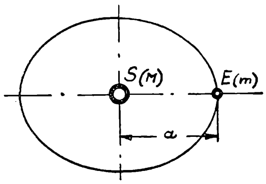
$$b_r = \left(\frac{(R - at)\, v_0 + v_0 t a}{(R - at)^2}\right)^2 \cdot r$$

$$b_r = \frac{R^2 \cdot v_0^2}{r^3}$$

Fadenkraft:

$$\underline{\underline{T = \frac{m v_0^2 R^2}{r^3}}}$$

Lösung 747



$R = 637 \cdot 10^6$ cm

$T = 365{,}25$ Tage $\triangleq 365{,}25 \cdot 24 \cdot 3600$ sek

$a = 149 \cdot 10^{11}$ cm

$\varrho_E = 5{,}5\ \mathrm{g/cm^3}$

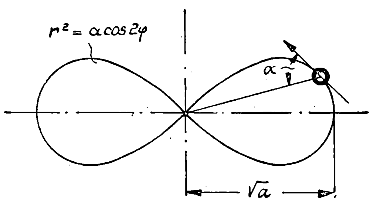
Nach Newton ist: $k = \Gamma \cdot \frac{m \cdot M}{a^2}$; Gravitationskonst. $\Gamma = \frac{R^2 \cdot g}{m}$

Ferner gilt: $k = m \cdot a\,\omega^2$, oder mit $\omega T = 2\pi$: $k = m \cdot a \cdot \frac{4\pi^2}{T^2}$

Somit: $m \cdot a \cdot \frac{4\pi^2}{T^2} = \Gamma \cdot \frac{m \cdot M}{a^2}$; $M = \frac{4\pi^2 a^3 m}{T^2 R^2 g}$; $m = \frac{4}{3}\pi R^3 \cdot \varrho$

also: $M = \frac{16\pi^3 a^3 R \cdot \varrho}{3 \cdot T^2 \cdot g} = \underline{\underline{197 \cdot 10^{31}\,\mathrm{g}}}$

Lösung 748



Zentralkraft

$$F = -\frac{m \cdot c^2}{r^2}\left(\frac{d^2\left(\frac{1}{r}\right)}{d\varphi^2} + \frac{1}{r}\right)$$

$c =$ Doppelte Sektorgeschw. $= r^2\dot{\varphi}$

$$c = r_0^2 \frac{v_0}{r_0} \sin\alpha = r_0 v_0 \sin\alpha$$

$$\frac{1}{r} = \frac{1}{\sqrt{a}} \cdot \frac{1}{\sqrt{\cos 2\varphi}}; \quad \frac{d\left(\frac{1}{r}\right)}{d\varphi} = \frac{1}{\sqrt{a}}\,\frac{\sin 2\varphi}{\cos 2\varphi^{3/2}}$$

$$\frac{d^2\left(\frac{1}{r}\right)}{d\varphi^2} = \frac{1}{\sqrt{a}} \cdot \frac{2\cos 2\varphi^{5/2} + 3\sin^2 2\varphi \cos 2\varphi^{1/2}}{\cos^3 2\varphi} = \frac{1}{r}\,(2 + 3\,\mathrm{tg}^2\, 2\varphi)$$

$$F = -\frac{m v_0^2 r_0^2 \sin^2\alpha \cdot 3\,(1 + \mathrm{tg}^2\, 2\alpha)}{r^3}; \quad \underline{\underline{F = -\frac{3 m v_0^2 r_0^2 a^2 \sin^2\alpha}{r^7}}}$$

Das Vorzeichen (—) besagt, daß F eine Anziehungskraft darstellt.

Lösung 749

Gleichgewichtsbedingung in Polarkoordinaten:

$$F - m(\ddot{r} - r\dot{\varphi}^2) = 0; \quad r^2\dot{\varphi} = h$$

$$v^2 = \dot{r}^2 + r^2\dot{\varphi}^2 = \frac{a^2}{r^2}; \quad \dot{r}^2 = \frac{a^2 - h^2}{r^2}$$

$$2\dot{r}\ddot{r} = 2\,\frac{h^2 - a^2}{r^3} \cdot \dot{r}; \quad \ddot{r} = \frac{h^2 - a^2}{r^3}$$

$$F = m\left[\frac{h^2-a^2}{r^3} - \frac{h^2}{r^3}\right]; \qquad \underline{\underline{F = -\frac{m\cdot a^2}{r^3}}};$$

Das Vorzeichen (—) besagt, daß F eine Anziehungskraft darstellt.

Bewegungsbahn:

$$-\frac{a^2}{r^3} - \ddot r + \frac{h^2}{r^3} = 0; \qquad \ddot r + \frac{1}{r^3}(h^2-a^2) = 0 \,\Big|\cdot \dot r; \qquad \ddot r \dot r + \frac{\dot r}{r^3}(h^2-a^2) = 0$$

$$\dot r^2 - \frac{1}{r^2}(h^2-a^2) = 0$$

$$\dot r = \frac{1}{r}\sqrt{h^2-a^2}; \qquad \frac{r\,dr}{\sqrt{h^2-a^2}} = dt = \frac{d\varphi\cdot r^2}{h}; \qquad \underline{\underline{\frac{h}{\sqrt{h^2-a^2}}\ln r = \varphi}}$$

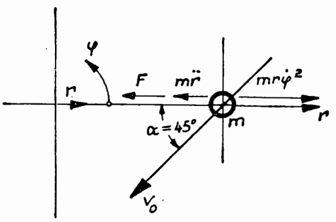
Die Bewegungsbahn ist also eine logarithmische Spirale.

Lösung 750

$m = 1\,\mathrm{g}$ $\qquad F = \frac{a}{r^3}$

$v_0 = 0{,}5\,\mathrm{m/sek}$ $\qquad a = 1\,\mathrm{dyn}$

$\alpha = 45°$ $\qquad r_0 = 2\,\mathrm{cm}$



$$F + m(\ddot r - r\dot\varphi^2) = 0$$

$$\frac{1}{r^3} + \ddot r - r\dot\varphi^2 = 0$$

Für eine Zentralbewegung gilt:

$$b_\varphi = r\ddot\varphi + 2\dot r\dot\varphi = \frac{1}{r}\,\frac{d}{dt}(r^2\dot\varphi) = 0$$

Somit: $\quad r^2\dot\varphi = C; \qquad \dot\varphi = \frac{C}{r^2}$

$$r = r_0; \qquad \dot\varphi = \frac{v_0}{r_0}\cdot\frac{1}{2}\sqrt{2}:$$

$$C = \frac{1}{2}\sqrt{2}\,r_0 v_0 = \frac{\sqrt{2}}{2}$$

$$\ddot r + \frac{1}{r^3}(1 - C^2) = 0; \qquad \ddot r + \frac{1}{r^3\,2} = 0 \,\Big|\cdot \dot r$$

$$\ddot r \dot r + \frac{\dot r}{r^3}\cdot\frac{1}{2} = 0; \qquad \dot r^2 - \frac{1}{2r^2} + C_1 = 0$$

$$r = r_0; \qquad \dot r = \frac{1}{2}\sqrt{2}\,v_0: \qquad C_1 = \frac{1}{2r_0^2} - \frac{1}{2}v_0^2 = 0$$

$$\dot r = \frac{\sqrt{2}}{2}\cdot\frac{1}{r}; \qquad 2r\,dr = dt; \qquad r^2 = \sqrt{2}\,t + C_2$$

$$t = 0; \qquad r = r_0: \qquad \underline{\underline{r^2 = \sqrt{2}\,t + 4}}$$

$$\dot\varphi = \frac{\sqrt{2}}{2r^2}; \qquad \varphi = \frac{\sqrt{2}}{2}\int\frac{dt}{\sqrt{2}\,t + 4} + C_3; \qquad \varphi = \frac{1}{2}\ln C_3 r^2$$

$$r = r_0; \qquad \varphi = 0: \qquad r = r_0\cdot e^{\varphi}; \qquad \underline{\underline{r = 2e^{\varphi}}}$$

Lösung 751

$$r = \frac{p}{1 + e\cos\varphi}; \qquad \dot{r} = \frac{p\,e\sin\varphi}{(1 + e\cos\varphi)^2}\cdot\dot{\varphi} = \frac{e\sin\varphi}{p}\cdot r^2\cdot\dot{\varphi}$$

$$r^2\cdot\dot{\varphi} = \text{konst.} = h: \qquad \dot{r} = \frac{e\sin\varphi}{p}\cdot h$$

$$\ddot{r} = \frac{h^2 e}{r^2 p}\cos\varphi = \frac{h^2}{r^2 p}\left(\frac{p}{r} - 1\right)$$

$$F_r = m(\ddot{r} - r\dot{\varphi}^2) = m\left[\frac{h^2}{r^2 p}\left(\frac{p}{r} - 1\right) - \frac{h^2}{r^3}\right] = \underline{\underline{-\frac{m h^2}{r^2 p}}}$$ (F_r positiv in positiver r-Richtung)

da $\ddot{\varphi} = 0$: $\underline{\underline{F_\varphi = 0}}$

Lösung 752

$$\ddot{r} - r\dot{\varphi}^2 = -\frac{\gamma}{r^2} \qquad (1)$$

$$r\ddot{\varphi} + 2\dot{r}\dot{\varphi} = 0 \qquad (2)$$

aus (2): $$r^2\dot{\varphi} = h = \text{konst.} \qquad (3)$$ Flächensatz

(3) in (1): $$\ddot{r} - \frac{h^2}{r^3} = -\frac{\gamma}{r^2} \qquad (4)$$

(4) $\cdot\,\dot{r}$ und integriert: $$\dot{r}^2 + \frac{h^2}{r^2} = \frac{2\gamma}{r} + k \qquad (5)$$

aus (5): $$\dot{r} = \sqrt{k + \frac{2\gamma}{r} - \frac{h^2}{r^2}} \qquad (6)$$

aus (3): $$r^2\frac{d\varphi}{dr}\cdot\dot{r} = h \qquad (7)$$

aus (7): $$d\varphi = \frac{dr}{r\sqrt{\frac{k}{h^2}r^2 + \frac{2\gamma}{h^2}r - 1}} \qquad (8)$$

(8) integriert: $$\varphi = \arccos\frac{h^2 - \gamma r}{r\gamma\sqrt{1 + \frac{k h^2}{\gamma^2}}} + \alpha \qquad (9)$$

aus (9): $$r = \frac{h^2}{\gamma}\cdot\frac{1}{1 + e\cos(\varphi - \alpha)} \qquad (10)$$

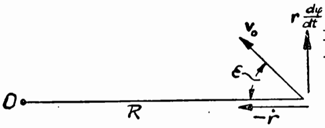
mit: $$e = \sqrt{1 + \frac{k h^2}{\gamma^2}} \qquad (11)$$

aus (6) folgt mit $h = 0$ und $v_{(r=\infty)} = 0$

$$v_{(r=R)} = \sqrt{\frac{2\gamma}{R}} \qquad (12)$$

Randbedingungen für $\varphi = 0$; $r = R$:

aus (10): $$R = \frac{h^2}{\gamma} \, \frac{1}{1 + e\cos\alpha} \tag{13}$$



nach Skizze und (3) und (6): $$\operatorname{ctg}\varepsilon = -\frac{\dot r}{r\dfrac{d\varphi}{dt}} = -\frac{\sqrt{k + \dfrac{2\gamma}{R} - \dfrac{h^2}{R^2}}}{\dfrac{h}{R}} \tag{14}$$

aus (13) und (11): $$\cos\alpha = \frac{\dfrac{h^2}{R} - \gamma}{\sqrt{k h^2 + \gamma^2}} \tag{15}$$

aus (15): $$\operatorname{tg}\alpha = \frac{\sqrt{1 - \cos^2\alpha}}{\cos\alpha} = \frac{\sqrt{k + \dfrac{2\gamma}{R} - \dfrac{h^2}{R^2}}}{\dfrac{h}{R}} \cdot \frac{\dfrac{h^2}{R\gamma}}{\dfrac{h^2}{R\gamma} - 1} \tag{16}$$

aus (14) und (16): $$\operatorname{tg}\alpha = \frac{\dfrac{h^2}{R\gamma}\operatorname{ctg}\varepsilon}{1 - \dfrac{h^2}{R\gamma}} \tag{17}$$

aus (14): $$K = \frac{h^2}{R^2} \cdot \frac{1}{\sin^2\varepsilon} - \frac{2\gamma}{R} \tag{18}$$

aus (11): $$e^2 = 1 + \frac{k h^2}{\gamma^2} = 1 + \frac{h^4}{R^2\gamma^2} \cdot \frac{1}{\sin^2\varepsilon} - \frac{2h^2}{R\gamma} \tag{19}$$

Diskussion der Bahnform:

aus (3) und (6): $$v_0 = \sqrt{k + \frac{2\gamma}{R}} \tag{20}$$

aus (11): $$k = \frac{\gamma^2}{h^2}(e^2 - 1) \tag{21}$$

Ellipse für $e < 1$ d. h. $k < 0$ d. h. $v_0 < \sqrt{\dfrac{2\gamma}{R}}$

Parabel für $e = 1$ d. h. $k = 0$ d. h. $v_0 = \sqrt{\dfrac{2\gamma}{R}}$

Hyperbel für $e > 1$ d. h. $k > 0$ d. h. $v_0 > \sqrt{\dfrac{2\gamma}{R}}$

Lösung 753

$$F + m(\ddot r - r\dot\varphi^2) = 0; \quad \dot\varphi = \frac{h}{r^2}; \quad r_0 = 2\,\text{cm} \quad \dot\varphi_0 = \frac{v_0}{r_0} = 0{,}25$$

$$v_0 = 0{,}5\,\text{cm} \quad h = \dot\varphi_0 \cdot r_0^2 = 1$$

$$F = \frac{a}{r^5}; \quad a = 8\,\text{dyn cm}^5$$

$$\frac{8}{r^5} + \ddot r - \frac{1}{r^3} = 0 \,\Big|\, \cdot \dot r; \quad \frac{8\dot r}{r^5} + \ddot r \dot r - \frac{\dot r}{r^3} = 0$$

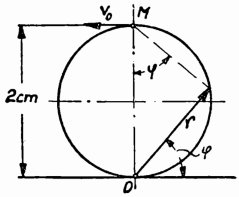
$$-\frac{2}{r^4} + \frac{\dot r^2}{2} + \frac{1}{2r^2} = C$$

$$r = 2\,\text{cm}; \quad \dot{r} = 0: \quad C = 0; \quad \dot{r} = \frac{\sqrt{4 - r^2}}{r^2}$$

$$\dot{\varphi} = \frac{1}{r^2} = \frac{d\varphi}{dr} \cdot \dot{r}; \quad d\varphi = \frac{dr}{\sqrt{4 - r^2}} = \frac{1}{2} \frac{dr}{\sqrt{1 - \left(\frac{r}{2}\right)^2}}$$

$$\varphi = \arcsin \frac{r}{2}; \quad \underline{\underline{r = 2 \sin \varphi}}$$

Dies ist die Gleichung eines Kreises vom Radius 1 cm, dessen senkrechter Durchmesser den Koordinatenursprung berührt.

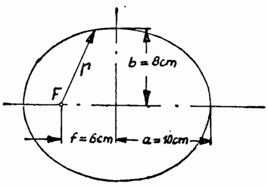


Lösung 754

Nach Aufgabe 752 Gleichung (6) gilt:

$$dt = \frac{r\,dr}{\sqrt{k r^2 + 2\gamma r - h^2}}$$

$$\oint dt = 2\pi \frac{\gamma}{\sqrt{-k^2}} = T \quad \text{(Umlaufzeit)}$$



$$f = \sqrt{a^2 - b^2} = 6\ \text{cm}$$

wegen $v_r = \frac{dr}{dt} = 0$ für $r = a \pm f$

gilt:

$$k(a+f)^2 + 2\gamma(a+f) - h^2 = 0$$
$$\underline{k(a-f)^2 + 2\gamma(a-f) - h^2 = 0}$$
$$4kaf + 4\gamma f = 0$$

$$k = -\frac{\gamma}{a}$$

$$T = 2\pi \sqrt{\frac{a^3}{\gamma}}; \quad \gamma = \frac{4\pi^2 a^3}{T^2}$$

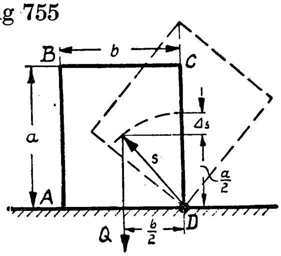
$$F = \frac{m\gamma}{r^2} = \frac{4\pi^2 m a^3}{T^2 r^2}; \quad \text{Mit } m = 20\,\text{g}; \quad a = 10\,\text{cm}; \quad T = 50\,\text{sek}:$$

$$F_{(r=16\,\text{cm})} = \underline{\underline{19{,}7\ \text{dyn}}}$$

$$F_{(r=4\,\text{cm})} = \underline{\underline{1{,}2\ \text{dyn}}}$$

29. Arbeit und Leistung

Lösung 755

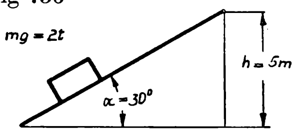


$$A = Q \cdot \Delta s$$

$$s = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}\,; \quad \Delta s = s - \frac{a}{2}$$

$$A = Q\left[\sqrt{\frac{a^2}{4} + \frac{b^2}{4}} - \frac{a}{2}\right] = \underline{\underline{4000 \text{ mkg}}}$$

Lösung 756



$$A = mgh + mg\cos\alpha \cdot \frac{h}{\sin\alpha} \cdot \mu$$

$$A = mgh\left(1 + \frac{\cos\alpha}{\sin\alpha} \cdot \mu\right)$$

$$A = \underline{\underline{18660 \text{ mkg}}}$$

Lösung 757

Notwendige Arbeit: $A = Q \cdot \Delta s = V \cdot \gamma \cdot \Delta s\,[\text{mkg}]$

Pumpennutzleistung: $N'_{\text{PS}} = N_{\text{PS}} \cdot \eta\,[\text{PS}]$; $N^* = N'_{\text{PS}} \cdot k\,[\text{mkg/sek}]$

$$k = 75\left[\frac{\text{mkg}}{\text{sek}} \Big/ \text{PS}\right]$$

$$v \cdot \gamma \cdot \Delta s = N^* \cdot \Delta t\,; \quad \Delta t = \frac{v \cdot \gamma \cdot \Delta s}{N^*} = \frac{v \cdot \gamma \cdot \Delta s}{N_{\text{PS}} \cdot \eta \cdot k}$$

$$\Delta t = \frac{5000 \cdot 1000 \cdot 3}{2 \cdot 0{,}8 \cdot 75 \cdot 3600} = 34{,}722 \text{ h}$$

$$\Delta t = \underline{\underline{34 \text{h } 43' \, 12''}}$$

Lösung 758

$G = 200$ kg

$s = 84 \cdot 0{,}75$ m

$\eta = 0{,}7$

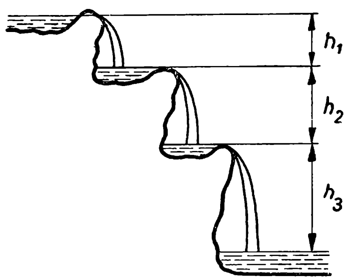
$t = 60$ sek

$$N^* = \frac{G \cdot s}{\eta \cdot t} \; \frac{\text{mkg}}{\text{sek}}$$

$$N_{\text{PS}} = \frac{N^*}{75} = \frac{200 \cdot 84 \cdot 0{,}75}{75 \cdot 60 \cdot 0{,}7} = \underline{\underline{4 \text{ PS}}}$$

$$N_{\text{kW}} = \frac{N^*}{101{,}9} = \frac{4 \cdot 75}{101{,}9} = \underline{\underline{2{,}94 \text{ kW}}}$$

Lösung 759



$$h_1 = 12 \text{ m}$$
$$h_2 = 12{,}8 \text{ m}$$
$$h_3 = 15 \text{ m}$$
$$\Sigma h = 39{,}8 \text{ m}$$

$$N_{\text{PS}} = \frac{G \cdot \Sigma h}{\Delta t \cdot 75}\,; \quad \frac{G}{\Delta t} = V \cdot \gamma$$

$$N_{\text{PS}} = \frac{V \cdot \gamma \cdot \Sigma h}{75} = \frac{75{,}4 \cdot 1000 \cdot 39{,}8}{75}$$

$$N_{\text{PS}} = \underline{\underline{40000 \text{ PS}}}$$

$1360 \text{ PS} \mathrel{\hat{=}} 1 \text{ MW}$; $N_{\text{MW}} = \underline{\underline{29{,}4 \text{ MW}}}$

Lösung 760

$n = 45$

$G = 10000$ kg

$N_v = 0{,}05\,N$ (Leitungsverluste)

$v = \frac{12}{3{,}6}$ m/sek

$\mu = 0{,}02$

$$N^* = P \cdot v + N_v^*$$

$$N^* = \frac{nG \cdot \mu \cdot v}{1 - 0{,}05} \, \frac{\text{mkg}}{\text{sek}}$$

$$N_{\text{PS}} = \frac{45 \cdot 10000 \cdot 12 \cdot 0{,}02}{0{,}95 \cdot 75 \cdot 3{,}6} = \underline{\underline{420\ \text{PS}}}$$

$$N_{\text{kW}} = \frac{N^*}{101{,}9} = \underline{\underline{309\ \text{kW}}}$$

Lösung 761

$P_E = 200$ kg

$P_N = 1000$ kg

$Q_F = 600$ t

$h = 10$ m

$\Delta t = 12$ h Umschlagzeit

$\frac{Q_F}{P_N} = B$ Anzahl der Hübe

$$A = (P_E + P_N) \cdot B \cdot h = N^* \cdot \Delta t$$

$$N^* = N_{\text{PS}} \cdot 75 \, \frac{\text{m kg}}{\text{sek}}$$

$$N_{\text{PS}} = \frac{(P_E + P_N) \cdot B \cdot h}{\Delta t \cdot 75} = \frac{1200 \cdot 600 \cdot 10}{12 \cdot 3600 \cdot 75}$$

$$N_{\text{PS}} = \underline{\underline{2{,}2\ \text{PS}}}; \quad N_{\text{kW}} = \underline{\underline{1{,}63\ \text{kW}}}$$

Lösung 762

$$A = G(\sin\alpha + \mu \cos\alpha) \cdot l$$

$$A = 20 \cdot 6\,(0{,}5 + 0{,}01 \cdot 0{,}866) = \underline{\underline{61{,}04\ \text{mkg}}}$$

Lösung 763

$v = 15$ Knoten $\triangleq 15 \cdot 0{,}5144$ m/sek

$N_{\text{PS}} = 5133$ PS

$\eta = 0{,}4$

$$75 \cdot N_{\text{PS}} \cdot \eta = P \cdot v; \quad P = \frac{75 \cdot N_{\text{PS}} \cdot \eta}{v \cdot 1000}\ \text{t}$$

$$\underline{\underline{P = 20\ \text{t}}}$$

Lösung 764

$$N^* = P \cdot v \cdot \eta; \quad P = p_m \cdot F; \quad v = \frac{n \cdot s}{60}\ \text{m/sek}; \quad N^* = N_{\text{PS}} \cdot 75 \, \frac{\text{mkg}}{\text{sek}}$$

$$N^* = N_{\text{kW}} \cdot 101{,}9 \, \frac{\text{mkg}}{\text{sek}}$$

$$N_{\text{PS}} = \frac{p_m \cdot F \cdot n \cdot s \cdot \eta}{75 \cdot 60} = \frac{5 \cdot 300 \cdot 120 \cdot 0{,}4 \cdot 0{,}9}{75 \cdot 60}$$

$$N_{\text{PS}} = \underline{\underline{14{,}4\ \text{PS}}}; \quad N_{\text{kW}} = N_{\text{PS}} \cdot \frac{75}{101{,}9} = \underline{\underline{10{,}6\ \text{kW}}}$$

Lösung 765

$$P \cdot \mu = U; \quad U \cdot v = N^* = N_{PS} \cdot 75; \quad v = \frac{D \cdot \pi \cdot n}{60}$$

$$P = \frac{60 \cdot N_{PS} \cdot 75}{\mu \cdot \pi \cdot n \cdot D} = \frac{60 \cdot 1{,}6 \cdot 75}{0{,}2 \cdot \pi \cdot 120 \cdot 0{,}6} = \underline{\underline{159\,\mathrm{kg}}}$$

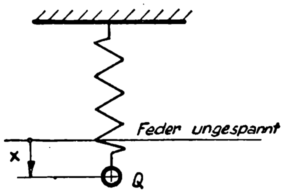
Lösung 766

$$N_{PS} = \frac{P_s \cdot s}{t_s \cdot \eta \cdot 75} = \frac{1200 \cdot 2}{10 \cdot 0{,}8 \cdot 75} = \underline{\underline{4\,\mathrm{PS}}}$$

Lösung 767

$$U \cdot v = N_{PS} \cdot 75; \quad v = \frac{d \cdot \pi \cdot n}{60}; \quad U = \frac{60 \cdot N_{PS} \cdot 75}{d \cdot \pi \cdot n} = \frac{60 \cdot 75}{8 \cdot \pi \cdot 6} = \underline{\underline{29{,}9\,\mathrm{kg}}}$$

Lösung 768



$$W = \underbrace{\frac{Q}{2g}\dot{x}^2}_{\text{Kinetische Energie}} + \underbrace{\frac{c}{2}x^2 - Q \cdot x}_{\text{Potentielle Energie}} = \text{const}$$

Lösung 769

$s = 20000\,\mathrm{m}$

$a = 0{,}08\,\mathrm{m}$

$T = 4\,\mathrm{sek}$

$G = 80\,\mathrm{kg}$

$\mu = 0{,}05$

$\Delta t = 1{,}5\,\mathrm{h}$

Anzahl der Perioden: $\nu = \dfrac{\Delta t \cdot 3600}{T} = 1350\,\mathrm{H}$

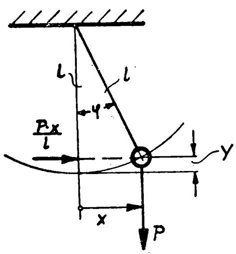
Gesamtarbeit: $A = A_{\text{Reibung}} + A_{\text{Heben+Senken}}$

$$A = G\,[\mu \cdot s + 2 \cdot 1{,}4 \cdot a \cdot \nu]\,\mathrm{mkg}$$

$$A = \underline{\underline{104600\,\mathrm{mkg}}}$$

$$N_{PS} = \frac{A}{\Delta t \cdot 75} = \frac{104600}{1{,}5 \cdot 3600 \cdot 75} = \underline{\underline{0{,}258\,\mathrm{PS}}}$$

Lösung 770



$$A_1 = \underline{\underline{P \cdot y}}; \quad A_2 = \frac{Px}{l} \cdot \frac{x}{2} = \underline{\underline{\frac{Px^2}{2l}}}$$

$$y = l\,(1 - \cos\varphi); \quad \sin\varphi = \frac{x}{l}$$

$$y = l\left(1 - \sqrt{1 - \frac{x^2}{l^2}}\right)$$

$$y = \frac{x^2}{l} + \frac{y^2}{l}$$ Bei Vernachlässigung von y^2 wird $A_1 = A_2$

Lösung 771

Umfangskraft $P_u = S_1 - S_2 = Q - P$

$$N_{\mathrm{PS}} = \frac{P_u \cdot v}{75} = (Q-P)\,\frac{d\pi n}{60 \cdot 75} = 3 \cdot \frac{0{,}636 \cdot \pi \cdot 120}{60 \cdot 75} = \underline{\underline{0{,}16\ \mathrm{PS}}}$$

$$N_{\mathrm{W}} = N_{\mathrm{PS}} \cdot \frac{75}{0{,}1019} = \underline{\underline{117{,}8\ \mathrm{Watt}}}$$

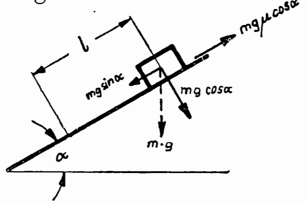
Lösung 772

$$N_{\mathrm{PS}} = \frac{(T-t)}{75} \cdot v = \frac{(T-t)}{75} \cdot \frac{2r\pi n}{60}; \quad T = 2t$$

$$t = \frac{N_{\mathrm{PS}} \cdot 75 \cdot 60}{2r\pi \cdot n} = \frac{20 \cdot 75 \cdot 60}{2 \cdot 0{,}5 \cdot \pi \cdot 150} = \underline{\underline{191\ \mathrm{kg}}}; \quad T = \underline{\underline{382\ \mathrm{kg}}}$$

30. Energiesatz des Massenpunktes

Lösung 773

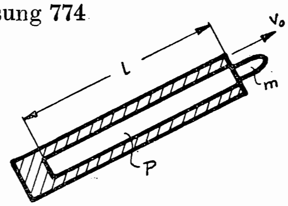


$$mg \sin\alpha \cdot l = m\,\frac{v^2}{2} + mg \cos\alpha \cdot \mu \cdot l$$

$$v = \sqrt{2gl\,(\sin\alpha - \cos\alpha \cdot \mu)}$$

$$v = \sqrt{2 \cdot 9{,}81 \cdot 2\,(0{,}5 - 0{,}0866)} = \underline{\underline{4{,}02\ \mathrm{m/sek}}}$$

Lösung 774



$$\frac{mv_0^2}{2} = P_m \cdot l; \quad P_m = \frac{mv_0^2}{2l}$$

$$P_m = \frac{24 \cdot 500^2}{9{,}81 \cdot 2 \cdot 2} = 152\,900\ \mathrm{kg}$$

$$P_m = \underline{\underline{152{,}9\ \mathrm{t}}}$$

Lösung 775

$$v_1 = -5\ \mathrm{m/sek}$$

$$v_2 = +55\ \mathrm{m/sek}$$

$$v = v_2 - v_1 = 60\ \mathrm{m/sek}$$

$$P = m \cdot b; \quad b = \frac{v}{t} = \frac{60}{30} = 2\ \mathrm{m/sek^2}$$

$$P = \frac{3}{9{,}81} \cdot 2 = \underline{\underline{0{,}612\ \mathrm{kg}}}$$

$$A = -\frac{mv_1^2}{2} + \frac{mv_2^2}{2} = \frac{m}{2}\,(v_2^2 - v_1^2) = \underline{\underline{459\ \mathrm{mkg}}}$$

Lösung 776

$$\frac{mv^2}{2} + mg \cdot s \cdot \alpha = mg \cdot s \cdot \mu; \quad s = \frac{v^2}{2g(\mu - \alpha)} = \frac{\left(\frac{36}{3,6}\right)^2}{2 \cdot 9,81 \cdot (0,1 - 0,008)}$$

$$s = \underline{\underline{55,3 \text{ m}}}$$

$$s = \frac{vt}{2}; \quad t = \frac{2s}{v} = \frac{2 \cdot 55,3}{10} = \underline{\underline{11,06 \text{ sek}}}$$

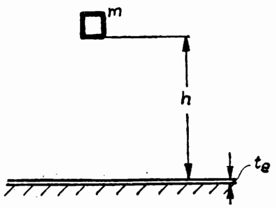
Lösung 777

$$P = m \cdot b + R; \quad v = bt + v_0$$

$$N_{\text{PS}} = \frac{P \cdot v}{75} = \frac{mb + R}{75}(v_0 + b \cdot t); \quad N_{\text{PS}} = \underline{\underline{1620 \text{ PS}}}$$

$$N_{\text{kW}} = \underline{\underline{1192 \text{ kW}}}$$

Lösung 778



$$G(h + t_e) = W \cdot F \cdot t_e$$

$$W = \frac{G(h + t_e)}{F \cdot t_e} = \frac{60 \cdot 101}{12 \cdot 100 \cdot 1} = \underline{\underline{5,05 \text{ kg/cm}^2}}$$

Lösung 779

Energiegleichung am Ende der Beschleunigungsperiode:

$P = 25$ kg

$R = 15$ kg

$s_0 = 20$ m

$mg = 6000$ kg

$$\frac{m v_{\max}^2}{2} = (P - R)\, s_0$$

$$v_{\max} = \underline{\underline{0,808 \text{ m/sek}}}$$

Ausrollen des Wagens: $\dfrac{m v_{\max}^2}{2} = R \cdot s_x$

$$s_x = \frac{m v_{\max}^2}{2R} = 13,33 \text{ m}$$

Gesamte Fahrstrecke: $s = s_0 + s_x = \underline{\underline{33,33 \text{ m}}}$

Lösung 780

Gewicht des Hammers: $\dfrac{P \cdot v^2}{g \cdot 2} = R \cdot l$

$R = 70$ kg

$l = 0,15$ cm

$v = 1,25$ m/sek

$$P = \frac{R \cdot l \cdot g \cdot 2}{v^2} = \frac{70 \cdot 0,0015 \cdot 9,81}{1,25^2}$$

$$P = \underline{\underline{1,37 \text{ kg}}}$$

Lösung 781

$$\frac{mv^2}{2} = Fl; \qquad v = \sqrt{2\,\frac{F \cdot l}{m}} = \sqrt{2 \cdot \frac{50\,000 \cdot 1{,}875 \cdot 9{,}81}{39}} = \underline{\underline{217 \text{ m/sek}}}$$

$$\text{Fallhöhe } H\text{:} \quad v = \sqrt{2gH}; \qquad H = \frac{v^2}{2g} = \underline{\underline{2400 \text{ m}}}$$

Lösung 782

$$mg = 500 \text{ t} \qquad m\ddot{x} + 51\dot{x} + 765 = 0; \quad \ddot{x} = \dot{x}\,\frac{d\dot{x}}{dx}$$

$$R = 765 + 51\dot{x} \text{ kg}$$

$$v_0 = 15 \text{ m/sek} \qquad m\,\frac{\dot{x}\,d\dot{x}}{51\dot{x} + 765} = -dx$$

$$\frac{m}{51}\left[\int d\dot{x} - 765 \int \frac{d\dot{x}}{51\dot{x} + 765}\right] = -x + x_0$$

$$\frac{m}{51}\left[\dot{x} - \frac{765}{51}\ln(51\dot{x} + 765)\right] = -x + x_0;$$

$$\begin{matrix}\dot{x} = v_0\\ x = 0\end{matrix}: \quad x_0 = \frac{m}{51}\left[v_0 - 15\ln(51v_0 + 765)\right]$$

$$\begin{matrix}\dot{x} = 0\\ x = s\end{matrix}: \quad s = \frac{m}{51}\left[v_0 + 15\ln\frac{765}{51v_0 + 765}\right]; \qquad s = \underline{\underline{4600 \text{ m}}}$$

Lösung 783

$$v_B = \sqrt{2 \cdot g \cdot 2\,\overline{MO}} = \sqrt{4 \cdot 9{,}81 \cdot 0{,}981} = \underline{\underline{6.2 \text{ m/sek}}}$$

Lösung 784

Federkonstante $c = 0{,}4$ t/cm

$$U = \int P \cdot dx; \quad P = c \cdot x; \quad U = \int cx\,dx + \text{const} = \frac{cx^2}{2} + \text{const}$$

$$U = \underline{\underline{0{,}2\,x^2 + \text{const tcm}}}$$

Lösung 785

$$\frac{mv_0^2}{2} = \frac{c \cdot s^2}{2}; \quad v_0 = s\sqrt{\frac{c}{m}} = 0{,}1\sqrt{\frac{20 \cdot 9{,}81}{0{,}03}} = \underline{\underline{8{,}1 \text{ m/sek}}}$$

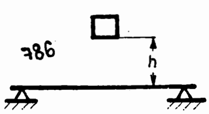
Lösung 786

$$mg = Q \qquad c \cdot \eta_0 = Q$$

$$h = 10 \text{ cm}$$

$$\eta_0 = 2 \text{ mm} \qquad c = \frac{Q}{\eta_0} = \frac{Q}{0{,}2} = 5Q \text{ kg/cm}$$

786



1. Die Last wird auf den ungebogenen Träger ohne Anfangsgeschwindigkeit aufgesetzt:

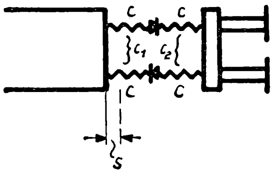
$$\frac{c \cdot \eta^2}{2} = Q \cdot \eta; \quad \eta = \frac{2}{5} \text{ cm}; \quad \eta = \underline{\underline{4 \text{ mm}}}$$

2. Die Last fällt von 10 cm Höhe ohne Anfangsgeschwindigkeit auf den Träger:

$$\frac{c \cdot \eta^2}{2} = \frac{m v^2}{2} + Q \cdot \eta; \quad v = \sqrt{2 g h}$$

$$\eta^2 - \frac{2}{5}\eta = \frac{2}{5} h; \quad \eta = 2{,}21 \text{ cm} \mathrel{\hat{=}} \underline{\underline{22{,}1 \text{ mm}}}$$

Lösung 787



$$c_1 = c_2 = 2c$$

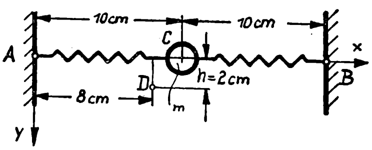
$$\frac{1}{c_{ges}} = \frac{1}{c_1} + \frac{1}{c_2} = \frac{2}{2c}; \quad c_{ges} = c$$

$$\frac{m v^2}{2} = P \cdot s = \frac{c_{ges} \cdot s^2}{2}$$

$$s = \sqrt{\frac{m v^2}{c_{ges}}} = \sqrt{\frac{16\,000 \cdot 4}{9{,}81 \cdot 500\,000}} = 0{,}114 \text{ m}$$

$$s_{Puffer} = \frac{s}{2} = \underline{\underline{5{,}7 \text{ cm}}}$$

Lösung 788



$$c_1 = 2 \text{ kg/cm}; \quad c_2 = 4 \text{ kg/cm}$$

$$m g = 1{,}962 \text{ kg}; \quad v_0 = 2 \text{ m/sek}$$

$$\frac{m v_0^2}{2} + m g h = \frac{m v^2}{2} + c_1 \frac{(\Delta l_1)^2}{2} + c_2 \frac{(\Delta l_2)^2}{2}; \quad \Delta l_1 = 10 - \sqrt{8^2 + 2^2}$$

$$= 1{,}75 \text{ cm}$$

$$\Delta l_2 = 10 - \sqrt{12^2 + 2^2}$$

$$= -2{,}17 \text{ cm}$$

$$v = \sqrt{v_0^2 + 2 g h - \frac{1}{m}\left[c_1 (\Delta l_1)^2 + c_2 (\Delta l_2)^2\right]}$$

$$v = 178 \text{ cm/sek}; \quad v = \underline{\underline{1{,}78 \text{ m/sek}}}$$

Lösung 789

Potentielle Energie: $U = \underline{\underline{P \cdot l (1 - \sin\varphi)}}$

Kinetische Energie: $T = \frac{m v^2}{2}; \quad v = \sqrt{2 g l \sin\varphi}; \quad m g = P$

$$T = \underline{\underline{P \cdot l \sin\varphi}}$$

$$T + U = \underline{\underline{P \cdot l = \text{konst.}}}$$

Lösung 790

$$x = a\sin(kt+\beta); \qquad x = 0: \; kt+\beta = 0; \quad U = 0$$

$$T_{x=0} = T_{\max} = \frac{m}{2}\cdot\dot{x}_{(kt+\beta=0)} = \frac{mk^2}{2}a^2$$

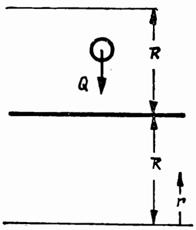
$$T = \frac{mk^2}{2}a^2 - U; \qquad U = \frac{cx^2}{2} = \underline{\underline{\frac{mk^2}{2}}}\,x^2;$$

Aus der Schwingungsgleichung folgt

$$k = \sqrt{\frac{c}{m}}$$

$$\underline{\underline{T = \frac{mk^2}{2}(a^2 - x^2)}}$$

Lösung 791



$$Q = \frac{c}{r^2}; \quad r = R; \quad Q = P: \quad C = PR^2$$

$$Q = \frac{PR^2}{r^2}$$

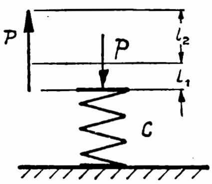
Nach Aufgabenstellung soll sein:

$$\frac{mv^2}{2} = \int_{r=R}^{2R} Q\cdot dr = Q_x\cdot R$$

$$P\cdot R^2\left[\frac{1}{R} - \frac{1}{2R}\right] = Q_x\cdot R$$

$$\underline{\underline{Q_x = \frac{P}{2}}}$$

Lösung 792



$$\frac{c}{2}l_1^2 + P(l_1 + l_2) = \frac{c}{2}l_2^2; \quad cl_1 = P$$

$$\frac{l_1}{2} + l_1 + l_2 = \frac{l_2^2}{2l_1};$$

$$\frac{l_2^2}{l_1^2} - 2\frac{l_2}{l_1} - 3 = 0; \quad \frac{l_2}{l_1} = 1 \overset{+}{\underset{(-)}{}} \sqrt{1+3} = \underline{\underline{3}}$$

Lösung 793

$$\frac{mv_0^2}{2} = \int_R^{R+H} Q\cdot dr; \quad Q = \frac{mg\cdot R^2}{r^2}$$

$$\frac{v_0^2}{2} = gR^2\left[\frac{1}{R} - \frac{1}{R+H}\right]; \quad H = \frac{2gR^2}{2gR - v_0^2} - R; \quad H = \underline{\underline{\frac{Rv_0^2}{2gR - v_0^2} = 51\ \text{km}}}$$

Lösung 794

$$\frac{mv^2}{2} = \int_{r=5}^{r=\infty} F\cdot dr = q_1 q_2 \int_5^\infty \frac{dr}{r^2} = \frac{q_1 q_2}{5}; \quad v = \sqrt{\frac{2q_1q_2}{m\cdot 5}}$$

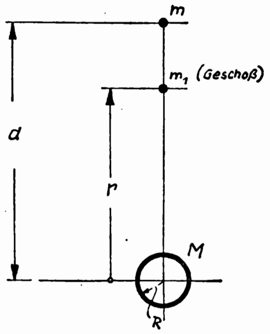
$$v = \sqrt{\frac{2\cdot 100\cdot 10}{1\cdot 5}} = \underline{\underline{20\ \text{cm/sek}}}$$

Lösung 795

Nach Aufgabe 791 gilt: $\dfrac{mv^2}{2} = mgR^2\left[\dfrac{1}{R} - \dfrac{1}{2R}\right]$

$$v = \sqrt{g \cdot R} = \underline{\underline{7{,}9 \text{ km/sek}}}$$

Lösung 796



Potentielle Energie: $U = \displaystyle\int_{r=R}^{P=0} P \cdot dr$

$$P = m_1 f\left[\frac{M}{r^2} - \frac{m}{(d-r)^2}\right]$$

$$U = m_1 f\left|-\frac{M}{r} - \frac{m}{(d-r)}\right|_{r=R}^{r_0}$$

Gleichgewicht bei: $\dfrac{M m_1}{r_0^2} \cdot f = \dfrac{m m_1}{(d-r_0)^2} \cdot f$

$$M(d-r_0)^2 = m r_0^2$$

$$r_0 = \frac{d}{M-m}\left[M \overset{(+)}{-} \sqrt{Mm}\right]$$

$$U = m_1 f\left[\frac{M}{R} + \frac{m}{(d-R)} - \frac{M}{r_0} - \frac{m}{d-r_0}\right]; \quad f = \frac{g}{\dfrac{M}{R^2} - \dfrac{m}{(d-R)^2}};$$

Energiebedingung: $\dfrac{m_1 v_0^2}{2} = U$

$$v_0^2 = 2g\,\frac{\dfrac{M}{R} + \dfrac{m}{(d-R)} - \dfrac{M}{r_0} - \dfrac{m}{d-r_0}}{\dfrac{M}{R^2} - \dfrac{m}{(d-R)^2}}; \quad v_0^2 = 2g\,\frac{\dfrac{M}{R} + \dfrac{m}{d-R} - \dfrac{m}{d}\left(1 + \sqrt{\dfrac{M}{m}}\right)^2}{\dfrac{M}{R^2} - \dfrac{m}{(d-R)^2}}$$

$$v_0^2 = \frac{2gR(d-R)}{d} \cdot \frac{\dfrac{M}{m}(d-R)^2 + R^2 - 2\sqrt{\dfrac{M}{m}}(d-R)\,R}{\dfrac{M}{m}(d-R)^2 - R^2}$$

$$\underline{\underline{v_0^2 = \frac{2gR(d-R)}{d} \cdot \frac{\sqrt{\dfrac{M}{m}}(d-R) - R}{\sqrt{\dfrac{M}{m}}(d-R) + R}}}; \quad v_0^2 = \frac{59}{30}\,\frac{1-\alpha}{1+\alpha} \cdot gR; \qquad \alpha = \frac{1}{59\sqrt{80}}$$

$$\underline{\underline{v_0 = 10{,}75 \text{ km/sek}}}$$

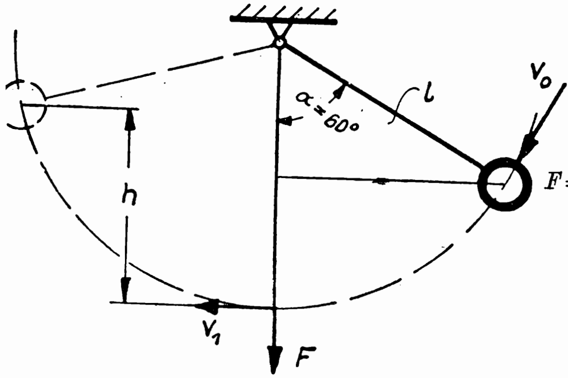
Lösung 797

$$\frac{mv_0^2}{2} + mg \cdot s = F \cdot s; \quad P = mg$$

$$F = P\left(1 + \frac{v_0^2}{2gs}\right) = 6\left(1 + \frac{144}{2 \cdot 9{,}81 \cdot 10}\right) = \underline{\underline{10{,}3 \text{ t}}}$$

31. Gemischte Aufgaben

Lösung 798



$$\frac{mv_0^2}{2} + mgl(1 - \cos 60°) = \frac{mv_1^2}{2}$$

$$F = \frac{mv_1^2}{l} + mg$$

$$F = mg + \frac{G}{gl}\,[2gl(1 - \cos 60°) + v_0^2]$$

$$F = mg\left(2 + \frac{v_0^2}{gl}\right)$$

$$F = 1\left(2 + \frac{210^2}{981 \cdot 50}\right) = \underline{\underline{2{,}9\ \text{kg}}}$$

$$h = l(1 - \cos 60°) + \frac{v_0^2}{2g} = 25 + \frac{210^2}{2 \cdot 981} = \underline{\underline{47{,}5\ \text{cm}}}$$

Lösung 799

Im höchsten Bahnpunkt muß die Fliehkraft gleich dem Gewicht sein.

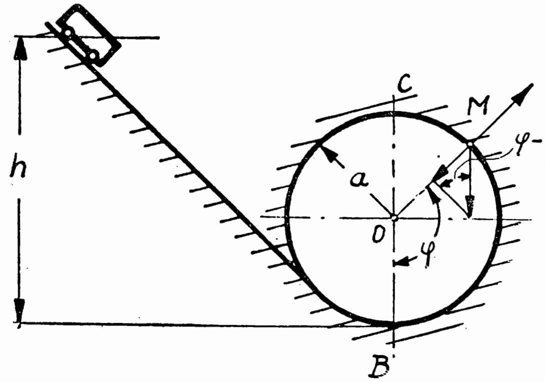
$$\frac{mv_2^2}{l} = mg; \quad v_2^2 = gl$$

Energiegleichung: $\dfrac{mv_2^2}{2} + mg \cdot 2l = \dfrac{mv_0^2}{2} + mgl(1 - \cos 60°)$

$$v_0 = \sqrt{2gl\left(\frac{3}{2} + \cos 60°\right)};$$

$$v_0 \geq \underline{\underline{4{,}43\ \text{m/sek}}}$$

Lösung 800



Bahndruck:

$$N = \frac{mv_\varphi^2}{a} - mg \cdot \sin\left(\varphi - \frac{\pi}{2}\right)$$

$$\sin\left(\varphi - \frac{\pi}{2}\right) = -\cos\varphi$$

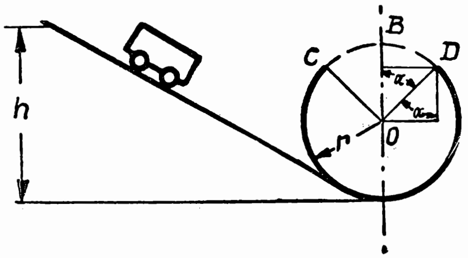
$$v_\varphi^2 = 2g[h - a(1 - \cos\varphi)]; \quad mg = P$$

$$\underline{\underline{N = P\left[\frac{2h}{a} - 2 + 3\cos\varphi\right]}}$$

für $N \geqq 0$ gilt $\varphi = \pi$:

$$\underline{\underline{h = \frac{5}{2}a}}$$

Lösung 801



$$v_0^2 = 2g\,[h - r\,(1 + \cos\alpha)]$$

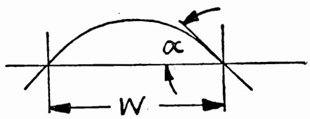
Wurfparabel: $y = x \operatorname{tg}\alpha - \dfrac{g}{2} \cdot \dfrac{x^2}{v_0^2 \cos^2\alpha}$

$$\frac{dy}{dx} = 0: \quad x_{(y\,\max)} = \frac{W}{2} = \frac{v_0^2 \sin 2\alpha}{2g}$$

Um die Symmetrie der Wurfparabel zu erhalten, gilt

$$\frac{W}{2} = \frac{v_0^2 \sin\alpha \cos\alpha}{g} = r \sin\alpha$$

$$r = \frac{2g}{g}\,[h - r\,(1 + \cos\alpha)] \cdot \cos\alpha$$

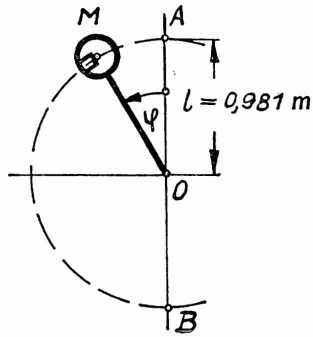


$$\underline{\underline{h = r\left[1 + \cos\alpha + \frac{1}{2\cos\alpha}\right]}}; \quad \frac{dh}{d\alpha} = 0 = -\sin\alpha + \frac{1}{2}\,\frac{\sin\alpha}{\cos^2\alpha}; \qquad \cos^2\alpha = \frac{1}{2}$$

$$\cos\alpha = \sqrt{\frac{1}{2}}$$

also: $h_{\min}$ $\underline{\underline{\text{für } \alpha = 45^\circ}}$

Lösung 802

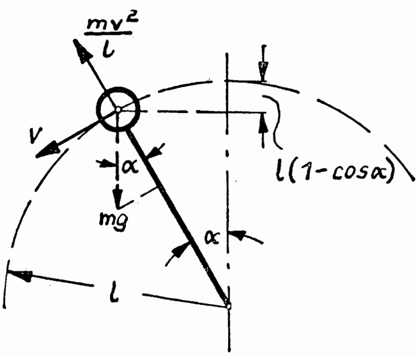


$$\frac{m}{2}\,v_B^2 = mg \cdot 2l; \quad v_B^2 = 4lg$$

$$P_B = mg + \frac{m v_B^2}{l} = mg\,(1 + 4)$$

$$P_B = \underline{\underline{100\ \text{kg}}}$$

Lösung 803



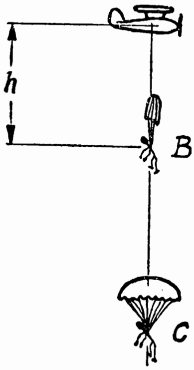
$$v^2 = 2gl\,(1 - \cos\alpha)$$

$$N = \frac{mv^2}{l} - mg\cos\alpha$$

$$N = 0: \quad 2g\,(1 - \cos\alpha) = g\cos\alpha$$

$$\underline{\underline{\alpha = \operatorname{arc}\cos\frac{2}{3}}}$$

Lösung 804



$$v_B = \sqrt{2gh} = \sqrt{2 \cdot 9{,}81 \cdot 100} = 44{,}3 \text{ m/sek}$$

$$v_B - v_c = b \cdot t; \qquad b = \frac{v_B - v_c}{t}$$

$$b = \frac{44{,}3 - 4{,}3}{5} = 8 \text{ m/sek}^2$$

$$P = m \cdot b + mg = 70\left(\frac{8}{9{,}81} + 1\right)$$

$$\underline{\underline{P = 127{,}4 \text{ kg}}}$$

Lösung 805

$$\frac{m}{2} v_0^2 - mg \cdot h - W \cdot s = P \cdot s; \qquad P = \frac{mv_0^2}{2s} - \frac{mgh}{s} - W$$

$$m \cdot g = 1000 \text{ t} \qquad P = \underline{\underline{8690 \text{ kg}}}$$

$$W = 2 \text{ t}$$

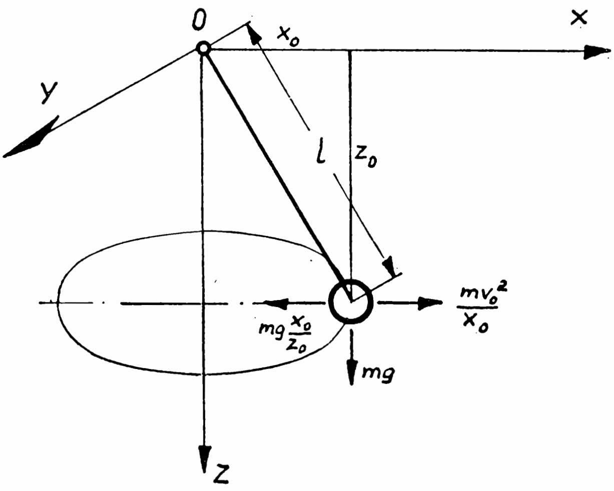
$$s = 500 \text{ m}$$

$$h = 2 \text{ m}$$

Lösung 806

$$t = 0: \quad x = x_0; \quad y = 0; \quad z = z_0$$

$$\dot{x} = 0; \quad \dot{y} = v_0; \quad \dot{z} = 0$$



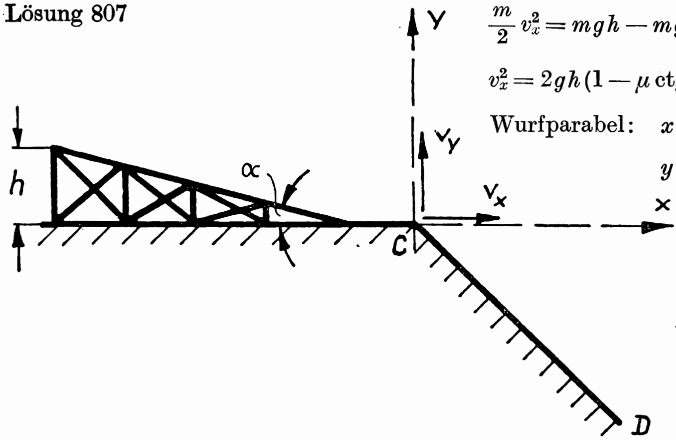
$$\frac{mv_0^2}{x_0} - mg \cdot \frac{x_0}{z_0} = 0$$

$$\underline{\underline{v_0 = x_0 \sqrt{\frac{g}{z_0}}}}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{v_0} \cdot x_0$$

$$\underline{\underline{T = 2\pi \sqrt{\frac{z_0}{g}}}}$$

Lösung 807



$$\frac{m}{2} v_x^2 = mgh - mg \frac{\cos\alpha}{\sin\alpha} \cdot \mu \cdot h$$

$$v_x^2 = 2gh(1 - \mu \operatorname{ctg}\alpha) = 152 \frac{\mathrm{m}^2}{\mathrm{sek}^2}$$

Wurfparabel: $x = v_x t$

$$y = v_y \cdot t - \frac{g}{2} t^2$$

$$y = \frac{v_y}{v_x} \cdot x - \frac{g}{2} \frac{x^2}{v_x^2}$$

Gleichung der Strecke CD:

$$y = -x$$

Beide Gleichungen müssen für y_0 und x_0 übereinstimmen, also:

$$-x_0 = \frac{v_y}{v_x} x_0 - \frac{g}{2} \frac{x_0^2}{v_x^2};$$

$$x_0 = \frac{v_x^2 \cdot 2}{g}\left(1 + \frac{v_y}{v_x}\right) = \frac{152 \cdot 2}{9{,}81}\left(1 + \frac{1}{12{,}3}\right) = 33{,}4\ \mathrm{m}$$

Die Entfernung des Landungspunktes von C aus auf CD ist somit

$$s = x_0 \sqrt{2} = \underline{\underline{47{,}4\ \mathrm{m}}}$$

Lösung 808

$$m\ddot{x} + cx = mg; \qquad \text{mit } x(0) = 0; \qquad \dot{x}(0) = \sqrt{2gH}:$$

$$x = c_1 \cos\sqrt{\frac{c}{m}}\, t + c_2 \sin\sqrt{\frac{c}{m}}\, t + \frac{mg}{c}; \qquad c_1 = -\frac{mg}{c}$$

$$c_2 = \sqrt{\frac{2gHm}{c}}$$

$$x = \frac{mg}{c}\left(1 - \cos\sqrt{\frac{c}{m}}\, t\right) + \sqrt{\frac{2gHm}{c}} \sin\sqrt{\frac{c}{m}}\, t$$

$$\dot{x} = g\sqrt{\frac{m}{c}} \sin\sqrt{\frac{c}{m}}\, t + \sqrt{2gH} \cos\sqrt{\frac{c}{m}}\, t$$

$$\dot{x} = 0; \qquad t = T: \qquad \operatorname{tg}\sqrt{\frac{c}{m}}\, T = -\sqrt{\frac{2cH}{mg}}$$

Energie: $\quad mg(H + h) = \frac{c}{2} h^2; \qquad c = \frac{2mg(H+h)}{h^2}$

$$\operatorname{tg}\frac{\sqrt{2g(H+h)}}{h} \cdot T = -\frac{2\sqrt{H(H+h)}}{h}$$

$$\underline{\underline{T = \frac{h}{\sqrt{2g(H+h)}}\left[\frac{\pi}{2} + \operatorname{arc\,tg}\frac{h}{2\sqrt{H(H+h)}}\right]}}$$

$$S = \int_0^T P_F \cdot dt = \int_0^T c \cdot x \cdot dt$$

$$S = \left| mg\left(t - \sqrt{\frac{m}{c}}\sin\sqrt{\frac{c}{m}}\,t\right) - m\sqrt{2gH}\cos\sqrt{\frac{c}{m}}\,t \right|_0^T$$

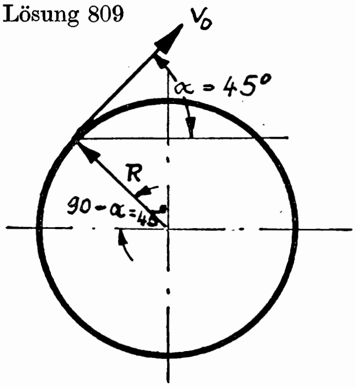
$$S = mg\left(T - \frac{h}{\sqrt{2g(H+h)}}\sin\sqrt{\frac{c}{m}}\,T\right) + m\sqrt{2gH}\left(1 - \cos\sqrt{\frac{c}{m}}\,T\right)$$

$$S = mg\left(T + \sqrt{\frac{2H}{g}}\cos\sqrt{\frac{c}{m}}\,T\right) + m\sqrt{2gH}\left(1 - \cos\sqrt{\frac{c}{m}}\,T\right)$$

$$\underline{\underline{S = mgT + m\sqrt{2gH}}}$$

Dieses Ergebnis erhält man ebenfalls, wenn man in die in der Aufgabensammlung angegebene Lösung die Konstanten einsetzt.

Lösung 809



Winkel der größten Wurfweite: $\alpha = 45°$

$$x_{max} = \frac{v_0^2}{g} = s \quad \text{(vgl. Aufg. 801)}$$

$$v_0 = \sqrt{g \cdot s} = \underline{\underline{52{,}5\ \text{m/sek}}}$$

$$n = \frac{v_0 \cdot 60}{2R\pi} = \underline{\underline{286\ \text{U/min}}}$$

Lösung 810

Energie: $mg(2r - y_0) = c\,\frac{(2r-a)^2}{2} + \frac{mv_B^2}{2}$

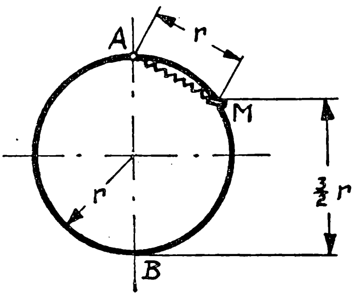
Kräfte: $\frac{mv_B^2}{r} + mg = c(2r - a); \quad mv_B^2 = cr(2r-a) - mg$

$r = a = 20$ cm: $\quad y_0 = a\cos 60° = 0{,}5a$

$$0 = -2mg \cdot \frac{a}{2r} + 5mg - c\left(\frac{(2r-a)^2}{r} + 2r - a\right)$$

$$\underline{\underline{c = 0{,}5\ \text{kg/cm}}}$$

Lösung 811



Energie im Punkt M = Energie im Punkt B

$$\frac{3}{2}mg\cdot r+\frac{c}{2}\cdot\left(\frac{r}{2}\right)^2=\frac{mv_B^2}{2}+\frac{c}{2}\left(2r-\frac{r}{2}\right)^2$$

$$\frac{mv_B^2}{2}=mg\frac{3}{2}r-cr^2$$

Daraus die Zentrifugalkraft: $\frac{mv_B^2}{r}=3mg-2cr$

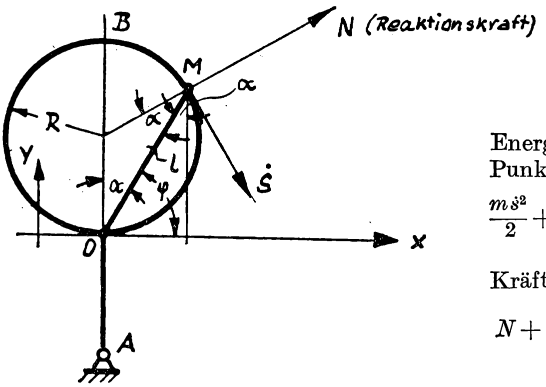
$$=1\text{ kg}$$

$$\frac{mv_B^2}{r}+mg-N-\frac{3}{2}r\cdot c=0$$

$$N=1+7-15=\underline{\underline{-7\text{ kg}}}\text{ (Reaktion)}$$

Der Druck der Last auf den Ring ist somit nach oben gerichtet.

Lösung 812



Energie im Punkt M = Energie im Punkt B

$$\frac{m\dot{s}^2}{2}+mgl\cos\alpha+\frac{c}{2}l^2=mg2R+2cR^2$$

Kräfte in M:

$$N+\frac{m\dot{s}^2}{R}-cl\cos\alpha-mg\cos2\alpha=0$$

$$l=2R\cos\alpha$$

$$4mgR+4cR^2-4mgR\cos^2\alpha-4cR^2\cos^2\alpha=m\dot{s}^2$$

$$2cR^2\cos^2\alpha+Rmg\cos2\alpha-NR=m\dot{s}^2\qquad(-)$$

$$\begin{aligned}4mgR+4cR^2-4mgR\cos^2\alpha&-mgR\cos2\alpha\\-4cR^2\cos^2\alpha&+NR\\-2cR^2\cos^2\alpha&\qquad=0\end{aligned}$$

$$4(mg+cR)-2\cos^2\alpha(3cR+2mg)-mg\cos2\alpha+N=0;\qquad\varphi=\frac{\pi}{2}-\alpha$$

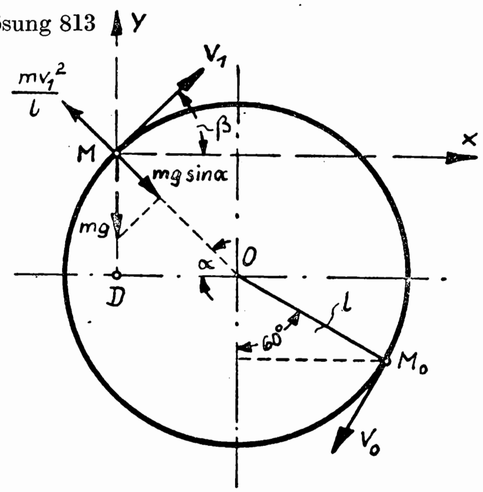
$$4(mg+cR)-2\sin^2\varphi(3cR+2mg)-mg(\sin^2\varphi-\cos^2\varphi)+N=0;$$

$$mg=Q$$

$$\underline{\underline{N=-[2Q+cR+3(Q+cR)\cos2\varphi]}}$$

Der Druck des Gewichtes auf den Ring hat entgegengesetztes Vorzeichen.

Lösung 813



1. Energie im Punkt M_0:

$$\frac{mv_0^2}{2} + mgl(1 - \cos 60°) = E_1$$

Energie im Punkt M:

$$\frac{mv_1^2}{2} + mgl(1 + \sin\alpha) = E_2$$

Der Auflagedruck im Punkt M soll Null sein:

$$\frac{mv_1^2}{l} = mg\sin\alpha$$

$E_2 = E_1$:

$$\frac{mv_1^2}{2} + mgl + mv_1^2 = \frac{mv_0^2}{2} + \frac{mgl}{2}$$

$$\frac{3}{2}v_1^2 = \frac{v_0^2}{2} - \frac{gl}{2}$$

$$v_1 = \sqrt{\frac{v_0^2 - gl}{3}} = \underline{\underline{157 \text{ cm/sek}}}$$

$$\sin\alpha = \frac{v_1^2}{l \cdot g}; \quad \overline{MD} = l\sin\alpha$$

$$\overline{MD} = \frac{v_1^2}{g} = \underline{\underline{25 \text{ cm}}}$$

2. $y = v_1 t\sin\beta - \frac{g}{2}t^2$

$x = v_1 t\cos\beta$

$y = x \cdot \operatorname{tg}\beta - \frac{gx^2}{2v_1^2\cos^2\beta}$; $\quad \beta = 90 - \alpha = 60°$, $\quad \operatorname{tg}\beta = \sqrt{3}$

$\underline{\underline{y = x\sqrt{3} - 0{,}08x^2}}$ cm

$t = \frac{x}{v_1\cos\beta}$;

Gleichung des Kreises: ($\alpha = 30°$; $r = l$; $v_1^2 = gl\sin\alpha$)

$$\left(y + \frac{r}{2}\right)^2 + \left(x - \frac{\sqrt{3}}{2}r\right)^2 = r^2$$

Gleichung der Parabel: $y = x\sqrt{3} - \frac{4x^2}{r}$

Ineinander eingesetzt:

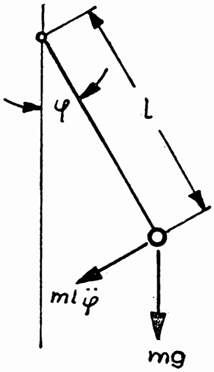
$$\left(x\sqrt{3} - \frac{4x^2}{r} + \frac{r}{2}\right)^2 + \left(x - \frac{\sqrt{3}}{2}r\right)^2 = r^2$$

$$3x^2 + \frac{16x^4}{r^2} + \frac{r^2}{4} - \frac{8\sqrt{3}\,x^3}{r} + \sqrt{3}\,rx - 4x^2 + x^2 + \frac{3}{4}r^2 - \sqrt{3}\,rx = r^2$$

$$\frac{16x^4}{r^2} = \frac{8\sqrt{3}\,x^3}{r}; \qquad \underline{\underline{x = \frac{\sqrt{3}}{2} r}}$$

$$t = \frac{x}{v_1 \cos\beta} = \frac{\sqrt{3}\cdot r}{2 v_1 \cos\beta} = \frac{\sqrt{3}\cdot 50\cdot 2}{2\cdot 157} = \underline{\underline{0{,}55\,\text{sek}}}$$

Lösung 814



Mathematisches Pendel:

$$ml^2\ddot{\varphi} + mgl\varphi = 0; \qquad \ddot{\varphi} + \frac{g}{l}\varphi = 0$$

$$\underline{\omega^2 = \frac{g}{l}} \qquad \underline{\underline{T = 2\pi\sqrt{\frac{l}{g}}}}$$

$$g = \frac{g_0}{h^2} R^2$$

$$T_{1;\,h=R} = 2\pi\sqrt{\frac{l_1}{g_0}}; \quad T_{2;\,h=R+H} = 2\pi\sqrt{\frac{l_2(R+H)^2}{g_0 R^2}}$$

$$\frac{l_1}{g_0} = \frac{l_2(R+H)^2}{g_0 R^2}$$

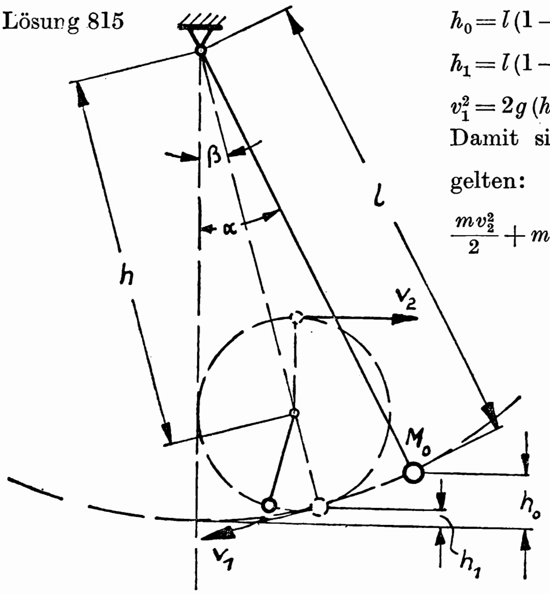
$$l_2 = l_1 \cdot \frac{R^2}{(R+H)^2}$$

$$l_2 = l_1 \cdot 0{,}9968712$$

Die Pendellänge in $H = 10$ km Höhe muß um

$\underline{\underline{0{,}0031288\, l_1 \text{ verkürzt werden.}}}$

Lösung 815



$$h_0 = l(1-\cos\alpha)$$

$$h_1 = l(1-\cos\beta); \quad r = l - h = l\left(1-\frac{h}{l}\right)$$

$$v_1^2 = 2g(h_0 - h_1) = 2gl(\cos\beta - \cos\alpha)$$

Damit sich der Faden **aufwindet, muß** gelten: $\dfrac{mv_2^2}{r} = mg$

$$\frac{mv_2^2}{2} + mg\cdot 2r = \frac{mv_1^2}{2} + mgr(1-\cos\beta)$$

$$v_2^2 = v_1^2 - 2gr(1+\cos\beta)$$

$$gr = v_1^2 - 2gr(1+\cos\beta)$$

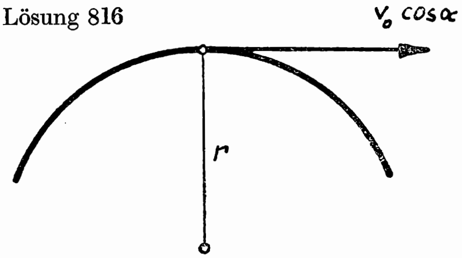
$$0 = 2gl(\cos\beta - \cos\alpha) - 2gl\left(1-\frac{h}{l}\right)\left(\frac{3}{2}+\cos\beta\right)$$

$$\underline{\underline{\alpha = \arccos\left[\frac{h}{l}\left(\frac{3}{2}+\cos\beta\right) - \frac{3}{2}\right]}}$$

$$v_1^2 = 2gl\left(\frac{3}{2}+\cos\beta\right)\left(\frac{l-h}{l}\right); \quad \Delta P_z = mv_1^2\left(\frac{1}{l-h}-\frac{1}{l}\right)$$

$$\underline{\underline{\Delta P_z = 2mg\cdot\frac{h}{l}\left(\frac{3}{2}+\cos\beta\right)}}$$

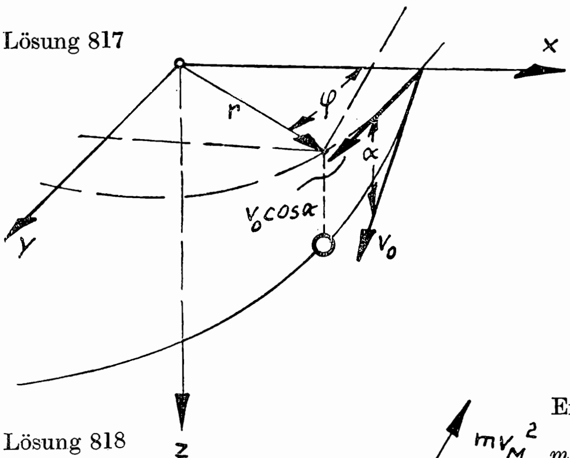
Lösung 816



Der Druck auf die Zylinderwand ist gleich der Zentrifugalkraft.

$$\underline{\underline{N = \frac{mv_0^2\cos^2\alpha}{r}}}$$

Lösung 817



$$x = r\cos\varphi; \quad y = r\sin\varphi$$

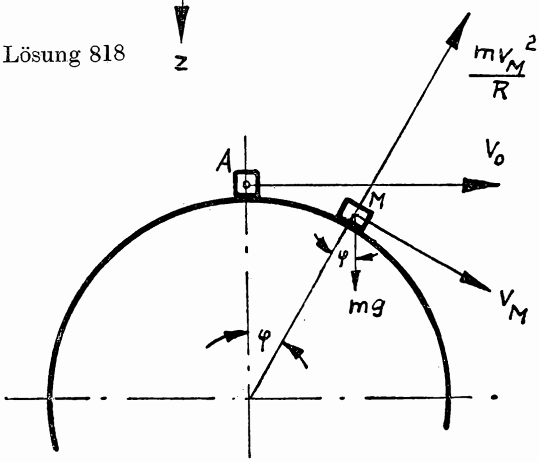
$$\varphi = \omega\cdot t; \quad \omega = \frac{v_0\cos\alpha}{r}$$

$$\underline{\underline{x = r\cos\left[\frac{v_0\cos\alpha}{r}\,t\right]}}$$

$$\underline{\underline{y = r\sin\left[\frac{v_0\cos\alpha}{r}\,t\right]}}$$

$$\underline{\underline{z = v_0\,t\sin\alpha + \frac{g}{2}\,t^2}}$$

Lösung 818



Energie in A = Energie in M

$$mgR + \frac{m}{2}v_0^2 = mgR\cos\varphi + \frac{m}{2}v_M^2$$

Bei der Ablösung des Steines von der Kugel gilt:

$$\frac{mv_M^2}{R} = mg\cos\varphi$$

Somit: $mgR + \frac{m}{2}v_0^2$

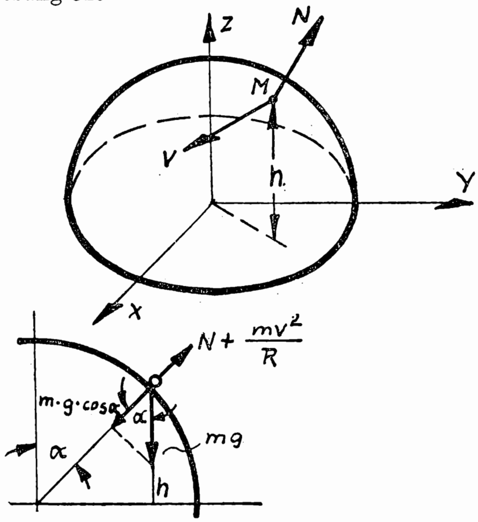
$$= mgR\cos\varphi + \frac{1}{2}mgR\cos\varphi$$

$$gR + \frac{v_0^2}{2} = \frac{3}{2}gR\cos\varphi$$

$$\underline{\underline{\varphi = \arccos\left[\frac{2}{3}+\frac{v_0^2}{3gR}\right]}}$$

Soll die Ablösung in A erfolgen, gilt: $\frac{mv_0^2}{2} = mg$; $\underline{\underline{v_0 \geqq \sqrt{gR}}}$

Lösung 819



$$\frac{mv_0^2}{2} + mgh_0 = \frac{mv^2}{2} + mgh$$

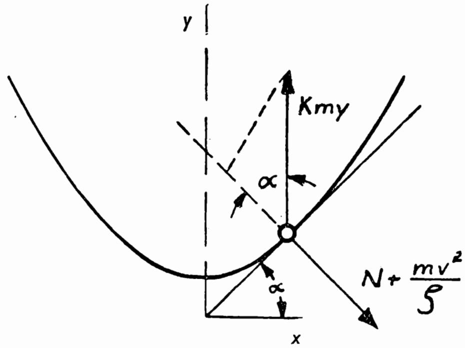
$$v^2 = v_0^2 + 2g(h_0 - h)$$

$$N = mg\cos\alpha - \frac{mv^2}{R}; \quad \cos\alpha = \frac{h}{r}$$

$$N = mg\cdot\frac{h}{R} - \frac{m}{R}[v_0^2 + 2g(h_0 - h)]$$

$$\underline{\underline{N = \frac{mg}{R}\left[3h - 2h_0 - \frac{v_0^2}{g}\right]}}$$

Lösung 820



$$y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right) = a\,\mathrm{Cof}\,\frac{x}{a}$$

$$N + \frac{mv^2}{\varrho} - kmy\cos\alpha = 0$$

$$\varrho = \frac{y^2}{a}; \quad v^2 = \dot{x}^2 + \dot{y}^2$$

$$m\ddot{x} = 0; \quad \dot{x} = C = 1\ \mathrm{m/sek}$$

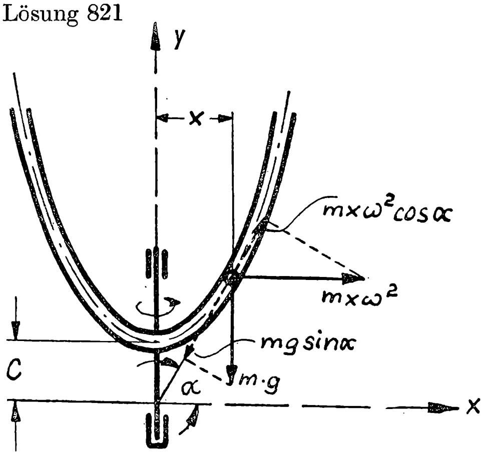
$$\dot{y} = \frac{dy}{dx}\cdot\frac{dx}{dt} = y'\cdot\dot{x}; \quad y' = \mathrm{Sin}\,\frac{x}{a}$$

$$N = \frac{km\cdot y}{\sqrt{1 + y'^2}} - \frac{(1 + y'^2)\,am}{y^2}$$

$$N = kma\,\frac{\mathrm{Cof}\,\frac{x}{a}}{\sqrt{1 + \mathrm{Sin}^2\frac{x}{a}}} - \frac{1 + \mathrm{Sin}^2\frac{x}{a}}{a^2\,\mathrm{Cof}^2\frac{x}{a}}\cdot am; \quad \mathrm{Cof}^2\frac{x}{a} = 1 + \mathrm{Sin}^2\frac{x}{a}$$

$$N = kma - \frac{m}{a}; \quad \underline{\underline{N = 0}}; \quad \underline{\underline{x = (t+1)\,\mathrm{m}}}$$

Lösung 821



$$\frac{dy}{dx} = \operatorname{tg}\alpha$$

$$m g \sin\alpha - m x \omega^2 \cos\alpha = 0$$

$$\operatorname{tg}\alpha = y' = \frac{\omega^2}{g} x$$

$$\underline{\underline{y = \frac{\omega^2}{2g} x^2 + c}}$$

Lösung 822

Kinetische Energie: $T = \frac{m}{2}(\dot r^2 + \dot z^2 + r^2 \dot\varphi^2)$

Potentielle Energie: $U = -\frac{c}{2}(r^2 + z^2)$; $\quad z^2 = r^2$

$$U = -c r^2; \quad T = \frac{m}{2}(2 \dot r^2 + r^2 \dot\varphi^2)$$

Lagrangesche Funktion $L = T - U$

$$\left(\frac{\partial L}{\partial \dot\varphi}\right)^{\cdot} - \frac{\partial L}{\partial \varphi} = 0: \quad r^2 \dot\varphi = \text{const} = h; \quad \dot\varphi \cdot r = v_0; \quad r = a \frac{\sqrt{2}}{2}$$

$$h = \frac{\sqrt{2}}{2} \cdot a v_0 = 2\sqrt{2}\,\frac{\text{cm}^2}{\text{sek}}$$

$$\left(\frac{\partial L}{\partial \dot r}\right)^{\cdot} - \frac{\partial L}{\partial r} = 0: \quad 2 m \ddot r - 2 c r - m r \dot\varphi^2 = 0$$

$$(1) \quad \ddot r - \frac{c r}{m} - \frac{h^2}{2 r^3} = 0 \,\Big|\, \cdot \dot r; \quad \dot r \ddot r - \frac{c r \dot r}{m} - \frac{h^2 \dot r}{2 r^3} = 0$$

$$\frac{\dot r^2}{2} - \frac{c r^2}{2m} + \frac{h^2}{4 r^2} = k_1; \quad r = a \frac{\sqrt{2}}{2}; \quad \dot r = 0:$$

$$k_1 = 0$$

$$\dot r^2 = \frac{c}{m} r^2 - \frac{h^2}{2} \frac{1}{r^2}$$

$$\text{Aus (1)}: \quad r \ddot r = \frac{c}{m} r^2 + \frac{h^2}{2} \frac{1}{r^2}$$

$$\frac{1}{2}(r^2)^{\cdot\cdot} = \frac{2c}{m} r^2;$$

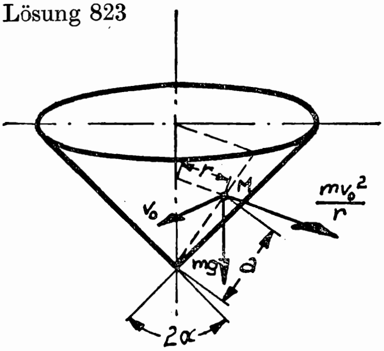
$$\underline{\underline{r^2 = \frac{a^2}{2} \mathfrak{Cof} \sqrt{\frac{4c}{m}}\, t}}; \qquad \underline{r^2 = e^{2t} + e^{-2t}}$$

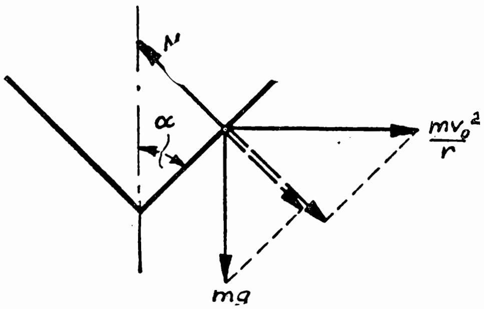
$$\dot{\varphi} = \frac{a v_0}{\sqrt{2} \cdot r^2} = \frac{\sqrt{2}\, v_0}{a} \cdot \frac{1}{\mathfrak{Cof} \sqrt{\frac{4c}{m}}\, t}; \quad \varphi = \sqrt{\frac{2m}{c}} \frac{v_0}{a} \left(\operatorname{arc\,tg} e^{\sqrt{\frac{4c}{m}}\, t} + k_2 \right)$$

$$\varphi = 0; \quad t = 0; \quad k_2 = -\frac{\pi}{4}; \quad \underline{\underline{\operatorname{tg}\left(\frac{a}{v_0} \sqrt{\frac{c}{2m}}\, \varphi + \frac{\pi}{4}\right) = e^{\sqrt{\frac{4c}{m}}\, t}}}$$

$$\underline{\operatorname{tg}\left(\frac{\varphi}{\sqrt{2}} + \frac{\pi}{4}\right) = e^{2t}}$$

Lösung 823





$$N = mg \sin\alpha + \frac{m v_0^2}{r} \cdot \cos\alpha$$

$$r = a \sin\alpha;$$

$$N = m \sin\alpha \left[g + \frac{v_0^2 \cos\alpha}{a \sin^2\alpha} \right]$$

$$N = m \sin\alpha \left[g + \frac{v_0^2 \sin 2\alpha}{2a \sin^3\alpha} \right]$$

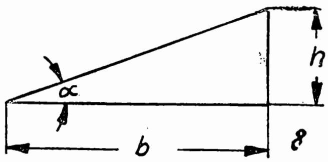
$$\underline{\underline{N = m \sin\alpha \left[g + \frac{a^2 v_0^2}{2 r^3} \sin(2\alpha) \right]}}$$

Lösung 824

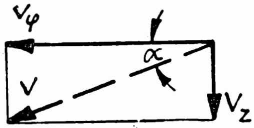
Schraubenlinie: $r = r_0$

Steigung: $h = a \cdot 2\pi$

Umfang des Zylinders: $b = r_0 \cdot 2\pi$



$$\operatorname{tg}\alpha = \frac{h}{b} = \underline{\underline{\frac{a}{r_0}}}$$

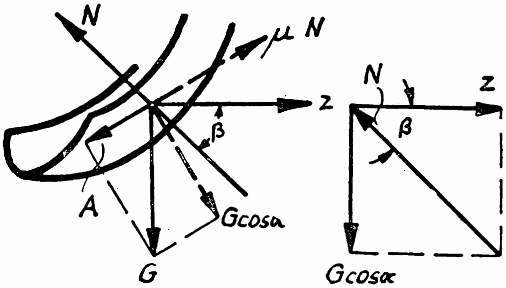


Die Geschwindigkeit auf der Bahn setzt sich zusammen aus der „Drehgeschwindigkeit" v_φ und der senkrecht nach unten gerichteten Geschwindigkeit v_z.

Zentrifugalkraft: $Z = \frac{m v_\varphi^2}{r_0} = \frac{m v^2 \cos^2\alpha}{r_0}$

Gewicht: $G = mg$

Bewegungsgleichung bezogen auf die Tangente der Schraubenlinie:



(1) $\mu \cdot N = A; \quad A = G \cdot \sin\alpha$

(2) $Z = N \cdot \cos\beta;$ Bewegungsgleichung bezogen auf die Normale.

(3) $G\cos\alpha = N\sin\beta;$ Bewegungsgleichung bezogen auf die Binormale.

Daraus $\operatorname{tg}\beta = \frac{G\cdot\cos\alpha}{Z}$

Geometrisch gilt: (4) $\operatorname{tg}\beta = \frac{1}{f'(r_0)\cos\alpha}; \quad \operatorname{ctg}\beta = f'(r_0)\cos\alpha$

Aus (2); (3) u. (4): $\frac{G\cos\alpha}{Z} = \frac{1}{f'(r_0)\cos\alpha}; \quad \frac{mg\cos\alpha\cdot r_0}{mv^2\cos^2\alpha} = \frac{1}{f'(r_0)\cos\alpha}$

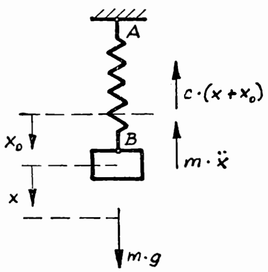
$$\underline{\underline{v = \sqrt{g\cdot r_0 \cdot f'(r_0)}}}$$

Aus (1) und (4): $\frac{\mu\cdot mg\cos\alpha}{\sin\beta} = mg\sin\alpha$

$$\operatorname{tg}\alpha - \frac{\mu}{\sin\beta} = 0; \quad \underline{\underline{\operatorname{tg}\alpha - \mu\sqrt{1+f'^2(r_0)\cos^2\alpha} = 0}}; \quad \operatorname{tg}\alpha = \frac{v_z}{v_\varphi} = \underline{\underline{\frac{a}{r_0}}}$$

32. Schwingende Bewegungen

Lösung 825



$$m\ddot{x} + c(x+x_0) - mg = 0; \quad cx_0 = mg$$

$$\ddot{x} + \frac{c}{m}x = 0; \quad \frac{c}{m} = \omega^2$$

Ansatz: $x = A\sin\omega t + B\cos\omega t$

$t = 0; \quad x = -x_0: \quad B = -x_0$

$t = 0; \quad \dot{x} = 0: \quad \dot{x} = \omega A\cos\omega t - \omega B\sin\omega t$

$A = 0$

$$\underline{x = -x_0\cos\sqrt{\frac{c}{m}}\,t}$$

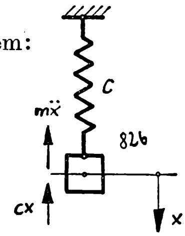
$x_0 = \frac{G}{c}$: $c = 20$ g/cm; $x_0 = \frac{100}{20} = 5$ cm; $\omega = \sqrt{\frac{c}{m}} = \sqrt{\frac{20 \cdot 981}{100}} = 14$ 1/sek

$\underline{\underline{x = -5\cos(14t)}}$; $x = 0$ für $\cos\omega t = 0$; Somit Zeit einer vollen Schwingung

$$T = \frac{2\pi}{\omega} = \underline{\underline{0{,}45 \text{ sek}}}$$

Lösung 826

Ersatzsystem:



$$m\ddot{x} + cx = 0; \quad \ddot{x} + \frac{c}{m}x = 0$$

$$x = A\sin\omega t + B\cos\omega t$$

$$\dot{x} = A\omega\cos\omega t - B\omega\sin\omega t$$

$$\ddot{x} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$$

$$t = 0; \quad x = 0: \quad B = 0$$

$$\dot{x} = v_0: \quad v_0 = A\omega$$

$$x_{\max} = \frac{v_0}{\omega}$$

Dynamische Zusatzbelastung: $P = c \cdot x_{\max} = c \cdot \frac{v_0}{\sqrt{c}}\sqrt{m} = v_0\sqrt{mc}$

$$P = v_0\sqrt{\frac{Q}{g} \cdot c}; \quad P = 45{,}1 \text{ t}$$

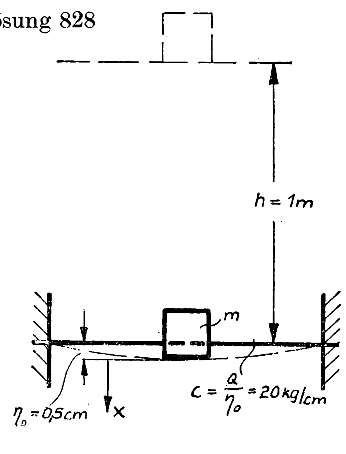
$$\underline{\underline{F = Q + P = 47{,}1 \text{ t}}}$$

Lösung 827

$$F = Q + v_0\sqrt{m \cdot c^*}; \quad \frac{1}{c^*} = \frac{1}{c_1} + \frac{1}{c} = 2{,}75; \quad c^* = 0{,}364 \text{ t/cm}$$

$$F = 2 + 500\sqrt{\frac{2 \cdot 0{,}364}{981}} = \underline{\underline{15{,}6 \text{ t}}}$$

Lösung 828



$$m\ddot{x} + cx = 0$$

$$\frac{Q}{g}\ddot{x} + \frac{Q}{\eta_0}x = 0$$

$$\ddot{x} + 2gx = 0; \quad \omega^2 = 2g$$

$$\omega = 44{,}3 \text{ 1/sek}$$

$$x = A\sin\omega t + B\cos\omega t$$

$$t = 0; \quad x = -0{,}5: \quad B = -0{,}5$$

$$\dot{x} = \sqrt{2gh} = 443:$$

$$A\omega = 443$$

$$A = 10 \text{ cm}$$

Somit:

$$\underline{\underline{x = 10\sin 44{,}3t - 0{,}5\cos 44{,}3t \text{ cm}}}$$

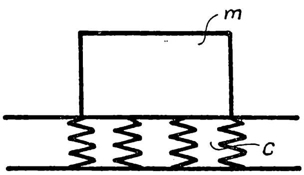
Lösung 829

Es gilt die Schwingungsgleichung $m\ddot{x} + cx = 0$ und somit:

$$\omega T = 2\pi; \quad T = 2\pi\sqrt{\frac{m}{c}}; \quad \frac{m}{c} = \frac{P}{g \cdot c} = \frac{\eta \cdot c}{g \cdot c}$$

$$T = 2\pi\sqrt{\frac{\eta}{g}} = 2\pi\sqrt{\frac{0{,}5}{981}}; \quad \underline{\underline{T = 0{,}45\ \text{sek}}}$$

Lösung 830



$$m = \frac{90}{9{,}81} = 9{,}18\ \text{tsek}^2/\text{m}$$

$$c = \lambda \cdot S = 45 \cdot 10^3\ \text{t/m}$$

$$m\ddot{x} + cx = 0; \quad \omega^2 = \frac{c}{m}$$

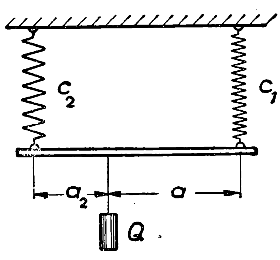
$$T = 2\pi \cdot \frac{1}{\omega} = 2\pi\sqrt{\frac{m}{c}}; \quad T = \underline{\underline{0{,}09\ \text{sek}}}$$

Lösung 831

Auftrieb des Schiffes: $A = S \cdot x \cdot \gamma; \quad m\ddot{x} + Sx\gamma = 0; \quad \gamma = 1;$

$$m = \frac{P}{g}; \quad \underline{\underline{T = 2\pi\sqrt{\frac{P}{g \cdot S}}}}$$

Lösung 832

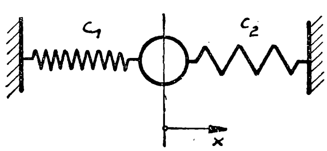


$$P_F = x(c_1 + c_2) = x \cdot c; \quad \underline{\underline{c = c_1 + c_2}}$$

$$T = 2\pi\sqrt{\frac{m}{c}} = \underline{\underline{2\pi\sqrt{\frac{Q}{g(c_1 + c_2)}}}}$$

$$a_1 c_1 x = a_2 c_2 x; \quad \underline{\underline{\frac{a_1}{a_2} = \frac{c_2}{c_1}}}$$

Lösung 833



$$P_F = x_1 \cdot c_1 + x_2 \cdot c_2$$

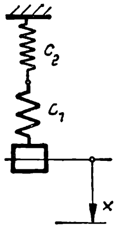
Da die Federkraft für Druck und Zug gleich ist und gleiche Federwege zurückgelegt werden, gilt:

$$P_F = x(c_1 + c_2) = cx$$

$$c = c_1 + c_2$$

$$\underline{\underline{T = 2\pi\sqrt{\frac{Q}{g(c_1 + c_2)}}}}$$

Lösung 834



$$P_F = c_1 x_1 + c_2 x_2 = c \cdot x$$

$$x = x_1 + x_2$$

$$x_1 = \frac{P_F}{c_1}; \quad x_2 = \frac{P_F}{c_2}$$

$$x = \frac{P_F}{c_1} + \frac{P_F}{c_2} = \frac{P_F}{c}; \quad \frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2}$$

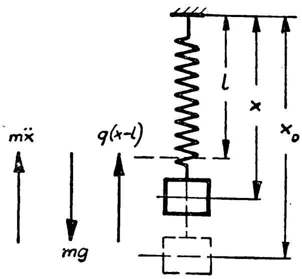
$$c = \frac{c_1 c_2}{c_1 + c_2}$$

$$T = 2\pi \sqrt{\frac{m}{c}} = 2\pi \sqrt{\frac{Q(c_1 + c_2)}{g\, c_1 c_2}}$$

Lösung 835

$$\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \cdots \frac{1}{c_n}; \quad c = \frac{1}{\sum\limits_{i=1}^{i=n} \frac{1}{c_i}} \quad \text{(vgl. Aufgabe 834)}$$

Lösung 836



$$m\ddot{x} + q(x - l) = mg; \quad q = \text{Federkonst. g/cm}$$

$$(x - l) = \frac{P}{q} + c_1 \cos \sqrt{\frac{gq}{P}}\, t + c_2 \sin \sqrt{\frac{gq}{P}} \cdot t$$

$$t = 0; \quad \dot{x} = 0: \quad c_2 = 0$$

$$t = 0; \quad x = x_0: \quad x_0 - l = \frac{P}{q} + c_1$$

$$c_1 = x_0 - l - \frac{P}{q}$$

$$x = l + \frac{P}{q} + \left(x_0 - l - \frac{P}{q}\right) \cos \sqrt{\frac{gq}{P}} \cdot t$$

$x_{\min}$ für $\cos \sqrt{\frac{gq}{P}}\, t = -1$

$$x_{\min} = 2l + \frac{2P}{q} - x_0 \geqq l$$

Somit:

$$l \leqq x_0 \leqq l + \frac{2P}{q}$$

Lösung 837

$$N_1 = mg \cdot \frac{l - x}{2l}; \qquad R = \mu (N_2 - N_1) = \frac{mg \cdot \mu \cdot x}{l}$$

$$N_2 = mg \cdot \frac{l + x}{2l}; \qquad m\ddot{x} + \frac{mg\mu x}{l} = 0$$

$$\ddot{x} + \frac{g \cdot \mu}{l} \cdot x = 0; \qquad \frac{g \cdot \mu}{l} = \omega^2$$

Bei $t = 0$; $x = x_0$

$$\dot{x} = 0 \quad \text{gilt:} \quad x = x_0 \cos \omega t = \underline{\underline{x_0 \cos \sqrt{\frac{g\mu}{l}} \cdot t}}$$

$$\omega T = 2\pi; \qquad \omega^2 = \frac{4\pi^2}{T^2} = \frac{g\mu}{l}; \qquad \underline{\underline{\mu = \frac{4\pi^2 l}{gT^2}}} = \frac{4\pi^2 \cdot 0{,}25}{9{,}81 \cdot 4} = \underline{\underline{0{,}25}}$$

Lösung 838

$$m\ddot{x} + cx = 0; \qquad \text{bei } t = 0; \quad x = -x_0 = -\frac{mg}{c}$$

$$\dot{x} = 0 \quad \text{gilt:} \quad x = -\frac{mg}{c} \cos \omega t$$

$$\omega^2 = \frac{c}{m}$$

1. $mg = p$

$$\omega = \sqrt{\frac{cg}{p}}: \qquad \underline{\underline{x_1 = -\frac{p}{c} \cos \sqrt{\frac{c \cdot g}{p}}\, t}}; \qquad T_1 = 2\pi \sqrt{\frac{p}{g \cdot c}}$$

2. $mg = 3p$

$$\omega = \sqrt{\frac{cg}{3p}}; \qquad \underline{\underline{x_2 = -\frac{3p}{c} \cos \sqrt{\frac{cg}{3p}}\, t}}; \qquad T_2 = 2\pi \sqrt{\frac{3p}{g \cdot c}}$$

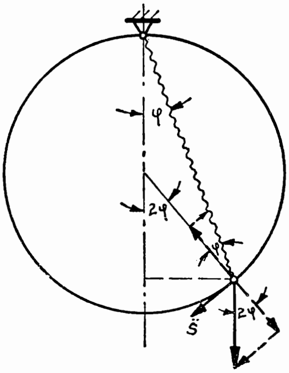
$$\underline{\underline{\frac{T_2}{T_1} = \sqrt{3}}}$$

Lösung 839

$$T = 2\pi \sqrt{\frac{Q}{g \cdot c}}; \qquad T_1 = 2\pi \sqrt{\frac{(Q + Q_1)}{g \cdot c}}; \qquad \underline{\underline{T_1 = T \sqrt{\frac{(Q + Q_1)}{Q}}}}$$

$$T = \frac{45}{100} = 0{,}45 \text{ sek}; \qquad Q = 12 \text{ kg}; \qquad Q_1 = 6 \text{ kg}; \qquad \underline{\underline{T_1 = 0{,}55 \text{ sek}}}$$

Lösung 840



$$M\ddot{s} + Mg \cdot \sin 2\varphi - c\,(l \cos \varphi - a) \sin \varphi = 0$$

$$c = \frac{Mg}{b}; \qquad s = l\varphi; \qquad \cos \varphi \approx 1$$

$$\sin \varphi = \varphi$$

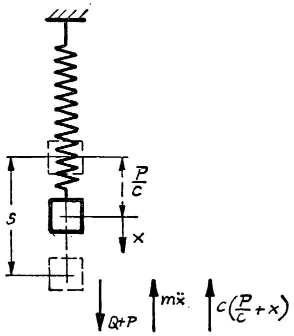
$$l\ddot{\varphi} + 2g\varphi - \frac{g}{b}(l - a)\,\varphi = 0$$

$$\ddot{\varphi} + \varphi \left(\frac{2g}{l} - \frac{g}{b} + \frac{g}{lb} \cdot a \right) = 0$$

$$l = a + b: \qquad \ddot{\varphi} + \frac{g}{l}\varphi = 0$$

$$\underline{\underline{T = 2\pi \sqrt{\frac{l}{g}}}}$$

Lösung 841



Koordinatenursprung: Statisches Gleichgewicht von P, Beginn der Zeitmessung: Vom Wirken der Kraft Q an

$$\frac{m v_0^2}{2} + P \cdot s - \frac{c s^2}{2} = 0$$

$$s^2 - \frac{m v_0^2}{c} - \frac{2P}{c} \cdot s = 0$$

$$s = \frac{P}{c} \underset{(-)}{+} \sqrt{\left(\frac{P}{c}\right)^2 + \frac{m v_0^2}{c}}$$

$$m\ddot{x} + c\left(\frac{P}{c} + x\right) - (Q + P) = 0; \qquad m = \frac{P}{g}$$

$$m\ddot{x} + c x = Q; \qquad \ddot{x} + \frac{c}{m} x = \frac{Q}{m}; \qquad \frac{c}{m} = \omega^2$$

$$x = A \sin \omega t + B \cos \omega t + \frac{Q}{c}$$

$$t = 0; \qquad \dot{x} = 0: \quad A = 0$$

$$x = s - \frac{P}{c}: \quad s - \frac{P}{c} = B + \frac{Q}{c}$$

$$B = \sqrt{\frac{m v_0^2}{c} + \left(\frac{P}{c}\right)^2} - \frac{Q}{c}$$

$$\underline{\underline{x = \frac{Q}{c} + \left[\sqrt{\frac{v_0^2 \cdot P}{c \cdot g} + \left(\frac{P}{c}\right)^2} - \frac{Q}{c}\right] \cos \sqrt{\frac{c \cdot g}{P}}\, t}}$$

$$\underline{\underline{T = 2\pi \sqrt{\frac{P}{cg}}}}$$

Lösung 842

Statische Durchsenkung: $P_1 = c \cdot l_1 = m_1 \cdot g$

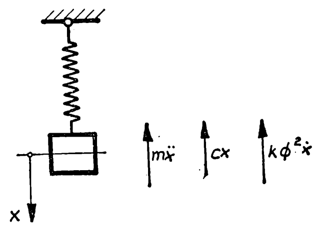
$P_2 = c \cdot l_2 = m_2 \cdot g$

Schwingungszeit: $T_1 = 2\pi \sqrt{\frac{m_1 + m_F}{c}}; \quad m_F = \frac{T_1^2 \cdot c}{4\pi^2} - m_1$

$T_2 = 2\pi \sqrt{\frac{m_2 + m_F}{c}}; \quad m_F = \frac{T_2^2 \cdot c}{4\pi^2} - m_2$

$$\frac{T_1^2 c - T_2^2 c}{4\pi^2} = m_1 - m_2 = \frac{c l_1 - c l_2}{g}; \qquad \underline{\underline{g = 4\pi^2 \frac{l_1 - l_2}{T_1^2 - T_2^2}}}$$

Lösung 843



$$m\ddot{x} + k\Phi^2 \dot{x} + c x = 0; \qquad c = 20 \cdot 981 \text{ g/sek}^2$$

$$\ddot{x} + \frac{k\Phi^2}{m} \dot{x} + \frac{c}{m} x = 0; \qquad m = 100 \text{ g}$$

Allgemein:

$$\ddot{x} + 2n\dot{x} + \nu^2 x = 0$$

Ansatz: $x = A e^{pt}$ in die Differentialgleichung eingesetzt:

$$p^2 + 2np + \nu^2 = 0$$

$$p_{1,2} = -n \pm \sqrt{n^2 - \nu^2}$$

In der Aufgabe gilt:

$$2n = \frac{k\Phi^2}{m}; \qquad n^2 = \left(\frac{k\Phi^2}{m\cdot 2}\right)^2 = \left(\frac{10^{-4}\cdot 5\cdot 10^6}{100\cdot 2}\right)^2 = (2{,}5)^2 = 6{,}25$$

$$\nu^2 = \frac{c}{m} = \frac{20\cdot 981}{100} = 196; \qquad \sqrt{n^2-\nu^2} = 13{,}78\,i$$

Da $n < \nu$, sind die Wurzeln von p imaginär $\sqrt{n^2-\nu^2} = \pm i\omega$

$$x = A_1\cdot e^{-nt}\cdot e^{+i\omega t} + A_2 e^{-nt}\cdot e^{-i\omega t}$$

$$x = e^{-nt}(C_1\cos\omega t + C_2\sin\omega t)$$

Anfangsbedingungen: $t=0$; $x = \frac{mg}{c} = \frac{100\cdot 981}{c\cdot 981} = 5\text{ cm}$

$$C_1 = 5$$

$$t=0:\quad \dot{x}=0:\quad \dot{x} = e^{-nt}(-C_1\omega\sin\omega t + C_2\omega\cos\omega t) - n e^{-nt}(C_1\cos\omega t + C_2\sin\omega t)$$

$$0 = C_2\omega - C_1 n = C_2\cdot 13{,}78 - 5\cdot 2{,}5$$

$$C_2 = \frac{12{,}5}{13{,}78} = 0{,}907$$

$\underline{\underline{x = e^{-2{,}5t}(0{,}907\sin 13{,}78\,t + 5\cos 13{,}78\,t)}}$ schwache Dämpfung

Lösung 844

Nach Aufgabe 843 gilt mit $\Phi = 10000$:

$$n^2 = \left(\frac{\Phi k^2}{m\cdot 2}\right)^2 = \left(\frac{10^{-4}\cdot 10^8}{100\cdot 2}\right)^2 = 50^2 = 2500$$

$$n^2-\nu^2 = 2500 - 196 = 2304; \qquad p_{1,2} = -50 \pm 48; \qquad p_1 = -98, \quad p_2 = -2$$

$$x = A_1 e^{p_1 t} + A_2 e^{p_2 t}$$

$$\dot{x} = p_1 A_1 e^{p_1 t} + p_2 A_2 e^{p_2 t}$$

Anfangsbedingungen: $t=0$; $x = \frac{mg}{c} = 5\text{ cm}$:

$$5 = A_1 + A_2$$

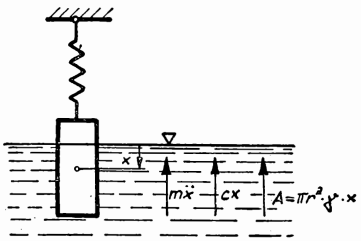
$$t=0;\quad \dot{x}=0:\quad p_1 A_1 + p_2 A_2 = 0$$

$$A_2 = -\frac{p_1}{p_2}A_1 = -49A_1; \qquad 5 = A_1 - 49A_1$$

$$A_1 = -\frac{5}{48}$$

$\underline{\underline{x = -\frac{5}{48}e^{-98t}(49e^{96t}-1)}}$ Starke Dämpfung, die Bewegung verläuft ohne Schwingung asymptotisch gegen die Lage $x=0$.

Lösung 845



$$\ddot{x} + \frac{c + \gamma \pi r^2}{m} x = 0$$

$$k^2 = \frac{c + \gamma \pi r^2}{P} g$$

Lösungsansatz:

$$x = A \sin kt + B \cos kt$$

Anfangsbedingungen:

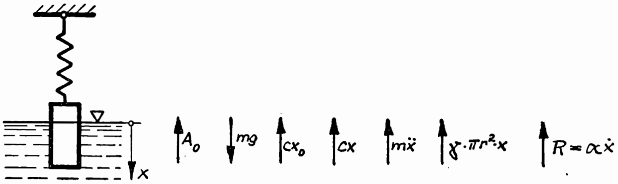
$$x/_{t=0} = \frac{h}{6} = B$$

$$\dot{x}/_{t=0} = 0 = A$$

somit:

$$x = \frac{h}{6} \cos kt$$

Lösung 846



$mg = cx_0 + A_0$ Statisches Gleichgewicht

$$m\ddot{x} + \alpha \dot{x} + (c + \gamma \pi r^2) x = 0; \qquad \ddot{x} + \frac{\alpha}{m} \dot{x} + \left(\frac{c}{m} + \frac{\gamma \pi r^2}{m}\right) x = 0$$

Abkürzungen: $n = \frac{\alpha}{2m}$; $k^2 = \frac{c}{m} + \frac{\gamma \pi r^2}{m}$; $\omega = \sqrt{k^2 - n^2}$

Somit: $\ddot{x} + 2n\dot{x} + k^2 x = 0$; Schwingende Bewegung tritt auf, wenn

$$n^2 - k^2 < 0$$

bzw.

$$\frac{c}{m} + \frac{\gamma \pi r^2}{m} - \left(\frac{\alpha}{2m}\right)^2 > 0$$

Lösungsansatz: $x = Ce^{-nt} \sin(\omega t + \beta)$

$$\dot{x} = C[e^{-nt} \omega \cos(\omega t + \beta) - ne^{-nt} \sin(\omega t + \beta)]$$

Anfangsbedingungen: $t = 0$; $x = \frac{h}{6}$:

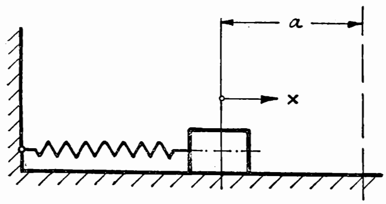
$$\frac{h}{6} = C \sin \beta = C \frac{\operatorname{tg} \beta}{\sqrt{1 + \operatorname{tg}^2 \beta}}$$

$t = 0$; $\dot{x} = 0$: $\omega \cos \beta - n \sin \beta = 0$

$$\operatorname{tg} \beta = \frac{\omega}{n} = \frac{1}{n} \sqrt{k^2 - n^2}$$

$$x = \frac{h}{6} \sqrt{\frac{k^2}{k^2 - n^2}} \cdot e^{-nt} \cdot \sin\left(\sqrt{k^2 - n^2}\, t + \beta\right)$$

Lösung 847



$$m\ddot{x} + cx \pm R = 0; \quad \frac{R}{c} = \frac{G \cdot \mu}{c} = x_0$$

$$m(x \pm x_0)^{\cdot\cdot} + c(x \pm x_0) = 0$$

$\omega^2 = \frac{c}{m};$ Zeit einer vollen Schwingung:

$$T = 2\pi \cdot \frac{1}{\omega}$$

Zeit der Bewegung zwischen zwei Bewegungsnullpunkten

$$\tau = \frac{T}{2} = \underline{\underline{0{,}141 \text{ sek}}}$$

Lösungsansatz: $(x \underset{-}{(+)} x_0) = A \sin \omega t + B \cos \omega t$

Anfangsbedingungen: $t = 0$: $\dot{x} = 0$: $A = 0$

$x = 3 \text{ cm} = a$: $a \pm x_0 = B$

Der Körper bewegt sich dabei in negativer Richtung, also:

$$B = a - x_0$$

$$x - x_0 = (a - x_0) \cos \omega t$$

Die erste Amplitude wird durch $\dot{x} = 0$ festgelegt:

$$(x - x_0)^{\cdot} = -\omega (a - x_0) \sin \omega t = 0; \quad t = \frac{\pi}{\omega}$$

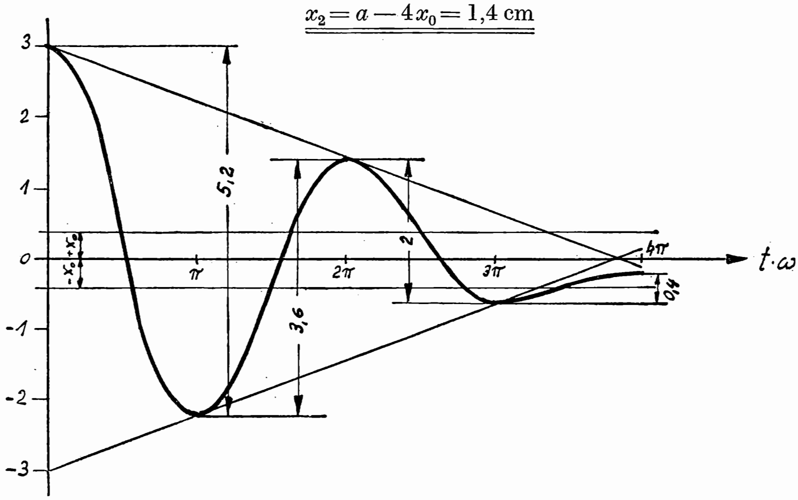
Somit: $\underline{\underline{x_1 = -(a - 2x_0) = -2{,}2 \text{ cm}}}$ (von 0 aus gerechnet)

Für die nächste Schwingung gelten neue Anfangsbedingungen:

$$t = \frac{\pi}{\omega}; \quad x_t = -(a - 2x_0): \quad x \underset{(-)}{+} x_0 = C \sin \omega t + D \cos \omega t$$

$$C = 0: \quad D = a - 3x_0$$

$$\underline{\underline{x_2 = a - 4x_0 = 1{,}4 \text{ cm}}}$$



Entsprechend ergibt sich: $x_3 = -(a - 6x_0) = -0{,}6$
$x_4 = (a - 8x_0) = -0{,}2$

Die Masse vollführt also vier halbe Schwingungen mit den Nullpunktsentfernungen von 5,2 cm; 3,6 cm; 2,0 cm; 0,4 cm (vgl. Abb.)

Lösung 848

$$m\ddot{x} + k\dot{x} + cx = 0; \qquad x = e^{-nt}(A\sin\omega t + B\cos\omega t)$$

$$\omega = \sqrt{\frac{c}{m} - \left(\frac{k}{2m}\right)^2}; \qquad n = \frac{k}{2m}$$

$$e^{-\frac{k}{2m}T} = 0{,}9; \qquad k = \frac{2m}{T}\ln\frac{10}{9}$$

$$R = k \cdot v = \frac{2vm}{T}\ln\frac{10}{9} = \frac{2\cdot 1\cdot 1}{0{,}5\cdot 981}\cdot\ln\frac{10}{9}$$

$$\underline{\underline{R = 0{,}00043\ \text{g/g}}}$$

Lösung 849

1. Schwingung in Luft: $m\ddot{x} + cx = 0; \qquad T_1 = 2\pi\sqrt{\frac{m}{c}}$

2. Schwingung in Flüssigkeit: $m\ddot{x} + 2S\eta\cdot\dot{x} + cx = 0$

$$T_2 = \frac{2\pi}{\sqrt{\frac{c}{m} - \left(\frac{S\eta}{m}\right)^2}}$$

Daraus: $\left(\frac{T_2}{T_1}\right)^2 = \frac{c}{c - \frac{\eta^2 S^2}{m}}; \qquad m = \frac{P}{g}; \qquad \underline{\underline{\eta = \frac{\pi P}{gST_1T_2}\sqrt{T_2^2 - T_1^2}}}$

Lösung 850

$$\ddot{x} + \frac{k}{m}\dot{x} + \frac{c}{m}x = 0; \qquad x = e^{-nt}(A\sin\omega t + B\cos t)$$

$$\omega = \sqrt{\frac{c}{m} - \left(\frac{k}{2m}\right)^2} \qquad n = \frac{k}{2m}$$

Für $t = 0$ und $t = 4T = \frac{4\cdot 2\pi}{\omega}$ gilt: $x_0 = 1 = B$

$$x_{4T} = \frac{1}{12} = e^{-n4T}B$$

$\frac{1}{12} = e^{-4nT}$; Logarithmisches Dekrement: $\frac{nT}{2} = \frac{1}{8}\ln 12 = \underline{\underline{0{,}316}}$

Schwingungszeit: $\frac{kT}{4m} = \frac{\pi}{\sqrt{\frac{c}{m} - \frac{k^2}{4m^2}}}\cdot\frac{k}{2m} = 0{,}316$

$\frac{\pi^2 k^2}{4m^2} = 0{,}316^2\left(\frac{c}{m} - \frac{k^2}{4m^2}\right); \qquad \frac{k^2}{4m^2} = 3{,}8: \qquad T = \underline{\underline{0{,}319\ \text{sek}}}$

Lösung 851

Ohne Dämpfung gilt: $T = 2\pi\sqrt{\frac{m}{c}}$

Mit geschwindigkeitsproportionaler Dämpfung gilt: $m\ddot{x} + \mu\dot{x} + cx = 0$

$$k = \mu \cdot \dot{x} \text{ kg}$$

$$x = e^{-\frac{\mu \cdot t}{m \cdot 2}}\left[C_1 \cos\sqrt{\frac{c}{m} - \left(\frac{\mu}{2m}\right)^2}\, t + C_2 \sin\sqrt{\frac{c}{m} - \left(\frac{\mu}{2m}\right)^2}\, t\right]$$

$$T_1 = \frac{2\pi}{\sqrt{\frac{c}{m} - \frac{\mu^2}{4m^2}}}; \quad \underline{\underline{\mu^2 = 16\pi^2 m^2\left(\frac{1}{T^2} - \frac{1}{T_1^2}\right)}}; \quad \mu = 3{,}6$$

Damit wird: $k = 3{,}6 \cdot 0{,}01 = \underline{\underline{0{,}036 \text{ kg}}}$

$$\omega = \sqrt{\frac{c}{m} - \frac{\mu^2}{4m^2}}; \quad \frac{c}{m} = \frac{4\pi^2}{T^2}; \quad \omega = 4 \text{ 1/sek}$$

Somit: $x = e^{-3t}\{C_1 \cos 4t + C_2 \sin 4t\}$ bzw. $x = Ce^{-3t} \sin(4t + \varphi)$

Anfangsbedingungen: $t = 0$; $x = 4$ cm: $4 = C \sin\varphi$

$$\dot{x} = 0: \quad 0 = 4\cos\varphi - 3\sin\varphi$$

$$\operatorname{tg}\varphi = \frac{4}{3}; \quad C = 5$$

$$\underline{\underline{x = 5e^{-3t} \sin\left(4t + \operatorname{arc\,tg}\frac{4}{3}\right)}}$$

Lösung 852

$$m\ddot{x} + k\dot{x} + cx = 0; \quad m = \frac{1{,}96}{981}; \quad c = \frac{1}{20}; \quad k = 0{,}02$$

$$\frac{c}{m} = \frac{k^2}{4m^2} \text{ (aperiodischer Grenzfall)}$$

Hierfür gilt der Ansatz: $x = e^{-nt}(C_1 t + C_2)$; $n = \frac{k}{2m}$

$$\dot{x} = -ne^{-nt}(C_1 t + C_2) + C_1 e^{-nt}$$

Anfangsbedingungen: $t = 0$; $x = 5$: $5 = C_2$

$$\dot{x} = 0: \quad 0 = -nC_2 + C_1$$

$$n = \frac{k}{2m} = 5; \quad C_1 = 25$$

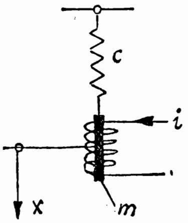
$$\underline{\underline{x = 5e^{-5t}(5t + 1) \text{ cm}}}$$

Lösung 853

$$m\ddot{x} + cx = F; \quad F = 16\pi \cdot 20 \sin 8\pi t$$

Ansatz: $x = A \sin \omega t + B \cos \omega t + C \sin 8\pi t$

Die erzwungene Schwingung wird durch das partikuläre Integral dargestellt, die Anteile der freien Schwingung werden dabei nicht beachtet.



Somit: $x = C \sin 8\pi t$

$$\ddot{x} = -64\pi^2 C \sin 8\pi t$$

$$\left(-64\pi^2 + \frac{c}{m}\right) C \cdot \sin 8\pi t = \frac{320 \cdot \pi}{m} \cdot \sin 8\pi t$$

$$C = \frac{320\pi}{20 - 64\,\frac{100}{981}\pi^2} \quad \frac{\text{dyn cm}}{\text{g}}$$

$$C = \frac{320\pi}{981\left(20 - \frac{6400}{981}\pi^2\right)} = -0{,}023 \text{ cm}$$

$$\underline{\underline{x = -0{,}023 \sin 8\pi t \text{ cm}}}$$

Lösung 854

$$m = m_1 + m_2; \quad m\ddot{x} + k\Phi^2\dot{x} + cx = 320\pi \sin 8\pi t$$

Partikuläres Integral: $x = D \sin \alpha t + E \cos \alpha t; \quad \alpha = 8\pi$

$$\dot{x} = \alpha D \cos \alpha t - \alpha E \sin \alpha t$$

$$\ddot{x} = -\alpha^2 D \sin \alpha t - \alpha^2 E \cos \alpha t$$

Die beiden Konstanten D und E werden durch Koeffizientenvergleich bestimmt:

$$\sin \alpha t\,[-m\alpha^2 D - k\Phi^2 \alpha E + cD] = 320\pi \sin 8\pi t$$

$$\cos \alpha t\,[-m\alpha^2 E + k\Phi^2 \alpha D + cE] = 0$$

$$D[c - m\alpha^2] - Ek\Phi^2\alpha = 320\pi$$

$$Dk\Phi^2\alpha + E(c - m\alpha^2) = 0$$

$$D = \frac{320\pi(c - m\alpha^2)}{(c - m\alpha^2)^2 + (k\Phi^2\alpha)^2}; \quad E = \frac{-320\pi k\Phi^2\alpha}{(c - m\alpha^2)^2 + (k\Phi^2\alpha)^2}$$

Andere Form des Ansatzes: $x = B \sin(\alpha t - \beta)$

$$x = B[\sin \alpha t \cos \beta - \cos \alpha t \sin \beta]$$

Somit:

$$B \cos \beta = D; \quad -B \sin \beta = E; \quad B = \sqrt{D^2 + E^2}$$

$$\beta = -\operatorname{arc\,tg} \frac{E}{D}$$

$$B = \frac{320\pi}{\sqrt{(c - m\alpha^2)^2 + (k\Phi^2\alpha)^2}} = \frac{320\pi}{\sqrt{(20 \cdot 981 - 64\pi^2 \cdot 100)^2 + (10^{-4} \cdot 5 \cdot 10^6 \cdot 8 \cdot \pi)^2}}$$

$$B = 0{,}022 \text{ cm}; \quad \operatorname{tg} \beta = -0{,}288; \quad \beta = 2{,}861 = 0{,}91\,\pi$$

Aus $B\cos\beta = D$ und $-B\sin\beta = E$ folgt, daß $\cos\beta$ negativ und $\sin\beta$ positiv sein müssen (D ist negativ, da $m\alpha^2 > c$).

Der Winkel β liegt also im 2. Quadranten

$$\underline{\underline{x = 0{,}022\sin(8\pi t - 0{,}91\,\pi)\ \text{cm}}}$$

Lösung 855

$$m\ddot{x} + cx = c\cdot a\sin nt; \qquad \ddot{x} + \frac{c}{m}x = \frac{ac}{m}\sin nt$$

Das partikuläre Integral lautet: $x = C\sin nt$

$$\ddot{x} = -n^2 C\sin nt$$

$$\left(-n^2 + \frac{c}{m}\right)C = \frac{ac}{m}; \qquad C = \frac{ac}{c - mn^2} = \frac{2\cdot 40}{40 - \dfrac{400\cdot 49}{981}} = 4\ \text{cm}$$

$$\underline{\underline{x = 4\sin 7t\ \text{cm}}}$$

Lösung 856

$$m\ddot{x} + cx - ca\sin kt = 0; \qquad \text{Ansatz:} \quad x = A\sin\omega t + B\cos\omega t + D\sin kt$$

Anfangsbedingungen: $t = 0$; $x = 0$: $B = 0$

$$\dot{x} = 0: \quad 0 = A\omega + Dk; \qquad -A = \frac{Dk}{\omega}$$

Bestimmung von D der partikulären Lösung: $x_p = D\sin kt$

$$\ddot{x}_p = -Dk^2\sin kt$$

$$-mDk^2 + cD = ca$$

$$D = \frac{ac}{c - mk^2}; \qquad \text{somit} \quad A = -\frac{kac}{(c - mk^2)}\sqrt{\frac{m}{c}}$$

Es ist also:

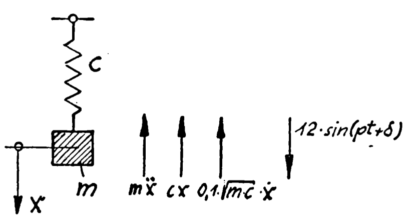
$$x = -D\left(\frac{k}{\omega}\sin\omega t - \sin kt\right); \qquad \omega = \sqrt{\frac{c}{m}}; \qquad \frac{mg}{\delta} = c$$

$$\underline{\underline{x = \frac{ag}{\delta k^2 - g}\left[k\sqrt{\frac{\delta}{g}}\sin\sqrt{\frac{g}{\delta}}\cdot t - \sin kt\right]}} \quad \text{für} \quad k \neq \sqrt{\frac{g}{\delta}}$$

Bei $k = \sqrt{\frac{g}{\delta}}$ ist $x = \frac{0}{0}$; Grenzwert:

$$\underline{\underline{\lim_{\sqrt{\frac{g}{\delta}} \to k} x = \frac{a}{2}\left[\sin kt - kt\cos kt\right]}}$$

Lösung 857



$$m\ddot{x} + cx + 0{,}1\sqrt{mc}\,\dot{x} = 12\sin(pt + \delta)$$

$$\ddot{x} + \frac{c}{m}x + 0{,}1\sqrt{\frac{c}{m}}\,\dot{x} = \frac{12}{m}\sin(pt + \delta)$$

A_{max} tritt bei Resonanz ein, also

$$\omega_e = p$$

$$\omega_e = \sqrt{\frac{c}{m} - \left[\frac{0{,}1}{2}\sqrt{\frac{c}{m}}\right]^2} \quad \text{(vgl. Aufg. 843)}$$

$$\omega_e = p = \underline{\underline{1{,}72\ 1/\text{sek}}}$$

Ansatz für die partikuläre Lösung: $x = C \sin pt + B \cos pt; \quad \frac{12}{m} = a$

$$C = \frac{(\omega^2 - p^2)\, a \cos\delta + 0{,}1\,\omega p \sin\delta}{(\omega^2 - p^2)^2 + (0{,}1\,\omega p)^2}; \qquad B = \frac{(\omega^2 - p^2)\, a \sin\delta - 0{,}1\, a p \omega \cos\delta}{(\omega^2 - p^2)^2 + (0{,}1\,\omega p)^2}$$

$$\underset{\omega_e = p}{A_{\max}} = \sqrt{C^2 + B^2} = \frac{a}{0{,}1\,\omega_e^2}; \qquad \underline{\underline{A_{\max} = 20{,}0 \text{ cm}}}$$

Lösung 858

$$k = \text{Erregerfrequenz}; \quad k = \frac{v}{L} \cdot 2\pi \cdot \frac{1}{3{,}6} \; [v \text{ in km/h}]$$

$$\omega = \text{Eigenfrequenz}; \quad \omega^2 = \frac{c}{m} = \frac{mg}{m\,\Delta l_{\text{st.}}}$$

Es muß sein: $\omega^2 = k^2; \quad \frac{g}{\Delta l_{\text{st.}}} = \frac{v^2 \cdot 4\pi^2}{L^2 \cdot 3{,}6^2}$

$$v = \frac{3{,}6}{2\pi} \cdot L \sqrt{\frac{2}{\Delta l_{\text{st.}}}} \text{ km/h}; \qquad \underline{\underline{v = 96 \text{ km/h}}}$$

Lösung 859

Dampfkraft $P = p \cdot F = F\left(4 + 3 \sin\frac{2\pi}{T} t\right)$

$T = \frac{2\pi}{\omega}$; Erregerfrequenz $k = \frac{2\pi}{T} = 2\pi \cdot n = 2\pi \cdot 3 = 6\pi$

$$m\ddot{x} + cx = F(4 + 3 \sin 6\pi t)$$

$$\ddot{x} + \frac{c}{m} x = \frac{16}{m} + \frac{12}{m} \sin 6\pi t$$

Ansatz für die partikuläre Lösung: $x = A \sin 6\pi t + B$

$$\ddot{x} = -36\pi^2 A \sin 6\pi t$$

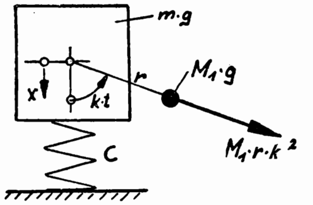
$$A\left(-36\pi^2 + \frac{c}{m}\right) = \frac{12}{m}; \qquad \frac{c}{m} B = \frac{16}{m}$$

$$A = \frac{12}{c - 36\pi^2 m}; \quad B = \frac{16}{c}$$

$$x = 4{,}5 \sin 6\pi t + \frac{16}{3}$$

Die Amplitude der erzwungenen Schwingung ist somit $\underline{\underline{\alpha = 4{,}5 \text{ cm}}}$

Lösung 860



$$m\ddot{x} + cx = M_1 k^2 \cdot r \cdot \cos kt$$

$$x = A \sin \alpha t + B \cos \alpha t + D \cos kt$$

$t = 0; \quad x = 0; \quad \dot{x} = 0: \quad D + B = 0$

$$A = 0$$

$$D = \frac{M_1 k^2 r}{c - mk^2}$$

$$x = \frac{M_1 k^2 r}{c - mk^2} [\cos kt - \cos \alpha t]$$

$$\alpha^2 = \frac{c}{m} = \frac{30 \cdot 981}{32{,}7} = 30^2; \quad k = 30\ 1/\text{sek}; \quad \text{Es herrscht also Resonanz}$$

$$\lim_{\alpha \to k} x = \frac{2 M_1 k r (\cos k t - \cos \alpha t) + M_1 k^2 t r \sin k t}{2 m k}$$

$$\lim_{\alpha \to k} x = 0{,}12 t \sin k t = \underline{\underline{0{,}12 t \sin 30 t \text{ cm}}}$$

Lösung 861

$$m\ddot{x} + a\dot{x} + c x = H \sin (62{,}6 t + \beta)$$

$$\ddot{x} + \frac{a}{m}\dot{x} + \frac{c}{m} x = \frac{H}{m} \sin (p t + \beta)$$

Ansatz für die partikuläre Lösung:

$$x = A \sin p t + B \cos p t$$

Nach Differentiation in die Differentialgleichung eingesetzt:

$$A\left(\frac{c}{m} - p^2\right) - B\left(\frac{ap}{m}\right) = \frac{H}{m} \cos \beta$$

$$A\left(\frac{ap}{m}\right) + B\left(\frac{c}{m} - p^2\right) = \frac{H}{m} \sin \beta$$

$$A = \frac{\frac{H}{m}\left[\left(\frac{c}{m} - p^2\right)\cos\beta + \frac{ap}{m}\sin\beta\right]}{\left(\frac{c}{m} - p^2\right)^2 + \left(\frac{ap}{m}\right)^2}; \quad B = \frac{\frac{H}{m}\left[\left(\frac{c}{m} - p^2\right)\sin\beta - \frac{ap}{m}\cos\beta\right]}{\left(\frac{c}{m} - p^2\right)^2 + \left(\frac{ap}{m}\right)^2}$$

$$\text{Amplitude:} \quad x_{\max} = \sqrt{A^2 + B^2} = \frac{H}{m}\,\frac{1}{\sqrt{\left(\frac{c}{m} - p^2\right)^2 + \left(\frac{ap}{m}\right)^2}}$$

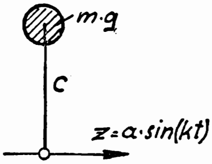
$$\frac{x_{\max}}{x_{\max}^*} = \sqrt{\frac{\left(\frac{c}{m} - p^2\right)^2 + \left(\frac{3ap}{m}\right)^2}{\left(\frac{c}{m} - p^2\right)^2 + \left(\frac{ap}{m}\right)^2}}; \quad \frac{c}{m} = \frac{12 \cdot 981}{3} = 3920$$

$$p^2 = 3920; \quad \text{also Resonanz.}$$

$$\underline{\underline{x_{\max}^* = \frac{1}{3}\, x_{\max}}}$$

33. Relativbewegungen

Lösung 862

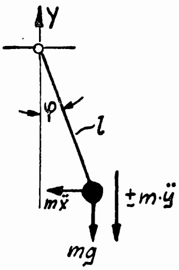


$$m\ddot{x} + c x = c a \sin k t; \quad k = \frac{2\pi}{T}$$

$$x_{\text{part.}} = D \sin k t$$

$$D = \frac{ac}{c - mk^2} = \frac{0{,}1 \cdot 0{,}1}{0{,}1 - \frac{2{,}5}{981}\frac{4\pi^2}{1{,}1^2}} = \underline{\underline{5{,}9 \text{ cm}}}$$

Lösung 863



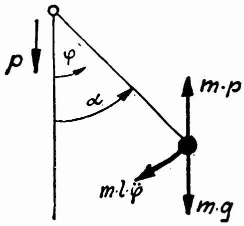
Bewegung nach oben: (+)

$$m l^2 \ddot{\varphi} + m g l \varphi \pm m \ddot{y} l \varphi = 0 \qquad \dot{y} = p$$

$$\ddot{\varphi} + \varphi \frac{(g \pm p)}{l} = 0; \qquad \omega^2 = \frac{g \pm p}{l}$$

$$\underline{\underline{T = 2\pi \sqrt{\frac{l}{g \pm p}}}}$$

Lösung 864



$$m l^2 \ddot{\varphi} + l \sin\varphi \, m (g - p) = 0$$

$$\ddot{\varphi} + \frac{g-p}{l} \sin\varphi = 0 \qquad \ddot{\varphi} = \frac{d(\dot{\varphi})}{d\varphi} \dot{\varphi}$$

$$\dot{\varphi} \, d(\dot{\varphi}) + \frac{g-p}{l} \sin\varphi \, d\varphi = 0$$

$$\frac{\dot{\varphi}^2}{2} - \frac{g-p}{l} \cos\varphi + C = 0 \qquad \varphi = \alpha; \quad \dot{\varphi} = 0:$$

$$C = \frac{g-p}{l} \cos\alpha$$

1. $g = p$: $\dot{\varphi} = 0$; $\varphi = \text{konst.} = \alpha$: $\underline{\underline{s = 0}}$

2. $g \neq p$: $\dot{\varphi}^2 = \frac{2(g-p)}{l} (\cos\varphi - \cos\alpha)$

Die Bedingung $0 = \cos\varphi - \cos\alpha$

wird erfüllt für $\varphi = \alpha \quad \varphi = 2\pi - \alpha$

somit

$$s = l \cdot \Delta\varphi = \underline{\underline{2l(\pi - \alpha)}}$$

Lösung 865

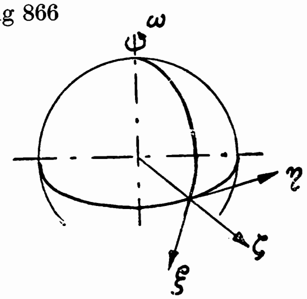
Coriolisbeschleunigung: $b = 2\,\omega\,u \sin\varphi$

$$\omega = 2\pi \cdot \frac{1}{24 \cdot 60 \cdot 60}$$

$$b = \frac{2 \cdot 2\pi \cdot 15 \cdot 0{,}866}{24 \cdot 3600} = 1{,}9 \cdot 10^{-3}\ \text{m/sek}^2$$

Seitendruck: $k = m b = \frac{2 \cdot 10^6 \cdot 1{,}9 \cdot 10^{-3}}{9{,}81} = \underline{\underline{384\ \text{kg}}}$ jeweils auf die rechte Schiene

Lösung 866



$$\ddot{\eta} = 2 g t \omega \cos\varphi; \qquad \ddot{\zeta} = -g$$

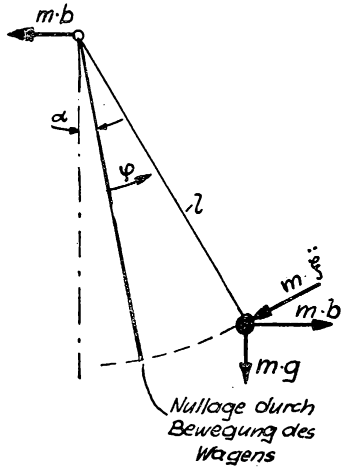
$$\eta = 2 \omega g \frac{t^3}{6} \cos\varphi; \qquad \zeta = h - \frac{g t^2}{2}$$

Für $\zeta = 0$ ist $t^2 = \frac{2h}{g}$

Damit: $\underline{\underline{\eta = \frac{2}{3} \omega h \sqrt{\frac{2h}{g}} \cos\varphi}}$

$\eta = \underline{\underline{12\ \text{cm}}}$

Lösung 867



$$m g l \sin(\varphi + \alpha) + m \ddot{\xi} - m b l \cos(\varphi + \alpha) = 0$$

$$\xi = l \cdot \varphi$$

$$g \sin(\varphi + \alpha) + l \ddot{\varphi} - b \cos(\varphi + \alpha) = 0$$

Ist das Pendel in Ruhe, so gilt: $b = g \operatorname{tg} \alpha$

$$b = \underline{\underline{103\ \text{cm/sek}^2}}$$

Unter Verwendung des Additionstheorems und der Vereinfachung für kleine Winkel

$\sin\varphi = \varphi$; $\cos\varphi = 1$ gilt:

$$\ddot{\varphi} + \varphi\{g\cos\alpha + b\sin\alpha\}\frac{1}{l} = \frac{1}{l}[b\cos\alpha - g\sin\alpha]$$

$$\omega_1^2 = \frac{g\cos\alpha + b\sin\alpha}{l}$$

$$T_1 = 2\pi\sqrt{\frac{l}{g\cos\alpha + g\dfrac{\sin^2\alpha}{\cos\alpha}}} = T\sqrt{\cos\alpha}$$

$$T - T_1 = T\left(1 - \sqrt{\cos\alpha}\right) = \underline{\underline{0{,}0028\,T}}$$

Lösung 868

$$m l \ddot{\varphi} + m g \varphi = m a p^2 \sin pt; \quad \ddot{\varphi} + \frac{g}{l}\cdot\varphi = \frac{a p^2}{l}\sin pt; \quad \frac{g}{l} = k^2$$

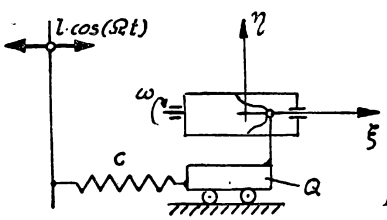
Ansatz: $\varphi = A \sin kt + B \cos kt + D \sin pt$; $D = \dfrac{a p^2}{l\left(\dfrac{g}{l} - p^2\right)} = \dfrac{a p^2}{l(k^2 - p^2)}$

Anfangsbedingungen: $t = 0$; $\varphi = 0$: $B = 0$

$\dot{\varphi} = 0$: $Ak + Dp = 0$; $A = -D\dfrac{p}{k}$

Somit: $\underline{\underline{\varphi = \dfrac{a p^2}{l(k^2 - p^2)}\left[\sin pt - \dfrac{p}{k}\sin kt\right]}}$

Lösung 869



$$\eta = \underline{\underline{\omega \cdot r \cdot t}}$$

$$m\ddot{\xi} + c\,\xi = c \cdot l \cdot \cos\Omega t$$

$$\ddot{\xi} + \frac{cg}{Q}\xi = \frac{c \cdot l \cdot g}{Q}\cos\Omega t$$

$$\lambda^2 = \frac{cg}{Q}$$

Ansatz:

$$\xi = A\cos\lambda t + B\sin\lambda t + C\cos\Omega t$$

Bestimmung von C:

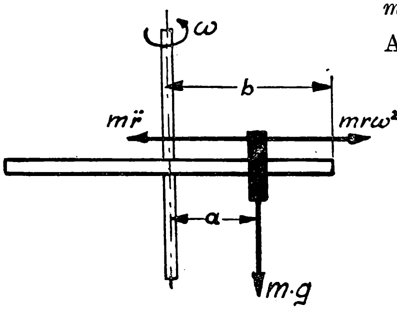
$$-C\,\Omega^2 \cos\Omega t + \lambda^2 \cdot C \cos\Omega t = l\,\lambda^2 \cos\Omega t$$

$$C = \frac{l\lambda^2}{\lambda^2 - \Omega^2} = \frac{l c g}{c g - \Omega^2 Q}$$

Somit:

$$\underline{\underline{\xi = A \cos\sqrt{\frac{cg}{Q}} \cdot t + B \sin\sqrt{\frac{cg}{Q}} \cdot t + \frac{lcg}{cg - \Omega^2 Q} \cos\Omega t}}$$

Lösung 870



$$m\ddot{r} = m\omega^2 \cdot r$$

Ansatz: $r = C_1 e^{\lambda t} + C_2 e^{-\lambda t}$

$$\ddot{r} = C_1 e^{\lambda t} \cdot \lambda^2 + C_2 e^{-\lambda t} \cdot \lambda^2$$

$$\lambda^2 \cdot r = \omega^2 \cdot r; \quad \lambda = \omega$$

$$r = C_1 e^{\omega t} + C_2 e^{-\omega t}$$

Anfangsbedingungen:

$$t = 0; \quad r = a: \quad C_1 + C_2 = a$$

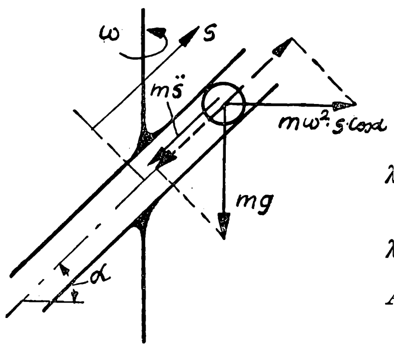
$$\dot{r} = 0: \quad 0 = C_1 - C_2$$

$$C_1 = C_2 = \frac{a}{2}$$

$$r = \frac{a}{2}(e^{\omega t} + e^{-\omega}); \quad \frac{r}{a} = \mathfrak{Cof}\,\omega t; \quad \omega t = \mathfrak{Ar}\,\mathfrak{Cof}\,\frac{r}{a}$$

$$\omega t = \ln\left[\frac{r}{a} + \sqrt{\left(\frac{r}{a}\right)^2 - 1}\right]; \quad \begin{matrix} r = b \\ t = t_1 \end{matrix}: \quad \underline{\underline{t_1 = \frac{1}{2\pi}\ln 3 = 0{,}175\,\text{sek}}}$$

Lösung 871



$$\ddot{s} - s\omega^2 \cos^2\alpha + g \sin\alpha = 0$$

Ansatz: $s = A + B e^{-\lambda t} + C e^{\lambda t}$

$$\ddot{s} = \lambda^2 B e^{-\lambda t} + \lambda^2 C e^{\lambda t}$$

$$\lambda^2 C e^{\lambda t} + \lambda^2 B e^{-\lambda} - A\omega^2 \cos^2\alpha - \omega^2 \cos^2\alpha\, B e^{-\lambda t}$$
$$- \omega^2 \cos^2\alpha\, C e^{\lambda t} + g \sin\alpha = 0$$

$$\lambda^2 - \omega^2 \cos^2\alpha = 0; \qquad \lambda = \omega \cos\alpha$$

$$A\omega^2 \cos^2\alpha = g \sin\alpha; \quad A = \frac{g}{\omega^2} \cdot \frac{\sin\alpha}{\cos^2\alpha}$$

Anfangsbedingungen: $t = 0$; $s = a$; $\dot{s} = 0$:

$$a = A + B + C; \quad 0 = -\lambda B + \lambda C; \quad C = B = \frac{1}{2}\left(a - \frac{g}{\omega^2} \cdot \frac{\sin\alpha}{\cos^2\alpha}\right)$$

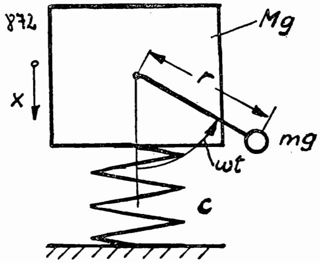
Somit:

$$s=\frac{g}{\omega^2}\cdot\frac{\sin\alpha}{\cos^2\alpha}+\frac{1}{2}\left(a-\frac{g}{\omega^2}\,\frac{\sin\alpha}{\cos^2\alpha}\right)\left(e^{-\omega\cos\alpha t}+e^{\omega\cos\alpha t}\right)$$

$\alpha=\frac{\pi}{2}$;

$$\underline{\underline{s=\frac{g\sqrt{2}}{\omega^2}+\frac{1}{2}\left(a-\frac{g\sqrt{2}}{\omega^2}\right)\left(e^{\frac{-\omega\sqrt{2}}{2}t}+e^{\frac{\omega\sqrt{2}}{2}t}\right)}}$$

Lösung 872



$$M\ddot{x}+c\,x=m\,\omega^2\,r\cos\omega t$$

Partikuläre Lösung: $x=A\cos\omega t$

$$A=\frac{m\,\omega^2 r}{c-M\,\omega^2}=\underline{\underline{0{,}41\text{ mm}}}$$

Die kritische Drehzahl tritt bei der Eigenfrequenz des Systems auf:

$$\omega_e=\sqrt{\frac{c}{M}};\quad n_k=\frac{\omega_e\cdot 60}{2\pi}=\frac{60}{2\pi}\sqrt{\frac{c}{M}}$$

$$\underline{\underline{n_k=950\text{ U/min}}}$$

Lösung 873

$$\frac{p+Q}{g}\,\ddot{y}+k\,\dot{y}+c\,y=\frac{\alpha^2 p}{g}\cdot r\sin\alpha t$$

$$\ddot{y}+\frac{kg}{p+Q}\,\dot{y}+\frac{cg}{p+Q}\cdot y=\frac{\alpha^2\cdot p}{p+Q}\,r\sin\alpha t$$

$$\omega_{\text{eigen}}=\sqrt{\frac{cg}{p+Q}-\left(\frac{kg}{2\,(p+Q)}\right)^2}\ \text{(Vergl. Aufg. 843)}$$

Zur Bestimmung von k wird das logarithmische Dekrement angewendet.

$$\ln\frac{A\,n}{A\,n+1}=\ln\frac{10}{g}=\frac{k\pi g}{2\,(p+Q)\,\omega_{\text{eigen}}}$$

$$\underline{\underline{k=0{,}322\text{ kg sek/cm}}}$$

Partikuläre Lösung:

$$y=a\sin\alpha t+b\cos\alpha t$$
$$\dot{y}=\alpha\,a\cos\alpha t-\alpha\,b\sin\alpha t$$
$$\ddot{y}=-\alpha^2 a\sin\alpha t-\alpha^2 b\cos\alpha t$$

In die Differentialgleichung eingesetzt ergibt sich durch Koeffizientenvergleich:

$$a=\frac{\alpha^2(\omega_0^2-\alpha^2)\,\frac{pr}{p+Q}}{(\omega_0^2-\alpha^2)^2+\alpha^2\left(\frac{kg}{p+Q}\right)^2}=0{,}193;\quad \omega_0=\frac{c\cdot g}{p+Q}$$

$$b=-\frac{\alpha^3\,\frac{kpr}{(p+Q)^2}}{(\omega_0^2-\alpha^2)^2+\alpha^2\left(\frac{k\cdot g}{p+Q}\right)^2}=-0{,}162;\quad \alpha=\frac{\pi\cdot n}{30}$$

$$A^2 = a^2 + b^2; \quad \underline{\underline{A = 0{,}253 \text{ cm}}}$$

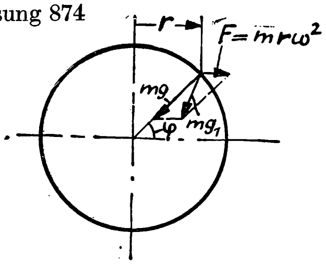
$$y = a \sin \alpha t + b \cos \alpha t = A \sin(\alpha t + \varepsilon)$$

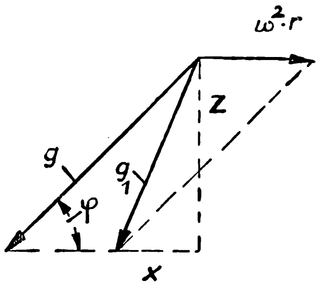
$$= A \{\sin \alpha t \cos \varepsilon + \cos \alpha t \sin \varepsilon\}$$

$$a = A \cos \varepsilon; \quad b = A \sin \varepsilon; \quad \operatorname{tg} \varepsilon = \frac{b}{a};$$

$$\underline{\underline{\varepsilon = 137^\circ}}$$

Lösung 874





Gewicht = Erdanziehungskraft — Fliehkraft

$$z^2 + x^2 = g_1^2$$

$$\sin \varphi = \frac{z}{g}; \quad z^2 = g^2 \sin^2 \varphi$$

$$\cos \varphi = \frac{x + \omega^2 r}{g}; \quad x^2 = (g \cos \varphi - \omega^2 r)^2$$

$$g_1^2 = g^2 \sin^2 \varphi + g^2 \cos^2 \varphi - 2g \cos \varphi \cdot \omega^2 r + \omega^4 r^2$$

ω^4 wird wegen seiner Kleinheit vernachlässigt

$$g_1 = g \sqrt{1 - \frac{2 \cos \varphi \cdot \omega^2 r}{g}}; \quad r = R \cos \varphi$$

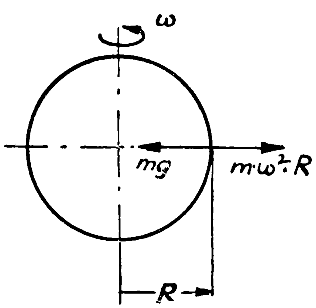
Dieser Wurzelausdruck wird in eine Reihe entwickelt, deren Glieder höherer Ordnung (ω^4; ω^6 usw.) wieder vernachlässigt werden.

Somit: $g_1 = g \left(1 - \frac{\omega^2 R}{g} \cos^2 \varphi\right)$

mit $g = 9{,}832$ ergibt sich:

$$\underline{\underline{g_1 = g \left(1 - \frac{\cos^2 \varphi}{292}\right)}}$$

Lösung 875



$$\omega = k\, \omega_{\text{Erde}}$$

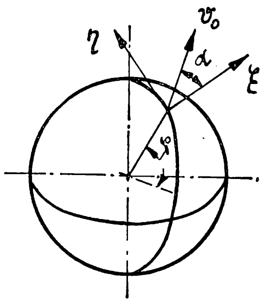
$$m \cdot g = m \omega^2 R$$

$$\frac{g}{R} = k^2 \cdot \omega_{\text{Erde}}^2$$

$$k = \sqrt{\frac{g}{R}} \cdot \frac{1}{\omega_{\text{Erde}}}; \quad \omega_{\text{Erde}} = \frac{2\pi \cdot 1}{24 \cdot 60 \cdot 60}$$

$$\underline{\underline{k = 17{,}1}}$$

Lösung 876



Coriolisbeschleunigung:

$$\ddot{\xi}_c = 2\,v_0 \cos\alpha\, \omega \sin\varphi$$

$$\ddot{\eta}_c = 2\,v_0 \sin\alpha\, \omega \sin\varphi$$

$$b_c = \sqrt{\ddot{\xi}_c^2 + \ddot{\eta}_c^2} = 2\,v_0\, \omega \sin\varphi$$

Die Ablenkung der Schußrichtung ist also unabhängig vom Winkel α.

Ablenkung: $\ddot{s} = b_c$

$$s = v_0\, \omega\, t^2 \sin\varphi$$

Schußweite: $l = v_0 t;\quad t = \frac{l}{v_0}\ (l = 18\,\mathrm{km})$

$$s = \frac{\omega\, l^2}{v_0} \sin\varphi = \underline{\underline{22{,}7\ \mathrm{m}}}$$

Das Geschoß wird nach rechts abgelenkt.

Lösung 877

Die Erde dreht sich mit der Winkelgeschwindigkeit $\omega_E = \frac{2\pi}{24}\,\frac{1}{h}$

Am Breitengrad $\varphi = 60°$ ist: $\omega = \omega_E \sin 60°$

Die Erde dreht sich unter dem Pendel weg, für eine Viertelumdrehung der Pendelebene benötigt sie:

$$T^* = \frac{1}{4}\cdot\frac{2\pi}{\omega} = \frac{2\pi\cdot 24}{4\cdot 2\pi\cdot 0{,}866} = 6{,}93\ \mathrm{h}$$

Die Pendelebene steht also immer nach $T^*\cdot n = T$ Stunden in der Nord-Süd-Richtung ($n = 1;\ 3;\ 5\cdots$)

oder: $n = 2k + 1;\quad k = 1;\ 2;\ 3;\ 4\cdots$

$$T = (2k+1)\,T^* = 2\,T^*(k + 0{,}5) = \underline{\underline{13{,}86\,(0{,}5 + k)\ \mathrm{h}}}$$

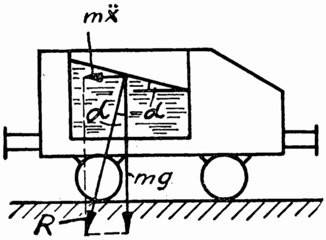
IX. Dynamik des materiellen Systems

34. Grundlagen der Kinetostatik

Lösung 878

$$\omega = \frac{v}{R} = 20\,\frac{1}{\text{sek}}\,; \quad F = m\,\omega^2 \cdot r = \frac{200}{9{,}81} \cdot 400 \cdot 0{,}3 = \underline{\underline{2{,}45\ \text{t}}}$$

Lösung 879



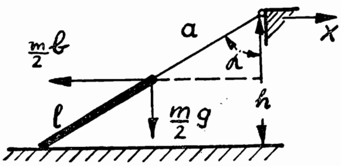
Da der Tender gleichförmig beschleunigt wird, gilt:

$$\operatorname{tg}\alpha = \frac{m\ddot{x}}{mg} = \frac{v}{t \cdot g}$$

$$\operatorname{tg}\alpha = \frac{72}{3{,}6 \cdot 20 \cdot 9{,}81} = 0{,}102$$

$$\underline{\underline{\alpha = 5^\circ\, 50'}}$$

Lösung 880



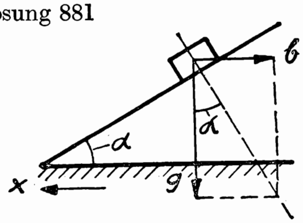
$$\frac{m}{2}\,b \cdot a \cdot \cos\alpha - \frac{mg}{2}\,a \sin\alpha = 0$$

$$b = g \cdot \operatorname{tg}\alpha$$

$$\operatorname{tg}\alpha = \frac{\sqrt{(l+a)^2 - h^2}}{h}$$

$$\underline{\underline{b = \frac{g}{h}\sqrt{(l+a)^2 - h^2}}}$$

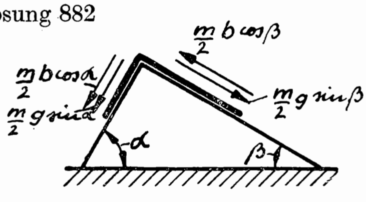
Lösung 881



$$\operatorname{tg}\alpha = \frac{b}{g}$$

$$\underline{\underline{b = g \operatorname{tg}\alpha}}$$

Lösung 882



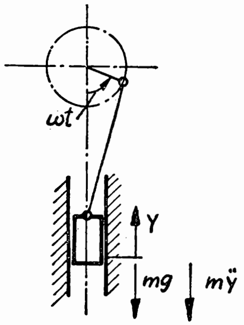
$$b \cos\alpha + g \sin\alpha + b \cos\beta - g \sin\beta = 0$$

$$b\,(\cos\alpha + \cos\beta) = g\,(\sin\beta - \sin\alpha)$$

$\underline{\underline{b = g \operatorname{tg}\frac{\beta - \alpha}{2}}}$ Bei Bewegung nach rechts

$\underline{\underline{b = g \operatorname{tg}\frac{\alpha - \beta}{2}}}$ Bei Bewegung nach links

Lösung 883



Nach Aufgabe 408 gilt:

$$\ddot{y} = r\,\omega^2\left(\cos\omega t + \frac{r}{l}\cos\omega t\right)$$

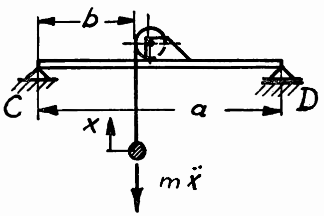
Die durch die Beschleunigung zusätzlich hervorgerufene Kraft beträgt:

$$K^* = m\ddot{y}$$

Die Gesamtkraft ist somit:

$$\underline{\underline{K = p\left\{1 + \frac{r\,\omega^2}{g}\left(\cos\omega t + \frac{r}{l}\cos 2\,\omega t\right)\right\}}}$$

Lösung 884



Zusätzliche dynamische Belastung:

$$P_C \cdot a = m\ddot{x}\,(a-b)$$

$$P_C = m\ddot{x}\,\frac{(a-b)}{a} = \underline{\underline{63,75\text{ kg}}}$$

$$P_C + P_D = m\ddot{x}$$

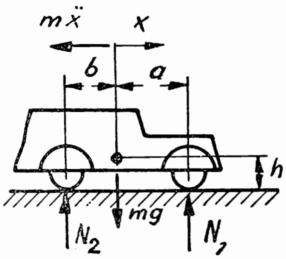
$$P_D = \underline{\underline{38,25\text{ kg}}}$$

Lösung 885

$$K = mb;\quad b = \frac{v^2}{2s};\quad K = \frac{G}{g}\cdot\frac{v^2}{2s} = \frac{7}{9,81}\cdot\frac{144}{2\cdot 3\cdot 3,6^2} = 1,32\text{ t}$$

$$\text{Seilkraft } T = \frac{K}{2} = \underline{\underline{0,66\text{ t}}}$$

Lösung 886



$$\Sigma M_{(1)} = 0:\quad N_2(a+b) = m\ddot{x}\cdot h + mga$$

$$\underline{\underline{N_2 = \frac{P}{g}\cdot\frac{(ag + h\ddot{x})}{a+b}}}$$

$$\Sigma M_{(2)} = 0:\quad N_1(a+b) + m\ddot{x}h = mg\cdot b$$

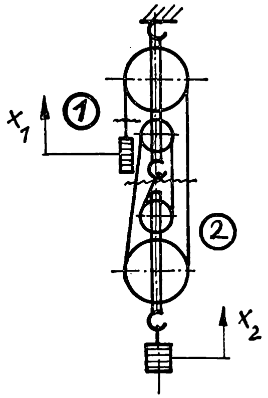
$$\underline{\underline{N_1 = \frac{P}{g}\cdot\frac{(bg - h\ddot{x})}{a+b}}}$$

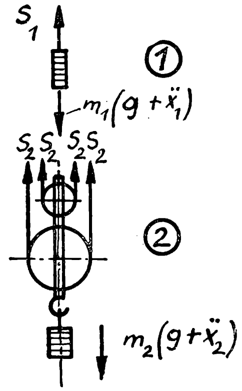
Für $N_1 = N_2$ gilt: $ag + h\ddot{x} = bg - h\ddot{x}$

$$\underline{\underline{\ddot{x} = -\frac{(a-b)\,g}{2\,h}}}$$

Eine negative Beschleunigung ist eine Verzögerung.

Lösung 887





Zwangsbedingungen: $x_1 = -4x_2$; $s_1 = s_2$

Somit lauten die Gleichgewichtsbedingungen: $4m_1(g + \ddot{x}_1) = m_2(g + \ddot{x}_2)$

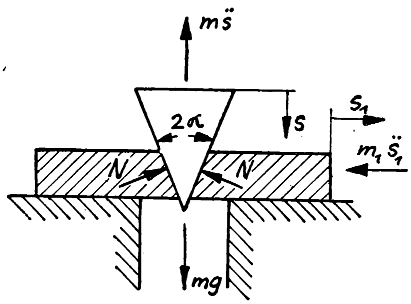
$$\ddot{x}_1 = b = -4g\,\frac{4m_1 - m_2}{m_2 + 16m_1} = \underline{\underline{-4g\,\frac{4P - Q}{16P + Q}}};$$

Das negative Vorzeichen entspricht dem Sinken der Last P

Für eine gleichförmige Lastbewegung gilt: $\ddot{x}_1 = 0$

$$4m_1 - m_2 = 0; \quad \underline{\underline{\frac{P}{Q} = \frac{1}{4}}}$$

Lösung 888



$$m\ddot{s} - mg + 2N\sin\alpha = 0$$

$$N\cos\alpha - m_1\ddot{s}_1 = 0; \quad \operatorname{tg}\alpha = \frac{s_1}{s}$$

Keil: $\ddot{s}_1 = b_1 = g\,\dfrac{P\operatorname{ctg}\alpha}{P\operatorname{ctg}\alpha + 2P_1\operatorname{tg}\alpha}$

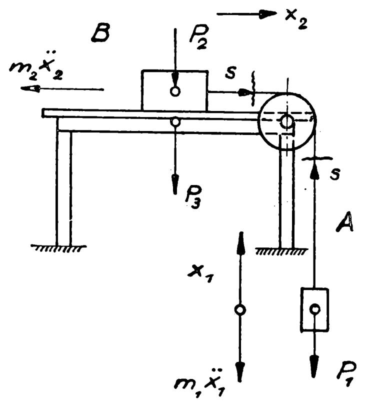
$$s = g\,\frac{P\cdot\operatorname{ctg}\alpha}{P\operatorname{ctg}\alpha + 2P_1\operatorname{tg}\alpha}\cdot\frac{t^2}{2}$$

Platte: $\ddot{s}_1 = b_1 = g\,\dfrac{P}{P\operatorname{ctg}\alpha + 2P_1\operatorname{tg}\alpha}$

$$s_1 = g\,\frac{Pt^2}{2(P\operatorname{ctg}\alpha + 2P_1\operatorname{tg}\alpha)}$$

$$\underline{\underline{N = \frac{PP_1}{P\operatorname{ctg}\alpha + 2P_1\operatorname{tg}\alpha}\cdot\frac{1}{\cos\alpha}}}$$

Lösung 889



Die gesamte auf den Boden drückende Last beträgt:

$$N = P_1 + P_2 + P_3 + \frac{P_1}{g}\ddot{x}_1$$

Ermittlung von $\ddot{x}_1$:

$$m_2\ddot{x}_2 = S; \quad m_1\ddot{x}_1 + m_1 g = S$$

$$\frac{P_2}{g}\ddot{x}_2 = \frac{P_1}{g}\ddot{x}_1 + P_1$$

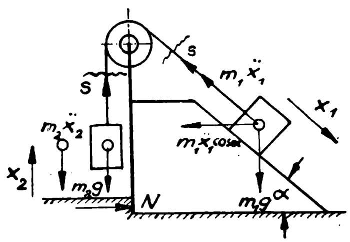
Zwangsbedingungen: $x_1 = -x_2$

$$\ddot{x}_1 = -\ddot{x}_2$$

somit: $\ddot{x}_1 = -\dfrac{P_1}{P_1 + P_2}\cdot g$

$$\underline{\underline{N = P_1 + P_2 + P_3 - \frac{P_1^2}{P_1 + P_2}}}$$

Lösung 890



$$x_1 = x_2; \quad \ddot{x}_1 = \ddot{x}_2$$

$$m_2\ddot{x}_2 + m_2 g = S$$

$$-m_1\ddot{x}_2 + m_1 g \sin\alpha = S$$

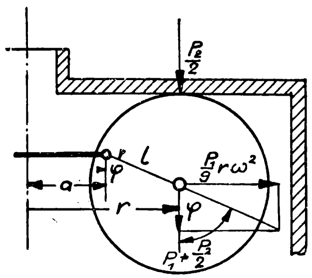
$$N = m_1\ddot{x}_1 \cdot \cos\alpha$$

$$m_2\ddot{x}_2 + m_2 g = -m_1\ddot{x}_1 + m_1 g \sin\alpha$$

$$\ddot{x}_1 = \frac{m_1 g \sin\alpha - m_2 g}{m_2 + m_1}$$

$$\underline{\underline{N = P_1 \cdot \frac{P_1 \sin\alpha - P_2}{P_1 + P_2} \cdot \cos\alpha}}$$

Lösung 891



$$\operatorname{tg}\varphi = \frac{\dfrac{P_1}{g}\cdot r\omega^2}{P_1 + \dfrac{P_2}{2}}; \quad r = a + l\sin\varphi$$

$$\underline{\underline{\omega^2 = g\,\frac{2P_1 + P_2}{2P_1(a + l\sin\varphi)}\cdot \operatorname{tg}\varphi}}$$

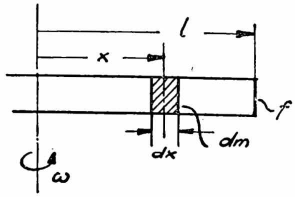
Lösung 892

Zentrifugalkraft: $F = m\omega^2 \cdot r$; $\quad r = 0{,}108\,\text{cm}$; $\quad \omega = \pi\cdot\dfrac{n}{30} = \pi\cdot\dfrac{910}{30}\,\dfrac{1}{\text{sek}}$

$$m = \frac{110}{981}\,\frac{\text{t}\,\text{sek}^2}{\text{cm}}$$

$$\underline{\underline{F = N = 109{,}7\,\text{t}}}$$

Lösung 893



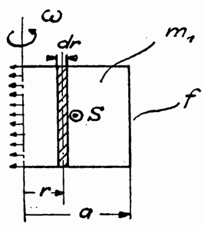
$$dF = dm \cdot x \cdot \omega^2; \quad dm = f \cdot \frac{\gamma}{g} \cdot dx$$

$$F = \frac{f \cdot \gamma \cdot \omega^2}{g} \int_{x=a}^{x=l} x\,dx$$

$$F = \frac{f \cdot \gamma \omega^2}{g \cdot 2}(l^2 - a^2); \quad f \cdot \gamma \cdot l = P$$

$$\underline{\underline{F = \frac{P(l^2 - a^2)}{2gl} \cdot \omega^2}}$$

Lösung 894



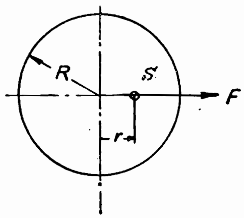
$$m_1 \cdot g = \frac{P}{2}$$

$$dF = dm_1 \cdot \omega^2 \cdot r; \quad dm_1 = \varrho \cdot f \cdot dr$$

$$F = \omega^2 \varrho f \int_0^a r\,dr = \omega^2 \varrho f \frac{a^2}{2} = m_1 \omega^2 \cdot \frac{a}{2}$$

$$\underline{\underline{F = \frac{P}{4g} a \omega^2}}$$

Lösung 895

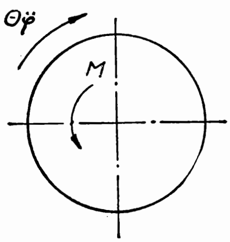


Schwerpunktsabstand einer Halbkreisscheibe:

$$r = \frac{4}{3} \cdot \frac{R}{\pi}$$

$$F = \frac{P}{2g} \cdot r\,\omega^2 = \underline{\underline{\frac{2PR\omega^2}{3\pi g}}}$$

Lösung 896



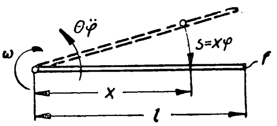
$$\Theta \ddot{\varphi} = M; \quad \varphi = 3t^2$$

$$\ddot{\varphi} = 6 \frac{1}{\text{sek}^2}$$

$$\Theta = \frac{G}{g} \cdot \frac{r^2}{2}$$

$$G = \frac{M \cdot 2g}{\ddot{\varphi} r^2} = \frac{4 \cdot 2 \cdot 981}{6 \cdot 400} = \underline{\underline{3{,}27\ \text{kg}}}$$

Lösung 897



Tangentialkraft:

$$dT = dm \cdot \ddot{s}; \qquad \ddot{s} = x \cdot \ddot{\varphi}$$

$$\varphi = at^2; \quad \ddot{\varphi} = 2a$$

$$dT = f \cdot \frac{\gamma}{g} \cdot \ddot{\varphi} \cdot x\,dx; \qquad T = f \cdot \frac{\gamma}{g} \cdot 2a \int_0^l x\,dx$$

$$\underline{\underline{T = m \cdot a \cdot l = \frac{P}{g} \cdot al}}$$

$$M = \Theta\ddot{\varphi}; \quad \Theta = m\frac{l^2}{3}$$

$$M = \frac{2}{3}\frac{P}{g}al^2; \quad M = T \cdot z$$

$$\underline{\underline{z = \frac{2}{3}l}}$$

(Entfernung der resultierenden Tangentialkraft T von der Drehachse)

Zentrifugalkraft:

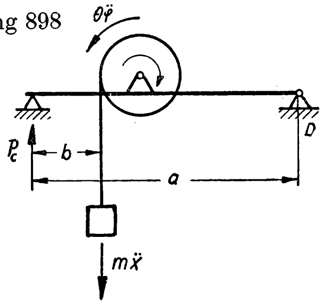
$$dF = dm \cdot x \cdot \omega^2; \quad \omega = \dot{\varphi} = 2at$$

$$dF = f \cdot \frac{\gamma}{g} \cdot x \cdot dx \cdot (2at)^2$$

$$F = \frac{(2at)^2 f \cdot \gamma}{g}\int_0^l x\,dx = \frac{(2at)^2 \cdot f \cdot \gamma}{g} \cdot \frac{l^2}{2}$$

$$\underline{\underline{F = \frac{2Pa^2t^2 \cdot l}{g}}}$$

Lösung 898



$$P_C a = m\ddot{x}(a-b) + \Theta\ddot{\varphi}$$

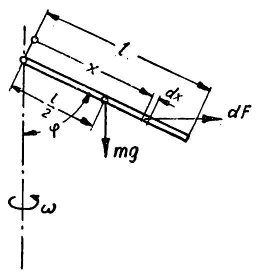
$$\ddot{\varphi} = \frac{\ddot{x}}{r}$$

$$\underline{\underline{P_C = 63{,}85\ \text{kg}}}$$

$$P_D \cdot a = m\ddot{x} \cdot b - \Theta \cdot \ddot{\varphi}$$

$$\underline{\underline{P_D = 38{,}15\ \text{kg}}}$$

Lösung 899



$$dF = f \cdot \varrho \cdot \sin\varphi \cdot \omega^2 x\,dx$$

Gleichgewicht: $\int dF \cdot x \cdot \cos\varphi = mg \cdot \frac{l}{2}\sin\varphi$

$$\cos\varphi \cdot f \cdot \varrho \cdot \sin\varphi\,\omega^2 \int_0^l x^2\,dx = m\frac{g \cdot l}{2}\sin\varphi$$

$$m\omega^2\frac{l^2}{3}\cos\varphi = m\frac{gl}{2}$$

$$\underline{\underline{\cos\varphi = \frac{3g}{2\omega^2 l}}}$$

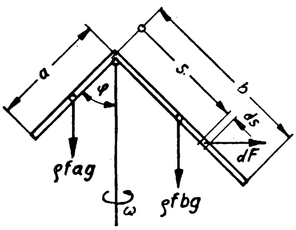
Auflagekraft: $N = \sqrt{F^2 + P^2}; \quad F = \frac{P}{g} \cdot \sin\varphi\, \omega^2 \frac{l}{2}$

$$\sin^2\varphi = 1 - \frac{9g^2}{4\omega^4 l^2}$$

$$N = \frac{1}{2}\,\frac{P}{g}\,\omega^2 l \sqrt{1 - \frac{9g^2}{4\omega^4 l^2} + \frac{g^2 \cdot 4}{\omega^4 l^2}}$$

$$\underline{\underline{N = \frac{1}{2}\,\frac{P}{g}\,\omega^2 l \sqrt{1 + \frac{7g^2}{4l^2\omega^4}}}}$$

Lösung 900



$$dM_F = dm\,\omega^2 \cdot s^2 \cdot \sin\varphi \cdot \cos\varphi$$

$$dm = \varrho \cdot f \cdot ds$$

Gleichgewicht:

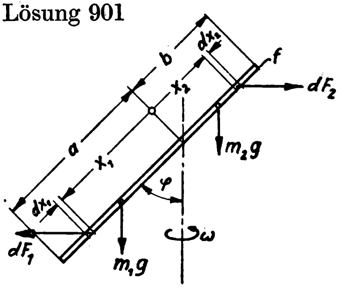
$$\frac{1}{2}\varrho f \omega^2 \sin 2\varphi \int_0^a s^2 ds - \varrho f a g \cdot \frac{a}{2}\sin\varphi$$

$$+ \varrho f \cdot b \cdot g \cdot \frac{b}{2}\cos\varphi - \frac{1}{2}\varrho f \omega^2 \sin 2\varphi \int_0^b s^2 ds = 0$$

$$\omega^2 \frac{a^3}{3}\sin 2\varphi - \omega^2 \frac{b^3}{3}\sin 2\varphi - a^2 g \sin\varphi + b^2 g \cos\varphi = 0$$

$$\underline{\underline{\omega^2 = 3g\,\frac{b^2\cos\varphi - a^2\sin\varphi}{(b^3 - a^3)\sin 2\varphi}}}$$

Lösung 901



$$\sum M_0 = 0: \quad \int x_1 dF_1 \cdot \cos\varphi + \int x_2 dF_2 \cos\varphi$$

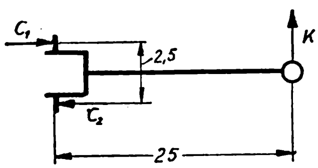
$$- m_1 g \frac{a\sin\varphi}{2} + m_2 g \frac{b\sin\varphi}{2} = 0$$

$$dF = f \cdot \varrho \cdot \omega^2 \sin\varphi\, x\, dx; \quad m_1 = \frac{a}{b} m_2$$

$$\cos\varphi = \frac{3g}{2\omega^2}\,\frac{(a^2 - b^2)}{(a^3 + b^3)}$$

$$\underline{\underline{\cos\varphi = \frac{3g}{2\omega^2}\,\frac{(a - b)}{a^2 - ab + b^2}}}$$

Lösung 902



Coriolisbeschleunigung:

$$b_C = 2\omega v \sin\varphi = 2 \cdot \frac{\pi \cdot 180}{30} \cdot 0{,}2 \cdot \frac{\sqrt{2}}{2}$$

$$b_C = 5{,}34 \text{ m/sek}^2$$

Corioliskraft: $K = m \cdot b_C$

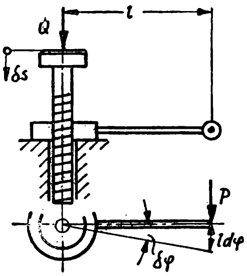
$$K = \frac{10}{9{,}81} \cdot 5{,}34 = 5{,}42 \text{ kg}$$

$$K \cdot 25 = C_1 \cdot 2{,}5; \quad C_1 = C_2$$

$$\underline{\underline{C_1 = C_2 = 54{,}2 \text{ kg}}}$$

35. Das Prinzip der virtuellen Verrückung

Lösung 903



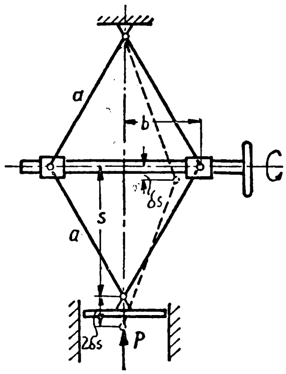
$$\Sigma \delta A = 0$$

$$Q \cdot \delta s - P\,(l\,\delta\varphi) = 0$$

$$Q = P \cdot \frac{l\,\delta\varphi}{\delta s} = 16 \cdot \frac{600 \cdot 2\pi}{12}$$

$$\underline{\underline{Q = 5020 \text{ kg}}}$$

Lösung 904



$$M \cdot \delta\varphi = 2P\,\delta s; \qquad \delta\varphi = \delta b \cdot \frac{2\pi}{h}$$

$$a^2 = b^2 + s^2$$

$$a^2 = (b - \delta b)^2 + (s + \delta s)^2$$

$$a^2 = b^2 - 2b\,\delta b + \delta b^2 + s^2 + 2s\,\delta s + \delta s^2$$

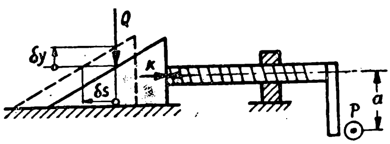
Unter Vernachlässigung der kleinen Glieder höherer Ordnung ergibt sich:

$$\delta s = \delta b \operatorname{tg}\alpha; \quad \operatorname{tg}\alpha = \frac{b}{s}$$

$$\frac{M \cdot \delta b \cdot 2\pi}{h} = 2P \cdot \delta b \cdot \operatorname{tg}\alpha;$$

$$\underline{\underline{P = M \cdot \frac{\pi}{h} \cdot \operatorname{ctg}\alpha}}$$

Lösung 905



$$P\,a\,\delta\varphi = K\,\delta s$$

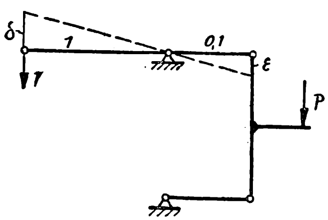
$$K\,\delta s = Q\,\delta y; \quad \delta y = \operatorname{tg}\alpha \cdot \delta s$$

$$\delta\varphi = \delta s \cdot \frac{2\pi}{h}$$

$$P \cdot a \cdot \delta\varphi = Q \cdot \operatorname{tg}\alpha \cdot \delta s$$

$$\underline{\underline{Q = \frac{P\,2\,a\,\pi}{h \operatorname{tg}\alpha}}}$$

Lösung 906

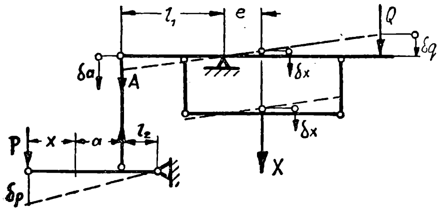


$$p\,\delta = P \cdot \varepsilon; \quad \frac{\varepsilon}{\delta} = \frac{0{,}1}{1}$$

$$p = P \cdot \frac{\varepsilon}{\delta}$$

$$p = 0{,}1 P; \quad p = \underline{\underline{10 \text{ kg}}}$$

Lösung 907



Gleichgewicht ohne die Kraft X:

$$P(a+l_2)-A\,l_2=0;\quad A\,l_1=Q\cdot b$$

$$Q=P\,\frac{l_1(a+l_2)}{b\,l_2}$$

Virtuelle Verrückung:

$$P\cdot\delta p+X\cdot\delta x+Q\,\delta q=0$$

Geometrische Zusammenhänge:

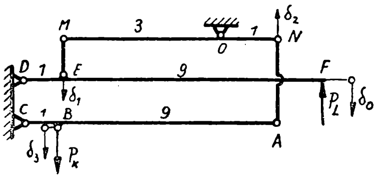
$$\delta q=\frac{b}{e}\,\delta x;\quad \delta p=-\frac{x+a+l_2}{l_2}\cdot\frac{l_1}{e}\cdot\delta x$$

Somit:

$$-P\,\delta x\,\frac{l_1(x+a\,l_2)}{e\,l_2}+X\,\delta x+P\,\delta x\,\frac{l_1(a+l_2)}{l_2\cdot e}=0$$

$$\underline{\underline{X=P\,\frac{l_1 x}{e\,l_2}}}$$

Lösung 908



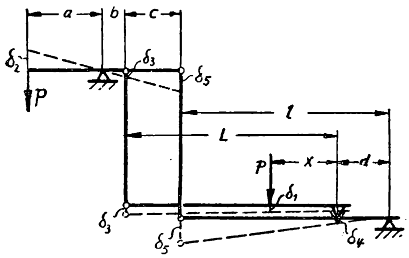
$$P_L\cdot\delta_0=P_K\cdot\delta_3$$

$$\frac{\delta_0}{\delta_1}=\frac{10}{1};\quad \frac{\delta_1}{\delta_2}=\frac{3}{1};\quad \frac{\delta_2}{\delta_3}=\frac{10}{1}$$

$$\frac{\delta_0}{\delta_3}=\frac{\delta_0}{\delta_1}\cdot\frac{\delta_1}{\delta_2}\cdot\frac{\delta_2}{\delta_3}=300$$

$$\underline{\underline{P_L=\frac{\delta_3}{\delta_0}\cdot P_K=\frac{1}{300}\,P_K}}$$

Lösung 909



Um eine Unabhängigkeit des Ergebnisses von der Lage der Last zu erhalten, muß die Gerade EG nach der Verschiebung parallel zur Ursprungslage liegen, also $\delta_3=\delta_4=\delta_1$

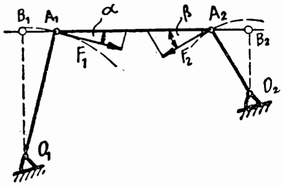
$$\frac{\delta_5}{\delta_4}=\frac{l}{d};\quad \frac{\delta_3}{\delta_5}=\frac{b}{b+c};$$

$$\delta_3=\delta_4:\quad \underline{\underline{\frac{b+c}{b}=\frac{l}{d}}}$$

$$P\,\delta_1=p\,\delta_2;\quad \frac{\delta_2}{\delta_5}=\frac{a}{b+c};\quad \frac{\delta_1}{\delta_5}=\frac{b}{b+c}$$

$$\underline{\underline{p=\frac{b}{a}\cdot P}}$$

Lösung 910



Es herrscht Gleichgewicht, wenn sich die Komponenten von F_1 und F_2 in Richtung $A_1 A_2$ aufheben:

$$\frac{F_1}{\cos\alpha} = \frac{F_2}{\cos\beta}; \quad \cos\alpha = \frac{B_1 O_1}{A_1 O_1}$$

$$\cos\beta = \frac{B_2 O_2}{A_2 O_2}$$

$$\underline{\underline{F_1 \cdot O_1 A_1 \cdot O_2 B_2 = F_2 \cdot O_2 A_2 \cdot O_1 B_1}}$$

Lösung 911

$$P\,\delta x = C \cdot l\,\delta\varphi; \quad M = C \cdot l$$

$$x = 2l\sin\varphi$$

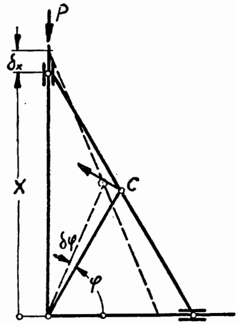
$$x + \delta x = 2l\sin(\varphi + \delta\varphi)$$

$$x + \delta x = 2l\,[\sin\varphi\cos\delta\varphi + \cos\varphi\sin\delta\varphi]$$

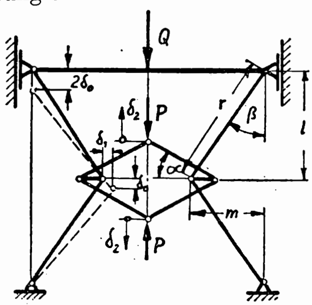
$$x + \delta x = 2l\sin\varphi + 2l\cos\varphi\,\delta\varphi$$

$$\frac{\delta x}{\delta\varphi} = 2l\cos\varphi$$

$$C = P \cdot \frac{\delta x}{\delta\varphi} \cdot \frac{1}{l}; \quad \underline{\underline{M = 2Pl\cos\varphi}}$$



Lösung 912



$$Q \cdot 2\delta_0 = 2P\delta_2$$

$$r^2 = l^2 + m^2$$

$$(l - \delta_0)^2 + (m + \delta_1)^2 = r^2$$

$$l\delta_0 = m\delta_1$$

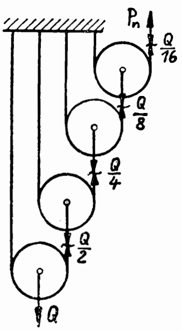
$$\frac{m}{l} = \operatorname{tg}\beta; \quad \frac{\delta_0}{\delta_1} = \operatorname{tg}\beta$$

Somit auch:

$$\frac{\delta_1}{\delta_2} = \operatorname{tg}\alpha$$

$$Q = P \cdot \frac{\delta_2}{\delta_0}; \quad \underline{\underline{Q = P\operatorname{ctg}\alpha\operatorname{ctg}\beta}}$$

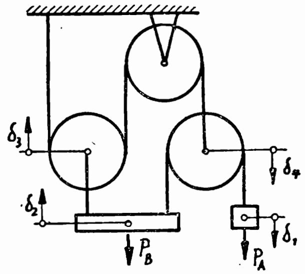
Lösung 913



$$P = \frac{Q}{2^n}$$

$$\underline{\underline{Q = P \cdot 2^n}}$$

Lösung 914



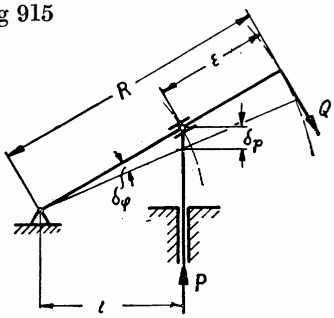
$$P_A \cdot \delta_1 = P_B \cdot \delta_2$$

$$\delta_4 = 2\,\delta_3; \quad \delta_2 = \delta_3$$

$$\delta_1 = \delta_4 + (\delta_2 + \delta_4) = 5\,\delta_2$$

$$\underline{\underline{P_B = 5\,P_A}}$$

Lösung 915

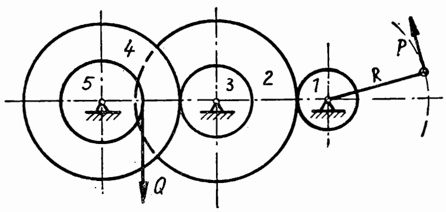


$$P \cdot \delta p = Q \cdot R \cdot \delta\varphi$$

$$\cos\varphi = \frac{(R-\varepsilon)\,\delta\varphi}{\delta p}; \quad \cos\varphi = \frac{l}{R-\varepsilon}$$

$$\delta\varphi = \frac{\delta p \cos^2\varphi}{l}; \quad \underline{\underline{Q = \frac{P \cdot l}{R\cos^2\varphi}}}$$

Lösung 916



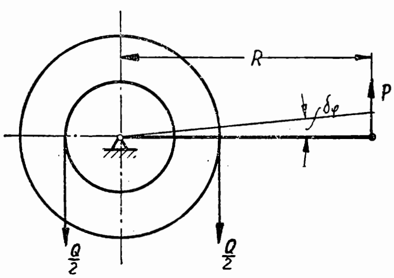
$$Q \cdot (r_5 \cdot \varphi_5) = P\,(R \cdot \varphi_1)$$

$$r_4 \cdot \varphi_5 = r_3 \cdot \varphi_3; \quad r_2 \cdot \varphi_3 = r_1 \cdot \varphi_1$$

$$P = \frac{r_5}{R} \cdot Q \cdot \frac{\varphi_5}{\varphi_1}$$

$$\underline{\underline{P = Q \cdot \frac{r_5}{R} \cdot \frac{r_3}{r_4} \cdot \frac{r_1}{r_2} = 5\ \text{kg}}}$$

Lösung 917

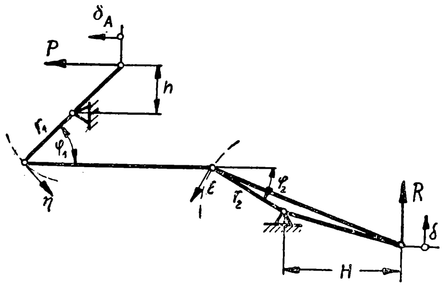


$$P \cdot R\,\delta\varphi + \frac{Q}{2}\,r_1\,\delta\varphi - r_2\,\frac{Q}{2}\,\delta\varphi = 0$$

$$PR = \frac{Q}{2}\,(r_2 - r_1)$$

$$\underline{\underline{P = Q\,\frac{(r_2 - r_1)}{2R} = 12\ \text{kg}}}$$

Lösung 918



$$P\delta_A = R \cdot \delta_D$$

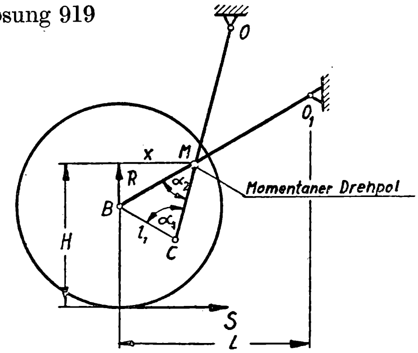
$$\frac{H}{\delta_D} = \frac{r_2}{\varepsilon}; \quad \delta_D = \frac{H}{r_2} \cdot \varepsilon$$

$$\frac{h}{\delta_A} = \frac{r_1}{\eta}; \quad \delta_A = \frac{h}{r_1} \cdot \eta$$

$$\varepsilon \cdot \sin\varphi_2 = \eta \sin\varphi_1$$

$$\underline{\underline{P = R \cdot \frac{H r_1}{h \cdot r_2} \cdot \frac{\sin\varphi_1}{\sin\varphi_2}}}$$

Lösung 919



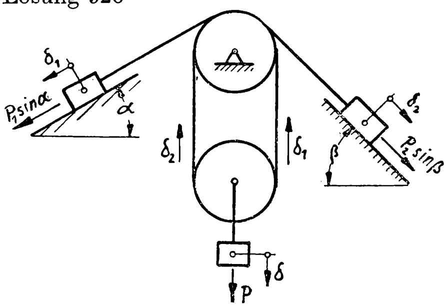
$$R\delta r + S\delta s = 0$$

$$\frac{\delta r}{x} = -\frac{\delta s}{H}; \quad \frac{\delta r}{\delta s} = -\frac{x}{H}$$

$$x = \frac{BM \cdot L}{l_2}; \quad BM = l_1 \cdot \frac{\sin\alpha_1}{\sin\alpha_2}$$

$$\underline{\underline{S = R \cdot \frac{L l_1}{H l_2} \frac{\sin\alpha_1}{\sin\alpha_2}}}$$

Lösung 920



$$\frac{1}{2}\delta_1 + \frac{1}{2}\delta_2 = \delta$$

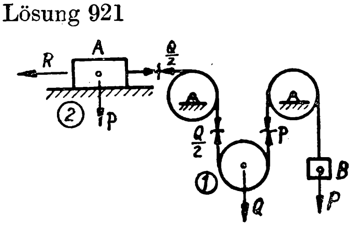
$$P_1 \sin\alpha \cdot \delta_1 + P_2 \sin\beta \cdot \delta_2 = P\delta$$

$$P_1 \sin\alpha = P_2 \sin\beta$$

$$P_1 \sin\alpha\,(\delta_1 + \delta_2) = P \cdot \delta$$

$$\underline{\underline{P_1 = \frac{P}{2\sin\alpha}}}; \quad \underline{\underline{P_2 = \frac{P}{2\sin\beta}}}$$

Lösung 921



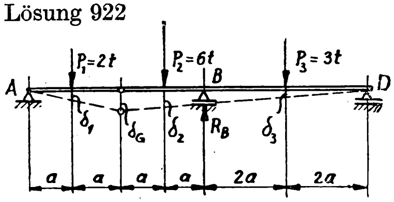
Aus Schnitt 1 folgt: $\underline{\underline{P = \frac{Q}{2}}}$

Aus Schnitt 2 folgt:

$$\frac{Q}{2} = R = \mu \cdot P$$

$$\underline{\underline{\mu = 1}}$$

Lösung 922

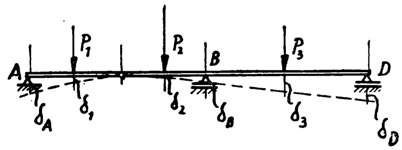


$$P_1\delta_1+P_2\delta_2+P_3\delta_3=R_B\,\delta_B$$

$$\frac{\delta_1}{\delta_G}=\frac{a}{2a};\quad \frac{\delta_2}{\delta_G}=\frac{5a}{6a};\quad \frac{\delta_B}{\delta_G}=\frac{4a}{6a}$$

$$\frac{\delta_3}{\delta_G}=\frac{2a}{6a};\quad R_B=\frac{3}{4}P_1+\frac{5}{4}P_2+\frac{1}{2}P_3$$

$$\underline{\underline{R_B=10{,}5\ \mathrm{t}}}$$



$$R_A\,\delta_A=P_1\delta_1;\quad \delta_A=2\,\delta_1$$

$$R_A=\frac{1}{2}P_1=\underline{\underline{1\ \mathrm{t}}}$$

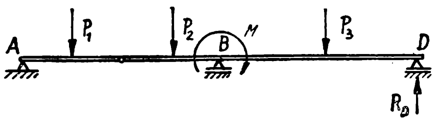
$$P_2\delta_2-R_B\,\delta_B+P_3\delta_3-R_D\,\delta_D=0$$

$$\frac{\delta_2}{\delta_D}=\frac{a}{6a};\quad \frac{\delta_B}{\delta_D}=\frac{2a}{6a};\quad \frac{\delta_3}{\delta_D}=\frac{4a}{6a}$$

$$R_D=P_2\cdot\frac{1}{6}-R_B\cdot\frac{2}{6}+P_3\cdot\frac{4}{6}$$

$$R_D=-\underline{\underline{0{,}5\ \mathrm{t}}}$$

Lösung 923



$$\sin\delta\varphi=\operatorname{tg}\delta\varphi=\delta\varphi=\frac{\delta_D}{6a}\ \text{bzw.}\ \frac{\delta_G}{6a}$$

Nach Aufgabe 922 gilt unter Hinzunahme eines Momentes:

1. $P_1\delta_1+P_2\delta_2+P_3\delta_3-R_B\,\delta_B-M\dfrac{\delta_G}{6a}=0$

$$3P_1+5P_2+2P_3-4R_B-\frac{M}{a}=0$$

2. $P_2\delta_2-R_B\,\delta_B+P_3\delta_3-R_D\,\delta_D+\dfrac{M}{6a}\delta_D=0$

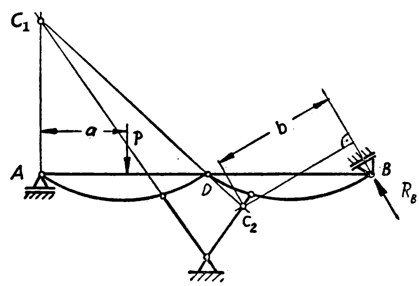
$$P_2-2R_B+4P_3-R_D+\frac{M}{a}=0$$

R_D soll Null werden, aus 1. und 2. folgt somit:

$$3P_1+3P_2-6P_3-3\frac{M}{a}=0$$

$$M=a\,[P_1+P_2-2P_3]=\underline{\underline{2a\ \mathrm{tm}}}$$

Lösung 924



$$P\cdot\delta_P+R_B\cdot\delta_B=0$$

$$\frac{\delta_P}{a}=\frac{\delta_D}{DC_1};\quad \frac{\delta_D}{DC_2}=\frac{\delta_B}{b}$$

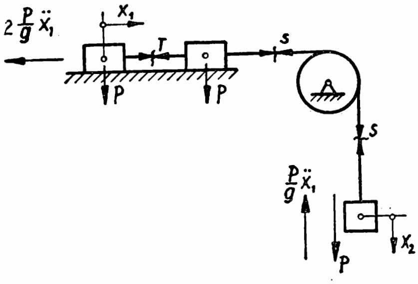
$$\delta_P=\frac{a\,DC_2}{b\,DC_1}\cdot\delta_B$$

$$R_B=-P\frac{\delta_P}{\delta_B}=\underline{\underline{-P\cdot\frac{a\,DC_2}{b\,DC_1}}}$$

Das Vorzeichen (—) besagt, daß R_B entgegen der angenommenen Richtung wirkt.

36. Allgemeine Gleichungen der Dynamik

Lösung 925



Gesamtes System:

$$2\frac{P}{g}\ddot{x}_1 - S = 0$$

$$P - \frac{P}{g}\ddot{x}_2 - S = 0; \quad x_1 - x_2 = 0$$

$$2\frac{P}{g}\ddot{x}_1 - P + \frac{P}{g}\ddot{x}_1 = 0$$

$$3\frac{\ddot{x}_1}{g} = 1; \quad \ddot{x}_1 = b = \underline{\underline{\frac{g}{3}}}$$

Belastung des Fadens:

$$\frac{P}{g}\ddot{x}_1 - T = 0$$

$$\frac{P}{g}\cdot\frac{g}{3} = T; \quad \underline{\underline{T = \frac{P}{3}}}$$

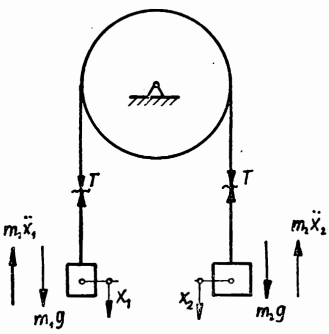
Lösung 926

Gleichgewichtsbedingung:

$$m_1 g - m_1\ddot{x}_1 - T = 0$$

$$m_2 g - m_2\ddot{x}_2 - T = 0$$

Zwangsbedingung:



$$x_1 + x_2 = 0$$

$$\ddot{x}_1 = -\ddot{x}_2$$

$$m_1 g - m_1\ddot{x}_1 - T = 0$$

$$m_2 g + m_2\ddot{x}_1 - T = 0$$

$$g(m_1 - m_2) - \ddot{x}_1(m_1 + m_2) = 0$$

$$\ddot{x}_1 = g\frac{m_1 - m_2}{m_1 + m_2}$$

Der Betrag der Beschleunigung ist somit:

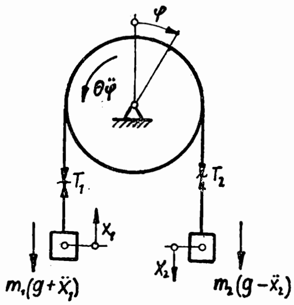
$$\underline{\underline{b = \frac{P_2 - P_1}{P_1 + P_2}\cdot g}}$$

Seilkraft:

$$T = m_1 g - m_1 g\frac{m_1 - m_2}{m_1 + m_2}$$

$$\underline{\underline{T = \frac{2P_1P_2}{P_1 + P_2}}}$$

Lösung 927



Zwangsbedingung:

$$x_1 = x_2 = r\varphi$$

Gleichgewichtsbedingungen:

$$T_2 + m_2\ddot{x}_2 - m_2 g = 0$$
$$T_1 - m_1(g + \ddot{x}_1) = 0$$
$$T_2 r - T_1 r - \Theta\ddot{\varphi} = 0; \quad \Theta = m r^2$$

$$T_2 + m_2\ddot{x}_1 - m_2 g = 0$$
$$T_1 - m_1\ddot{x}_1 - m_1 g = 0$$
$$T_2 - T_1 = m\ddot{x}_1$$

$$m\ddot{x}_1 + T_1 + m_2\ddot{x}_1 = m_2 g$$
$$-m_1\ddot{x}_1 + T_1 = m_1 g$$

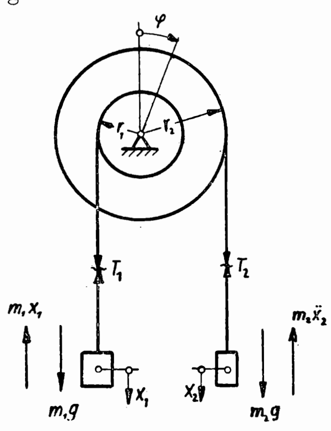
$$\ddot{x}_1(m + m_1 + m_2) = g(m_2 - m_1)$$

$$\underline{\underline{b = \ddot{x}_2 = \ddot{x}_1 = g\,\frac{P_2 - P_1}{P_1 + P_2 + P}}}$$

Seilkräfte: $T_2 - T_1 = P\,\dfrac{P_2 - P_1}{P + P_1 + P_2}$

$$\underline{\underline{T_1 = \frac{P_1(P + 2P_2)}{P + P_1 + P_2}}}; \qquad \underline{\underline{T_2 = \frac{P_2(P + 2P_1)}{P + P_1 + P_2}}}$$

Lösung 928



Gleichgewichtsbedingungen:

$$m_2 g - m_2\ddot{x}_2 - T_2 = 0$$
$$m_1 g - m_1\ddot{x}_1 - T_1 = 0$$
$$T_1 r_1 - T_2 r_2 = 0$$

Zwangsbedingungen: $r_2\varphi = x_2$

$r_1\varphi = -x_1$

Daraus:

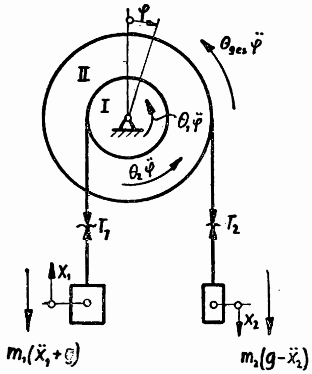
$$\underline{\underline{\ddot{\varphi} = \varepsilon = g\,\frac{P_2 r_2 - P_1 r_1}{P_1 r_1^2 + P_2 r_2^2}}}$$

Lösung 929

Zwangsbedingungen: $r_2\varphi = x_2$

$r_1\varphi = x_1$

$\Theta_{\text{ges}} = m_{\text{I}} r_1^2 + m_{\text{II}} r_2^2; \quad P_{\text{II}} = 2P_{\text{I}}$



Gleichgewichtsbedingungen:

$$T_1 - m_1(\ddot{x}_1 + g) = 0$$
$$T_2 + m_2(\ddot{x}_2 - g) = 0$$
$$T_2 \cdot r_2 - \Theta_{\text{ges}} \cdot \ddot{\varphi} - T_1 r_1 = 0$$

$$T_1 - m_1 r_1 \ddot{\varphi} = m_1 g$$
$$T_2 + m_2 r_2 \ddot{\varphi} = m_2 g$$
$$-r_1 T_1 + r_2 T_2 - \Theta_{\text{ges}} \cdot \ddot{\varphi} = 0$$

Somit:

$$\ddot{\varphi} = \varepsilon = g \frac{P_2 r_2 - P_1 r_1}{P_2 r_2^2 + P_1 r_1^2 + P_{\text{I}}(r_1^2 + 2 r_2^2)}$$

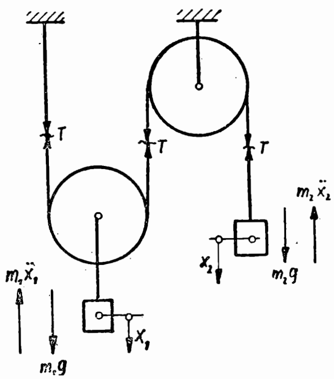
$$T_1 = \frac{P_1 P_2 (r_2^2 + r_1 r_2) + P_1 P_{\text{I}}(r_1^2 + 2 r_2^2)}{P_2 r_2^2 + P_1 r_1^2 + P_{\text{I}}(r_1^2 + 2 r_2^2)}$$

$$T_2 = \frac{P_1 P_2 (r_1^2 + r_1 r_2) + P_2 P_{\text{I}}(r_1^2 + 2 r_2^2)}{P_2 r_2^2 + P_1 r_1^2 + P_{\text{I}}(r_1^2 + 2 r_2^2)}$$

Mit den gegebenen Zahlenwerten ergibt sich

$$\varepsilon = 49\ 1/\text{sek}^2$$
$$T_1 = 25\ \text{kg}$$
$$T_2 = 17\ \text{kg}$$

Lösung 930



Zwangsbedingungen: $x_2 + 2x_1 = 0$

$$\ddot{x}_1 = -\frac{1}{2}\ddot{x}_2$$

Gleichgewichtsbedingungen:

$$m_2 g - m_2 \ddot{x}_2 - T = 0$$
$$m_1 g - m_1 \ddot{x}_1 - 2T = 0$$

$$2 m_2 g - 2 m_2 \ddot{x}_2 - 2T = 0$$
$$m_1 g + 0{,}5 m_1 \ddot{x}_2 - 2T = 0$$

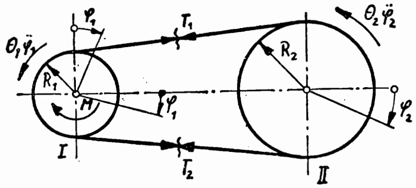
$$g(2m_2 - m_1) - \ddot{x}_2\left(2m_2 + \frac{1}{2} m_1\right) = 0$$

$$\ddot{x}_2 = \frac{(2m_2 - m_1)g}{2m_2 + \frac{1}{2} m_1}$$

$$b_2 = \frac{(2 \cdot 8 - 10)}{2 \cdot 8 + 5} \cdot 9{,}81 = \underline{\underline{2{,}8\ \text{m/sek}^2}}$$

$$T = m_2(g - \ddot{x}_2) = \frac{8}{9{,}81}(9{,}81 - 2{,}8) = \underline{\underline{5{,}72\ \text{kg}}}$$

Lösung 931



$$\text{I: } M + T_1 R_1 - T_2 R_1 - \Theta_{\mathrm{I}}\ddot{\varphi}_1 = 0$$

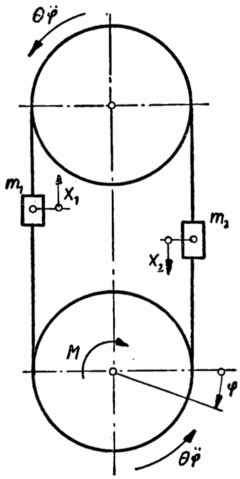
$$\text{II: } T_1 R_2 + \Theta_2\ddot{\varphi}_2 - T_2 R_2 = 0$$

$$\text{I u. II: } M = \Theta_{\mathrm{I}}\ddot{\varphi}_1 + \Theta_{\mathrm{II}}\ddot{\varphi}_1 \frac{R_2^2}{R_1^2}$$

$$\Theta_{\mathrm{I}} = m_1 R_1^2; \quad \Theta_{\mathrm{II}} = m_2 R_2^2; \quad m = \frac{P}{g}$$

$$\ddot{\varphi}_1 = \varepsilon_1 = \frac{M g}{(P_1 + P_2)\, R_1^2}$$

Lösung 932



$$x_1 = x_2 = x = r\varphi; \quad \Theta = \frac{Q}{g} \cdot \frac{r^2}{2}$$

Kinetische Energie:

$$T = 2 \cdot \frac{\Theta\dot{\varphi}^2}{2} + \frac{\dot{x}^2}{2}(m_1 + m_2)$$

Potentielle Energie:

$$U = g x (m_1 - m_2)$$

Erteilte Energie:

$$A = M \cdot \varphi = M \frac{x}{r}$$

Lagrangesche Funktion:

$$L = \frac{Q \dot{x}^2}{g 2} + (m_1 + m_2)\frac{\dot{x}^2}{2} - g x (m_1 - m_2)$$

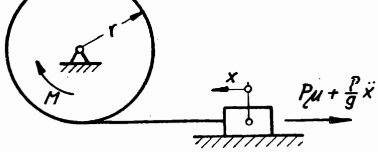
$$\left(\frac{\partial L}{\partial \dot{x}}\right)^{\cdot} - \frac{\partial L}{\partial x} = \frac{\partial A}{\partial x}$$

$$\ddot{x}\left[\frac{Q}{g} + m_1 + m_2\right] - g\,[m_2 - m_1] = \frac{M}{r}$$

$$\ddot{x} = \frac{\frac{M}{r} + g\,[m_2 - m_1]}{m_1 + m_2 + \frac{Q}{g}}$$

$$b = \ddot{x} = \frac{M + (P_2 - P_1)\, r}{(P_1 + P_2 + Q)\, r} \cdot g$$

Lösung 933

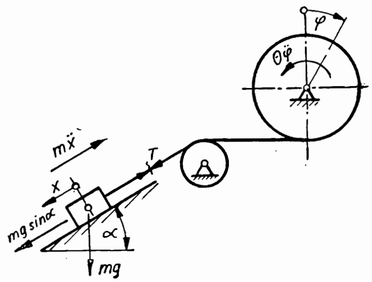


Unter Vernachlässigung des Trägheitsmomentes der Welle gilt:

$$M - r P \mu - \frac{P}{g}\ddot{x} r = 0$$

$$\ddot{x} = b = g\,\frac{M - P r \mu}{P \cdot r}$$

Lösung 934



Gleichgewichtsbedingungen:

$$m\ddot{x} + T - mg\sin\alpha = 0$$

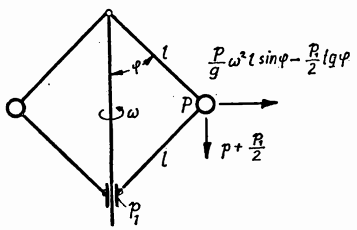
$$\Theta\ddot{\varphi} - T\cdot r = 0; \qquad \Theta = \frac{Q}{g}\cdot\frac{r^2}{2}$$

Zwangsbedingung: $\varphi\cdot r = x$

$$\ddot{\varphi}\left(mr + \frac{Qr}{2g}\right) = mg\sin\alpha$$

$$\underline{\underline{\varepsilon = \ddot{\varphi} = \frac{2P\sin\alpha\cdot g}{r(2P+Q)}}}$$

Lösung 935



$$\sum M_0 = 0:$$

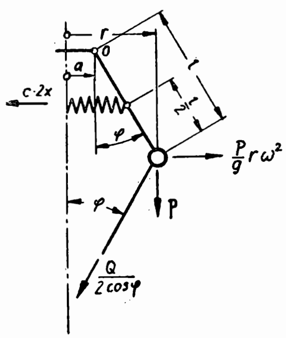
$$\frac{p}{g}\,\omega^2 l^2\sin\varphi\cos\varphi - \frac{p_1}{2}\,l\operatorname{tg}\varphi\cos\varphi$$

$$-\left(p + \frac{p_1}{2}\right) l\sin\varphi = 0$$

$$\frac{p}{g}\,\omega^2 l\cos\varphi - \frac{p_1}{2} - \left(p + \frac{p_1}{2}\right) = 0$$

$$\underline{\underline{\cos\varphi = g\,\frac{p_1 + p}{pl\omega^2}}}$$

Lösung 936



$$\sum M_0 = 0: \qquad x = \frac{l}{2}\sin\varphi$$

$$r = a + l\sin\varphi$$

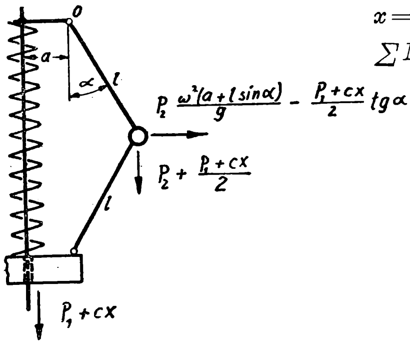
$$-2c\,\frac{l}{2}\sin\varphi\,\frac{l}{2}\cos\varphi - P\cdot l\sin\varphi - \frac{Q}{2\cos\varphi}\cdot l\cdot\sin 2\varphi$$

$$+\,\omega^2 r\cdot\frac{P}{g}\cdot l\cos\varphi = 0$$

Daraus:

$$\underline{\underline{\omega^2 = g\,\frac{P + Q + \dfrac{c\cdot l}{2}\cos\varphi}{P(a + l\sin\varphi)}\operatorname{tg}\varphi}}$$

Lösung 937



$$x = 2l(1 - \cos\alpha)$$

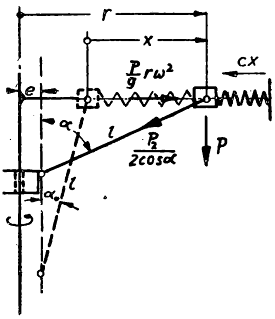
$$\sum M_0 = 0:$$

$$\left[P_2 \frac{\omega^2}{g}(a + l\sin\alpha) - \frac{P_1 + cx}{2}\,\mathrm{tg}\,\alpha\right] l\cos\alpha$$

$$-\left[P_2 + \frac{P_1 + cx}{2}\right] l\sin\alpha = 0$$

$$\underline{\underline{\omega^2 = g\,\frac{P_1 + P_2 + 2cl(1 - \cos\alpha)}{P_2(a + l\sin\alpha)} \cdot \mathrm{tg}\,\alpha}}$$

Lösung 938



$$P = P_A = P_B$$

$$x = l(\sin\alpha - \sin\alpha_0)$$

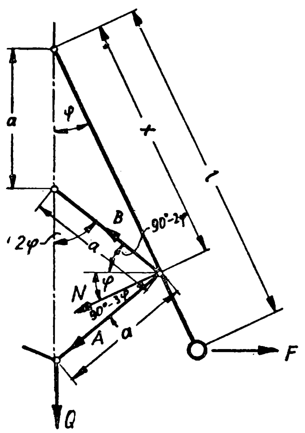
$$r = e + l\sin\alpha$$

$$\frac{P_2}{2}\,\mathrm{tg}\,\alpha + cx = \frac{P}{g}\,\omega^2 r$$

$$\omega^2 = 388 \ \ 1/\mathrm{sek}^2$$

$$\underline{\underline{n = 188 \ \ \mathrm{U/min}}}$$

Lösung 939



Wenn das System im Gleichgewicht sein soll, so gilt:

$$N \cdot x = F \cdot l\cos\varphi$$

$$F = 2m\omega^2 \cdot l\sin\varphi$$

$$x = a\,\frac{1 + \cos 2\varphi}{\cos\varphi}$$

$$N = A\cos(90 - 3\varphi) + B\cos(90 - \varphi)$$

$$B\sin(90 - \varphi) = A\sin(90 - 3\varphi)$$

$$A = \frac{Q}{2} \cdot \frac{1}{\cos 2\varphi}$$

$$N = \frac{Q\sin 3\varphi}{2\cos 2\varphi} + \frac{Q\sin\varphi\cos 3\varphi}{2\cos 2\varphi\cos\varphi}$$

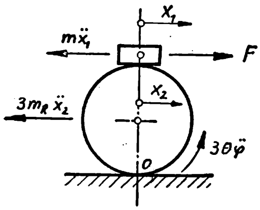
$$\frac{(1 + \cos 2\varphi)(\sin 3\varphi\cos\varphi + \sin\varphi\cos 3\varphi)}{\cos^3\varphi\cos 2\varphi \cdot \sin\varphi} = \frac{4m\omega^2 l^2}{Q \cdot a}$$

$$\frac{(1 + \cos 2\varphi) \cdot 2 \cdot \sin 2\varphi\cos 2\varphi}{\cos^2\varphi\cos 2\varphi\sin\varphi\cos\varphi} = \frac{4m\omega^2 l^2}{Q \cdot a}$$

$$2\cos^2\varphi = 1 + \cos 2\varphi$$

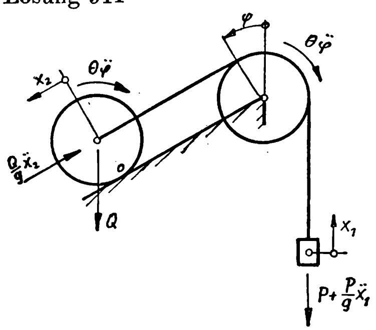
Somit:
$$8 = \frac{4P\omega^2 l^2}{Q \cdot g \cdot a}$$
$$\omega = \sqrt{\frac{2gQa}{Pl^2}}$$

Lösung 940



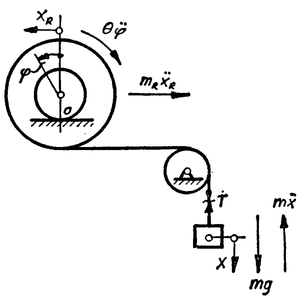
$\sum M_0 = 0$:
$$m\ddot{x}_1 \cdot 2r - F \cdot 2r + 3m_R \cdot \ddot{x}_2 \cdot r + 3\Theta\ddot{\varphi} = 0$$
$$x_1 = 2x_2$$
$$\varphi = \frac{x_2}{r}$$
$$\ddot{x}_1 = b = \frac{8gF}{8Q + 9P}$$

Lösung 941



$$\Theta\ddot{\varphi} + \frac{Q}{g}\ddot{x}_2 \cdot r + \Theta\ddot{\varphi} + P \cdot r + \frac{P}{g}\ddot{x}_1 r$$
$$- Qr\sin\alpha = 0$$
$$x_1 = x_2 = r \cdot \varphi; \quad \Theta = \frac{Q}{g} \cdot \frac{r^2}{2}$$
$$\frac{Q}{g} \cdot \frac{r}{2} \cdot \ddot{x}_2 + \frac{Q}{g} r\ddot{x}_2 + \frac{Q}{g}\,\frac{r}{2}\ddot{x}_2 + \frac{P}{g} r\ddot{x}_2$$
$$+ P \cdot r - Qr\sin\alpha = 0$$
$$\ddot{x}_2 = b = g\frac{Q\sin\alpha - P}{2Q + P}$$

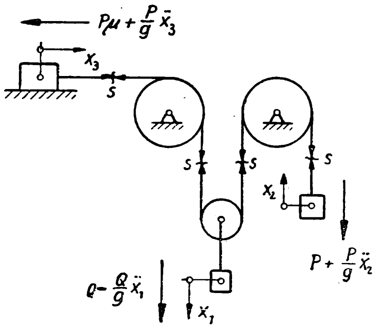
Lösung 942



Gleichgewichtsbedingungen:
$$m\ddot{x} + T - mg = 0$$
$$m_R\ddot{x}_R \cdot r - T(R - r) + \Theta\ddot{\varphi} = 0; \quad \Theta = m_R\varrho^2$$

Zwangsbedingungen:
$$x_R = r \cdot \varphi$$
$$x = (R - r)\varphi \quad x_R = \frac{r}{R - r} \cdot x$$
$$m\ddot{x} - mg + T = 0$$
$$m_R \cdot \frac{r^2}{(R - r)^2} \cdot \ddot{x} + \Theta\frac{\ddot{x}}{(R - r)^2} - T = 0$$
$$\ddot{x}\left[m + m_R\frac{r^2 + \varrho^2}{(R - r)^2}\right] = mg$$
$$\ddot{x} = b = g\frac{P(R - r)^2}{P(R - r)^2 + Q(r^2 + \varrho^2)}$$

Lösung 943



Zwangsbedingung:

$$2x_1 - x_2 - x_3 = 0$$

Gleichgewichtsbedingungen:

$$2S + \frac{Q}{g}\ddot{x}_1 - \quad Q = 0$$

$$S - \frac{P}{g}\ddot{x}_2 - \quad P = 0$$

$$S - \frac{P}{g}\ddot{x}_3 - \mu P = 0$$

Es besteht somit folgendes Gleichungssystem:

$$\begin{array}{llllll} 2S & +\frac{Q}{g}\ddot{x}_1 & & & = Q \\ S & & -\frac{P}{g}\ddot{x}_2 & & = P \\ S & & & -\frac{P}{g}\ddot{x}_3 & = \mu P \\ & 2\ddot{x}_1 & -\ddot{x}_2 & -\ddot{x}_3 & = 0 \end{array}$$

Daraus:

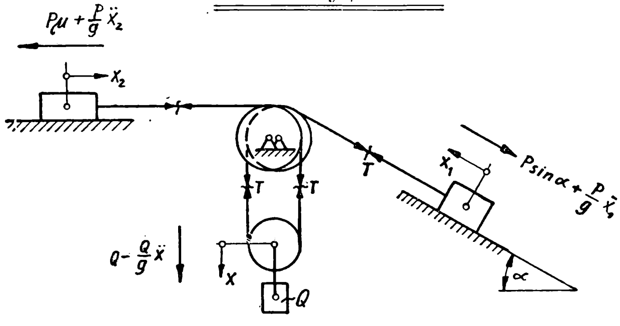
$$\ddot{x}_1 = b = \frac{\begin{vmatrix} 2 & Q & 0 & 0 \\ 1 & P & -\frac{P}{g} & 0 \\ 1 & \mu P & 0 & -\frac{P}{g} \\ 0 & 0 & -1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & \frac{Q}{g} & 0 & 0 \\ 1 & 0 & -\frac{P}{g} & 0 \\ 1 & 0 & 0 & -\frac{P}{g} \\ 0 & 2 & -1 & -1 \end{vmatrix}} = \underline{\underline{\frac{Q - P(\mu + 1)}{Q + 2P} \cdot g}}$$

Abwärtsbewegung von Q tritt nur ein, wenn $\underline{\underline{Q > P(\mu + 1)}}$

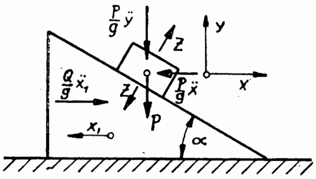
Lösung 944

Die Aufgabe entspricht der Aufgabe 943, wenn man an der Last D die parallel zur Schräge wirkende Kraft $P \sin\alpha$ an Stelle der Kraft P der Last A (Aufgabe 943) setzt.

Somit: $\quad Q > P(\sin\alpha + \mu); \quad \underline{\underline{b = g\frac{Q - P(\sin\alpha + \mu)}{Q + 2P}}}$



Lösung 945



Gleichgewichtsbedingungen:

1. $\frac{P}{g}\ddot{x} - Z\sin\alpha = 0$

2. $\frac{P}{g}\ddot{y} + P - Z\cos\alpha = 0$

3. $\frac{Q}{g}\ddot{x}_1 - Z\sin\alpha = 0$

4. $\operatorname{tg}\alpha = -\frac{y}{x + x_1}$ (Zwangsbedingung)

Aus 1. und 3.: $\ddot{x} = \frac{Q}{P}\ddot{x}_1$

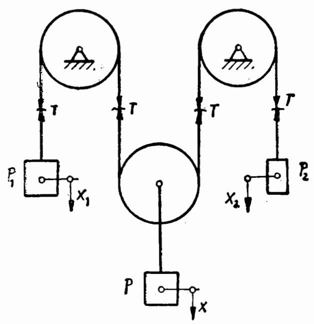
Aus 2. und 4.:

$$-\frac{P}{g}\operatorname{tg}\alpha\left(\ddot{x}_1 + \frac{Q}{P}\ddot{x}_1\right) + P - \operatorname{ctg}\alpha\cdot\frac{Q}{g}\ddot{x}_1 = 0$$

$$\ddot{x}_1 = \frac{-P}{-\frac{Q}{g}\operatorname{ctg}\alpha - \frac{P}{g}\operatorname{tg}\alpha - \frac{Q}{g}\operatorname{tg}\alpha}$$

$$\underline{\underline{\ddot{x}_1 = b = g\,\frac{P\sin 2\alpha}{2(Q + P\sin^2\alpha)}}}$$

Lösung 946



Das aus Zwangs- und Gleichgewichtsbedingungen aufgebaute Gleichungssystem hat die Form:

$$\begin{array}{lllll}
-\frac{P}{g}\ddot{x} & & & -2T & = -P \\
& -\frac{P_1}{g}\ddot{x}_1 & & -T & = -P_1 \\
& & -\frac{P_2}{g}\ddot{x}_2 & -T & = -P_2 \\
2\ddot{x} & +\ \ddot{x}_1 & +\ddot{x}_2 & & = 0
\end{array}$$

Daraus:

$$\ddot{x} = \frac{\begin{vmatrix} -P & 0 & 0 & -2 \\ -P_1 & -P_1/g & 0 & -1 \\ -P_2 & 0 & -P_2/g & -1 \\ 0 & 1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} -P/g & 0 & 0 & -2 \\ 0 & -P_1/g & 0 & -1 \\ 0 & 0 & -P_2/g & -1 \\ 2 & 1 & 1 & 0 \end{vmatrix}}$$

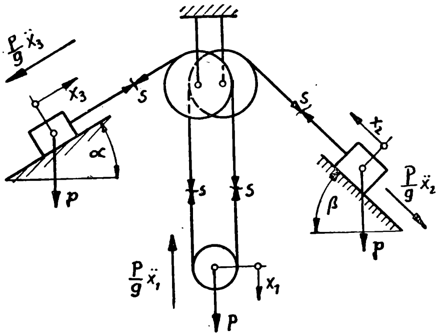
$$\ddot{x} = \frac{P(P_1 + P_2) - 4P_1P_2}{P(P_1 + P_2) + 4P_1P_2}\cdot g$$

$\ddot{x} = b = \underline{\underline{-\frac{1}{11}}}g$ (nach oben)

Ebenso erhält man: $\ddot{x}_1 = b_1 = \underline{\underline{-\frac{1}{11}g}}\begin{pmatrix}\text{nach}\\ \text{oben}\end{pmatrix}$

$\ddot{x}_2 = b_2 = \underline{\underline{\frac{3}{11}g}}\begin{pmatrix}\text{nach}\\ \text{unten}\end{pmatrix}$

Lösung 947



Gleichgewichtsbedingungen:

$$2S + \frac{P}{g}\ddot{x}_1 - P = 0$$

$$S - \frac{p}{g}\ddot{x}_2 - p\sin\beta = 0$$

$$S - \frac{p}{g}\ddot{x}_3 - p\sin\alpha = 0$$

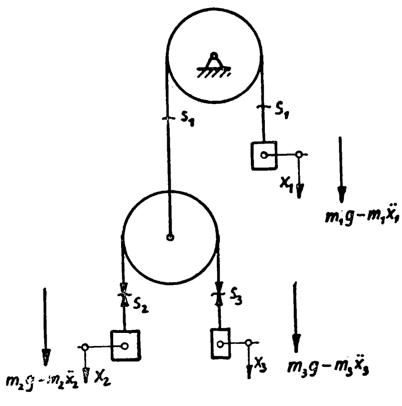
Zwangsbedingung:

$$2x_1 - x_2 - x_3 = 0$$

$$\ddot{x}_1 = b = \frac{\begin{vmatrix} 2 & P & 0 & 0 \\ 1 & p\sin\beta & -p/g & 0 \\ 1 & p\sin\alpha & 0 & -p/g \\ 0 & 0 & -1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & P/g & 0 & 0 \\ 1 & 0 & -p/g & 0 \\ 1 & 0 & 0 & -p/g \\ 0 & 2 & -1 & -1 \end{vmatrix}}$$

$$b = \frac{2\frac{p}{g}\left[P - p(\sin\alpha + \sin\beta)\right]}{2\frac{p}{g}\left[2\frac{p}{g} + \frac{P}{g}\right]}; \qquad \underline{\underline{b = g\,\frac{P - p(\sin\alpha + \sin\beta)}{2p + P}}}$$

Lösung 948



Gleichgewichtsbedingungen:

$$S_1 + m_1\ddot{x}_1 - m_1 g = 0; \qquad S_2 = S_3$$

$$S_2 + m_2\ddot{x}_2 - m_2 g = 0; \qquad S_1 = S_2 + S_3$$

$$S_3 + m_3\ddot{x}_3 - m_3 g = 0; \qquad S_1 = 2S_2$$

Zwangsbedingung:

$$2x_1 + x_2 + x_3 = 0$$

$$\ddot{x}_1 = \frac{\begin{vmatrix} 2 & m_1 g & 0 & 0 \\ 1 & m_2 g & m_2 & 0 \\ 1 & m_3 g & 0 & m_3 \\ 0 & 0 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & m_1 & 0 & 0 \\ 1 & 0 & m_2 & 0 \\ 1 & 0 & 0 & m_3 \\ 0 & 2 & 1 & 1 \end{vmatrix}}$$

$$\ddot{x}_1 = \frac{\left[-4m_2m_3 + m_1(m_2 + m_3)\right]\cdot g}{4m_2m_3 + m_1(m_2 + m_3)}$$

Damit m_1 abwärts sinkt, muß $m_1(m_2 + m_3) - 4m_2m_3 > 0$ sein, d. h.

$$\underline{\underline{m_1 > \frac{4m_2m_3}{m_2 + m_3}}}$$

Lösung 949

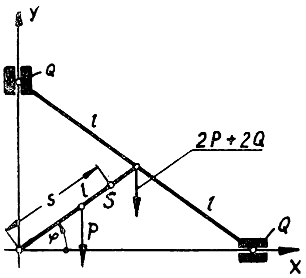
Das aus Zwangs- und Gleichgewichtsbedingungen aufgebaute Gleichungssystem hat die Form (nach 948):

$$\begin{aligned} 2S_2 + m_1\ddot{x}_1 \quad\quad\quad\quad &= m_1 g; \\ S_2 \quad\quad + m_2\ddot{x}_2 \quad\quad &= m_2 g; \\ S_2 \quad\quad\quad\quad + m_3\ddot{x}_3 &= m_3 g; \\ +2\ddot{x}_1 + \ddot{x}_2 + \ddot{x}_3 &= 0; \end{aligned} \qquad S_1 = 2S_2$$

$$T = S_1 = 2 \cdot \frac{g \begin{vmatrix} m_1 & m_1 & 0 & 0 \\ m_2 & 0 & m_2 & 0 \\ m_3 & 0 & 0 & m_3 \\ 0 & 2 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & m_1 & 0 & 0 \\ 1 & 0 & m_2 & 0 \\ 1 & 0 & 0 & m_3 \\ 0 & 2 & 1 & 1 \end{vmatrix}}; \qquad \underline{\underline{T = \frac{8 m_1 m_2 m_3 \cdot g}{m_1(m_2 + m_3) + 4 m_2 m_3}}}$$

37. Schwerpunktsatz

Lösung 950

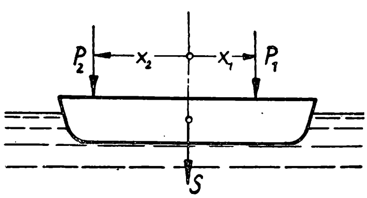


Der Schwerpunkt liegt auf der Kurbel. Er beschreibt einen Kreis mit dem Radius s.

$$s(3P + 2Q) = 2l(P + Q) + P\frac{l}{2}$$

$$\underline{\underline{s = \frac{l}{2} \cdot \frac{5P + 4Q}{3P + 2Q}}}$$

Lösung 951



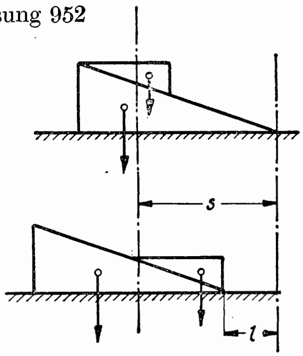
Das Moment um den Schwerpunkt muß stets Null bleiben.

$$P_2 x_2 = P_1 x_1$$

$$x_2 = \frac{P_1}{P_2} x_1 = \underline{\underline{1{,}43\ \mathrm{m}}}$$

Da der erste Mann sich an die rechte Bootsseite setzt, muß der zweite an der linken sitzen.

Lösung 952



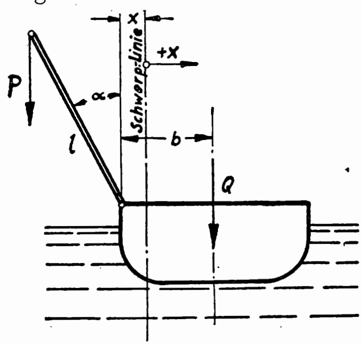
Der Schwerpunkt des Systems bleibt in horizontaler Ruhe.

$$1.\quad 4P\cdot s = 3P\,\frac{2}{3}\,a + P\left(a - \frac{2}{3}\,b\right)$$

$$2.\quad 4P\cdot s = 3P\left[l + \frac{2}{3}\,a\right] + P\left[l + \frac{b}{3}\right]$$

$$\underline{\underline{\frac{a-b}{4} = l}}$$

Lösung 953



$$P\,(l\sin\alpha + x) = Q\,(b - x)$$

$$x = \frac{Q\cdot b - P\,l\sin\alpha}{P+Q}$$

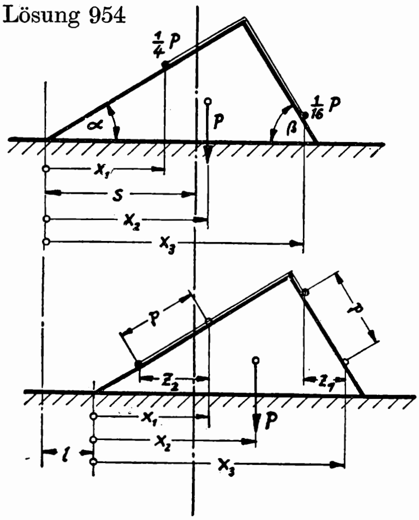
Der Schwerpunkt wandert von links nach rechts um:

$$|\Delta x| = \frac{Q\cdot b - P\,l\sin 30^\circ}{P+Q} - \frac{Q\cdot b}{P+Q}$$

$$|\Delta x| = \frac{P\cdot l\sin 30^\circ}{P+Q} = \underline{\underline{0{,}36\ \mathrm{m}}}$$

Das Schiff bewegt sich um den gleichen Betrag nach links.

Lösung 954



$$\frac{21}{16}\,P\cdot s = \frac{1}{4}\,P x_1 + P x_2 + \frac{1}{16}\,P x_3$$

$$\frac{21}{16}\,P\cdot s = \frac{1}{4}\,P\,[l + x_1 - z_2] + P\,[l + x_2]$$

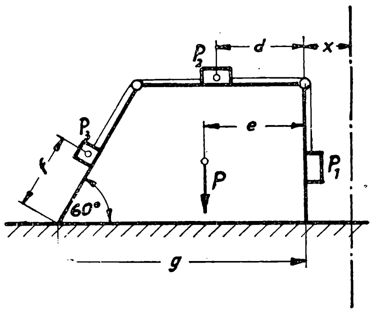
$$+ \frac{P}{16}\,[l + x_3 - z_1]$$

$$z_2 = \frac{h}{\operatorname{tg}\alpha}; \qquad z_1 = \frac{h}{\sin\alpha}\cdot\cos\beta$$

Beide Gleichungen gleichgesetzt ergibt:

$$\underline{\underline{l = 3{,}77\ \mathrm{cm}}}$$

Lösung 955



Vor der Verschiebung:

$$P_3(g+x-f\cos 60°)+P_2(d+x)+P(e+x)+P_1x=0$$

Nach der Verschiebung:

$$P_3(g+x_1-(f+1)\cos 60°)+P_2(d-1+x_1)+P(e+x_1)+P_1x_1=0$$

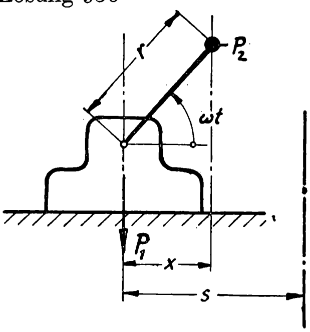
Daraus:

$$(x-x_1)(P_3+P_2+P+P_1)=-(P_3\cos 60°+P_2)$$

$$x-x_1=-\frac{P_2+P_3\cdot\frac{1}{2}}{P+P_1+P_2+P_3}$$

$$\underline{\underline{\Delta x=13{,}8\text{ cm nach links}}}$$

Lösung 956



Schwerpunktsatz:

$$P_2(s-x)+P_1\cdot s=0;\qquad x=r\sin\omega t$$

$$s=\frac{P_2}{P_1+P_2}\cdot r\sin\omega t$$

$$\omega=\pi\frac{n}{30}=\pi\cdot\frac{240}{30}=8\pi\ 1/\text{sek}$$

$$\underline{\underline{s=3\sin 8\pi t}}$$

Lösung 957

$mv=mv_s+m_kv_k;$ Kolbenbewegung: $s=\frac{s_0}{2}\sin\omega t$

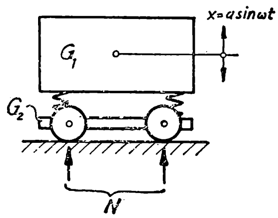
Kolbengeschwindigkeit $\dot{s}=v_k=\omega\cdot\frac{s_0}{2}\cos\omega t$

$$v=v_s+\frac{m_k}{m}\omega\cdot\frac{s_0}{z}\cos\omega t;\qquad \omega=\frac{\pi n}{30}=4\pi$$

$$\underline{\underline{v=10+0{,}00314\cos 4\pi t}}$$

Das Vorzeichen ist von den Anfangsbedingungen abhängig.

Lösung 958



$$N=(G_1+G_2)\pm\frac{G_1}{g}\ddot{x}$$

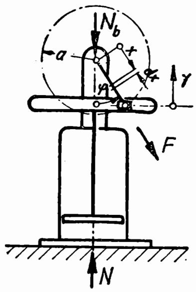
$$x=a\sin\omega t;\quad T=\frac{2\pi}{\omega}$$

$$\ddot{x}=-a\omega^2\sin\omega t;\quad \omega=\frac{2\pi}{T}=4\pi$$

$$\frac{G_1}{g}\ddot{x}_{\max}=\frac{10\cdot 2{,}5\cdot 16\cdot\pi^2}{981}=4{,}0\text{ t}$$

$$N_1=11-4=\underline{\underline{7\text{ t}}};\quad N_2=11+4=\underline{\underline{15\text{ t}}}$$

Lösung 959



$$y = a\cos\varphi = a\cos\omega t$$

$$\ddot{y} = -\omega^2 a\cos\omega t$$

$$N_b = F\cos\omega t - m_3\ddot{y}$$

$$N_b = \omega^2\cdot\cos\omega t\int_0^a \varrho f x\,dx + m_3\omega^2 a\cos\omega t$$

$$N = P_1 + P_2 + P_3 + N_b$$

$$\underline{\underline{N = P_1 + P_2 + P_3 + \frac{a\omega^2}{2g}(2P_3 + P_2)\cos\omega t}}$$

Lösung 960

$$T = m\ddot{x}_0 - (G + mg); \qquad mg = 981\ \mathrm{kg}$$

$$N = m\ddot{x}_u + (G + mg); \qquad G = 10000\ \mathrm{kg}$$

$$r = 30\ \mathrm{cm}$$

$$\lambda = \frac{1}{6}\ \text{(Schubstangenverhältnis)}$$

$$x = r\left[(1-\cos\omega t) + \frac{1}{2}\lambda\sin^2\omega t\right]; \qquad \ddot{x} = r\omega^2[\cos\omega t + \lambda\cos 2\omega t]$$

$\ddot{x}_{\max}$ herrscht bei $\dddot{x} = 0$: $\quad -\sin\omega t - 2\lambda\sin 2\omega t = 0$

$$\omega t = 0;\quad \pi$$

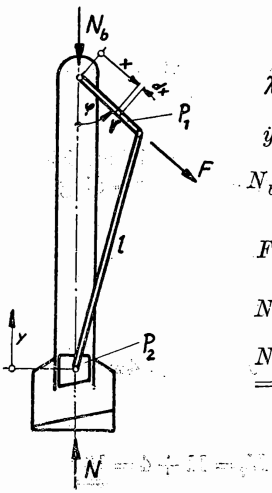
$$\ddot{x}_{\max} = r\omega^2[\pm 1 + \lambda] \qquad \ddot{x}_0 = 30\,(10\pi)^2\cdot\frac{7}{6} = 35\cdot 100\cdot\pi^2$$

$$\ddot{x}_u = 30\,(10\pi)^2\cdot\frac{5}{6} = 25\cdot 100\cdot\pi^2$$

$$T = 3500\pi^2 - 10981 = \underline{\underline{23.62\ \mathrm{t}}}$$

$$N = 2500\pi^2 + 10981 = \underline{\underline{35{,}68\ \mathrm{t}}}$$

Lösung 961



$$\lambda = \frac{r}{l}$$

$$\ddot{y} = r\omega^2(\cos\varphi + \lambda\cos 2\varphi) \quad \text{[vgl. 960]}$$

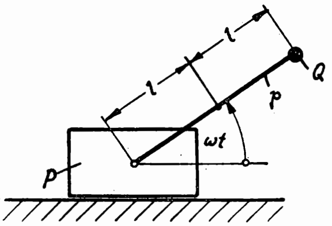
$$N_b = \frac{P_2}{g}\ddot{y} + F\cos\varphi$$

$$F = \omega^2\int_0^r \varrho\cdot f\cdot x\,dx$$

$$N = P_1 + P_2 + P_3 + N_b$$

$$\underline{\underline{N = P_1 + P_2 + P_3 + \frac{r\omega^2}{2g}\left[(P_1 + 2P_2)\cos\omega t + 2P_2\frac{r}{l}\cos 2\omega t\right]}}$$

Lösung 962



1. Horizontale Motorbewegung: x_p

$$\frac{p+Q+P}{g}\ddot{x}_P=\frac{p}{g}\ddot{x}_p+\frac{Q}{g}\ddot{x}_Q$$

$$x_Q=2l\cos\omega t$$

$$x_p=l\cos\omega t$$

$$\ddot{x}_Q=-2l\omega^2\cos\omega t$$

$$\ddot{x}_p=-l\omega^2\cos\omega t$$

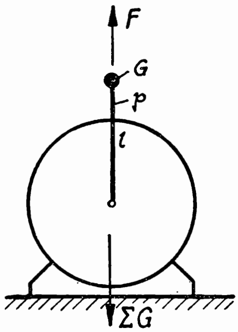
$$\underline{\underline{\ddot{x}_P=-\frac{p+2Q}{Q+P+p}\cdot l\omega^2\cos\omega t}}$$

$$\underline{\underline{x_P=\frac{p+2Q}{Q+P+p}\cdot l\cos\omega t}}$$

2. Horizontale Kraft

$$R_{\max}=\frac{p+Q+P}{g}\ddot{x}_P=\underline{\underline{\frac{(p+2Q)}{g}l\omega^2}}$$

Lösung 963

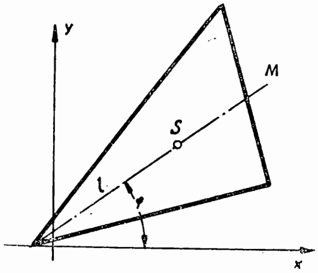


Bedingung: $\sum G-F\leqq 0$

$$P+p+Q\leqq\frac{Q}{g}\omega^2\cdot 2l+\omega^2\int_0^{2l}\varrho f x\,dx$$

$$\underline{\underline{\omega^2\geqq\frac{g}{l}\cdot\frac{P+p+Q}{p+2Q}}}$$

Lösung 964



Da das Dreieck reibungsfrei gleitet, bewegt sich der Schwerpunkt senkrecht nach unten.

$$x_s=\text{konst.}$$

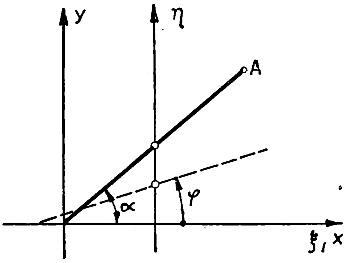
$$y=l\sin\varphi$$

$$x=x_s+\frac{l}{3}\cos\varphi$$

$$x-x_s=\frac{l}{3}\sqrt{1-\frac{y^2}{l^2}};\qquad l^2=90,\quad x_s=2$$

$$\underline{\underline{9(x-2)^2+y^2=90}}$$

Lösung 965



Der Schwerpunkt der Stange fällt senkrecht nach unten

$$\eta = 2l\sin\varphi$$
$$\xi = l\cos\varphi$$
$$\xi^2 + \frac{\eta^2}{4} = l^2$$

Aus der Anfangsbedingung $\varphi = \alpha$ folgt:

$$\xi = x - l\cos\alpha; \qquad \eta = y$$
$$\underline{\underline{(x - l\cos\alpha)^2 + \frac{y^2}{4} = l^2}}$$

38. Impulssatz

Die Bewegungsgröße ist das Produkt $\mathfrak{B} = m\mathfrak{v}$ aus Masse m und Geschwindigkeit $\mathfrak{v}$. Ist $\mathfrak{P}$ die wirkende Kraft, so gilt:

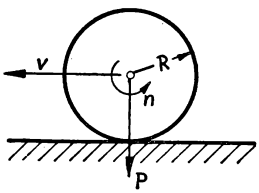
$$\mathfrak{P}\,dt = m\,d\mathfrak{v};$$
$$\int \mathfrak{P}\,dt = m\mathfrak{v}_2 - m\mathfrak{v}_1 = \mathfrak{B}_2 - \mathfrak{B}_1;$$

$\int \mathfrak{P}\,dt$ heißt Antrieb oder Impuls. Die Zunahme der Bewegungsgröße ist gleich dem Antrieb der Kraft.

Lösung 966

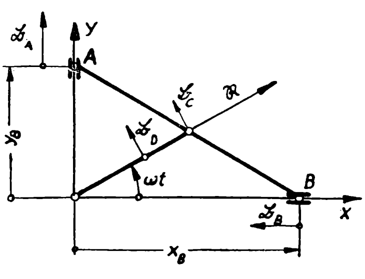
Da die Geschwindigkeit des Schwerpunktes Null ist, ist auch die Bewegungsgröße Null.

Lösung 967



$$v = \frac{2R\cdot\pi\cdot n}{60}$$
$$B = \frac{P}{g}\cdot v = \frac{P\cdot R\cdot\pi\cdot n}{g\cdot 30} = \underline{\underline{10{,}2\,\pi\,\text{kg sek}}}$$

Lösung 968



$$x_B = 2l\cos\omega t; \qquad y_B = 0$$
$$x_A = 0; \qquad y_A = 2l\sin\omega t$$
$$x_C = l\cos\omega t; \qquad y_C = l\sin\omega t$$
$$x_D = \frac{l}{2}\cos\omega t; \qquad y_D = \frac{l}{2}\sin\omega t$$
$$\mathfrak{B} = \mathfrak{B}_A + \mathfrak{B}_B + \mathfrak{B}_C + \mathfrak{B}_D$$
$$\mathfrak{B} = \mathfrak{i}\left[-m_B 2l\omega\sin\omega t - m_C l\omega\sin\omega t - \frac{l}{2}\,m_D\,\omega\sin\omega t\right]$$
$$+\,\mathfrak{j}\left[m_A\cdot 2l\omega\cos\omega t + m_C l\omega\cos\omega t + m_D\cdot\frac{l}{2}\cdot\omega\cos\omega t\right]$$

$$B_x = -\frac{\omega l}{g}\sin\omega t\left(2P_2 + 2P_1 + \frac{P_1}{2}\right)$$

$$B_y = \frac{\omega l}{g}\cos\omega t\left(2P_2 + 2P_1 + \frac{P_1}{2}\right)$$

$$\mathfrak{B} = \frac{4P_2 + 5P_1}{2g}\cdot\omega l(-\sin\omega t\,\mathfrak{i} + \cos\omega t\,\mathfrak{j})$$

Der Vektor der Bewegungsgröße steht also senkrecht zur Kurbel

Lösung 969

Die Koordinaten der jeweiligen Bewegungsbahn sind:

Schwerpunkt des Rades: $x_R = -r\sin\omega t$

$y_R = -r\cos\omega t$

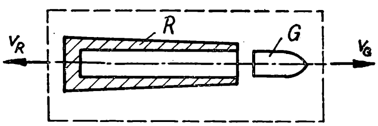
Schwerpunkt der Stange: $x_S = 0$

$y_S = -2r\cos\omega t$

Bewegungsgröße $B_x = -\frac{p}{g}r\,\omega\cos\omega t$

$$B_y = m_R\cdot\dot{y}_R + m_S\cdot\dot{y}_S = \frac{p}{g}r\,\omega(1+2k)\sin\omega t$$

Lösung 970

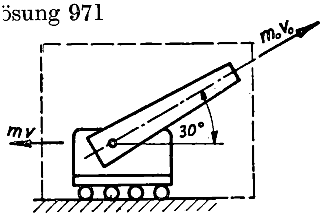


Nach dem Impulssatz gilt:

$$m_G\cdot v_G = m_R\cdot v_R$$

$$v_R = \frac{m_G}{m_R}\cdot v_G = \frac{54}{11\,000}\cdot 900 = 4{,}42\ \text{m/sek}$$

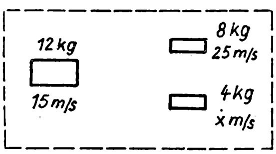
Lösung 971



$$m_R\cdot v_R = m_0 v_0\cdot\cos 30°$$

$$v_R = \frac{m_0}{m_R}\cdot v_0\cdot\cos 30° = 3{,}82\ \text{m/sek}$$

Lösung 972



$$mv = m_1 v_1 + m_2 v_2;$$
$$Gv = G_1 v_1 + G_2 v_2;$$
$$v_2 = \frac{G\cdot v - G_1 v_1}{G_2}$$
$$v_2 = -5{,}05\ \text{m/sek}$$

$v = 15$ m/sek

$G = 12$ kg

$v_1 = 25$ m/sek

$G_1 = 8$ kg

$G_2 = G - G_1$

$= 4$ kg

Lösung 973

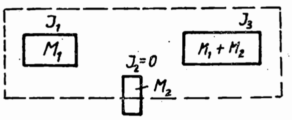
$$v_D \cdot m_D = v(m_D + m_K)$$

$$v = \frac{m_D}{m_D + m_K} \cdot v_D = \frac{600}{600 + 400} \cdot 1{,}5 = \underline{\underline{0{,}9 \text{ m/sek}}}$$

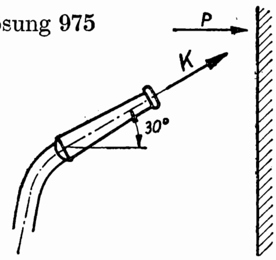
Lösung 974

$$m_W \cdot v_W = (m_M + m_W)v; \qquad v = \frac{m_W}{m_M + m_W} \cdot v_W = \frac{240}{240 + 50} \cdot 3{,}6$$

$$\underline{\underline{v = 2{,}98 \text{ km/h.}}}$$



Lösung 975



$K = m\dot{v}$; oder bei konst. Geschwindigkeit und veränderlicher Masse:

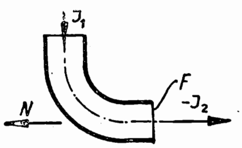
$$K = \dot{m} \cdot v$$

$$K = F \cdot \varrho \cdot v \cdot v$$

$$P = K \cos 30°$$

$$P = F \cdot \frac{\gamma}{g} v^2 \cdot \cos 30°; \quad \underline{\underline{P = 9{,}05 \text{ kg}}}$$

Lösung 976



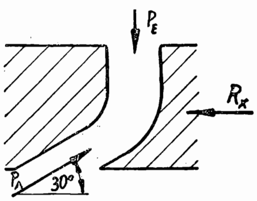
Entsprechend Aufgabe 975 gilt:

$$N \cdot t = mv; \quad N = \frac{m}{t} \cdot v$$

$$N = \frac{\gamma}{g} \cdot F v^2 = \frac{1 \cdot 1000 \cdot 0{,}3^2 \pi}{9{,}81 \quad 4} \cdot 2^2$$

$$\underline{\underline{N = 28{,}9 \text{ kg}}}$$

Lösung 977



Kraft des eintretenden Impulses:

$$P_E = \dot{m} v = F_0 \cdot \varrho_0 \cdot v_0^2$$

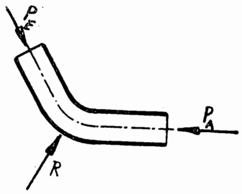
Kraft des austretenden Impulses:

$$P_A = F_0 \varrho_0 \cdot v_0 \cdot v_1 = 16{,}3 \text{ kg}$$

Reaktionskomponente in x-Richtung:

$$R_X = P_A \cdot \cos 30° = \underline{\underline{14{,}1 \text{ kg}}}$$

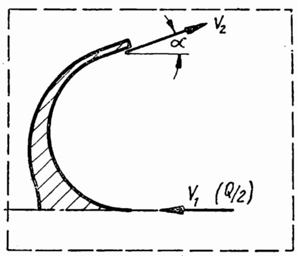
Lösung 978



$$R = P_E = P_A = \frac{\gamma}{g} \cdot F \cdot v^2$$

$$R = \frac{1000}{9{,}81} \cdot \frac{\pi}{4} \cdot 0{,}04 \cdot 16 = \underline{\underline{51{,}2\ \mathrm{kg}}}$$

Lösung 979



Kraft des eintretenden Impulses:

$$P_{XE} = \frac{Q}{2} \cdot \frac{\gamma}{g} \cdot v_1$$

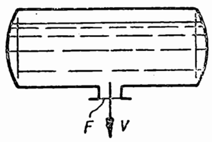
Kraft des austretenden Impulses:

$$P_{XA} = \frac{Q}{2}\ \frac{\gamma}{g} v_2 \cos\alpha$$

$$\sum P: \qquad 2(P_{XE} + P_{XA}) = N$$

$$\underline{\underline{N = \frac{Q}{g} \cdot \gamma (v_1 + v_2 \cos\alpha)}}$$

Lösung 980



Der Lagerdruck beträgt:

$$L = G - \frac{\gamma}{g} F \cdot v^2$$

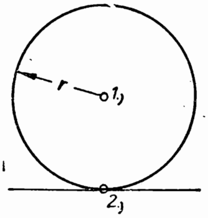
$$G = 10{,}35 + 15 = 25{,}35\ \mathrm{t}$$

$$\frac{\gamma}{g} F \cdot v^2 = \frac{\gamma}{g} F \left(H + \frac{p_0}{\gamma}\right) \cdot 2g = 25{,}35\ \mathrm{t}$$

Somit: $\underline{\underline{L = 0}}$

39. Drehimpulssatz; Physikalisches Pendel; Elementare Kreiseltheorie

Lösung 981



Der Drall ist das Moment der Bewegungsgröße:

$$dD = dm \cdot r \cdot v = r^2 dm \cdot \omega$$

$$\underline{\underline{D = \Theta \cdot \omega}}$$

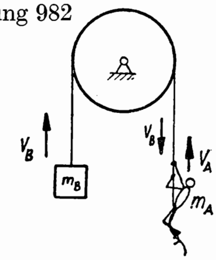
$$\Theta_1 = \frac{mr^2}{2}; \qquad \Theta_2 = \Theta_1 + mr^2 = \frac{3}{2} mr^2$$

$$\omega = \frac{2\pi \cdot 60}{60}$$

$$D_1 = mr^2\pi = \underline{\underline{1{,}44\ \mathrm{mkg\ sek}}}$$

$$D_2 = 3mr^2\pi = \underline{\underline{4{,}32\ \mathrm{mkg\ sek}}}$$

Lösung 982

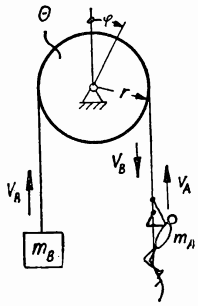


$$m_B v_B = m_A (v_A - v_B)$$

$$v_B = \frac{m_A \cdot v_A}{m_B + m_A}; \quad m_A = m_B$$

$$\underline{\underline{v_B = \frac{v_A}{2}}}$$

Lösung 983



$$\Theta = \frac{m_A}{4} r^2; \quad \dot{\varphi} \cdot r = v_B$$

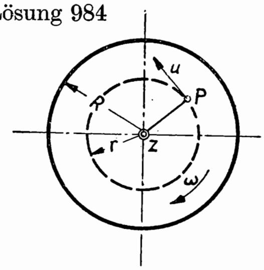
$$m_B \cdot v_B \cdot r + \Theta \dot{\varphi} = m_A \cdot r (v_A - v_B)$$

$$m_A = m_B = m$$

$$mr \left(v_B + v_B + \frac{1}{4} v_B \right) = mr v_A$$

$$\underline{\underline{v_B = \frac{4}{9} v_A}}$$

Lösung 984



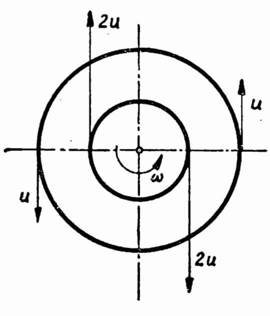
$$\frac{p}{g} (u - \omega r) r = \Theta \omega$$

$$\Theta = \frac{m R^2}{2}$$

$$\frac{p u r}{g} = \omega \left(\frac{p}{g} r^2 + \frac{P R^2}{2g} \right)$$

$$\underline{\underline{\omega = \frac{2pr}{2pr^2 + PR^2} \cdot u}}$$

Lösung 985

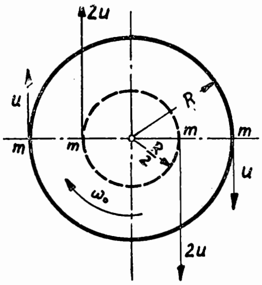


$$D = 2mR(u + \omega_0 R) + 2m \frac{R}{2} \left(\omega_0 \cdot \frac{R}{2} - 2u \right)$$

$$D = \frac{10}{4} m \omega_0 R^2$$

Die Relativbewegung der vier Männer erzeugt also keinen zusätzlichen Drall. Die Scheibe dreht sich mit derselben Winkelgeschwindigkeit ω_0 weiter.

Lösung 986



$$\Theta_{\text{ges}}\,\omega_0 = \Theta_S \cdot \omega_1 + 2mR(u + \omega_1 R) + 2m\frac{R}{2}\left(2u + \omega_1 \frac{R}{2}\right)$$

$$O_{\text{ges}} = \frac{4mR^2}{2} + 2mR^2 + 2m\left(\frac{R}{2}\right)^2 = \frac{9}{2}\,mR^2$$

$$\Theta_S = \frac{4mR^2}{2}$$

$$\frac{9}{2}\,mR^2\omega_0 = \frac{9}{2}\,mR^2\omega_1 + 4mRu$$

$$\underline{\underline{\omega_1 = \omega_0 - \frac{8}{9}\,\frac{u}{R}}}$$

$$\underline{\underline{\omega_1 = 0:}} \quad \underline{\underline{u = \frac{9}{8}\,R\omega_0}}$$

Lösung 987

Da kein Drall nach außen abgeleitet wird, gilt:

$$\Theta_1\dot{\varphi}_1 = \Theta_2\dot{\varphi}_2$$

$$\Theta_1 n_1 = \Theta_2 \cdot n_2; \qquad n_2 = \frac{\Theta_1}{\Theta_2}\,n_1 = \frac{0{,}8}{0{,}12}\cdot 15 = \underline{\underline{100\ \text{U/min}}}$$

Lösung 988

$$\Theta_1\omega_1 + \Theta_2\omega_2 = (\Theta_1 + \Theta_2)\,\omega$$

$$\underline{\underline{\omega = \frac{\Theta_1\omega_1 + \Theta_2\omega_2}{\Theta_1 + \Theta_2}}}$$

Lösung 989

$$\Theta_{\text{Stange}} = \frac{m_s \cdot (2L)^2}{3} = \frac{QL^2}{12g};$$

$$2\,\Theta_{\text{Kugel}} = 2m_k \cdot (2l)^2 = \frac{2Pl^2}{4g};$$

$$\left(\frac{QL^2}{3g} + \frac{2Pl_1^2}{g}\right)n_1 = \left(\frac{QL^2}{3g} + \frac{2Pl_2^2}{g}\right)n_2; \qquad n_2 = n_1\,\frac{QL^2 + 6Pl_1^2}{QL^2 + 6Pl_2^2} = \underline{\underline{34\ \text{U/min}}}$$

Lösung 990

$$r(T_1 - T_2) = \Theta \cdot \varepsilon + M_R; \qquad M_R = r(T_1 - T_2) - \frac{P}{g}\cdot\frac{r^2}{2}\cdot\varepsilon$$

$$M_R = 0{,}2\,(20 - 10) - \frac{3{,}27 \cdot 0{,}2^2 \cdot 1{,}5}{9{,}81 \cdot 2} = \underline{\underline{1\ \text{mkg}}}$$

Lösung 991

$$\Theta\omega_0 = Mt; \qquad \Theta = \frac{G}{g}\,i^2; \qquad M = \frac{G \cdot i^2\omega_0}{t \cdot g} = \frac{500 \cdot 2{,}25 \cdot \pi \cdot 240}{10 \cdot 60 \cdot 30 \cdot 9{,}81}$$

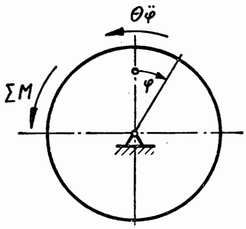
$$\underline{\underline{M = 4{,}8\ \text{mkg}}}$$

Lösung 992

$$\Theta\omega = P_r \cdot r \cdot t; \qquad P_r = \frac{G \cdot r \cdot \pi \cdot n}{g \cdot t \cdot 2 \cdot 30} = \frac{1 \cdot 10 \cdot \pi \cdot 100}{981 \cdot 1 \cdot 60 \cdot 2 \cdot 30 \cdot 2}$$

$$\underline{\underline{P_r = 0{,}44\ \mathrm{g}}}$$

Lösung 993



$$\Theta\ddot{\varphi} + k \cdot v + M_2 = 0; \qquad \dot{\varphi} = \frac{v}{D}$$

$$\Theta\frac{\dot{v}}{R} + kv + M_2 = 0$$

$$\frac{\Theta}{R} \cdot \frac{dv}{kv + M_2} + dt = 0$$

$$t = -\frac{\Theta}{kR} \ln(kv + M_2) + C$$

Anfangsbedingungen: $t = 0$; $v = \omega_0 R$: $\quad C = \frac{\Theta}{kR} \ln(\omega_0 kR + M_2)$

$$t = \frac{\Theta}{kR} \ln \frac{\omega_0 kR + M_2}{kv + M_2}$$

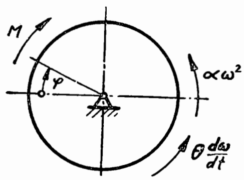
Nach $t = T$ ist $v = 0$:

$$\underline{\underline{T = \frac{2\Theta}{kD} \ln\left(1 + \frac{\omega_0 kD}{2M_2}\right)\ \mathrm{sek}}}$$

Lösung 994

$$M = \alpha\omega^2 + \Theta\frac{d\omega}{dt}; \qquad \int_0^t dt = \Theta \int_0^\omega \frac{d\omega}{M - \alpha\omega^2}$$

$$t = \Theta \frac{1}{2\sqrt{M\alpha}} \ln \frac{\sqrt{M\alpha} + \alpha\omega}{\sqrt{M\alpha} - \alpha\omega}; \qquad e^{\frac{2\sqrt{M\alpha}}{\Theta} \cdot t} = \frac{\sqrt{M\alpha} + \alpha\omega}{\sqrt{M\alpha} - \alpha\omega}; \qquad \beta = \frac{2\sqrt{M\alpha}}{\Theta}$$



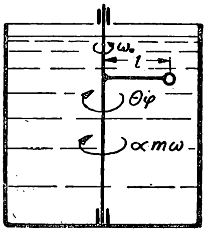
$$\underline{\underline{\omega = \sqrt{\frac{M}{\alpha}} \cdot \frac{e^{\beta t} - 1}{e^{\beta t} + 1}\ \frac{1}{\mathrm{sek}}}}$$

Lösung 995

$$M = \alpha\omega + \Theta\frac{d\omega}{dt}; \qquad \int_0^t dt = \Theta \int_0^\omega \frac{d\omega}{M - \alpha\omega}$$

$$t = -\frac{\Theta}{\alpha} \ln \frac{M - \alpha\omega}{M}; \qquad e^{-\frac{\alpha t}{\Theta}} = \left(1 - \frac{\alpha\omega}{M}\right); \qquad \underline{\underline{\omega = \frac{M}{\alpha}\left(1 - e^{-\frac{\alpha t}{\Theta}}\right)\frac{1}{\mathrm{sek}}}}$$

Lösung 993



$$\Theta\ddot{\varphi} + \alpha m l \dot{\varphi} = 0$$

$$l\ddot{\varphi} + \alpha\dot{\varphi} = 0; \quad \ddot{\varphi} = \frac{d(\dot{\varphi})}{d\varphi} \cdot \dot{\varphi}$$

$$\frac{d(\dot{\varphi})}{d\varphi} + \frac{\alpha}{l} = 0$$

$$\dot{\varphi} + \frac{\alpha}{l}\varphi + C = 0$$

Anfangsbedingung: $\varphi = 0; \quad \dot{\varphi} = \omega_0$

$$\omega_0 + C = 0$$

$$(\dot{\varphi} - \omega_0) + \frac{\alpha}{l}\varphi = 0$$

Bis zum Erreichen von $\dot{\varphi} = \frac{\omega_0}{2}$ werden n Umdrehungen gemacht.

$$\frac{\omega_0}{2} = \frac{\alpha}{l}\varphi; \quad n = \frac{\varphi}{2\pi} = \underline{\underline{\frac{l\omega_0}{4\pi\alpha}}} \text{ Umdrehungen}$$

Die dafür benötigte Zeit beträgt:

$$\frac{d\varphi}{dt} + \frac{\alpha}{l}\varphi - \omega_0 = 0; \quad \frac{d\varphi}{\omega_0 - \frac{\alpha}{l}\varphi} = dt; \quad t + C_0 = -\frac{l}{\alpha}\ln\left(\omega_0 - \frac{\alpha}{l}\varphi\right)$$

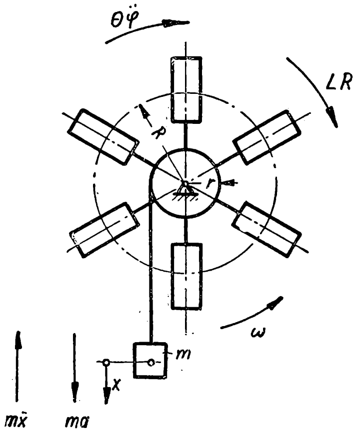
Anfangsbedingung: $t = 0; \quad \varphi = 0: \quad C_0 + t = -\frac{l}{\alpha}\ln\omega_0$

$$t = \frac{l}{\alpha}\left[\ln\omega_0 - \ln\left(\omega_0 - \frac{\alpha}{l}\varphi\right)\right]$$

$$t_{\varphi = \frac{l\omega_0}{2\alpha}} = T = \frac{l}{\alpha}\ln\frac{\omega_0}{\omega_0 - \frac{\alpha}{l}\cdot\frac{l\omega_0}{2\alpha}}$$

$$\underline{\underline{T = \frac{l}{\alpha}\ln 2 \text{ sek}}}$$

Lösung 997



$$L = nk\omega^2; \qquad \ddot{x} = r\ddot{\varphi}$$

$$\Theta\ddot{\varphi} + LR + m\ddot{x}r - mgr = 0$$

$$\ddot{\varphi} + \frac{nkR}{\Theta + mr^2}\dot{\varphi}^2 - \frac{mgr}{\Theta + mr^2} = 0$$

$$(\Theta + mr^2)\cdot\frac{d\dot{\varphi}}{mgr - nkR\dot{\varphi}^2} = dt$$

$$t = \frac{\Theta + mr^2}{2\sqrt{mgrnkR}}\left[\ln\frac{\sqrt{mgrnkR} + nkR\dot{\varphi}}{\sqrt{mgrnkR} - nkR\dot{\varphi}} + C\right]$$

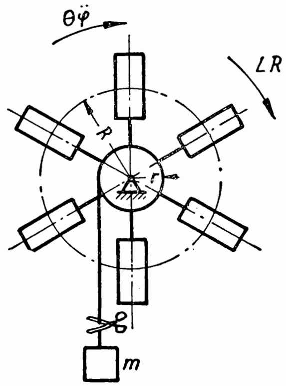
$$\frac{\Theta + mr^2}{2\sqrt{mgrnkR}} = \frac{1}{\alpha}; \quad \sqrt{mgrnkR} = \beta$$

Anfangsbedingungen: $t = 0; \quad \dot{\varphi} = 0: \quad C = 0$

$$e^{\alpha t}(\beta - nkR\omega) = \beta + nkR\omega$$

$$\omega = \frac{\beta(e^{\alpha t} - 1)}{nkR(e^{\alpha t} + 1)}; \quad \underline{\underline{\omega = \sqrt{\frac{mgr}{nkR}}\,\frac{e^{\alpha t} - 1}{e^{\alpha t} + 1}}}$$

Lösung 998



Nach dem Abschneiden des Gewichtes lautet die Differentialgleichung:

$$\Theta\ddot{\varphi} + knR\dot{\varphi}^2 = 0$$

$$\ddot{\varphi} + \frac{knR}{\Theta}\cdot\dot{\varphi}^2 = 0$$

$$\frac{\Theta}{knR}\cdot\frac{d\dot{\varphi}}{\dot{\varphi}^2} = -dt; \qquad t = \frac{\Theta}{knR}\cdot\frac{1}{\dot{\varphi}} + C$$

Anfangsbedingung: $t = 0$; $\dot{\varphi} = \omega_0$:

$$C = -\frac{\Theta}{knR}\cdot\frac{1}{\omega_0}$$

$$t = \frac{\Theta}{knR}\left[\frac{1}{\dot{\varphi}} - \frac{1}{\omega_0}\right]$$

$$d\varphi = \frac{dt}{\frac{1}{\omega_0} + \frac{knR}{\Theta}\cdot t}; \quad \varphi = \frac{\Theta}{knR}\left[\ln\left(\frac{1}{\omega_0} + \frac{knR}{\Theta}\cdot t\right) + C\right]$$

Anfangsbedingung: $t = 0$; $\varphi = 0$:

$$C_0 = -\ln\frac{1}{\omega_0}$$

$$\underline{\underline{\varphi = \frac{\Theta}{knR}\ln\left[1 + \frac{\omega_0 knR}{\Theta}\cdot t\right]}}$$

Lösung 999

Nach Aufgabe 997 gilt: $\dfrac{d\dot{\varphi}}{dt} + \dfrac{nkR}{\Theta + mr^2}\cdot\dot{\varphi} - \dfrac{mgr}{\Theta + mr^2} = 0$

$$(\Theta + mr^2)\cdot\frac{d\dot{\varphi}}{mgr - nkR\dot{\varphi}} = dt; \qquad \frac{nkR}{\Theta + mr^2} = \gamma; \qquad \frac{mgr}{nkR} = \sigma$$

$t = -\dfrac{1}{\gamma}\left[\ln(mgr - nkR\dot{\varphi}) + C\right]$; Anfangsbedingung: $t = 0$; $\dot{\varphi} = 0$:

$$C = -\ln mgr$$

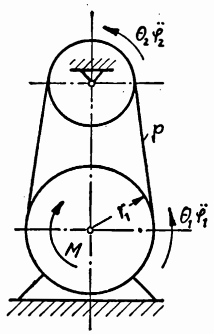
$$t = -\frac{1}{\gamma}\ln\left(1 - \frac{1}{\sigma}\cdot\dot{\varphi}\right)$$

$$e^{-\gamma t} = 1 - \frac{1}{\sigma}\dot{\varphi}; \qquad \dot{\varphi} = \sigma(1 - e^{-\gamma t}); \qquad \varphi = \sigma\left[t + \frac{1}{\gamma}e^{-\gamma t} + C_0\right]$$

Anfangsbedingung: $t = 0$; $\varphi = 0$: $C_0 = -\dfrac{1}{\gamma}$

$$\underline{\underline{\varphi = \sigma\left[t + \frac{1}{\gamma}(e^{-\gamma t} - 1)\right]}}$$

Lösung 1000

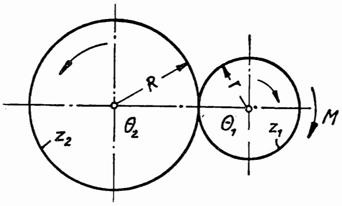


$$\Theta_1\ddot{\varphi}_1 + \Theta_2\ddot{\varphi}_2\cdot k + \frac{p}{g}\cdot r_1^2\ddot{\varphi}_1 = M$$

$$\ddot{\varphi}_2 = k\ddot{\varphi}_1$$

$$\underline{\underline{\ddot{\varphi}_1 = \varepsilon_1 = \frac{Mg}{(\Theta_1 + \Theta_2 k^2)\,g + pr_1^2}}}$$

Lösung 1001



$$M_1 = U \cdot r; \qquad M_2 = U \cdot R; \qquad \frac{M_2}{M_1} = \frac{R}{r} = k$$

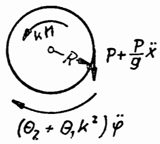
$$\ddot{x} = R\ddot{\varphi}$$

Gleichgewicht:

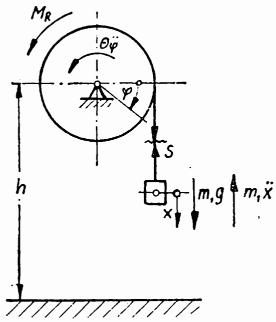
$$-\frac{P}{g}\ddot{x}R + kM - P \cdot R - (\Theta_2 + \Theta_1 k^2)\ddot{\varphi} = 0$$

$$\underline{\underline{\ddot{x} = b = g\,\frac{(kM - PR)\,R}{PR^2 + (\Theta_1 k^2 + \Theta_2)\,g}}}$$

Ersatzbild



Lösung 1002 Allgemein gilt:



$$\Theta\ddot{\varphi} + M_R = m(g - \ddot{x})R$$

$$\ddot{x}\left(mR + \frac{\Theta}{R}\right) = mgR - M_R$$

Für die verschiedenen Lasten gilt:

$$x_1 = \frac{m_1 g R - M_R}{m_1 R^2 + \Theta} \cdot R\,\frac{t_1^2}{2}$$

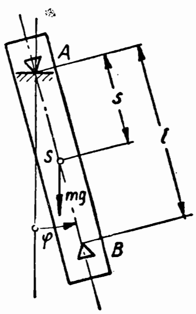
$$x_2 = \frac{m_2 g R - M_R}{m_2 R^2 + \Theta} \cdot R\,\frac{t_2^2}{2}$$

$$x_1 = x_2 = h: \quad t_1 = T_1; \quad t_2 = T_2$$

$$-M_R = \frac{2h}{RT_1^2}(m_1 R^2 + \Theta) - m_1 g R = \frac{2h}{RT_2^2}(m_2 R^2 + \Theta) - m_2 g R$$

$$\underline{\underline{\Theta = R^2 \cdot \frac{\frac{p_1 - p_2}{2h} - \frac{1}{g}\left(\frac{p_1}{T_1^2} - \frac{p_2}{T_2^2}\right)}{\frac{1}{T_1^2} - \frac{1}{T_2^2}}}} = \underline{\underline{1061 \text{ mkg sek}^2}}$$

Lösung 1003



Aufhängung in A:

$$\Theta_A\ddot{\varphi} + mgs\varphi = 0; \qquad T_A = 2\pi\sqrt{\frac{\Theta_A}{mgs}}$$

Aufhängung in B:

$$\Theta_B \cdot \ddot{\varphi} + mg\,(l - s)\varphi = 0; \quad T_B = 2\pi\sqrt{\frac{\Theta_B}{mg(l-s)}}$$

Es gilt: $T_A = T_B = T$

$$\Theta_A = \Theta_S + ms^2$$

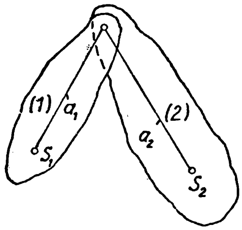
$$\Theta_B = \Theta_S + m(l - s)^2$$

Somit:

$$\Theta_S = mgs\left(\frac{T}{2\pi}\right)^2 - ms^2 = mg(l - s)\left(\frac{T}{2\pi}\right)^2 - m(l - s)^2$$

$$g = \frac{4\pi^2 l}{T^2}$$

Lösung 1004



Reduzierte Pendellängen:

$$l_1 = \frac{\Theta_1}{m_1 a_1}; \quad l_2 = \frac{\Theta_2}{m_2 a_2}$$

Gemeinsames Schwingen:

Gesamtschwerpunktsabstand:

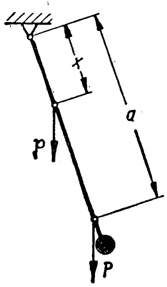
$$a = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$$

Reduzierte Pendellänge des Gesamtsystems:

$$l_z = \frac{\Theta_1 + \Theta_2}{(m_1 + m_2) \cdot a}$$

$$l_z = \frac{m_1 a_1 l_1 + m_2 a_2 l_2}{m_1 a_1 + m_2 a_2}; \quad \underline{\underline{l_z = \frac{p_1 a_1 l_1 + p_2 a_2 l_2}{p_1 a_1 + p_2 a_2}}}$$

Lösung 1005



$$l = \frac{\Theta}{m \cdot a}$$

$$\Theta_{\text{ges}} = \frac{P \cdot a \cdot l + p x^2}{g}$$

$$l_{\text{ges}} = \frac{P a l + p x^2}{(P \cdot a + p \cdot x)}$$

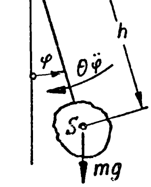
$$\Delta l = l_{\text{ges}} - l = \underline{\underline{\frac{p \cdot x (x - l)}{P \cdot a + p \cdot x}}}$$

$$p = \frac{P a \Delta l}{(\Delta l x + l x - x^2)}$$

$$\frac{dp}{dx} = 0: \quad \Delta l + l - 2 x_1 = 0$$

$$\underline{\underline{x_1 = \frac{1}{2} (l + \Delta l)}}$$

Lösung 1006



$$\Theta \ddot{\varphi} + m g h \varphi = 0$$

$$\Theta = \Theta_S + m h^2$$

$$\ddot{\varphi} + \frac{m g h}{\Theta_S + m h^2} \varphi = 0$$

Zeit einer halben Schwingung:

$$T = \pi \sqrt{\frac{\Theta_s + m h^2}{m g h}}$$

$$\underline{\underline{\Theta_S = p h \left[\frac{T^2}{\pi^2} - \frac{h}{g}\right]}}$$

Lösung 1007

$Q \cdot h = P \cdot l$; Schwerpunktsabstand: $h = \frac{P}{Q} \cdot l$

Schwingungsgleichung: $\Theta_0 \ddot{\varphi} + Q(r+h)\varphi = 0$

$$T = \pi \sqrt{\frac{\Theta_0}{Q(r+h)}}$$

$$\Theta_0 = \Theta_S + \frac{Q}{g}(r+h)^2; \quad \Theta_S = \frac{T^2}{\pi^2} Q(r+h) - \frac{Q}{g}(r+h)^2$$

mit $h = \frac{P}{Q} \cdot l$ wird: $\underline{\underline{\Theta_S = \frac{Pl + Qr}{g}\left\{\frac{gT^2}{\pi^2} - \frac{P}{Q} \cdot l - r\right\} = 1{,}77 \text{ kg m sek}^2}}$

Lösung 1008

Punkt C = Aufhängepunkt; $OC = h$

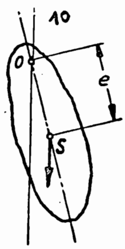
$$\Theta_0 \ddot{\varphi} + mgh \cdot \varphi = 0; \quad \Theta_0 = \frac{2}{5} mr^2 + mh^2; \quad T = \pi \sqrt{\frac{\frac{2}{5} mr^2 + mh^2}{mgh}}$$

$$\frac{T^2}{\pi^2} \cdot g \cdot h = \frac{2}{5} r^2 + h^2; \quad h^2 - \frac{T^2}{\pi^2} gh = -\frac{2}{5} r^2$$

$$h = \frac{1}{2\pi^2}\left[gT^2 + \sqrt{g^2T^4 - \frac{2}{5} r^2 4\pi^4}\right]$$

$$\underline{\underline{h = OC = \frac{1}{2\pi^2}\left[gT^2 + \sqrt{g^2T^4 - 1{,}6 r^2 \pi^4}\right]}}$$

Lösung 1009



Für das physikalische Pendel gilt:

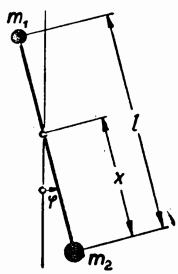
$$T = 2\pi \sqrt{\frac{mge}{\Theta_S + me^2}}; \quad T^2 = 4\pi^2 \frac{mge}{\Theta_S + me^2}$$

$$\frac{d(T^2)}{de} = 0: \quad \frac{(\Theta_S + me^2)mg - 2m^2e^2g}{(\Theta_S + me^2)^2} = 0$$

$$(\Theta_S + me^2)mg - 2m^2e^2g = 0$$

$\underline{\underline{e^2 = \frac{\Theta_S}{m}}}$. Dies ist der Trägheitsradius des Pendels.

Lösung 1010



$$[m_2x^2 + m_1(l-x)^2]\ddot{\varphi} + [m_2gx - m_1g(l-x)]\varphi = 0$$

$$\omega^2 = \frac{[(m_2+m_1)x - m_1 l] \cdot g}{x^2[m_2+m_1] - 2m_1 lx + m_1 l^2}$$

$T = 2\pi \cdot \frac{1}{\omega}$. Damit T ein Minimum wird, muß $\frac{dT}{dx} = 0$ bzw. $\frac{d(T^2)}{dx} = 0$ sein.

$$T^2 = \frac{4\pi^2}{g} \frac{(m_1+m_2)x^2 - 2m_1 lx + m_1 l^2}{(m_1+m_2)x - m_1 l}$$

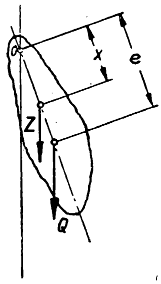
$$\frac{d(T^2)}{dx}=\frac{4\pi^2}{g}\left\{\frac{[2x(m_1+m_2)-2m_1l][(m_1+m_2)x-m_1l]-[(m_1+m_2)][(m_1+m_2)x^2-2m_1lx+m_1l^2]}{[(m_1+m_2)x-m_1l]^2}\right.$$

$$\frac{d(T^2)}{dx}=0:\quad [2x(m_1+m_2)-2m_1l][(m_1+m_2)x-m_1l]$$
$$-[m_1+m_2][(m_1+m_2)x^2-2m_1lx+m_1l^2]=0$$

$$\text{Daraus:}\quad x^2-\frac{2m_1l}{m_1+m_2}x+\frac{m_1l^2(m_1-m_2)}{(m_1+m_2)^2}=0$$

$$x=l\left[\frac{m_1(\pm)\sqrt{m_1m_2}}{m_1+m_2}\right];\qquad \underline{\underline{x=l\sqrt{m_1}\,\frac{\sqrt{m_1}+\sqrt{m_2}}{m_1+m_2}}}$$

Lösung 1011



$$\Theta_0\ddot{\varphi}+\frac{Z}{g}x^2\ddot{\varphi}+\varphi(Q\cdot e+Zx)=0$$

$$\omega^2=\frac{Q\cdot e+Z\cdot x}{\Theta_0+\dfrac{Z}{g}\cdot x^2}\quad \text{Kreisfrequenz mit Zusatzgewicht } Z$$

$$\omega_0^2=\frac{Q\cdot e}{\Theta_0}\quad \text{Kreisfrequenz ohne Zusatzgewicht } Z$$

$$\omega^2=\omega_0^2:\quad \frac{Q\cdot e}{\Theta_0}=\frac{Q\cdot e+Zx}{\Theta_0+\dfrac{Z}{g}x^2}$$

Daraus: $\underline{\underline{x=\dfrac{\Theta_0}{Q\cdot e}\cdot g}}$ Dies ist die reduzierte Pendellänge

Lösung 1012

Pendel mit Zusatzmasse:

$$\Theta\ddot{\varphi}+m\left(\frac{85}{72}l\right)^2\ddot{\varphi}+g\left(M\cdot\frac{l}{2}+m\frac{85}{72}l\right)\cdot\varphi=0$$

$$\Theta=\frac{1}{16}M\left(\frac{1}{9}l^2+\frac{16}{3}l^2\right)+\frac{Ml^2}{4}=\frac{85}{2\cdot 72}Ml^2$$

$$\ddot{\varphi}+\frac{\left(M\cdot\dfrac{l}{2}+m\dfrac{85}{72}\cdot l\right)\cdot g}{\dfrac{85}{72}l\left(\dfrac{M}{2}l+m\dfrac{85}{72}l\right)}\cdot\varphi=0$$

Schwingungsdauer: $T_m=2\pi\sqrt{\dfrac{85}{72}\,\dfrac{l}{g}}$

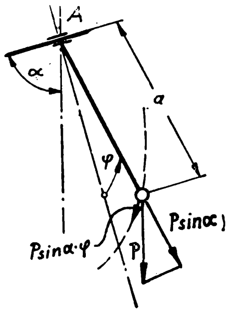
Pendel ohne Zusatzmasse: $\Theta\ddot{\varphi}+Mg\cdot\dfrac{l}{2}\varphi=0;\quad \Theta=\dfrac{85}{2\cdot 72}Ml^2$

$$\ddot{\varphi}+\frac{72\cdot g}{85\cdot l}\cdot\varphi=0$$

Schwingungsdauer: $T_0=2\pi\sqrt{\dfrac{85}{72}\cdot\dfrac{l}{g}}$

Die Schwingungsdauer ändert sich also nicht.

Lösung 1013

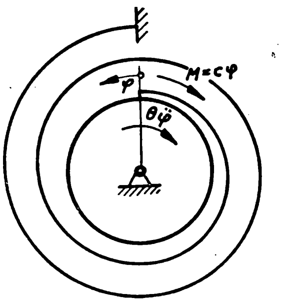


$$\Theta_A \ddot{\varphi} + P \sin\alpha \cdot a \cdot \varphi = 0$$

$$\Theta_A = \Theta_0 + \frac{P}{g} a^2$$

$$\underline{\underline{T = 2\pi \sqrt{\frac{\Theta_0 g + P a^2}{P g \sin\alpha \cdot a}}}}$$

Lösung 1014



$$\Theta \ddot{\varphi} + c \cdot \varphi = 0; \quad \omega = \sqrt{\frac{c}{\Theta}}$$

$$\Theta_{\text{Kugel}} = \frac{2}{5} m r^2$$

Ansatz: $\varphi = A \sin\omega t + B \cos\omega t$

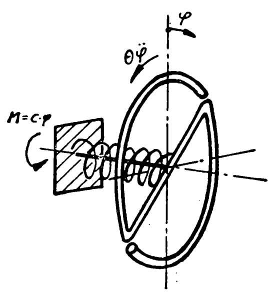
Anfangsbedingungen: $t = 0$: $\varphi = \varphi_0$

$\dot{\varphi} = 0$

Somit: $\varphi_0 = B$; $A = 0$

$$\underline{\underline{\varphi = \varphi_0 \cos\sqrt{\frac{5c}{2mr^2}} \cdot t}}$$

Lösung 1015



$$\Theta \ddot{\varphi} + c \cdot \varphi = 0$$

$$\ddot{\varphi} + \frac{c}{\Theta} \varphi = 0$$

Ansatz: $\varphi = A \sin\omega t + B \cos\omega t$

$$\omega = \sqrt{\frac{c}{\Theta}}$$

Anfangsbedingungen: $t = 0$: $\varphi = 0$

$\dot{\varphi} = \omega_0$

Damit: $B = 0$; $A = \frac{\omega_0}{\omega}$

$$\underline{\underline{\varphi = \omega_0 \sqrt{\frac{\Theta}{c}} \sin\sqrt{\frac{c}{\Theta}} \cdot t}}$$

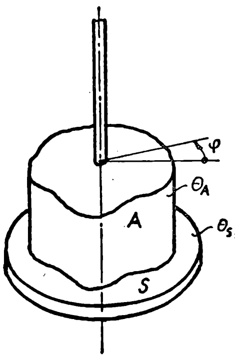
Lösung 1016

Schwingungszeit der Masse: $T_1 = 2\pi \sqrt{\frac{\Theta_z}{c}}$

Schwingungszeit der Scheibe: $T_2 = 2\pi \sqrt{\frac{P \cdot r^2}{g \cdot 2 \cdot c}}$

$\frac{T_1^2}{T_2^2} = \frac{\Theta_z \cdot 2g}{P r^2}$: $\quad \underline{\underline{\Theta_z = \frac{P r^2}{2g} \left(\frac{T_1}{T_2}\right)^2 = 0{,}117 \text{ kg cm sek}^2}}$

Lösung 1017



Schwingungszeit ohne Scheibe:

$$T_1 = 2\pi\sqrt{\frac{\Theta_z}{c}}$$

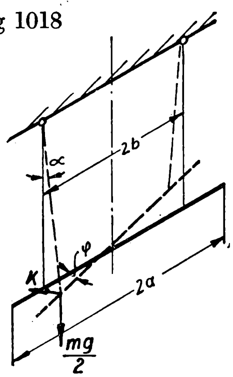
Schwingungszeit mit Scheibe:

$$T_2 = 2\pi\sqrt{\frac{\Theta_z + \Theta_s}{c}}$$

$$\frac{T_1^2}{T_2^2} = \frac{\Theta_z}{\Theta_z + \Theta_s}; \quad \Theta_s = \frac{P r^2}{2g}$$

$$\Theta_z = \frac{P r^2}{2g} \cdot \frac{T_1^2}{T_2^2 - T_1^2}$$

Lösung 1018



$$\Theta\ddot{\varphi} + M_k = 0; \quad M_k = \text{Rückstellmoment}$$

$$\alpha \cdot l = \varphi \cdot b; \quad \operatorname{tg}\alpha = \frac{2K}{mg} \approx \alpha$$

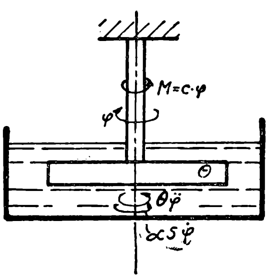
$$\varphi\frac{b}{l} = \frac{2K}{mg}; \quad K = \frac{mgb}{2l}\cdot\varphi \cdot \quad M_k = K \cdot 2b$$

$$\Theta\ddot{\varphi} + \frac{mgb^2}{l}\cdot\varphi = 0; \quad \Theta = m\frac{a^2}{3}$$

$$\ddot{\varphi} + \frac{3gb^2}{a^2 l}\cdot\varphi = 0$$

$$T = 2\pi\frac{a}{b}\sqrt{\frac{l}{3g}}$$

Lösung 1019



$$\Theta\ddot{\varphi} + \alpha S\dot{\varphi} + c\cdot\varphi = 0$$

$$\ddot{\varphi} + \frac{\alpha S}{\Theta}\dot{\varphi} + \frac{c}{\Theta}\varphi = 0$$

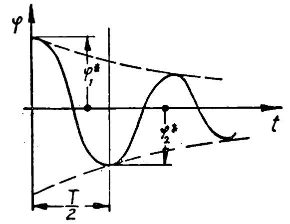
Lösung (vgl. Aufgabe 843):

$$\varphi = e^{-\frac{\alpha S}{2\Theta}t}\left(C_1\cos\sqrt{\frac{c}{\Theta} - \left(\frac{\alpha S}{2\Theta}\right)^2}\,t\right.$$

$$\left. + C_2\sin\sqrt{\frac{c}{\Theta} - \left(\frac{\alpha S}{2\Theta}\right)^2}\,t\right)$$

$$T = 2\pi\frac{2\Theta}{\sqrt{4\Theta c - \alpha^2 S^2}}$$

Lösung 1020

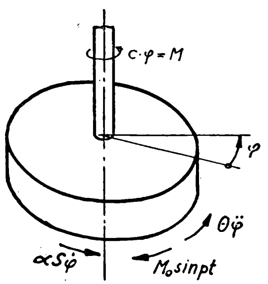


Aus der vorigen Aufgabe folgt:

$$\varphi_2^* = \varphi_1^* \cdot e^{-\frac{\alpha S}{2\Theta}\frac{T}{2}} = \varphi_1^* \cdot F$$

$$\underline{\underline{F = e^{-\frac{\pi\alpha S}{\sqrt{4\Theta c - \alpha^2 S^2}}}}}$$

Lösung 1021



$$\Theta\ddot{\varphi} + \alpha S\dot{\varphi} + c\varphi = M_0 \sin pt$$

Von der Lösung dieser Differentialgleichung interessiert hier nur das partikuläre Integral, da der homogene Lösungsanteil bei $t \to \infty$ verschwindet.

Ansatz: $\varphi_p = A \sin pt + B \cos pt$

Durch Einsetzen in die Differentialgleichung und Koeffizientenvergleich ergibt sich:

$$\begin{aligned} A(c - \Theta p^2) - \alpha S p B &= M_0; \\ A\alpha S p + B(c - \Theta p^2) &= 0; \end{aligned} \qquad A = \frac{\begin{vmatrix} M_0 & -\alpha S p \\ 0 & (c - \Theta p^2) \end{vmatrix}}{\begin{vmatrix} (c - \Theta p^2) & -\alpha S p \\ \alpha S p & (c - \Theta p^2) \end{vmatrix}}; \quad B = \frac{\begin{vmatrix} (c - \Theta p^2) & M_0 \\ \alpha S p & 0 \end{vmatrix}}{\begin{vmatrix} (c - \Theta p^2) & -\alpha S p \\ \alpha S p & (c - \Theta p^2) \end{vmatrix}}$$

$$\varphi_p = \frac{M_0(c - \Theta p^2)}{(c - \Theta p^2)^2 + \alpha^2 S^2 p^2} \sin pt + \frac{M_0 \alpha S p}{(c - \Theta p^2)^2 + \alpha^2 S^2 p^2} \cos pt$$

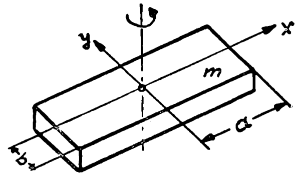
Amplitudenmaximum bei: $\dfrac{d\varphi_p}{dt} = 0$: $\operatorname{tg} pt = \dfrac{A}{B}$

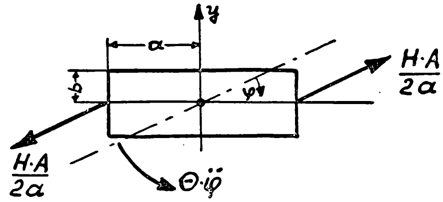
$$\varphi_0 = A \cdot \frac{A}{B\sqrt{1 + \left(\frac{A}{B}\right)^2}} + B \frac{1}{\sqrt{1 + \left(\frac{A}{B}\right)^2}} = \frac{A^2 + B^2}{\sqrt{A^2 + B^2}} = \sqrt{A^2 + B^2}; \quad \varphi_0 = \underline{\underline{\frac{M_0}{\sqrt{(c - \Theta p^2)^2 + \alpha^2 S^2 p^2}}}}$$

Die Eigenfrequenz des Systems ist bei der erzwungenen Schwingung:

$$p^2 = \omega^2 = \frac{c}{\Theta}; \qquad \text{somit:} \quad \underline{\underline{\alpha = \frac{M_0}{\varphi_0 S p}}}$$

Lösung 1022





$$\Theta \ddot{\varphi} + HA\,\varphi = 0;$$

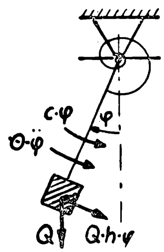
$$\ddot{\varphi} + \frac{HA}{\frac{m}{3}(a^2+b^2)} \cdot \varphi = 0;$$

$$\Theta = \Theta_x + \Theta_y = \frac{m}{4ab}\left(\frac{2a\,8b^3}{12} + \frac{8a^3\,2b}{12}\right)$$

$$\Theta = \frac{m}{3}(a^2+b^2)$$

$$\underline{\underline{T = 2\pi \sqrt{\frac{m(a^2+b^2)}{3AH}}}}$$

Lösung 1023

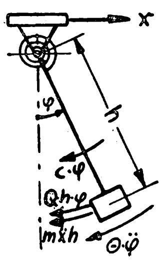


$$\Theta \ddot{\varphi} + \varphi(c + Q \cdot h) = 0$$

$$\ddot{\varphi} + \frac{(c + Q \cdot h)}{\Theta} \varphi = 0$$

$$\underline{\underline{T = 2\pi \sqrt{\frac{\Theta}{c + Q \cdot h}} = 0{,}5 \text{ sek}}}$$

Lösung 1024



$$\Theta \ddot{\varphi} + m\ddot{x}h + Qh\varphi + c\varphi = 0; \quad x = a \sin \omega t$$

$$\ddot{\varphi} + \left(\frac{c}{\Theta} + \frac{mgh}{\Theta}\right)\varphi = \frac{mha\omega^2}{\Theta} \sin \omega t$$

Das partikuläre Integral lautet:

$$\varphi_p = D \sin \omega t$$

$$-D\omega^2 + \left(\frac{c}{\Theta} + \frac{mgh}{\Theta}\right) D = \frac{mh\omega^2 \cdot a}{\Theta}$$

$$D = \varphi_{\max} = \frac{mh\omega^2 \cdot a}{\Theta\left[\frac{c}{\Theta} + \frac{mgh}{\Theta} - \omega^2\right]}$$

$$a = \frac{\varphi_{\max} \cdot [c + mgh - \omega^2 \Theta]}{mh\omega^2}$$

$$\varphi_{\max} = 6° \mathrel{\hat{=}} 0{,}1047$$

$$\omega = 60 \text{ 1/sek}$$

$$mh = \frac{4{,}5}{981} = 0{,}00459 \text{ kg sek}^4$$

$$\Theta = 0{,}03 \text{ kg cm sek}^2$$

$$c = 0{,}1 \text{ cm kg}$$

$$\underline{\underline{a = 6{,}5 \text{ mm}}}$$

Lösung 1025

Das logarithmische Dekrement ist:

$$\delta_i = \frac{\alpha_i \pi}{\sqrt{4c\Theta_i - \alpha_i^2}}; \qquad i = 1,\ 2 \tag{1}$$

Die Schwingungszeit einer halben Periode ist:

$$T_i = \frac{\pi}{\sqrt{\frac{c}{\Theta_i} - \frac{\alpha_i^2}{4\Theta_i^2}}} \qquad i = 1,\ 2 \tag{2}$$

Es bedeutet: $\Theta_1 = \Theta_0 + \Theta$; $\qquad \Theta_2 = \Theta$

$\Theta_0 = 2\Theta_{SK} + 2ma^2$; $\quad \Theta_{SK}$ = Trägheitsmoment der Kugel bezogen auf ihre Schwerachse

$\Theta_{SK} = \frac{2}{5} mr^2 \qquad \Theta$ = Trägheitsmoment des Rahmens

Aus Gl. (2):

$$T_1^2\left(\frac{c}{\Theta_1} - \frac{\alpha_1^2}{4\Theta_1^2}\right) = T_2^2\left(\frac{c}{\Theta_2} - \frac{\alpha_2^2}{4\Theta_2^2}\right) \tag{3}$$

Aus Gl. (3):

$$\delta_1^2(4c\Theta_1 - \alpha_1^2) = \alpha_1^2\pi^2; \qquad \alpha_1^2 = \frac{4\Theta_1 c\delta_1^2}{\pi^2 + \delta_1^2} \tag{4}$$

$$\alpha_2^2 = \frac{4\Theta_2 c\delta_2^2}{\pi^2 + \delta_2^2} \tag{5}$$

Gl. (4) und (5) in (3) eingesetzt:

$$\frac{T_1^2(\pi^2 + \delta_2^2)}{T_2^2(\pi^2 + \delta_1^2)} = \frac{\Theta_1}{\Theta_2} = \frac{\Theta_0 + \Theta}{\Theta}$$

$$\underline{\underline{\Theta = \frac{\Theta_0(\pi^2 + \delta_1^2)T_2^2}{(\pi^2 + \delta_2^2)T_1^2 - (\pi^2 + \delta_1^2)T_2^2}}} \tag{6}$$

Aus Gl. (2):

$$\left(\frac{c}{\Theta_1} - \frac{\alpha_1^2}{4\Theta_1^2}\right)T_1^2 = \pi^2; \qquad \left[\frac{c}{\Theta_1} - \frac{4\Theta_1 c\delta_1^2}{4\Theta_1^2(\pi^2 + \delta_1^2)}\right]T_1^2 = \pi^2$$

$$\left(c - \frac{c\delta_1^2}{\pi^2 + \delta_1^2}\right)\frac{T_1^2}{\Theta_1} = \pi^2; \qquad \underline{\underline{c = \frac{\Theta_1}{T_1^2}(\pi^2 + \delta_1^2)}} = \frac{\Theta}{T_2^2}(\pi^2 + \delta_2^2) \tag{7}$$

Gl. (7) in Gl. (4) und (5) eingesetzt:

$$\alpha_1^2 = \frac{4\Theta_1\delta_1^2\Theta_1(\pi^2 + \delta_1^2)}{(\pi^2 + \delta_1^2)T_1^2}$$

$$\alpha_1 = \frac{2\Theta_1\delta_1}{T_1} = \underline{\underline{\frac{2\delta_1}{T_1}(\Theta_0 + \Theta)}}$$

$$\alpha_2 = \frac{2\Theta\delta_2}{T_2}$$

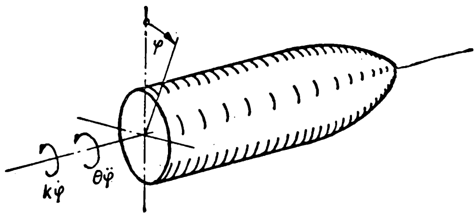
Zahlenwerte: $\Theta = 5{,}93 \cdot 10^{-6}$ cmkgsek²

$c = 2{,}92 \cdot 10^{-6}$ cmkg

$\alpha_1 = 0{,}85 \cdot 10^{-6}$ cmkgsek

$\alpha_2 = 0{,}79 \cdot 10^{-6}$ cmkgsek

Lösung 1026

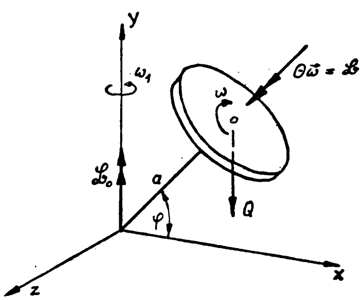


$$\Theta\ddot{\varphi}+k\dot{\varphi}=0$$
$$\ddot{\varphi}\dot{\varphi}+\frac{k}{\Theta}\dot{\varphi}^2=0$$
$$(\dot{\varphi}^2)^{\cdot}+\frac{2k}{\Theta}\dot{\varphi}^2=0$$
$$\ln(C\dot{\varphi})^2+\frac{2k}{\Theta}t=0;\quad C\dot{\varphi}=e^{-\frac{k}{\Theta}t}$$

Anfangsbedingung: $t=0;\quad \dot{\varphi}=\omega_0$:

$$C=\frac{1}{\omega_0}$$
$$\underline{\underline{\dot{\varphi}=\omega=\omega_0\, e^{-\frac{k}{\Theta}t}}}$$

Lösung 1027



Kreiselmoment: $\mathfrak{M}_k=\Theta\vec{\omega}\times\vec{\omega}_1$

$$\mathfrak{M}_k=\Theta\omega(-\mathfrak{i}\cos\varphi-\mathfrak{j}\sin\varphi)\times\omega_1\mathfrak{j}$$
$$\mathfrak{M}_k=\Theta\omega\omega_1\begin{vmatrix}\mathfrak{i} & \mathfrak{j} & \mathfrak{k}\\ -\cos\varphi & -\sin\varphi & 0\\ 0 & 1 & 0\end{vmatrix}=\Theta\omega\omega_1\cos\varphi\,\mathfrak{k}$$

Äußeres Moment: $\mathfrak{M}_A=-Q\cdot a\cos\varphi\,\mathfrak{k}$

$$\mathfrak{M}_A+\mathfrak{M}_k=0:\quad Qa=-\Theta\omega\omega_1$$
$$\omega_1=-\frac{Q\cdot a}{\Theta\cdot\omega}=-\frac{Q\cdot a}{\frac{Q}{g}i^2\cdot\omega}=-\frac{ga}{\omega i^2}=\underline{\underline{-0{,}49\,\frac{1}{\text{sek}}}}$$

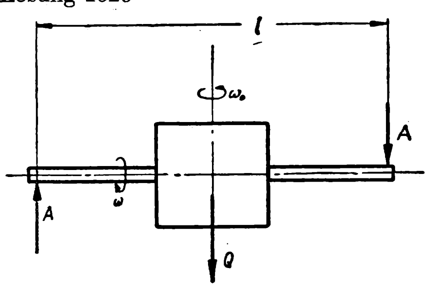
Das Vorzeichen (—) besagt, daß sich der Kreisel entgegen der angenommenen Richtung bewegt.

Lösung 1028

Nach Aufgabe 1027 gilt: $\omega_1=\dfrac{Q\cdot l}{\Theta\cdot\omega}=\dfrac{m\cdot g\cdot l\cdot 2}{\omega m r^2}=\dfrac{2gl}{\omega r^2}$

$$\underline{\underline{\omega_1=2{,}18\ 1/\text{sek}}}$$

Lösung 1029



ω_0 = Winkelgeschwindigkeit der Schiffsschwingung.

Da $\omega_0\perp\omega$ gilt:

$$M=\Theta\cdot\omega_0\cdot\omega$$
$$\omega=\frac{\pi n}{30}=50\pi\frac{1}{\text{sek}};\qquad \Theta=\frac{Q}{g}\cdot\varrho^2$$
$$\omega_0=\frac{\pi}{180}\cdot(\varphi^\circ)^{\cdot}=\frac{\pi}{18}\,\frac{1}{\text{sek}};$$
$$A=\frac{M}{l}=\frac{Q\cdot\varrho^2\cdot\omega_0\,\omega}{g\cdot l}=\underline{\underline{3090\ \text{kg}}}$$

Lösung 1030

Nach Aufgabe 1029 gilt: $A = \frac{Q \cdot \varrho^2 \cdot \omega \cdot \omega_0}{g \cdot l}$;

$\omega = \frac{\pi n}{30} = 100\,\pi \frac{1}{\text{sek}}$; $\quad \omega_0 = \dot{\varphi}_{0\max}$; $\quad \varphi_0 = \varphi_m \cdot \sin \alpha t$

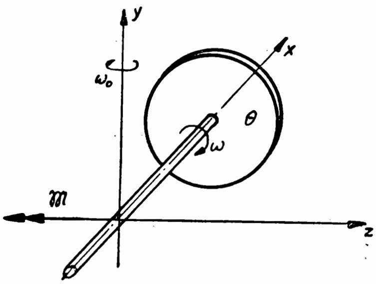
$\dot{\varphi}_0 = \alpha \cdot \varphi_m \cos \alpha t; \quad \alpha = \frac{2\pi}{T}$

$\dot{\varphi}_{0\max} = \frac{2\pi \cdot \pi \cdot 9}{180 \cdot 15} = \frac{\pi^2}{150}$

$A = \frac{3\,500 \cdot 0{,}36 \cdot 100\,\pi \cdot \pi^2}{9{,}81 \cdot 2 \cdot 150}$;

$\underline{\underline{A = 1\,320\ \text{kg}}}$

Lösung 1031



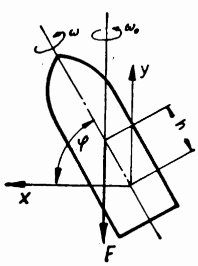
$\omega_0 =$ Winkelgeschwindigkeit in der Kurve (hier Rechtskurve)

$\omega_0 = \frac{v}{R} = \frac{40}{25}$ 1/sek

$M = \Theta\,\omega\,\omega_0 = \Theta \cdot 40\pi \cdot \frac{40}{25}$

$\underline{\underline{M = 160\ \text{mkg}}}$

Lösung 1032



$\mathfrak{M} = \Theta \vec{\omega} \times \vec{\omega}_0$

$\mathfrak{B} = \Theta \vec{\omega}; \quad |\mathfrak{B}| = 590\ \text{mkgsek}$

$\vec{\omega} = \omega (\cos\varphi\, \mathfrak{i} + \sin\varphi\, \mathfrak{j})$

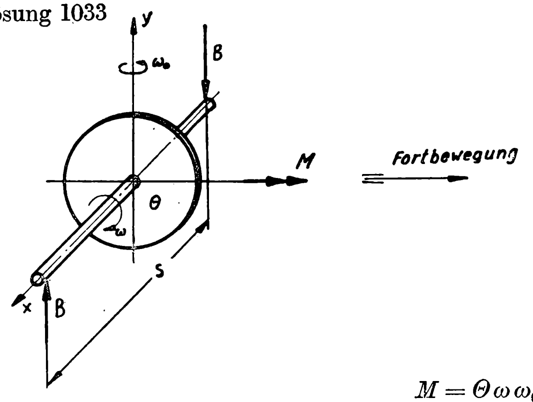
$\vec{\omega}_0 = \omega_0\, \mathfrak{j}$

$M = F \cdot h \cdot \cos\varphi; \quad F \cdot h \cos\varphi = \Theta\,\omega\,\omega_0 \cos\varphi$

$\omega_0 = \frac{2\pi}{T}$

$T = \frac{2\pi\Theta\omega}{F \cdot h} = \frac{2\pi \cdot 590}{2\,140 \cdot 0{,}2} = \underline{\underline{8{,}66\ \text{sek}}}$

Lösung 1033



$$B \cdot s - M = 0;$$

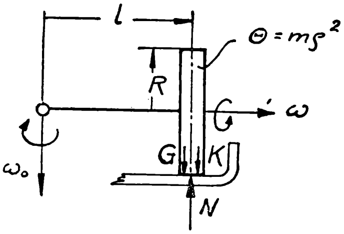
$$M = +\Theta \cdot \omega \cdot \omega_0$$

$$B = \frac{\Theta\,\omega\,\omega_0}{s}$$

$$= \frac{20}{1{,}5} \cdot \frac{1\,500\pi}{30} \cdot \frac{15}{250}$$

$$\underline{\underline{B = 126\,\text{kg}}}$$

Lösung 1034



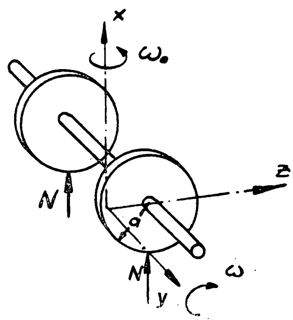
$$M = \Theta\,\omega\,\omega_0 = K \cdot l;$$

$$\omega_0 = \frac{\pi \cdot n}{30} = 2\pi; \qquad \omega = \omega_0 \cdot \frac{l}{R}$$

$$K \cdot l = m\,\varrho^2 \cdot \omega_0^2 \cdot \frac{l}{R}$$

$$K = m\,\frac{\varrho^2}{R}\,\omega_0^2 = 1540\,\text{kg}$$

$$N = K + G = \underline{\underline{2740\,\text{kg}}}$$



Lösung 1035

$$\Theta_y = \frac{P}{g}(\varrho^2 + a^2); \qquad \omega = \frac{v}{a}$$

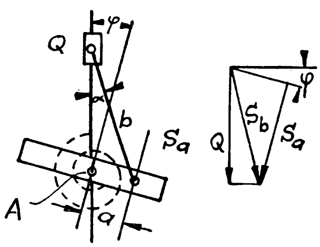
$$\omega_0 = \frac{v}{R}$$

$$M = \Theta_y \cdot \omega \cdot \omega_0$$

Schienendruck $N = \frac{P}{2} \pm \frac{\Theta_y \cdot \omega\,\omega_0}{s}$

$$\underline{\underline{N = 700 \pm 221\,\text{kg}}}$$

Lösung 1036



$$S_b = \frac{Q}{\cos\alpha}; \qquad S_a = S_b \cdot \sin(90° - \varphi - \alpha)$$

$$S_a = S_b \cdot \cos(\varphi + \alpha)$$

$$S_a = \frac{Q}{\cos\alpha}[\cos\varphi\cos\alpha - \sin\varphi\sin\alpha]$$

$$S_a = Q[\cos\varphi - \sin\varphi\,\mathrm{tg}\,\alpha]$$

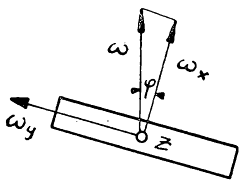
$$\sin\alpha = \frac{\sin(90 + \varphi) \cdot a}{b} = \frac{a}{b}\cos\varphi$$

$$\mathrm{tg}\,\alpha = \frac{\frac{a}{b}\cos\varphi}{\sqrt{1 - \frac{a^2}{b^2}\cos^2\varphi}}$$

Unter Vernachlässigung des quadratischen Gliedes von $\frac{a}{b}$ wird:

$$S_a = Q\cos\varphi\left[1 - \frac{a}{b}\sin\varphi\right]$$

Das Kreiselmoment um die z-Achse folgt aus den Eulerschen Gleichungen zu:



$$M_z = (\Theta_x - \Theta_y)\,\omega_x\omega_y \qquad (\dot{\omega}_z = 0)$$

$$\Theta_x = A; \qquad \omega_x = \omega\cos\varphi$$

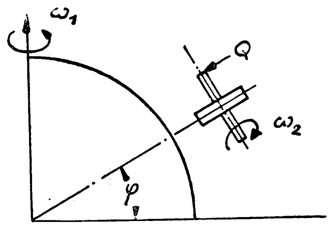
$$\Theta_y = C; \qquad \omega_y = \omega\sin\varphi$$

Momentengleichung um A:

$$S_a\cdot a + C(\varphi_0 - \varphi) - (C - A)\,\omega^2\cos\varphi\sin\varphi = 0$$

$$\underline{\underline{\omega^2 = \frac{c(\varphi_0 - \varphi) + Q\cdot a\left(1 - \frac{a}{b}\sin\varphi\right)\cos\varphi}{(C - A)\sin\varphi\cos\varphi}}}$$

Lösung 1037



$$\mathfrak{M} = \Theta\vec{\omega}\times\vec{\omega}_1$$

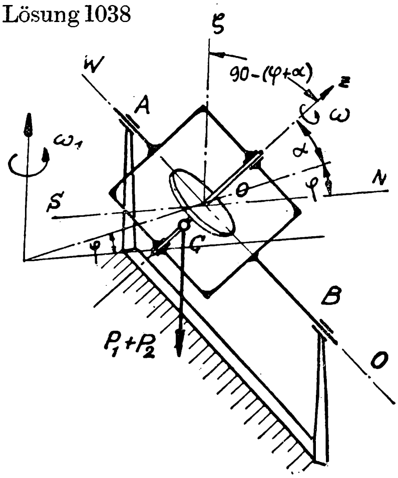
$$\vec{\omega} = \omega(-\mathfrak{i}\sin\varphi + \mathfrak{j}\cos\varphi)$$

$$\vec{\omega}_1 = \omega_1\mathfrak{j}$$

$$M = -\Theta\,\omega\,\omega_1\sin\varphi$$

$$Q\cdot a = \Theta\,\omega\,\omega_1\sin\varphi; \qquad \underline{\underline{Q = \frac{\Theta\,\omega\,\omega_1\sin\varphi}{a}}}$$

Lösung 1038



$$M = -\Theta\,\omega\,\omega_1\cos(\varphi + \alpha) + H\sin\alpha = 0$$

$$\frac{\cos(\varphi + \alpha)}{\sin\alpha} = \frac{H}{\Theta\,\omega\,\omega_1}$$

$$\omega_1 = \frac{2\pi}{24\cdot 60\cdot 60} = \frac{1}{13\,750}$$

$$\Theta = \frac{2000\cdot 16}{981\cdot 2} = 16{,}3\ \text{cmgsek}^2$$

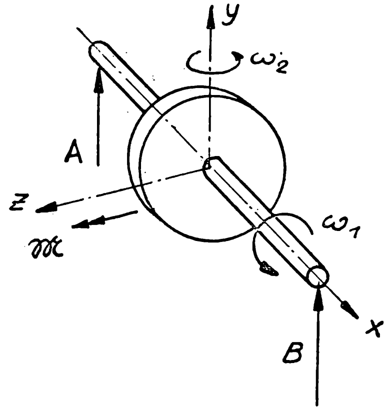
$$\frac{H}{\Theta\,\omega\,\omega_1} = 0{,}365$$

$$\cos\varphi\cdot\operatorname{ctg}\alpha - \sin\varphi = 0{,}365$$

$$\operatorname{ctg}\alpha = \frac{0{,}365 + 0{,}5}{0{,}866} = 1$$

$$\underline{\underline{\alpha = 45^\circ}}$$

Lösung 1039



$$M = \Theta\,\omega_1\omega_2; \quad \Theta = \frac{2p\cdot a^2}{g}$$

$$\sum M_B = 0; \quad N_A\cdot 2h - 2ph - \Theta\,\omega_1\omega_2 = 0$$

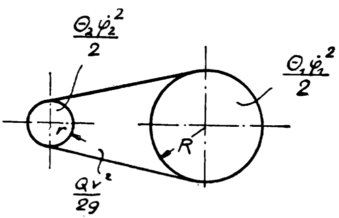
$$\underline{\underline{N_A = p\left(1 + \frac{a^2\omega_1\omega_2}{gh}\right)}}$$

$$N_A + N_B - 2p = 0$$

$$\underline{\underline{N_B = p\left(1 - \frac{a^2\omega_1\omega_2}{gh}\right)}}$$

40. Kinetische Energie des Massensystems

Lösung 1040



$$v = R\cdot\dot\varphi_1 = r\dot\varphi_2;$$

$$\dot\varphi_1 = \omega; \quad \dot\varphi_2 = \omega\cdot\frac{R}{r}$$

$$\underline{\underline{T = \frac{\omega^2}{2}\left[\frac{Q}{g}R^2 + \Theta_1 + \Theta_2\left(\frac{R}{r}\right)^2\right]}}$$

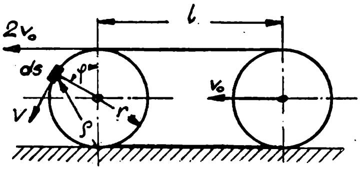
Lösung 1041

$$T_t = \frac{mv^2}{2}; \quad T_r = \frac{\Theta\,\omega^2}{2}; \quad \omega = 2\pi n$$

$$T_t = \frac{920\cdot 81\cdot 10^4}{9{,}81\cdot 2}; \quad T_r = \frac{2\cdot 4\pi^2\cdot 45^2}{2}$$

$$\frac{T_r}{T_t}\cdot 100 = \frac{8\pi^2\cdot 2025\cdot 2\cdot 9{,}81}{2\cdot 920\cdot 81\cdot 10^4}\cdot 100 = \underline{\underline{0{,}21\,\%}}$$

Lösung 1042



$$T = \frac{\gamma\cdot l\cdot 4v_0^2}{2g} + 2\cdot\int\frac{\gamma}{2g}\cdot v^2\,ds$$

$$ds = r\cdot d\varphi; \quad 2r\cos\frac{\varphi}{2} = \varrho$$

$$v = \frac{\varrho}{r}v_0$$

$$\frac{\gamma}{g}\int v^2\cdot ds = \frac{4r\gamma v_0^2}{g}\int_0^\pi \cos^2\frac{\varphi}{2}\,d\varphi = \frac{2r\gamma v_0^2\pi}{g}$$

$$\underline{\underline{T = \frac{2\gamma v_0^2}{g}\,[l + \pi r]}}$$

Lösung 1043

$$T = \Theta_0 \frac{\dot\varphi^2}{2} + m\frac{\dot x^2}{2}; \qquad x = a\cos\varphi; \qquad \dot x = -a\dot\varphi\sin\varphi$$

$$\underline{\underline{T}} = \Theta_0 \cdot \frac{\omega^2}{2} + m\frac{a^2\omega^2\sin^2\varphi}{2} = \underline{\underline{\frac{1}{2}\omega^2(\Theta_0 + m a^2 \sin^2\varphi)}}$$

Extremwerte von T treten auf bei $\frac{dT}{d\varphi} = 0$

$$\frac{dT}{d\varphi} = \frac{1}{2}\omega^2 m a^2 2\sin\varphi\cos\varphi = 0; \qquad \sin 2\varphi = 0$$

$$\varphi = 0; \quad \frac{\pi}{2}; \quad \pi; \quad \frac{3\pi}{2} \text{ usw.}$$

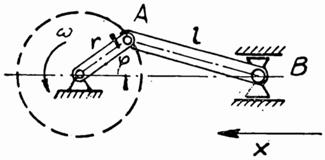
$$\frac{d^2T}{d\varphi^2} = \frac{1}{2}\omega^2 m a^2 \cdot 2[\cos^2\varphi - \sin^2\varphi] = C\cdot\cos 2\varphi$$

Es treten auf: Maxima für $\frac{d^2T}{d\varphi^2} < 0$; Minima für $\frac{d^2T}{d\varphi^2} > 0$

Also: Geringste kinetische Energie für $\varphi = 0;\ \pi \ldots$

Größte kinetische Energie für $\varphi = \frac{\pi}{2};\ \frac{3\pi}{2} \ldots$

Lösung 1044



$$\varphi = \omega t$$

$$T = \frac{1}{2}[\Theta_1\omega^2 + m_2\dot x^2]; \qquad \Theta_1 = \frac{1}{3}m_1 r^2$$

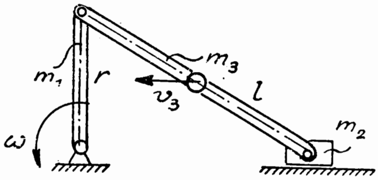
$$x \cong r\cos\omega t + l\sqrt{1 - \left(\frac{r}{l}\right)^2 \sin^2\omega t}$$

$$\dot x = -r\omega\left[\sin\omega t + \frac{r}{2l}\cdot\frac{\sin 2\omega t}{\sqrt{1-\left(\frac{r}{l}\right)^2\sin^2\omega t}}\right]$$

Somit wird:

$$\underline{\underline{T = \frac{1}{2}r^2\omega^2\left[\frac{1}{3}m_1 + m_2\left\{\sin\omega t + \frac{r}{2l}\frac{\sin 2\omega t}{\sqrt{1-\left(\frac{r}{l}\right)^2\sin^2\omega t}}\right\}^2\right]}}$$

Lösung 1045



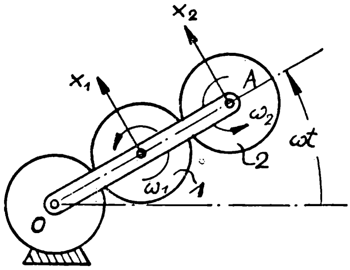
In dem betrachteten Augenblick gilt:

$$T = \Theta_1\frac{\omega^2}{2} + m_3\frac{v_3^2}{2} + m_2\frac{v_2^2}{2}$$

$$v_3 = v_2 = \omega r; \qquad \Theta_1 = \frac{1}{3}m_1 r^2$$

$$\underline{\underline{T = \frac{r^2\omega^2}{2}\left[\frac{1}{3}m_1 + m_3 + m_2\right]}}$$

Lösung 1046



$$T=\frac{m_1}{2}\dot{x}_1^2+\frac{m_2}{2}\dot{x}_2^2+\Theta_1\frac{\omega_1^2}{2}+\Theta_2\frac{\omega_2^2}{2}+\Theta_K\cdot\frac{\omega^2}{2}$$

$\omega_1=2\omega;\quad \dot{x}_1=2r\omega;$

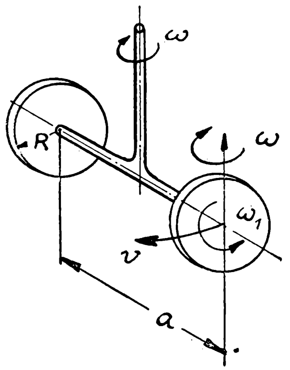
$\omega_2=0;\quad \dot{x}_2=4r\omega;$

$$\Theta_k=\frac{Q}{g}\cdot\frac{16r^2}{3};\quad \Theta_1=\Theta_2=\frac{P}{g}\cdot\frac{r^2}{2}$$

$$T=\frac{P}{2g}\cdot r^2\omega^2[4+16]+\frac{Pr^2\cdot 4\omega^2}{2g\cdot 2}+\frac{Q\cdot 16r^2}{3g\cdot 2}\cdot\omega^2$$

$$\underline{\underline{T=\frac{r^2\omega^2}{3g}[33P+8Q]}};$$ da $\omega_2=0$, gilt für Rad III: $T_r=0$

Lösung 1047



$$\omega=\frac{\pi\cdot n}{30}=\frac{2}{3}\pi$$

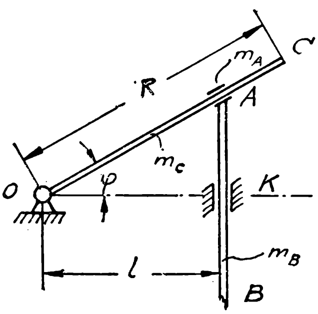
$$\omega_1=\omega\frac{a}{2R};\quad v=\omega\frac{a}{2}$$

$$T=2\left(\frac{mv^2}{2}+\frac{\Theta_1\omega_1^2}{2}+\frac{\Theta\omega^2}{2}\right)$$

$$\Theta_1=\frac{mR^2}{2};\quad \Theta=\frac{mR^2}{4}$$

$$\underline{\underline{T}}=\frac{P}{g}\omega^2\left[\frac{3a^2}{8}+\frac{R^2}{4}\right]=\underline{\underline{39\text{ kgm}}}$$

Lösung 1048



$\varphi=\omega t$

Energie der Kurbelbewegung:

$$T_c=\Theta_c\frac{\omega^2}{2};\quad \Theta_c=m_c\frac{R^2}{3}$$

$$T_c=\frac{m_cR^2\omega^2}{6}$$

Energie der Bewegung des Gleitstückes A und der Stange B

$$y=l\,\mathrm{tg}\,\omega t$$

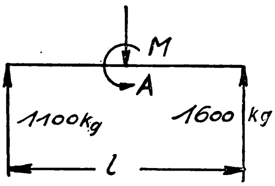
$$T_{AB}=\frac{(m_A+m_B)\cdot\dot{y}^2}{2};\quad \dot{y}=l\omega\frac{1}{\cos^2\omega t}$$

$$T_{AB}=\frac{(m_A+m_B)\,l^2\omega^2}{2\cos^4\omega t}$$

$$T=T_c+T_{AB}$$

$$\underline{\underline{T=\frac{\omega^2}{6\cos^4\varphi}[m_cR^2\cdot\cos^4\varphi+3l^2(m_A+m_B)]}}$$

Lösung 1049

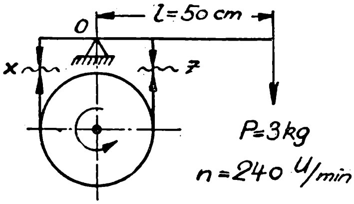


$$\eta \cdot N = \frac{M \cdot n}{716}$$

$$\sum M_A = 0: \quad M = 1600 \cdot 1 - 1100 \cdot 1 = 500 \,\text{mkg}$$

$$\underline{\underline{N}} = \frac{500 \cdot 1432}{0{,}8 \cdot 716} = \underline{\underline{1250 \,\text{PS}}}$$

Lösung 1050



$$N = \frac{K v}{75} \text{PS}; \qquad v = \frac{\pi \cdot n}{30} \cdot r$$

$$\sum M_0 = 0: \quad P \cdot l - X \cdot r + Z r = 0;$$

$$K = X - Z; \qquad K = P \cdot \frac{l}{r}$$

$$\underline{\underline{N}} = \frac{P \cdot l \cdot \pi \cdot n}{75 \cdot 30} = \frac{4 \pi}{25} = \underline{\underline{0{,}5 \,\text{PS} \triangleq 0{,}37 \,\text{kW}}}$$

Lösung 1051

Kinetische Energie:

a) Translation der Räder und des Kastens: $T_1 = \frac{(m_1 + 4 m_2)\, v^2}{2}$

$$T_1 = \frac{5800 \cdot 100}{2 \cdot 9{,}81} = 29\,500 \,\text{mkg}$$

b) Rotation der Räder: $T_2 = 4 \frac{\Theta \omega^2}{2}; \qquad \Theta = \frac{m_2 r^2}{2}; \qquad \omega = \frac{v}{r}$

$$T_2 = m_2 \cdot v^2 = \frac{200 \cdot 100}{9{,}81} = 2040 \,\text{mkg}$$

c) Raupe: $T_3 = 2 \cdot 2 \frac{\gamma}{g} (l + \pi r)\, v^2$ (vgl. Aufgabe 1042) $= 2 m_3 v^2$

$$T_3 = \frac{2 \cdot 500 \cdot 100}{9{,}81} = 10\,200 \,\text{mkg}$$

$$\underline{\underline{N}} = \frac{T_1 + T_2 + T_3}{t \cdot 75} = \frac{41\,740}{8 \cdot 75} = \underline{\underline{69{,}4 \,\text{PS}}}$$

Lösung 1052

Schwungradenergie:

$$T = \Theta \cdot \frac{\omega^2}{2} = m \cdot r^2 \cdot \frac{\pi^2 n^2}{2 \cdot 900}$$

Reibarbeit im Lager:

$$A = G \cdot \mu \cdot 2 \pi r_w \cdot u$$

$$A = T: \quad G \mu \pi u r_w = \frac{m r^2 \pi^2 n^2}{4 \cdot 900}$$

$$\mu = \frac{r^2 \pi n^2}{4 \cdot g \cdot u \cdot r_w \cdot 900} = \underline{\underline{0{,}067}}$$

Lösung 1053

Kinetische Energie des Schwungrades = Reibarbeit:

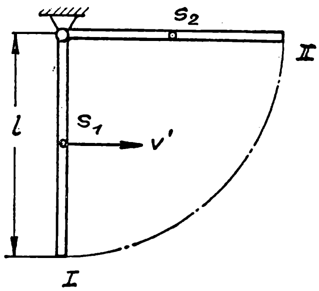
$$\frac{1}{2} m_s \cdot r_s^2 \cdot \omega^2 = \mu (G_s + G_w) r_w \cdot \varphi$$

Index s : Schwungrad
Index w: Welle

Zahl der Umdrehungen: $u = \dfrac{r_w \cdot \varphi}{2\pi r_w}$

$$\underline{\underline{u}} = \frac{\omega^2 m_s \cdot r_s^2}{2\mu (G_s + G_w) \cdot 2\pi r_w} = \underline{\underline{109{,}8 \text{ Umdrehungen}}}$$

Lösung 1054



Stellung I: $T = \dfrac{m}{2} v'^2 + \dfrac{1}{2} \Theta_s \cdot \dfrac{v'^2 \cdot 4}{l^2}$

$U = mg \cdot \dfrac{l}{2}$

Stellung II: $T = 0; \quad U = mgl$

$$T + U = \text{const}$$

$$\Theta_s = \frac{ml^2}{12}; \quad \frac{v'}{\frac{l}{2}} = \frac{v}{l}; \quad v = 2v'$$

$$mv'^2 \left[\frac{1}{2} + \frac{1}{6}\right] = mg \frac{l}{2}$$

$$v' = \frac{1}{2} \sqrt{3gl}; \quad \underline{\underline{v = \sqrt{3gl} = 9{,}81 \text{ m/sek}}}$$

Lösung 1055

Die kinetische Energie ist am Anfang und am Ende des Vorgangs Null. Somit muß die Summe der geleisteten Arbeit ebenfalls Null sein.

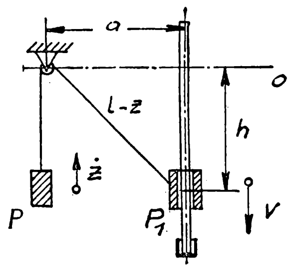
$$p_1 h - 2p\left(\sqrt{l^2 + h^2} - l\right) = 0; \qquad p_1 h + 2pl = 2p\sqrt{l^2 + h^2}$$

$$p_1^2 h^2 + 4p^2 l^2 + 4pp_1 lh = 4p^2 (l^2 + h^2)$$

$$h^2 (p_1^2 - 4p^2) + 4pp_1 lh = 0$$

$$\underline{\underline{h = \frac{4pp_1 l}{4p^2 - p_1^2}}}$$

Lösung 1056



Energie am Anfang:

$$T = 0; \quad U = -P(l - a)$$

Energie nach Abgleiten von P_1 um h:

$$T = \frac{P_1}{g} \cdot \frac{v^2}{2} + \frac{P}{g} \cdot \frac{\dot{z}^2}{2}$$

$$U = -P_1 h - Pz$$

$$(l - z)^2 = a^2 + h^2; \quad z = l - \sqrt{a^2 + h^2}$$

$$\dot{z} = -\frac{2h\dot{h}}{2\sqrt{a^2 + h^2}}$$

Somit: $-P(l-a)=\frac{1}{2}\left[\frac{P_1}{g}v^2+\frac{P}{g}\,\frac{h^2v^2}{a^2+h^2}\right]-P_1h-P\left[l-\sqrt{a^2+h^2}\right]$

$$\underline{\underline{v^2=2g\,(a^2+h^2)\,\frac{P_1h-P(\sqrt{a^2+h^2}-a)}{P_1\,(a^2+h^2)+Ph^2}}}$$

Lösung 1057

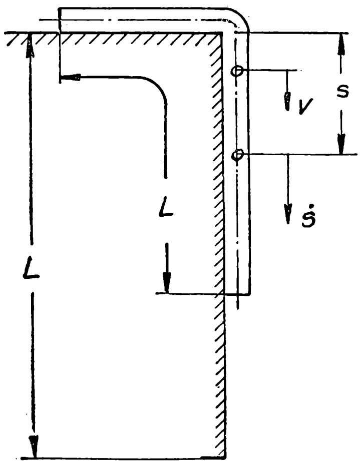
Energie vor dem Abheben: $(P+P_1)\,s_1-Q\mu s_1-\frac{P+P_1+Q}{2g}\,v^2=0 \quad \Big| \cdot(P+Q)$

Energie nach dem Abheben; $\frac{P+Q}{2g}\,v^2+Ps_2-Q\mu s_2=0 \quad \Big| \cdot(P+P_1+Q)$

Beide Gleichungen addiert ergibt:

$$\underline{\underline{\mu=\frac{s_1(P+P_1)\,(P+Q)+s_2P\,(P+P_1+Q)}{Q\,[s_1\,(P+Q)+s_2\,(P+P_1+Q)]}=0{,}2}}$$

Lösung 1058



$v=$ Geschwindigkeit des Gesamtschwerpunktes
$s=$ Schwerpunktsweg des herabhängenden Teiles

$$T=\frac{mv^2}{2}; \qquad v=2\dot{s}$$

$$U=mg\,\frac{2s}{L}\,(L-s)+mg\,\frac{L-2s}{L}\cdot L$$

$$U=\frac{mg}{L}\,(L^2-2s^2)$$

$$T+U=C: \qquad \dot{s}^2+\frac{g}{2L}\,(L^2-2s^2)=C$$

Anfangsbed.: $\quad \dot{s}=0; \qquad s=\frac{l}{2}$

$$C=\frac{g}{4L}\,(2L^2-l^2)$$

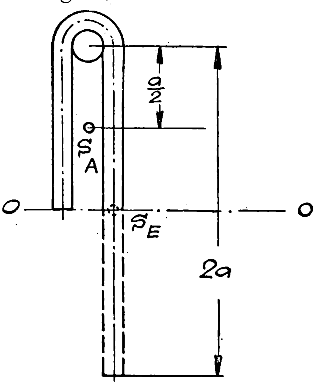
$$\frac{ds}{dt}=\sqrt{C+\frac{g}{2L}\,(2s^2-L^2)}$$

$$dt=\frac{ds}{\sqrt{s^2+\left(\frac{L\cdot C}{g}-\frac{L^2}{2}\right)}}\cdot\sqrt{\frac{L}{g}}$$

$$T=\sqrt{\frac{L}{g}}\int\limits_{\frac{l}{2}}^{\frac{L}{2}}\frac{ds}{\sqrt{s^2-\frac{l^2}{4}}}$$

$$\underline{\underline{T=\sqrt{\frac{L}{g}}\,\ln\frac{L+\sqrt{L^2-l^2}}{l}}}$$

Lösung 1059



Anfangswert:

Kinetische Energie: $T = \frac{mv_0^2}{2}$

Potentielle Energie: $U = mg\,\frac{a}{2}$

Endwert:

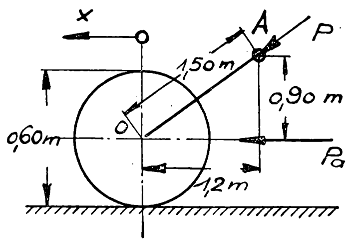
Kinetische Energie: $T = \frac{mv^2}{2}$

Potentielle Energie: $U = 0$

$$\frac{mv_0^2}{2} + mg\,\frac{a}{2} = \frac{mv^2}{2}$$

$$\underline{\underline{v = \sqrt{v_0^2 + ag}}}$$

Lösung 1060



$$\frac{P}{1{,}5} = \frac{P_a}{1{,}2}; \quad P = \frac{5}{4}\,P_a$$

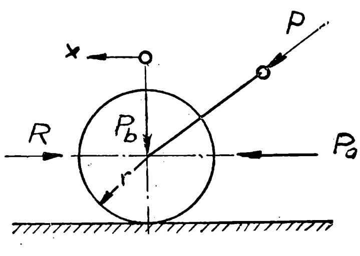
$$\frac{mv^2}{2} + \frac{\Theta\,\omega^2}{2} = \frac{4}{5}\,P \cdot x$$

$$\omega = \frac{v}{r}; \qquad \Theta = \frac{mr^2}{2}; \qquad x = 2\,\mathrm{m}$$

$$\frac{3\,mv^2}{4} = \frac{4}{5}\,P \cdot x; \quad P = \frac{15}{16} \cdot \frac{mv^2}{x}$$

$$\underline{\underline{P}} = \frac{15 \cdot 392 \cdot 6400}{16 \cdot 200 \cdot 980} = \underline{\underline{12\ \mathrm{kg}}}$$

Lösung 1061



$$P = \frac{5}{4}\,P_a; \qquad P_b = \frac{3}{5}\,P$$

$$\frac{mv^2}{2} + \frac{\Theta\,\omega^2}{2} + R \cdot x = \frac{4}{5}\,P \cdot x$$

$$R = \frac{(G + P_b) \cdot f}{r}$$

$$\frac{3}{4}\,mv^2 + \frac{Gf}{r}\,x = \frac{4}{5}\,P \cdot x - \frac{3}{5}\,\frac{P \cdot f}{r} \cdot x$$

$$P = \left(\frac{15}{4}\,\frac{mv^2}{x} + 5\,\frac{G \cdot f}{r}\right) \cdot \frac{1}{\left(4 - 3\,\frac{f}{r}\right)}$$

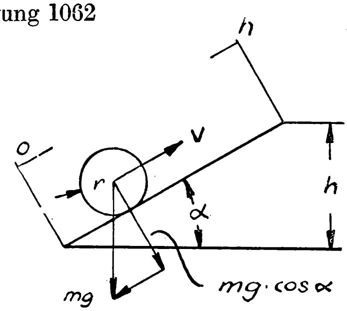
$$\underline{\underline{P = 20{,}4\ \mathrm{kg}}}$$

Soll keine Geschwindigkeitszunahme erfolgen, so hat der Mann nur die Reibungswiderstände zu überwinden.

$$\frac{4}{5}\,P' = \frac{(G + P_b') \cdot f}{r} = \frac{G \cdot f}{r} + \frac{3}{5}\,P'\,\frac{f}{r}; \qquad P' = \frac{5 \cdot G \cdot f}{r\left(4 - 3\,\frac{f}{r}\right)} = 8{,}27\ \mathrm{kg}$$

$$\underline{\underline{\Delta P}} = P - P' = \underline{\underline{12{,}13\ \mathrm{kg}}}$$

Lösung 1062



Anfangswert: $T_0 = \frac{mv^2}{2} + \Theta \frac{\omega^2}{2}$

$U_0 = 0$

Endwert: $T_h = 0$

$U_h = mgh; \quad A = mg\cos\alpha \cdot \frac{f}{r} \cdot \frac{h}{\sin\alpha}$

$\Theta = \frac{mr^2}{2}; \quad \omega = \frac{v}{r}$

$T_0 + U_0 = T_h + U_h + A$

$$\underline{\underline{v = \frac{2}{3}\sqrt{3gh\left(1 + \frac{f}{r}\operatorname{ctg}\alpha\right)}}}$$

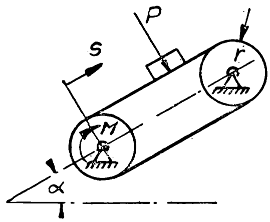
Lösung 1063

$$M_1 \cdot \varphi_1 = \frac{\frac{\Theta_1}{k_{12}^2} + \Theta_2}{2} \cdot \left(\frac{\pi n_2}{30}\right)^2; \qquad \varphi_1 = \frac{2\pi u_2}{k_{12}}$$

$$u_2 = \frac{\frac{\Theta_1}{k_{12}^2} + \Theta_2}{4\pi M_1} \cdot \left(\frac{\pi n_2}{30}\right)^2 \cdot k_{12} = \frac{\frac{500}{\left(\frac{3}{l}\right)^2} + 400}{4\pi \cdot 5000} \cdot \left(\frac{\pi \cdot 120}{30}\right)^2 \cdot \frac{3}{2}$$

$\underline{\underline{u_2 = 2{,}34 \text{ Umdrehungen}}}$

Lösung 1064



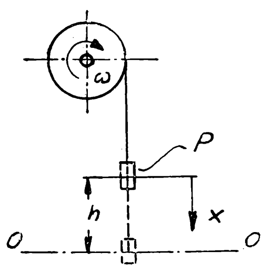
An das System gelieferte Arbeit = Von dem System verbrauchte Arbeit.

$$M \cdot \varphi = 2 \cdot \frac{Q}{g} \cdot \frac{r^2}{2} \cdot \frac{\dot{s}^2}{2r^2} + \frac{P}{2g}\dot{s}^2 + P \cdot s \cdot \sin\alpha$$

$$\frac{\dot{s}^2}{2g}(P + Q) = M \cdot \frac{s}{r} - P \cdot s \cdot \sin\alpha; \qquad \dot{s} = v$$

$$\underline{\underline{v = \sqrt{2g\frac{M - Pr\sin\alpha}{r(P+Q)} \cdot s}}}$$

Lösung 1065



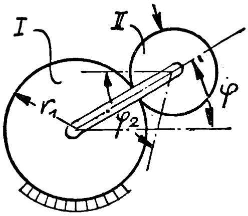
$$P \cdot h = \frac{P}{2g}v^2 + \frac{\Theta}{2}\omega^2; \qquad v = \omega \cdot r$$

$$\Theta = \frac{Q}{g} \cdot \frac{r^2}{2}$$

$$P \cdot h = \frac{Pv^2}{2g} + \frac{Q \cdot v^2}{4g}$$

$$\underline{\underline{v = 2\sqrt{\frac{ghP}{2P+Q}}}}$$

Lösung 1066



$$\Theta_2 \cdot \frac{\dot{\varphi}_2^2}{2} + \Theta_{\mathrm{St}} \cdot \frac{\dot{\varphi}^2}{2} + \frac{m_2}{2}\,\dot{\varphi}^2 (r_1 + r_2)^2 = M \cdot \varphi$$

$$\Theta_2 = \frac{P}{g} \cdot \frac{r_2^2}{2}; \quad \Theta_{\mathrm{St}} = \frac{Q}{g}\,\frac{(r_1+r_2)^2}{3}; \quad \varphi_2 = \varphi\left(\frac{r_1}{r_2} + 1\right)$$

$$\frac{P r_2^2}{g \cdot 2} \cdot \frac{\dot{\varphi}^2 (r_1 + r_2)^2}{2 r_2^2} + \frac{Q}{g}\,\frac{(r_1+r_2)^2 \dot{\varphi}^2}{3 \cdot 2} + \frac{P}{2g}\,\dot{\varphi}^2 (r_1 + r_2)^2 = M \cdot \varphi$$

$$\underline{\underline{\dot{\varphi} = \omega = \frac{2}{r_1 + r_2}\sqrt{\frac{3Mg}{gP + 2Q} \cdot \varphi}}}$$

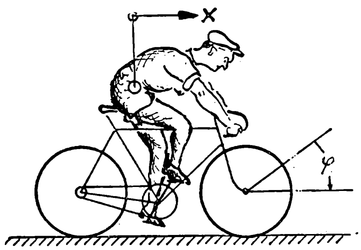
Lösung 1067

$$\Theta_0 = \frac{mr^2}{2} + me^2; \quad e = \frac{r}{2}; \quad \Theta_0 = \frac{3}{4}\,mr^2$$

$$\Theta_0 \cdot \frac{\omega^2}{2} = \frac{cx^2}{2}; \quad x = 2e = r; \quad \frac{3}{4}\,mr^2 \cdot \frac{\omega^2}{2} = c\,\frac{r^2}{2}$$

$$\underline{\underline{\omega = 2\sqrt{\frac{cg}{3p}}}}$$

Lösung 1068



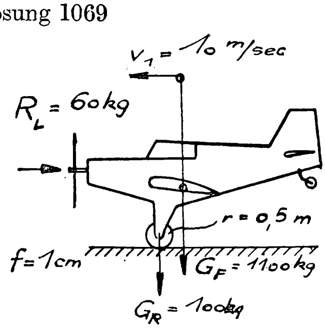
$$2 \cdot \Theta_{\mathrm{Rad}} \cdot \frac{\dot{\varphi}^2}{2} + 2 m_{\mathrm{Rad}} \cdot \frac{\dot{x}^2}{2} + \frac{m_{\mathrm{Fahrer}}}{2} \cdot \dot{x}^2 = (2 m_{\mathrm{Rad}} + m_{\mathrm{Fahrer}})\, g \cdot \frac{f}{r} \cdot x$$

$$\dot{x} = \dot{\varphi} r; \quad \Theta = m_{\mathrm{Rad}} \cdot r^2$$

$$\frac{\dot{x}^2 \left[2 G_R + \frac{G_F}{2}\right] \cdot r}{(2 G_R + G_F)\, g \cdot f} = x$$

$$\underline{\underline{x = 35{,}6\ \mathrm{m}}}$$

Lösung 1069



Hebelarm der rollenden Reibung $= f$

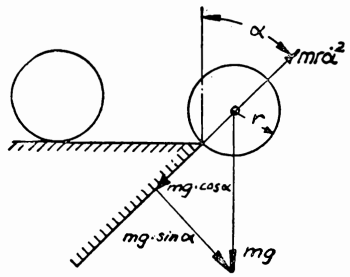
$$(R_L + R_B)\, x = \frac{G_F + G_R}{2g}\, v^2$$

$$R_B = \frac{(G_F + G_R) f}{r}$$

$$x = \frac{(G_F + G_R) \cdot v^2}{\left[\frac{(G_F + G_R) \cdot f}{r} + R_L\right] \cdot 2g}$$

$$\underline{\underline{x = 73\ \mathrm{m}}}$$

Lösung 1070



Energie beim Bewegungsbeginn:

$$T=0;\qquad U=mg\cdot r$$

Energie im Moment des Ablösens:

$$T=\frac{\Theta_B}{2}\dot\alpha^2;\qquad U=m\cdot g\cdot\cos\alpha\cdot r$$

Somit:

$$\frac{\Theta_B}{2}\dot\alpha^2+mgr\cos\alpha=mgr$$

Kräftegleichgewicht im Moment des Ablösens:

$$mr\dot\alpha^2=mg\cos\alpha$$

$$\Theta_B=\frac{3}{2}mr^2;$$

$$\dot\alpha^2=\frac{4g}{3r}(1-\cos\alpha)$$

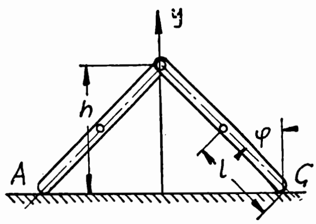
$$\dot\alpha^2=\frac{g}{r}\cos\alpha$$

Daraus:

$$\alpha=\operatorname{arc}\cos\frac{4}{7}$$

$$\dot\alpha=\omega=2\sqrt{\frac{g}{7r}}$$

Lösung 1071



Anfangswert: $U_1=\dfrac{mgh}{2}$

Endwert: $T_2=\Theta_c\dfrac{\dot\varphi^2}{2};\qquad U_2=\dfrac{mgy}{2}$

$$T_1+U_1=T_2+U_2$$

$$\frac{mgh}{2}=\frac{m}{2}\dot\varphi^2(l^2+k^2)+\frac{mgy}{2}$$

$$\cos\varphi=\frac{y}{2l};\qquad -\dot\varphi\sin\varphi=\frac{\dot y}{2l}$$

$$\dot\varphi^2=\frac{\dot y^2}{4l^2-y^2}$$

$$g(h-y)=\frac{(l^2+k^2)\dot y^2}{4l^2-y^2}$$

$$\dot y^2_{y=0}=\frac{4l^2gh}{l^2+k^2}$$

$$\underline{\underline{\dot y_{y=0}=v_1=2l\sqrt{\frac{gh}{l^2+k^2}}}}$$

$$\dot y_{y=\frac{h}{2}}=\sqrt{\frac{g\frac{h}{2}\left(4l^2-\frac{h^2}{4}\right)}{l^2+k^2}}$$

$$\underline{\underline{\dot y_{y=\frac{h}{2}}=v_2=\frac{1}{2}\sqrt{\frac{16l^2-h^2}{2(l^2+k^2)}gh}}}$$

Lösung 1072

Energie bei Bewegungsbeginn: $T_1=0;\quad mga=U_1$

Energie im Moment der Schwerpunktshöhe h: $T_2=m\dfrac{\dot h^2}{2}+\Theta\dfrac{\dot\varphi^2}{2}$

$$U_2=mgh$$

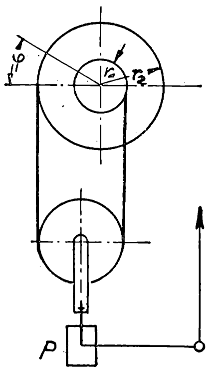
$$\cos\varphi = \frac{h}{a}; \quad -\sin\varphi\,\dot\varphi = \frac{\dot h}{a}; \quad \dot\varphi = -\frac{\dot h}{\sqrt{a^2-h^2}}; \quad \Theta = m\frac{a^2}{3}$$

$$T_1 + U_1 = T_2 + U_2$$

$$\frac{m\dot h^2}{2}\left[\frac{4a^2-3h^2}{3(a^2-h^2)}\right] + mgh = mga; \quad \dot h^2 = g(a-h)\frac{6(a^2-h^2)}{4a^2-3h^2}$$

$$\underline{\underline{\dot h = v = (a-h)\sqrt{\frac{6g(a+h)}{4a^2-3h^2}}}}$$

Lösung 1073



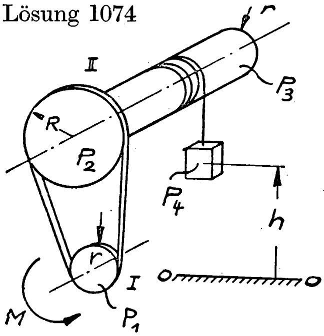
$$M\cdot\varphi = \frac{(\Theta_1+\Theta_2)}{2}\dot\varphi^2 + P\cdot s + \frac{P}{2g}\dot s^2$$

$$s = \frac{\varphi}{2}(r_2-r_1); \quad \dot s = \frac{\dot\varphi}{2}(r_2-r_1); \quad \varphi = \frac{2s}{(r_2-r_1)}$$

$$\dot\varphi^2\left[\frac{(\Theta_1+\Theta_2)}{2} + \frac{P}{2g}\,\frac{(r_2-r_1)^2}{4}\right] = M\frac{2s}{r_2-r_1} - P\cdot s$$

$$\underline{\underline{\dot\varphi = \omega = 2\sqrt{\frac{2gs\,[2M-P(r_2-r_1)]}{(r_2-r_1)\,[P(r_2-r_1)^2+4g(\Theta_1+\Theta_2)]}}}}$$

Lösung 1074



Gesucht ist $\dot x_4 = v$ bei $x_4 = h$

$$M\varphi_1 = \Theta_1\frac{\dot\varphi_1^2}{2} + \Theta_2\frac{\dot\varphi_2^2}{2} + \Theta_3\frac{\dot\varphi_2^2}{2} + \frac{P_4}{2g}\dot x_4^2 + P_4\cdot x_4$$

$$\varphi_2 = \frac{x_4}{r}; \quad \varphi_1 = \frac{R}{r}x_4; \quad x_4 = h: \quad \dot x_4 = v$$

$$\frac{v^2}{4g}\left[P_1\left(\frac{R}{r}\right)^2 + P_2\left(\frac{R}{r}\right)^2 + P_3 + 2P_4\right] + P_4\cdot h = M\frac{R}{r^2}h$$

$$\underline{\underline{v = 2\sqrt{\frac{gh\left(M\frac{R}{r^2}-P_4\right)}{P_1\left(\frac{R}{r}\right)^2+P_2\left(\frac{R}{r}\right)^2+P_3+2P_4}}}}$$

Lösung 1075

Wird in Aufgabe 1074 die Seilmasse berücksichtigt, so ergibt sich

Anfangswert: $T_A = 0; \quad U_A = -2ph^2$

Endwert: $T_E = \Theta_1\frac{\dot\varphi_1^2}{2} + \Theta_2\frac{\dot\varphi_2^2}{2} + \Theta_3\frac{\dot\varphi_2^2}{2} + \frac{P_4+pl}{2g}v^2$

$$U_E = P_4\cdot h - \frac{p}{2}h^2$$

$$A = M\cdot\varphi_1$$

Die Geschwindigkeit des Seiles ist gleich der Umfangsgeschwindigkeit der Trommel.

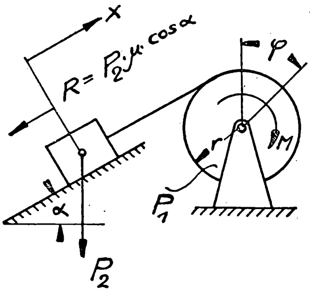
$A = T_E + U_E - (T_A + U_A)$; Mit $\dot\varphi_2 = \frac{v}{r}$; $\varphi_1 = \frac{R}{r}h$; $\Theta_1 = \frac{P_1}{g}\cdot\frac{r^2}{2}$; $\Theta_2 = \frac{P_2}{g}\cdot\frac{R^2}{2}$

$\Theta_3 = \frac{P_2}{g}\,\frac{r^2}{2}$ gilt:

$$\frac{v^2}{4g}\left[P_1\left(\frac{R}{r}\right)^2 + P_2\left(\frac{R}{r}\right)^2 + P_3 + 2P_4 + 2pl\right] + P_4\cdot h + \frac{3}{2}ph^2 = M\frac{R}{r^2}h$$

$$\underline{\underline{v = 2\sqrt{\frac{gh\left(M\frac{R}{r^2} - P_4 - \frac{3}{2}ph\right)}{P_1\left(\frac{R}{r}\right)^2 + P_2\left(\frac{R}{r}\right)^2 + P_3 + 2P_4 + 2pl}}}}$$

Lösung 1076



Anfangswert: $T_A = 0$; $U_A = 0$

Endwert: $T_E = \frac{P_2}{2g}\dot x^2 + \frac{P_1}{2g}\cdot\frac{r^2}{2}\omega^2$;

$U_E = P_2 x \sin\alpha$; $\frac{\dot x}{r} = \omega$

$A = M\Delta\varphi - P_2\mu\cos\alpha x$; $\frac{x}{r} = \Delta\varphi$

$A = T_E + U_E - (T_A + U_A)$;

$$\frac{P_2}{2g}r^2\omega^2 + \frac{P_1 r^2}{4g}\omega^2 + P_2\cdot\Delta\varphi\cdot r\sin\alpha = M\Delta\varphi - P_2\mu\Delta\varphi\cos\alpha$$

$$\underline{\underline{\omega = \frac{2}{r}\sqrt{g\frac{M - P_2 r(\sin\alpha + \mu\cos\alpha)}{P_1 + 2P_2}\Delta\varphi}}}$$

Lösung 1077

Anfangswert: $T_A = 0$; $U_A = -\frac{pa^2}{2}\sin\alpha$;

Endwert: $T_E = \frac{P_2 + pl}{2g}\dot x^2 + \frac{P_1}{g}\,\frac{r^2}{4}\omega^2$; $U_E = P_2 x\sin\alpha - \frac{p(a - r\Delta\varphi)^2}{2}\sin\alpha$;

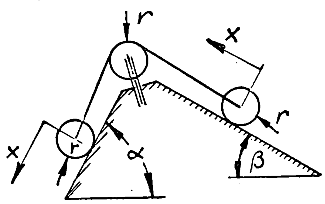
$\frac{\dot x}{r} = \omega$; $A = M\Delta\varphi - P_2\mu r\Delta\varphi\cos\alpha$

$A = T_E + U_E - (T_A + U_A)$:

$$M\Delta\varphi - P_2\mu r\Delta\varphi\cos\alpha = \omega^2\left[\frac{r^2}{4g}(2P_2 + P_1 + 2pl)\right] + \Delta\varphi\left[P_2 r\sin\alpha + \frac{pr}{2}\sin\alpha(2a - r\Delta\varphi)\right]$$

$$\underline{\underline{\omega = \frac{1}{r}\sqrt{2g\Delta\varphi\frac{2M - 2P_2 r(\sin\alpha + \mu\cos\alpha) - pr\sin\alpha(2a - r\Delta\varphi)}{P_1 + 2P_2 + 2pl}}}}$$

Lösung 1078



$T = 2m\frac{\dot x^2}{2} + 3\Theta\frac{\dot\varphi^2}{2}$; $\varphi = \frac{x}{r}$

$U = mgx(\sin\beta - \sin\alpha)$; $\Theta = m\frac{r^2}{2}$

$T + U = 0$:

$\frac{7}{4}m\dot x^2 + mgx(\sin\beta - \sin\alpha) = 0$

$$\underline{\underline{\dot x_{x=s} = v = 2\sqrt{\frac{1}{7}gs(\sin\alpha - \sin\beta)}}}$$

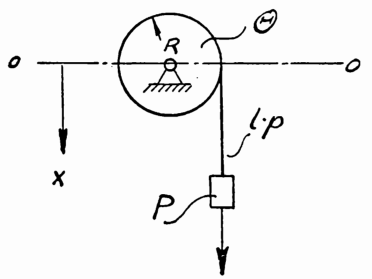
Lösung 1079

$$T+U=A; \qquad T+U=\frac{7}{4}m\dot{x}^2+mgx(\sin\beta-\sin\alpha); \text{ vgl. Aufgabe 1078}$$

$$A=-mg\frac{f}{r}x(\cos\alpha+\cos\beta)$$

$$\dot{x}_{x=s}=v=2\sqrt{\frac{1}{7}gs\left[\sin\alpha-\sin\beta-\frac{f}{r}(\cos\alpha+\cos\beta)\right]}$$

Lösung 1080



Anfangswert: $T_A=0$

$$U_A=-P\cdot x_0-x_0\cdot p\cdot\frac{x_0}{2}$$

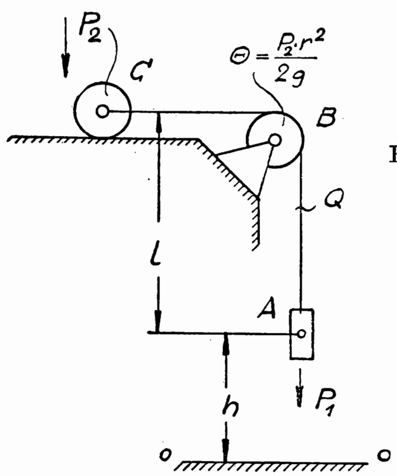
Endwert: $T_E=\frac{P}{g}\cdot\frac{\dot{x}^2}{2}+\frac{p\cdot l}{g}\cdot\frac{\dot{x}^2}{2}+\Theta\frac{\dot{x}^2}{2R^2}$

$$U_E=-P\cdot x-p\cdot x\cdot\frac{x}{2}$$

$T_A+U_A=T_E+U_E$; somit:

$$\dot{x}=v=R\sqrt{g\frac{[2P+p(x+x_0)](x-x_0)}{\Theta g+R^2(P+pl)}}$$

Lösung 1081



Anfangswert: $T_A=0$

$$U_A=P_1h+\frac{Q}{L}l\left(h+\frac{l}{2}\right)$$

$$+\frac{Q}{L}(L-l)(h+l)$$

Endwert: $T_E=\frac{P_2}{2g}v^2+2\frac{\Theta v^2}{2r^2}+\frac{P_1}{2g}v^2+\frac{Q}{2g}v^2$

$$U_E=\frac{Q}{2L}(h+l)^2+\frac{Q}{L}(h+l)[L-(h+l)]$$

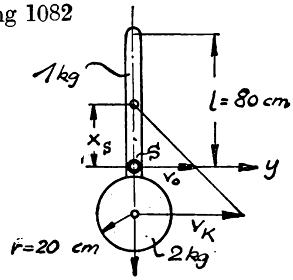
$$A=-\frac{P_2\cdot f}{r}\cdot h$$

$A=T_E+U_E-(T_A+U_A)$; somit:

$$v=\sqrt{2gh\frac{\left[P_1+\frac{Q}{2L}(h+2l)-P_2\frac{f}{r}\right]}{P_1+2P_2+Q}}$$

41. Ebene parallele Bewegung des starren Körpers

Lösung 1082



Gesamtschwerpunkt S:

$$x_S = \frac{2 \cdot (40 + 20)}{2 + 1} = 40 \text{ cm}$$

Bewegung des Schwerpunktes S:

$$x = g \frac{t^2}{2}; \quad y = v_0 \cdot t$$

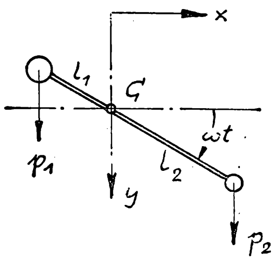
Nach dem Strahlensatz gilt:

$$v_0 = \frac{v_k}{60} \cdot 40 = \frac{2}{3} v_k$$

Somit: $x = \frac{g}{2} \frac{y^2}{v_0^2}$; $y^2 = \frac{2 v_0^2}{g} x$; $\underline{\underline{y^2 = 117{,}5\, x}}$

Drehbewegung: $\omega = \frac{v_k - v_0}{r} = \frac{v_k}{3r} = \underline{\underline{6 \text{ 1/sek}}}$

Lösung 1083



Lage des Schwerpunktes:

$$p_2 \cdot l - (p_1 + p_2) l_1 = 0$$

$$l_1 = \frac{p_2}{p_1 + p_2} \cdot l = \frac{1}{3} l; \quad l_2 = \frac{2}{3} l$$

Die Bewegung des Schwerpunktes ist senkrecht nach unten gerichtet

$$\ddot{y}_C = g; \quad \dot{y}_C = gt + C_0$$

$$y_C = \frac{gt^2}{2} + C_0 t + C_1$$

Anfangsbedingung für den Schwerpunkt:

$$t = 0: \quad y_C = 0; \quad \dot{y}_C = \frac{2}{3} v_1$$

Somit: $\underline{\underline{y_C = \frac{1}{2} g t^2 - \frac{2}{3} v_1 t}}$

$$\omega = \frac{v_1}{l} = \frac{60\pi}{60} = \underline{\underline{\pi \text{ 1/sek}}}$$

Für $t = 2$ sek gilt:

$$y_C = \frac{1}{2} g \cdot 4 - \frac{2}{3} \cdot 60\pi \cdot 2 = 1711 \text{ cm}$$

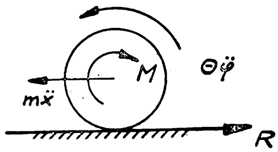
da bei $\omega t = 2\pi$ der Stab parallel zur x-Achse liegt, gilt:

$$\underline{\underline{h_1 = h_2 = 1711 \text{ cm}}}$$

Kraft im Stab:

$$\underline{\underline{T}} = m_1 \cdot \omega^2 \cdot l_1 = \frac{p_1 p_2 l}{g (p_1 + p_2)} \cdot \omega^2 = \underline{\underline{0{,}4 \text{ kg}}}$$

Lösung 1084

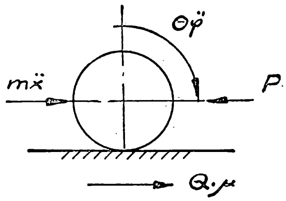


Für das Rollen mit Schlupf gilt:

$$\Theta\ddot{\varphi} - M + R\cdot r = 0$$
$$m\ddot{x} - R = 0$$
$$R = P\cdot\mu;\quad \Theta = m\varrho^2;\quad \varphi = \frac{x}{r}$$

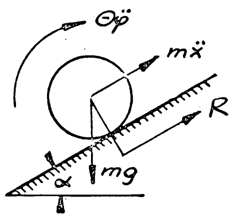
Somit: $\underline{\underline{M \leqq P\cdot\mu\cdot\frac{\varrho^2 + r^2}{r}}}$

Lösung 1085



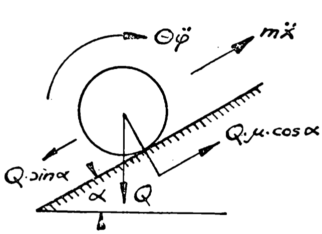
$$\Theta\ddot{\varphi} - Q\mu\cdot r = 0$$
$$\frac{Q}{g}\ddot{x} + Q\mu - P = 0$$
$$\Theta = \frac{Q}{g}\varrho^2;\quad \varphi = \frac{x}{r}$$
$$\underline{\underline{P \leqq Q\mu\frac{r^2+\varrho^2}{\varrho^2}}}$$

Lösung 1086



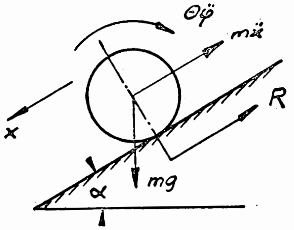
$$\Theta\ddot{\varphi} - R\cdot r = 0$$
$$m\ddot{x} + R - mg\sin\alpha = 0$$
$$\Theta = \frac{2}{5}mr^2;\quad R = mg\cos\alpha\cdot\mu;\quad \varphi = \frac{x}{r}$$
$$\frac{2}{5}mr^2\frac{\ddot{x}}{r} - mg\mu r\cos\alpha = 0$$
$$m\ddot{x} + mg(\mu\cos\alpha - \sin\alpha) = 0$$
$$\operatorname{tg}\alpha = \frac{7}{2}\mu;\quad \underline{\underline{\alpha \leqq \operatorname{arc}\operatorname{tg}\frac{7}{2}\mu}}$$

Lösung 1087



$$\Theta\ddot{\varphi} + \frac{Q}{g}\ddot{x}r - Qr\sin\alpha = 0$$
$$\varphi = \frac{x}{r};\quad \Theta = \frac{Q}{g}\frac{r^2}{2}$$
$$\ddot{x}\left(\frac{Q}{g}\frac{r^2}{2r} + \frac{Q}{g}r\right) = Qr\sin\alpha$$
$$\underline{\underline{\ddot{x} = b = \frac{2}{3}g\sin\alpha}}$$
$$Q\sin\alpha - Q\mu\cos\alpha - \frac{Q}{g}\ddot{x} = 0$$
$$\sin\alpha - \mu\cos\alpha - \frac{2}{3}\sin\alpha = 0$$
$$\underline{\underline{\alpha \leqq \operatorname{arc}\operatorname{tg} 3\mu}}$$

Lösung 1088

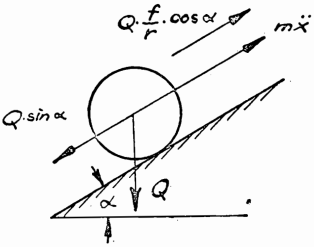


$$m\ddot{x} + R - mg\sin\alpha = 0$$
$$R = mg\,\mu\cos\alpha$$
$$\underline{\underline{\ddot{x} = b = g(\sin\alpha - \mu\cos\alpha)}}$$

Ohne Schlupf gilt:

$$\Theta\ddot{\varphi} = mg\,\mu r\cos\alpha$$
$$\frac{mr^2}{2}\cdot\frac{g}{r}(\sin\alpha - \mu\cos\alpha) = mgr\,\mu\cos\alpha$$
$$\operatorname{tg}\alpha = 3\mu$$
$$\underline{\underline{\alpha \lesseqgtr \operatorname{arc\,tg} 3\mu \quad \begin{matrix}\text{ohne Schlupf}\\ \text{mit Schlupf}\end{matrix}}}$$

Lösung 1089



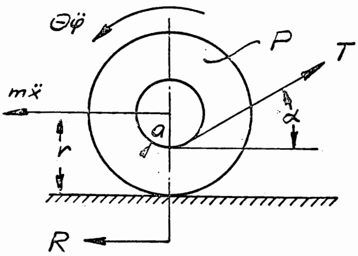
Bei gleichförmiger Bewegung gilt:

$$\ddot{x} = 0$$

Demnach:

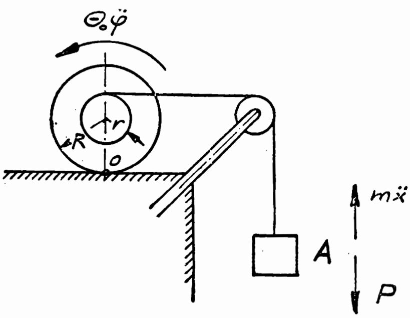
$$Q\sin\alpha = \frac{Q\cdot f}{r}\cos\alpha$$
$$\underline{\underline{f = r\operatorname{tg}\alpha}}$$

Lösung 1090



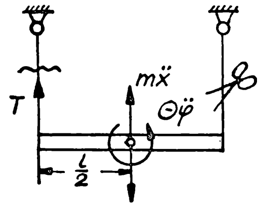
$$R + m\ddot{x} - T\cos\alpha = 0$$
$$R\cdot r - Ta - \Theta\ddot{\varphi} = 0$$
$$\left.\begin{aligned} m\ddot{x} - T\cos\alpha + R &= 0 \\ -m\frac{\varrho^2\ddot{x}}{r} - T\cdot a + Rr &= 0 \end{aligned}\right| \cdot(-r)$$
$$-m\ddot{x}\left(r + \frac{\varrho^2}{r}\right) + T\cos\alpha\left(r - \frac{a}{\cos\alpha}\right) = 0$$
$$\ddot{x} = \frac{T}{P}\cdot\frac{(r\cos\alpha - a)\,r}{(r^2 + \varrho^2)}\,g$$
$$\underline{\underline{x = \frac{T}{P}\,\frac{rg\,(r\cos\alpha - a)}{2\,(\varrho^2 + r^2)}\cdot t^2}}$$

Lösung 1091



$$\Theta_0\ddot{\varphi} - (P - m\ddot{x})(R + r) = 0; \quad \varphi = \frac{x}{(R + r)}$$
$$\frac{Q}{g}\,\frac{(R^2 + \varrho^2)\,\ddot{x}}{(R + r)} - \left(P - \frac{P}{g}\ddot{x}\right)(R + r) = 0$$
$$\underline{\underline{b = \ddot{x} = g\,\frac{P\,(R + r)^2}{Q\,(R^2 + \varrho^2) + P\,(R + r)^2}}}$$

Lösung 1092



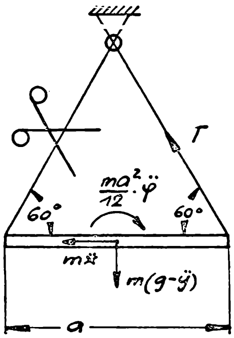
$$m\ddot{x} + T - mg = 0$$

$$T \cdot \frac{l}{2} - \Theta\ddot{\varphi} = 0; \quad \Theta = m\frac{l^2}{12}; \quad \varphi = \frac{x \cdot 2}{l}$$

$$m\ddot{x} = 3T$$

$$3T + T - P = 0; \quad \underline{\underline{T = \frac{P}{4}}}$$

Lösung 1093



Gleichgewichtsbedingungen:

$$T\frac{\sqrt{3}}{2} = m(g - \ddot{y})$$

$$T\frac{1}{2} = -m\ddot{x}$$

$$T\frac{a}{4}\sqrt{3} = \frac{ma^2}{12}\ddot{\varphi}$$

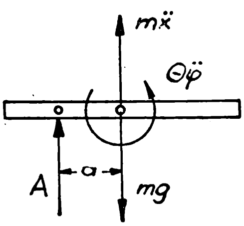
Zwangsbedingung: $\ddot{x}\frac{\sqrt{3}}{3} = \frac{a}{2}\ddot{\varphi} - \ddot{y}$

Somit: $-T\frac{\sqrt{3}}{6} = \frac{3\sqrt{3}}{2}T + T\frac{\sqrt{3}}{2} - mg$

$$mg = T\left(\frac{\sqrt{3}\cdot 13}{6}\right)$$

$$\underline{\underline{T = 0{,}266\,P}}$$

Lösung 1094



$$A + m\ddot{x} - mg = 0$$

$$\Theta\ddot{\varphi} - A \cdot a = 0$$

$$\Theta = \frac{m(2l)^2}{12} = \frac{ml^2}{3}; \quad \varphi = \frac{x}{a}$$

$$m\ddot{x} = A \cdot \frac{3a^2}{l^2};$$

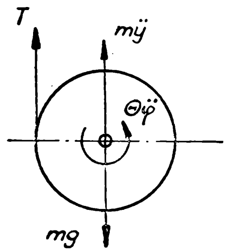
$$A\left(1 + \frac{3a^2}{l^2}\right) - mg = 0$$

$$A = \frac{l^2}{3a^2 + l^2} \cdot mg$$

$$\Delta A = A - A_{\text{Stat.}} = A - \frac{mg}{2}$$

$$\underline{\underline{\Delta A = \frac{l^2 - 3a^2}{2(3a^2 + l^2)} \cdot P}}$$

Lösung 1095



$$\Theta\ddot{\varphi} + m\ddot{y}r - mgr = 0$$

$$\varphi = \frac{y}{r}; \qquad \Theta = m\frac{r^2}{2}$$

$$\ddot{y} = \frac{2}{3}g$$

$$\dot{y} = \frac{2}{3}gt + v_0$$

$$y = \frac{1}{3}gt^2 + v_0 t + y_0$$

Anfangsbedingungen: $t = 0$: $y = 0$; $\dot{y} = 0$

Somit: $y = \frac{1}{3}gt^2$; $\dot{y} = \frac{2}{3}gt$

$$y = \frac{1}{3}g\dot{y}^2 \cdot \frac{9}{4g^2}$$

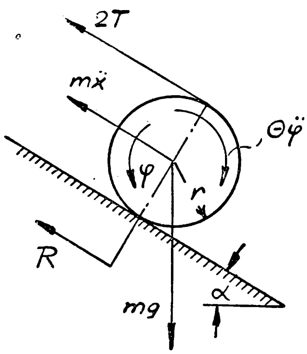
$$\dot{y} = \frac{2}{3}\sqrt{3gy}; \qquad \dot{y}_{y=h} = \underline{\underline{v = \frac{2}{3}\sqrt{3gh}}}$$

Fadenspannung: $T + m\ddot{y} - mg = 0$

$$T = mg\left(1 - \frac{2}{3}\right)$$

$$\underline{\underline{T = \frac{1}{3}mg}}$$

Lösung 1096



$$2T + m\ddot{x} + R - mg\sin\alpha = 0$$

$$\Theta\ddot{\varphi} + R\cdot r - 2Tr = 0$$

$$x = r\varphi; \qquad \Theta = \frac{mr^2}{2}$$

$$\underline{\underline{T = \frac{1}{6}P(\sin\alpha + \mu\cos\alpha)}}$$

$$\ddot{x} = \frac{2}{3}g(\sin\alpha - 2\mu\cos\alpha)$$

$$\underline{\underline{x = s = \frac{g}{3}(\sin\alpha - 2\mu\cos\alpha)t^2}}$$

Der Zylinder bleibt in Ruhe für $\ddot{x} = 0$

$$0 = \sin\alpha - 2\mu\cos\alpha$$

$$\underline{\underline{\operatorname{tg}\alpha \leq 2\mu}}$$

Lösung 1097

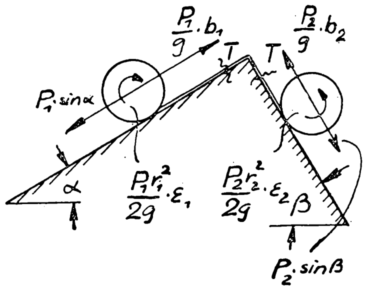
$$P_1\left(\frac{b_1}{g} - \sin\alpha\right) + T = 0 \tag{1}$$

$$P_2\left(\frac{b_2}{g} - \sin\beta\right) + T = 0 \tag{2}$$

$$\frac{P_1 r_1^2}{2g}\varepsilon_1 - Tr_1 = 0 \tag{3}$$

$$\frac{P_2 r_2^2}{2g}\varepsilon_2 - Tr_2 = 0 \tag{4}$$

$$r_1\varepsilon_1 + r_2\varepsilon_2 - b_1 - b_2 = 0 \tag{5}$$



Aus (3) u. (4): $r_1\varepsilon_1 + r_2\varepsilon_2 - T\,2g\left(\frac{1}{P_1}+\frac{1}{P_2}\right) = 0$ (6)

Aus (5) u. (6): $b_1 + b_2 = T\,2g\left(\frac{1}{P_1}+\frac{1}{P_2}\right)$ (7)

Aus (1) u. (2): $b_1 + b_2 = -Tg\left(\frac{1}{P_1}+\frac{1}{P_2}\right) + g\sin\alpha + g\sin\beta$ (8)

Aus (7) u. (8): $\underline{\underline{T = \frac{P_1 P_2 (\sin\alpha + \sin\beta)}{3(P_1+P_2)}}}$ (9)

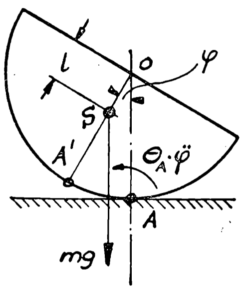
$$b_{\text{Faden}} = b_1 - r_1\varepsilon_1$$

Aus (1) u. (3): $b_1 - g\sin\alpha - r_1\varepsilon_1 + \frac{3Tg}{P_1} = 0$

$$b_F = g\left(\sin\alpha - \frac{3T}{P_1}\right)$$

$$\underline{\underline{b_F = g\,\frac{(P_1\sin\alpha - P_2\sin\beta)}{P_1+P_2}}}$$

Lösung 1098



$$\Theta_A\ddot\varphi + mgl\varphi = 0; \qquad l = \frac{4R}{3\pi}$$

$$\ddot\varphi + \frac{mgl}{\Theta_A}\cdot\varphi = 0; \qquad \ddot\varphi + \omega^2\varphi = 0$$

Für kleine Ausschläge gilt: $T = 2\frac{\pi}{\omega}$

$$\Theta_A = \Theta_A' = \Theta_0 - ml^2 + m(R-l)^2$$

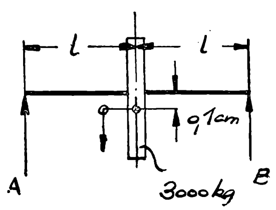
$$\Theta_A = mR^2\cdot\frac{9\pi-16}{6\pi}$$

$$T = 2\pi\sqrt{\frac{R(9\pi-16)}{8g}}$$

$$\underline{\underline{T = \frac{\pi}{2g}\sqrt{2gR(9\pi-16)}}}$$

42. Zusätzliche Kräfte auf die Drehachse rotierender Körper

Lösung 1099



$$R_A = R_B$$

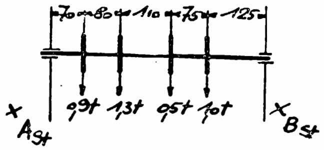
$$R_A = \frac{G}{2} + \frac{1}{2}m\omega^2\cdot r$$

$$R_A = \frac{3000}{2} + \frac{3000}{2\cdot 981}\cdot 0{,}1\cdot\frac{\pi^2\cdot 1200^2}{30^2}$$

$$\underline{\underline{R_A = R_B = 1500 + 2400\ \text{kg}}}$$

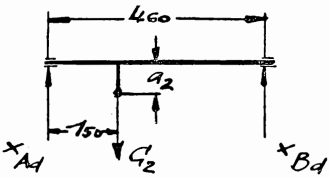
Lösung 1100

statische Belastung

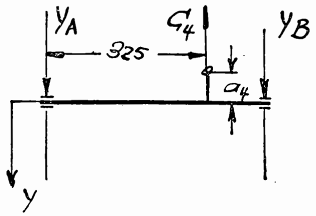


dynamische Belastung

xz-Ebene



yz-Ebene



Statische Belastung:

$$\sum M_A = 0:$$

$$X_{B_{St}} \cdot 460 = 1 \cdot 335 + 0{,}5 \cdot 260 + 1{,}3 \cdot 150 + 0{,}9 \cdot 70 + 1{,}3 \cdot 230$$

$$X_{B_{St}} = \frac{1023}{460} = 2{,}22 \text{ t}$$

$$X_{A_{St}} = [0{,}9 + 1{,}3 + 1{,}3 + 0{,}5 + 1{,}0] - X_{B_{St}} = 2{,}78 \text{ t}$$

Dynamische Belastung:

xz-Ebene:

$$\omega = \frac{\pi n}{30} = \pi \cdot 100$$

$$C_2 = \frac{1{,}3}{981} \cdot \pi^2 \cdot 100^2 \cdot 0{,}1 = 13{,}1 \text{ t}$$

$$\sum M_A = 0: \quad X_{Bd} \cdot 460 = 13{,}1 \cdot 150$$

$$X_{Bd} = 4{,}26 \text{ t}$$

$$X_{Ad} = C_2 - X_{Bd} = 8{,}84 \text{ t}$$

yz-Ebene:

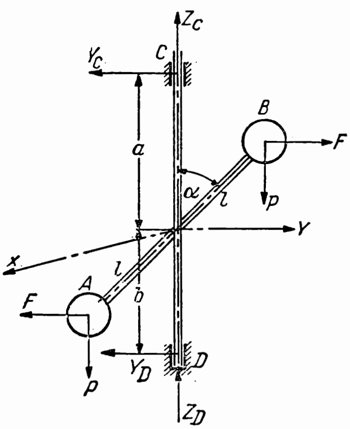
$$C_4 = \frac{1{,}0}{981} \cdot 100^2 \pi^2 \cdot 0{,}1 = 10{,}5 \text{ t}$$

$$Y_{Bd} \cdot 460 = 335 \cdot 10{,}5$$

$$Y_{Bd} = 7{,}33 \text{ t}$$

$$Y_{Ad} = C_4 - Y_{Bd} = 2{,}73 \text{ t}$$

Lösung 1101



Da in der x-Richtung keine Aktionen wirken, werden auch keine Reaktionen hervorgerufen, also: $X_C = 0$; $X_D = 0$

$$\sum M_D = 0:$$

$$Y_C(a+b) - \frac{P}{g}\omega^2 \cdot l \sin\alpha\,(b + l\cos\alpha) + \frac{P}{g}\omega^2 l \sin\alpha\,(b - l\cos\alpha) + P \cdot l \sin\alpha - P \cdot l \sin\alpha = 0$$

$$Y_C = \frac{P\omega^2 l \sin\alpha \cdot 2l\cos\alpha}{g(a+b)}$$

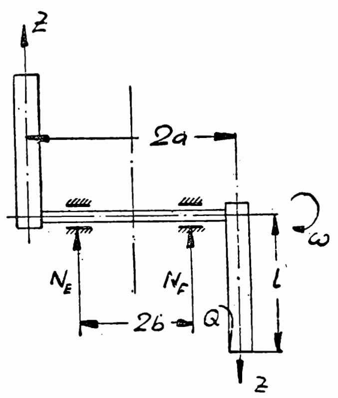
$$Y_C = \frac{P\omega^2 l^2 \sin 2\alpha}{g(a+b)}$$

$$\sum P_y = 0: \quad Y_C + Y_D = 0$$

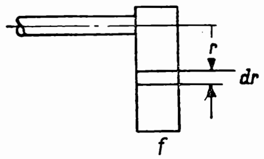
$$Y_C = -Y_D$$

$$\sum P_z = 0: \quad Z_D - 2P = 0; \quad Z_D = 2P$$

Lösung 1102



Fliehkraft einer Kurbel:



$$dZ = dm\,\omega^2 \cdot r$$

$$dm = \varrho \cdot f \cdot dr$$

$$Z = \varrho f \omega^2 \cdot \frac{l^2}{2} = m\omega^2 \cdot \frac{l}{2}$$

Dynamische Belastung:

$\sum M_E = 0$:

$$-Z(a-b) + N_{Fd} \cdot 2b - Z(a+b) = 0$$

$$N_{Fd} = \frac{Z \cdot a}{b}$$

$\sum P_x = 0$: $\quad N_{Fd} + N_{Ed} - Z + Z = 0$

$$N_{Ed} = -\frac{Z \cdot a}{b}$$

Statische Belastung:

$$N_{F_{st}} = N_{E_{st}} = Q + \frac{P}{2}$$

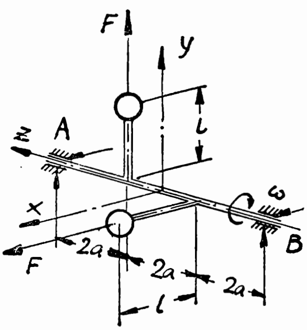
Gesamte Belastung (Lagerreaktionen):

$$\underline{\underline{N_E = Q + \frac{P}{2} - \frac{Q\omega^2 \cdot la}{2gb}}}$$

$$\underline{\underline{N_F = Q + \frac{P}{2} + \frac{Q \cdot \omega^2 la}{2gb}}}$$

Die Lageraktionen sind entgegengesetzt gerichtet.

Lösung 1103



Dynamische Belastung:

xz-Ebene:

$\sum M_B = 0$: $\quad X_A \cdot 6a + m\omega^2 \cdot l \cdot 2a = 0$

$$X_A = -\frac{1}{3}\, m\omega^2 l$$

yz-Ebene:

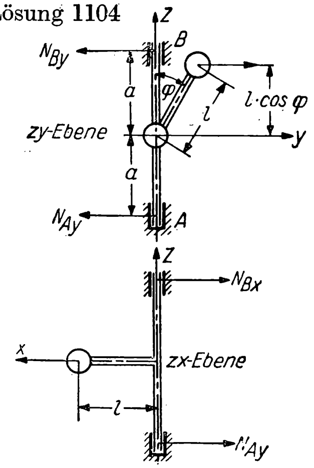
$\sum M_B = 0$: $\quad Y_A \cdot 6a + m\omega^2 l \cdot 4a = 0$

$$Y_A = -\frac{2}{3}\, m\omega^2 l$$

$$\underline{\underline{N_A}} = \sqrt{X_A^2 + Y_A^2} = \underline{\underline{\frac{\sqrt{5}}{3} \cdot m\omega^2 \cdot l}}$$

$$\underline{\underline{N_A = -N_B}}$$

Lösung 1104



zy-Ebene:

$$\sum M_A = 0:\quad m\omega^2\cdot l\sin\varphi\,(a + l\cos\varphi) - N_{By}\cdot 2a = 0$$

$$\underline{\underline{N_{By} = \frac{m l\omega^2\,(a + l\cos\varphi)\sin\varphi}{2a}}}$$

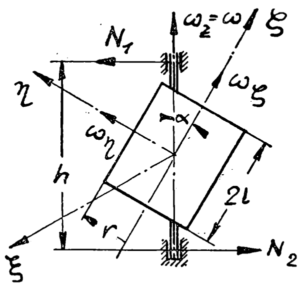
$$\sum P_y = 0:\quad N_{By} + N_{Ay} - m\omega^2\cdot l\sin\varphi = 0$$

$$\underline{\underline{N_{Ay} = \frac{m l\omega^2\,(a - l\cos\varphi)\sin\varphi}{2a}}}$$

xz-Ebene:

$$\underline{\underline{N_{Ax} = N_{Bx} = \frac{m\omega^2 l}{2}}}$$

Lösung 1105



$\omega_\zeta = \omega_z\cdot\cos\alpha;\quad \omega_\eta = \omega_z\cdot\sin\alpha$

Nach den Eulerschen Gleichungen ergibt sich für die Kreiselmomente:

$$M_\zeta = 0;\quad M_\eta = 0;\quad M_\xi = (\Theta_\zeta - \Theta_\eta)\,\omega_\zeta\cdot\omega_\eta$$

$$\Theta_\zeta = \frac{P r^2}{2g};\quad \Theta_\eta = \frac{1}{16}\cdot\frac{P}{g}\left(4r^2 + \frac{4\cdot 4 l^2}{3}\right)$$

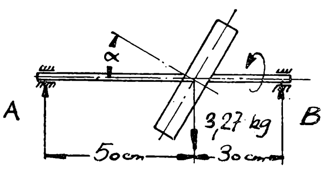
$$M_\xi = -\frac{\omega^2\sin\alpha\cos\alpha\cdot P}{g}\left[\frac{1}{3}l^2 - \frac{1}{4}r^2\right]$$

$$\sum M_2 = 0:\quad N_1\cdot h + M_\xi = 0$$

$$\underline{\underline{N_1 = \frac{\omega^2\cdot\sin 2\alpha\cdot P}{2gh}\left[\frac{1}{3}l^2 - \frac{1}{4}r^2\right]}}$$

$$\sum P_N = 0:\quad \underline{\underline{N_1 = N_2}}$$

Lösung 1106



Statische Belastung:

$$\underline{\underline{N_{A\,\mathrm{st}}}} = \frac{3{,}27\cdot 30}{50+30} = \underline{\underline{1{,}23\ \mathrm{kg}}}$$

$$\underline{\underline{N_{P\,\mathrm{st}}}} = 3{,}27 - 1{,}23 = \underline{\underline{2{,}04\ \mathrm{kg}}}$$

Dynamische Belastung:

$$\omega = \frac{\pi\cdot 30\,000}{30} = 1000\,\pi$$

Unter Verwendung von Aufgabe 1105 gilt bei folgenden Berücksichtigungen:

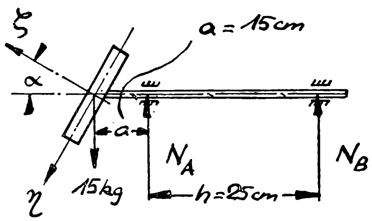
$$\frac{\sin 2\alpha}{2} = \sin\alpha\cos\alpha = 0{,}02 \qquad \alpha = \text{kleiner Winkel}$$

$$l = 0;\quad h = 80$$

$$N_{Ad} = \frac{1000^2\cdot\pi^2\cdot 0{,}02\cdot 3{,}27}{981\cdot 80}\cdot\frac{20^2}{4}$$

$$\underline{\underline{N_{Ad} = -N_{Bd} = 822\ \mathrm{kg}}}$$

Lösung 1107



Statische Belastung (Lagerreaktionen):

$$\underline{\underline{N_A}} = \frac{15(15+25)}{25} = \underline{\underline{24\ \mathrm{kg}}}$$

$$\underline{\underline{N_B}} = P - N_A = \underline{\underline{-9\ \mathrm{kg}}}$$

Dynamische Belastung (Lagerreaktionen):

Nach Aufgabe 1105 gilt mit

$$\Theta_\eta = 0; \quad \Theta_\xi = \Theta; \quad \omega_\xi \cdot \omega_\eta = \omega^2 \cdot \alpha:$$

$$M_\xi = \omega^2 \cdot \alpha \cdot \Theta$$

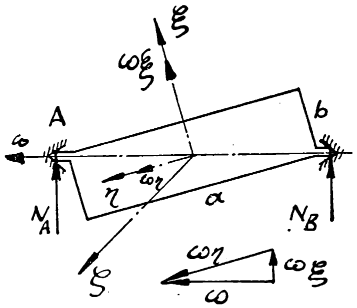
$$N_{Ad} = \frac{M_\xi}{h} = \left(\frac{3000\,\pi}{30}\right)^2 \cdot \frac{0{,}015 \cdot 0{,}5}{0{,}25}$$

$$\underline{\underline{N_{Ad} = 2960\ \mathrm{kg}}}$$

$$\underline{\underline{N_{Bd} = -N_{Ad}}}$$

Die Lageraktionen sind entgegengesetzt gerichtet.

Lösung 1108



$$\omega_\xi = \omega \frac{b}{\sqrt{a^2+b^2}}; \qquad \omega_\eta = \omega \frac{a}{\sqrt{a^2+b^2}}$$

$$\Theta_\xi = \frac{1}{12}\,\frac{P}{g}\,a^2; \qquad \Theta_\eta = \frac{1}{12}\,\frac{P}{g}\,b^2$$

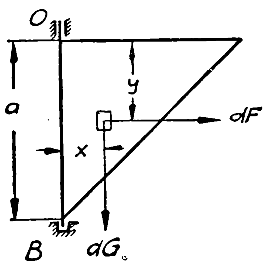
Eulersche Gleichung:

$$M_\zeta = (\Theta_\xi - \Theta_\eta) \cdot \omega_\xi \cdot \omega_\eta$$

$$\underline{\underline{N_{Ay}}} = \frac{M_\zeta}{\sqrt{a^2+b^2}} = \underline{\underline{\frac{P a b \omega^2 (a^2-b^2)}{12\, g\,(a^2+b^2)^{3/2}}}}$$

$$\underline{\underline{N_{By} = -N_{Ay}}}$$

Lösung 1109



$$\Sigma M_0 = 0: \quad G\frac{a}{3} = \varrho \cdot s \cdot \frac{a^2}{2} \cdot \frac{a}{3} \cdot g$$

$$d\,M_F = \omega^2 \cdot \varrho \cdot s \cdot x \cdot y \cdot dx\,dy$$

$s =$ Dicke der Platte

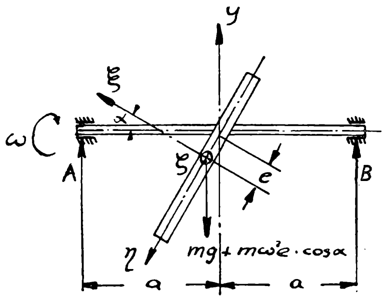
$$M_F = \omega^2 \cdot \varrho \cdot s \int_{x=0}^{a} \int_{y=0}^{x-a} x y\, dx\, dy$$

$$M_F = \frac{1}{24}\,\omega^2 \varrho \cdot s \cdot a^4$$

$$G\frac{a}{3} = M_F$$

$$\omega^2 = 4\,\frac{g}{a}; \quad \underline{\underline{\omega = 2\sqrt{\frac{g}{a}}}}$$

Lösung 1110



Statische Belastung:

$$\sum M_A = 0: \quad B' \cdot 2a = mg(a - e\sin\alpha)$$

$$\underline{\underline{B' = P\frac{a - e\sin\alpha}{2a}}}; \quad A' = P - B' = \underline{\underline{P\frac{a + e\sin\alpha}{2a}}}$$

Dynamische Belastung:

Hauptachsenmoment: $M_\zeta = (\Theta_\xi - \Theta_\eta)\,\omega_\xi\omega_\eta$

$$\omega_\xi = \omega\cos\alpha; \quad \omega_\eta = \omega\sin\alpha$$

$$\Theta_\xi = \frac{P}{g}\cdot\frac{r^2}{2}; \quad \Theta_\eta = \frac{1}{4}\,\frac{P}{g}\,r^2$$

$$M_\zeta = \frac{P\omega^2}{2g}\sin 2\alpha\cdot\frac{r^2}{4}$$

Fliehkraft der im Schwerpunkt angreifenden Masse: $F = m\,\omega^2 e\cos\alpha$

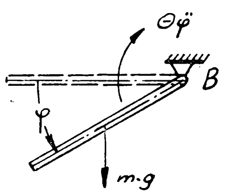
$$\sum M_A = 0: \quad B''\cdot 2a + \frac{P\omega^2}{2g}\sin 2\alpha\frac{r^2}{4} - \frac{P}{g}\,\omega^2 e\cos\alpha\,(a - e\sin\alpha) = 0$$

$$B'' = \frac{P\omega^2}{2g}\left[e\cos\alpha - \frac{\sin 2\alpha}{2a}\left(2e^2 + \frac{r^2}{4}\right)\right] \Bigg\|$$

$$A'' = \frac{P\omega^2}{2g}\left[e\cos\alpha + \frac{\sin 2\alpha}{2a}\left(2e^2 + \frac{r^2}{4}\right)\right] \Bigg\|$$

43. Gemischte Aufgaben

Lösung 1111



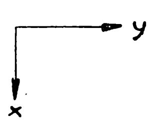
$$\Theta\ddot{\varphi} - lmg\cos\varphi = 0$$

$$\Theta\frac{d\omega}{d\varphi}\cdot\omega = lmg\cos\varphi$$

$$\Theta\omega\,d\omega = lmg\cos\varphi\,d\varphi; \quad \Theta = \frac{4}{3}ml^2$$

$$\omega^2\cdot\frac{2}{3}l = g\sin\varphi$$

$$\omega = \sqrt{\frac{3g\sin\varphi}{2l}}; \quad \underline{\underline{\omega_{\varphi=\frac{\pi}{2}} = \sqrt{\frac{3g}{2l}}}}$$



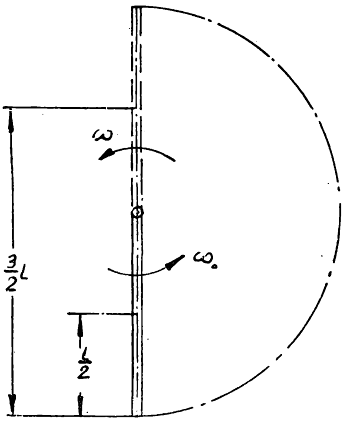
Bewegungsbahn des Schwerpunktes:

$$y = v_0 t; \quad v_0 = \omega_{\varphi=\frac{\pi}{2}}\cdot l$$

$$x = \frac{1}{2}gt^2 + l$$

$$\underline{\underline{y^2 = 3l\,(x - l)}}$$

Lösung 1112



$$T_A + U_A = T_E + U_E$$

$$\frac{\Theta\omega_0^2}{2} + mg\frac{l}{2} = \frac{\Theta\omega^2}{2} + mg\frac{3}{2}l$$

$$\Theta = m\frac{l^2}{3}$$

$$\omega^2 = \omega_0^2 - 6\frac{g}{l}; \qquad \underline{\underline{\omega = \sqrt{\frac{3g}{l}}}}$$

Freie Bewegung des Stabschwerpunktes:

$$\ddot{y}_C = -g, \qquad \dot{y}_C = -gt + \dot{y}_0$$

$$y_C = -\frac{1}{2}gt^2 + y_0; \qquad \text{Anfangsbedingungen:}$$

$$t = 0: \qquad \dot{y}_0 = 0$$

$$y_0 = \frac{l}{2}$$

$$\ddot{x}_C = 0; \qquad \dot{x}_C = -\omega_1\frac{l}{2}; \qquad x_C = -\omega_1\frac{lt}{2}$$

$$y_C = -\frac{g}{2}\cdot\frac{4x_C^2 l}{l^2\cdot 3g} + \frac{l}{2}$$

$$\underline{\underline{y_C = \frac{l}{2} - \frac{2}{3}\frac{x_C^2}{l}}}$$

Lösung 1113

$$T_A + U_A = T_E + U_E: \qquad \frac{\Theta}{2}\omega_0^2 = \frac{\Theta}{2}\omega^2 + mga; \qquad \Theta = \frac{4ma^2}{3}$$

$$\omega^2 = \omega_0^2 - \frac{3g}{2a}$$

Der zurückgelegte Schwerpunktsweg des freien Falles nach dem Ablösen:

$$-a = a\omega t - \frac{g}{2}t^2$$

Der Stab muß sich dabei um den Winkel $\omega t = \frac{(2k+1)\pi}{2}$ gedreht haben, um senkrecht aufzustoßen.

$$t = \frac{(2k+1)\pi}{2\omega}; \qquad -a = a\frac{(2k+1)\pi}{2} - \frac{g}{8}\,\frac{(2k+1)^2\pi^2}{\omega^2}$$

$$\omega^2 = \frac{g}{4a}\cdot\frac{(2k+1)^2\pi^2}{[2+(2k+1)\pi]}$$

$$\underline{\underline{\omega_0^2}} = \omega^2 + \frac{6}{4}\frac{g}{a} = \underline{\underline{\frac{g}{4a}\left[6 + \frac{\pi^2(2k+1)^2}{\pi(2k+1)+2}\right]}}$$

Lösung 1114

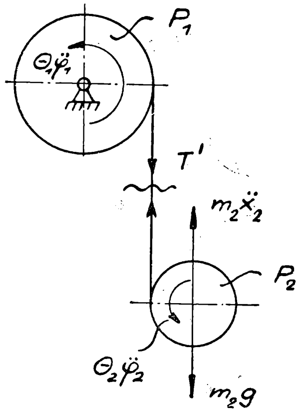
$$\Theta_1\ddot{\varphi}_1 - T'r_1 = 0$$

$$T'r_2 - \Theta_2\ddot{\varphi}_2 = 0$$

$$T' + m_2\ddot{x} - m_2 g = 0$$

$$x = r_1\varphi_1 + r_2\varphi_2; \qquad T' = 2T$$

$$\Theta = \frac{P\cdot r^2}{2g}$$



Somit:

$$\frac{P_1 r_1^2}{2g}\ddot{\varphi}_1 - T' r_1 = 0$$

$$-\frac{P_2 r_2^2}{2g}\ddot{\varphi}_2 + T' r_2 = 0$$

$$\frac{P_2 r_1}{g}\ddot{\varphi}_1 + \frac{P_2 r_2}{g}\ddot{\varphi}_2 + T' = m_2 g$$

Daraus:

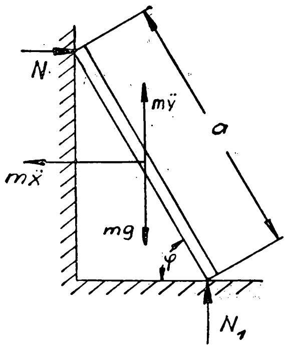
$$\underline{\underline{\ddot{\varphi}_1}} = \frac{2P_2 g}{r_1(3P_1+2P_2)}; \qquad \underline{\underline{\omega_1 = \frac{2P_2 g\cdot t}{r_1(3P_1+2P_2)}}}$$

$$\underline{\underline{\ddot{\varphi}_2}} = \frac{2P_1 g}{r_2(3P_1+2P_2)}; \qquad \underline{\underline{\omega_2 = \frac{2P_1 g\cdot t}{r_2(3P_1+2P_2)}}}$$

$$\underline{\underline{x}} = s = r_1\varphi_1 + r_2\varphi_2 = \underline{\underline{\frac{g(P_1+P_2)\cdot t^2}{3P_1+2P_2}}}$$

$$\underline{\underline{T = \frac{P_1 P_2}{2(3P_1+2P_2)}}};$$

Lösung 1115



Anfangswert: $T_A = 0$

$$U_A = mg\frac{a}{2}\sin\varphi_0$$

Endwert: $T_E = \frac{m}{2}v^2 + \frac{\Theta}{2}\dot{\varphi}^2$

$$U_E = mg\cdot\frac{a}{2}\sin\varphi$$

$$\Theta = \frac{ma^2}{12}; \quad v = \dot{\varphi}\cdot\frac{a}{2}$$

$T_A + U_A = T_E + U_E$:

$$\underline{\underline{\dot{\varphi} = \sqrt{\frac{3g}{a}(\sin\varphi_0 - \sin\varphi)}}}$$

$$2\dot{\varphi}\ddot{\varphi} = -\frac{3g}{a}\dot{\varphi}\cos\varphi$$

$$\underline{\underline{\ddot{\varphi} = -\frac{3g}{2a}\cos\varphi}}$$

Der Stab löst sich von der Wand, wenn $N = 0$ ist.

$$N = m\ddot{x}; \quad \text{somit } \ddot{x} = 0; \quad x = \frac{a}{2}\cos\varphi$$

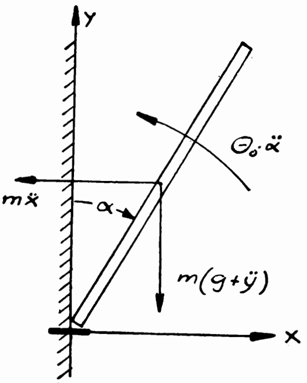
$$\dot{x} = -\frac{a}{2}\dot{\varphi}\sin\varphi$$

$$\ddot{x} = -\frac{a}{2}[\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi]$$

$$\ddot{x} = 0; \quad \frac{3g}{2a}\sin\varphi_1\cos\varphi_1 = \frac{3g}{a}(\sin\varphi_0 - \sin\varphi_1)\cdot\cos\varphi_1$$

$$\underline{\underline{\sin\varphi_1 = \frac{2}{3}\sin\varphi_0}}$$

Lösung 1116



Damit sich das Brett abhebt, muß sein:

$$g+\ddot{y}=0$$

$$y=l\cos\alpha$$

$$\dot{y}=-l\dot{\alpha}\sin\alpha$$

$$\ddot{y}=-l(\ddot{\alpha}\sin\alpha+\dot{\alpha}^2\cos\alpha)$$

$$\ddot{\alpha}\sin\alpha+\dot{\alpha}^2\cos\alpha=\frac{g}{l} \qquad (1)$$

$\sum M_0=0;$

$$\Theta_0\ddot{\alpha}=mgl\sin\alpha; \quad \Theta_0=\frac{4}{3}ml^2$$

$$\frac{4}{3}l\ddot{\alpha}=g\sin\alpha$$

$$\ddot{\alpha}=\frac{3}{4}\frac{g}{l}\sin\alpha$$

$$\frac{d\dot{\alpha}}{dt}=\frac{d\dot{\alpha}}{d\alpha}\cdot\dot{\alpha}=\frac{3}{4}\frac{g}{l}\sin\alpha$$

$$\frac{\dot{\alpha}^2}{2}=-\frac{3}{4}\frac{g}{l}\cos\alpha+C \qquad t=0:\ \alpha=0,\ \dot{\alpha}=0$$

$$\dot{\alpha}^2=\frac{3}{2}\frac{g}{l}(1-\cos\alpha)$$

Mit diesen Werten ergibt sich aus (1):

$$\underline{\underline{\alpha=\arccos\frac{1}{3}=70^\circ\,32'.}}$$

Lösung 1117

$$T_A=\frac{\Theta_1\omega_1^2}{2}+\frac{\Theta_2\omega_2^2}{2}; \quad T_E=(\Theta_1+\Theta_2)\frac{\omega^2}{2}$$

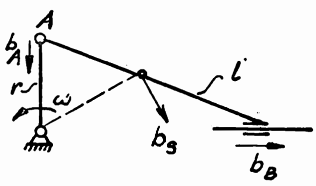
$$\Theta_1\omega_1+\Theta_2\omega_2=(\Theta_1+\Theta_2)\omega; \quad \omega=\frac{\Theta_1\omega_1+\Theta_2\omega_2}{(\Theta_1+\Theta_2)} \quad \text{(Drallsatz)}$$

$$\Delta T=\frac{1}{2}\left[\Theta_1\omega_1^2+\Theta_2\omega_2^2-\frac{(\Theta_1\omega_1+\Theta_2\omega_2)^2}{(\Theta_1+\Theta_2)}\right]$$

$$\underline{\underline{\Delta T=\frac{1}{2}\frac{\Theta_1\Theta_2}{(\Theta_1+\Theta_2)}(\omega_1-\omega_2)^2}}$$

Lösung 1118

1. Vertikale Lage der Kurbel:



$$\mathfrak{b}_B=\mathfrak{b}_A+\mathfrak{b}_{AB}$$

$$b_B=b_A\cdot\frac{r}{\sqrt{l^2-r^2}}$$

$$b_{AB}=b_A\cdot\frac{l}{\sqrt{l^2-r^2}}$$

$$\mathfrak{b}_S=\frac{\mathfrak{b}_A+\mathfrak{b}_B}{2}$$

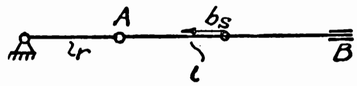
$$b_{SV}=\frac{b_A}{2}; \quad b_{SH}=\frac{b_B}{2}$$

$$b_S = \sqrt{\frac{b_A^2 + b_B^2}{4}} = \frac{b_A}{2}\sqrt{\frac{l^2}{l^2 - r^2}}$$

$$\underline{\underline{V}} = \frac{P}{g}\, b_S = \underline{\underline{\frac{P\omega_0^2}{2g}\cdot\frac{l\cdot r}{\sqrt{l^2 - r^2}}}}$$

$$\varepsilon_S = \frac{b_{AB}}{l} = \frac{b_A}{\sqrt{l^2 - r^2}}; \quad M_S = \Theta\cdot\varepsilon_S = \underline{\underline{\frac{P l^2 \omega_0^2 r}{12 g\sqrt{l^2 - r^2}}}}$$

2. Horizontale Lage der Kurbel:



$$\mathfrak{b}_B = \mathfrak{b}_A + \mathfrak{b}_{AB}$$

$$\overleftarrow{\omega^2 r\left(1 + \frac{r}{l}\right)} = \overleftarrow{\omega^2 r} + \overleftarrow{\frac{r^2\omega^2}{l}}$$

$$\mathfrak{b}_S = \frac{\mathfrak{b}_A + \mathfrak{b}_B}{2}$$

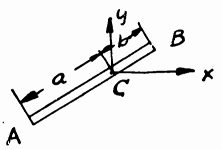
$$\mathfrak{b}_S = \omega^2 r\left(1 + \frac{r}{2l}\right)$$

$$\underline{\underline{V}} = \frac{P}{g}\cdot b_S = \underline{\underline{\frac{P\omega_0^2 r}{g}\left(1 + \frac{r}{2l}\right)}}$$

$$\underline{\underline{\varepsilon_S = 0; \quad M_S = 0}}$$

Lösung 1119

1. Stab mit der Masse m

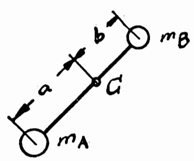


Trägheitskräfte:

$$K_1 = \sqrt{(m\ddot{x})^2 + (m\ddot{y})^2}$$

$$K_1 = m\sqrt{\ddot{x}^2 + \ddot{y}^2}$$

2. Masseloser Stab mit zwei entsprechenden Einzelmassen



$$m_A + m_B = m; \qquad m_A = \frac{mb}{a+b}$$

$$m_A\cdot a = m_B\cdot b; \quad m_B = \frac{ma}{a+b}$$

$$X = \left(\frac{mb}{a+b} + \frac{ma}{a+b}\right)\ddot{x} = m\ddot{x}$$

$$Y = \left(\frac{mb}{a+b} + \frac{ma}{a+b}\right)\ddot{y} = m\ddot{y}$$

$$K_2 = m\sqrt{\ddot{x}^2 + \ddot{y}^2}$$

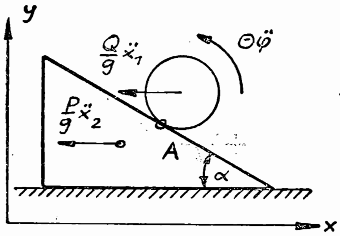
$$\underline{\underline{K_1 = K_2}}$$

Momente der Trägheit

$$M_1 = m\varrho^2\cdot\varepsilon \qquad\qquad M_2 = \left(\frac{mab^2}{a+b} + \frac{mba^2}{a+b}\right)\varepsilon$$

$$\underline{\underline{M_1 - M_2 = m\varepsilon\,(\varrho^2 - ab)}}$$

Lösung 1120



Schwerpunktsbedingung: $P\ddot{x}_2 + Q\ddot{x}_1 = 0$ (1)

Zwangsbedingung: $x_1 = x_2 + r\varphi\cos\alpha$ (2)

$\sum M_A = 0$: $\Theta_A\ddot{\varphi} = Qr\sin\alpha - \frac{Q}{g}\ddot{x}_2 r\cos\alpha$ (3)

$$\Theta_A = \frac{3}{2}\,\frac{Q}{g}\,r^2$$

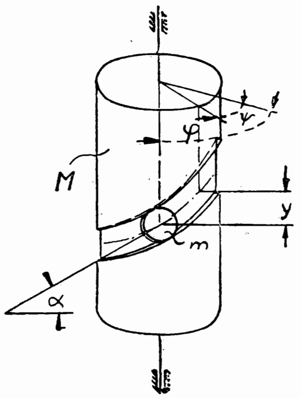
Aus (3): $\ddot{\varphi} = \frac{2}{3}g\frac{\sin\alpha}{r} - \frac{2}{3}\ddot{x}\frac{\cos\alpha}{r}$ (4)

Aus (2) u. (4): $\ddot{x}_1 = \ddot{x}_2 + \frac{1}{3}g\sin 2\alpha - \frac{2}{3}\ddot{x}\cos^2\alpha$ (5)

Aus (1) u. (5): $P\ddot{x}_2 + Q\left[\ddot{x}\left(1 - \frac{2}{3}\cos^2\alpha\right) + \frac{1}{3}g\sin 2\alpha\right] = 0$

$$\underline{\underline{\ddot{x}_2 = \frac{Q\sin 2\alpha}{3(P+Q) - 2Q\cos^2\alpha}\cdot g}}$$

Lösung 1121



$$y = R(\varphi - \psi)\,\mathrm{tg}\,\alpha$$

Energiesatz: $T + U = 0$

$$U = -mgy$$

$$T = \frac{MR^2}{4}\dot{\psi}^2 + \frac{m}{2}(R^2\dot{\varphi}^2 + \dot{y}^2)$$

Drallsatz: $mR^2\dot{\varphi} + \frac{M}{2}R^2\dot{\psi} = 0$

$$\varphi = -\frac{M}{2m}\psi$$

$$T + U = \frac{M}{4}R^2\dot{\psi}^2 + \frac{m}{2}\left\{R^2\frac{M^2}{4m^2}\dot{\psi}^2 + R^2\,\mathrm{tg}^2\alpha\,\dot{\psi}^2\left(\frac{M}{2m}+1\right)^2\right\} - mgy = 0$$

Mit $y = h$; $\dot{\psi} = \omega$ gilt:

$$\underline{\underline{\omega = \frac{2m\cos\alpha}{R}\sqrt{\frac{2gh}{(M+2m)(M+2m\sin^2\alpha)}}}}$$

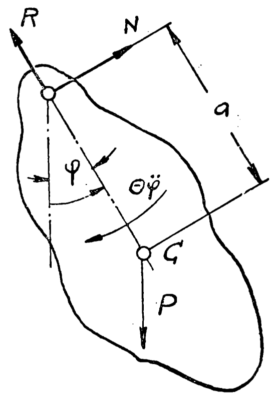
Lösung 1122

$\sum M_0 = 0$: $\Theta\ddot{\varphi} + Pa\sin\varphi = 0$; $\Theta = m(a^2 + \varrho^2)$

$$\ddot{\varphi} = -\frac{ga}{a^2+\varrho^2}\sin\varphi$$

$$\dot{\varphi}\,d\dot{\varphi} = -\frac{ga}{a^2+\varrho^2}\sin\varphi\,d\varphi$$

$$\frac{\dot{\varphi}^2}{2} = C + \frac{ga}{a^2+\varrho^2}\cos\varphi$$



Anfangsbedingungen: $t = 0 \quad \dot{\varphi} = 0$

$$\varphi = \varphi_0$$

$$C = -\frac{g a}{a^2 + \varrho^2} \cos \varphi_0$$

$$\dot{\varphi}^2 = 2 \frac{g a}{\varrho^2 + a^2} (\cos \varphi - \cos \varphi_0)$$

$$R = \frac{P}{g} \dot{\varphi}^2 \cdot a + P \cos \varphi$$

$$\underline{\underline{R = P \cos \varphi + \frac{2 P a^2}{\varrho^2 + a^2} (\cos \varphi - \cos \varphi_0)}}$$

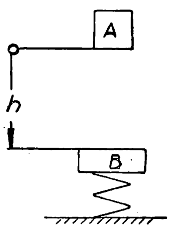
$$\sum M_C = 0: \quad a N + \Theta_S \ddot{\varphi} = 0$$

$$N = -\frac{P \varrho^2}{g \cdot a} \left(-\frac{g a}{a^2 + \varrho^2} \sin \varphi \right)$$

$$\underline{\underline{N = P \frac{\varrho^2}{\varrho^2 + a^2} \sin \varphi}}$$

44. Der Stoß

Lösung 1123



Impulssatz:

$$m_A v_A = (m_A + m_B) v$$

$$v_A = \sqrt{2 g h}$$

$$v = \frac{m_A \sqrt{2 g h}}{m_A + m_B} = \frac{10 \sqrt{2 \cdot 9{,}81 \cdot 4{,}905}}{15}$$

$$\underline{\underline{v = 6{,}54 \text{ m/sek}}}$$

Lösung 1124

Geschwindigkeit vor dem Stoß: v

Geschwindigkeit nach dem Stoß: c

Bei vollkommen elastischem Stoß erfolgt verlustfreie Energieumsetzung, also:

$$\frac{m}{2} (v_1^2 + v_2^2) = \frac{m}{2} (c_1^2 + c_2^2)$$

Impulssatz: $\quad m (v_1 + v_2) = m (c_1 + c_2)$

Daraus: $\quad v_1^2 - c_1^2 = -(v_2^2 - c_2^2) \qquad (1)$

$$v_1 - c_1 = -(v_2 - c_2) \qquad (2)$$

Hieraus durch Division beider Gleichungen: $v_1 + c_1 = v_2 + c_2 \qquad (3)$

Aus (3) und (2) folgt: $\underline{\underline{c_2 = v_1}}$ Die Kugeln wechseln also ihre Gechwindigkeit.

Lösung 1125

Geschwindigkeit vor dem Stoß: v

Geschwindigkeit nach dem Stoß: c

Ansatz für halbelastische Körper:

$$(m_1 + m_2)\,c_1 = m_1 v_1 + m_2 v_2 - m_2 (v_1 - v_2)\,k$$

Mit $m_1 = m_2$ und $c_1 = 0$ gilt: $v_1 + v_2 - (v_1 - v_2)\,k = 0$

$$\frac{v_1}{v_2} + 1 - \frac{v_1}{v_2}\cdot k + k = 0$$

$$\underline{\underline{\left|\frac{v_1}{v_2}\right| = \frac{v_A}{v_B} = \frac{1+k}{1-k}}}$$

Lösung 1126

Allgemein gilt:

$$c_1 = \frac{m_1 v_1 + m_2 v_2 - m_2 (v_1 - v_2)\,k}{m_1 + m_2}$$

$$c_2 = \frac{m_1 v_1 + m_2 v_2 + m_1 (v_1 - v_2)\,k}{m_1 + m_2}$$

zu 1. $v_1 = 0$; $c_2 = 0$: $m_2 v_2 - m_1 v_2 k = 0$; $\underline{\underline{\frac{m_2}{m_1} = k}}$

zu 2. $v_1 = -v_2$; $c_2 = 0$: $m_1 v_1 - m_2 v_1 + 2 m_1 v_1 k = 0$

$$\underline{\underline{\frac{m_2}{m_1} = 1 + 2k}}$$

Lösung 1127

Für vollkommen elastischen Stoß ist $k = 1$, also:

$$c_2 = \frac{(m_2 - m_1)\,v_2 + 2 m_1 v_1}{m_1 + m_2}$$

$$v_2 = 0: \quad c_2 = \frac{2 m_1 v_1}{m_1 + m_2}$$

Für die zweite und dritte Kugel gilt entsprechend:

$$c_3 = \frac{(m_3 - m_2)\,v_3 + 2 m_2 c_2}{m_2 + m_3}; \qquad v_3 = 0: \quad c_3 = \frac{2 m_2 \cdot 2 m_1 v_1}{(m_2 + m_3)\,(m_1 + m_2)}$$

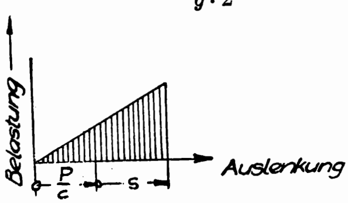
$$\frac{d c_3}{d m_2} = 0: \quad 4 m_1 v_1 (m_1 + m_2)(m_2 + m_3) - 4 m_1 m_2 v_1 (m_1 + 2 m_2 + m_3) = 0$$

$$\underline{\underline{m_2 = \sqrt{m_1 m_3}}}$$

Lösung 1128

Geschwindigkeit nach dem unelastischen Stoß: $v = \dfrac{P\sqrt{2gh}}{P+p}$

Energiesatz: $\dfrac{(P+p)\,v^2}{g\cdot 2} + (P+p)\,s = \dfrac{c}{2}\left[\left(\dfrac{p}{c} + s\right)^2 - \left(\dfrac{p}{c}\right)^2\right]$



$$s^2 - \frac{2P\cdot s}{c} = \frac{(P+p)\cdot P^2\cdot 2gh\cdot 2}{(P+p)^2\cdot 2\cdot c\cdot g}$$

$$\underline{\underline{s = \frac{P}{c} + \sqrt{\left(\frac{P}{c}\right)^2 + \frac{2P^2 h}{(P+p)\cdot c}}}}$$

Lösung 1129

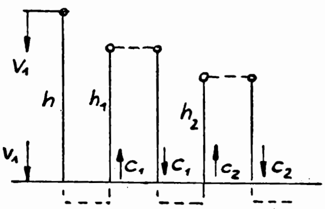
$$c_1 = \frac{m_1 v_1 + m_2 v_2 - m_2 (v_1 - v_2) k}{m_1 + m_2};$$

In der Aufgabe ist:

$$m_2 \to \infty$$
$$v_2 = 0$$
$$v_1 = \sqrt{2 g h_1}$$
$$c_1 = -\sqrt{2 g h_2}$$

Somit: $$\underline{\underline{k = \sqrt{\frac{h_2}{h_1}} = 0{,}95}}$$

Lösung 1130



$$c_1 = k \cdot v_1; \qquad v_1 = \sqrt{2 g h}$$
$$h_1 = \frac{c_1^2}{2g} = \frac{k^2 \cdot 2gh}{2g} = k^2 \cdot h$$
$$c_2 = k \cdot c_1$$
$$h_2 = \frac{c_2^2}{2g} = \frac{k^2 (k \sqrt{2gh})^2}{2g} = k^4 h$$
$$c_3 = k c_2$$
$$h_3 = \frac{c_3^2}{2g} = \frac{k^2 \cdot k^2 \cdot k^2 \cdot 2gh}{2g} = k^6 \cdot h \text{ usw.}$$

Der zurückgelegte Weg ist:

$$s = h + 2h_1 + 2h_2 + \cdots 2h_n$$
$$s = -h + 2h (1 + k^2 + k^4 + \cdots k^{2n})$$

Da $k^2 < 1$, ist die Summe der unendlichen geometrischen Reihe

$2h(1 + k^2 + k^4 + \cdots k^{2n})$: $\quad S = \dfrac{2h}{1 - k^2}$

Somit: $s = \dfrac{2h}{1 - k^2} - h$;

$$\underline{\underline{s = \frac{1 + k^2}{1 - k^2} \cdot h}}$$

Lösung 1131

Geschwindigkeit beider Massen nach dem Aufschlagen des Hammers:

$$c = \frac{m_1 v_1}{m_1 + m_2}$$

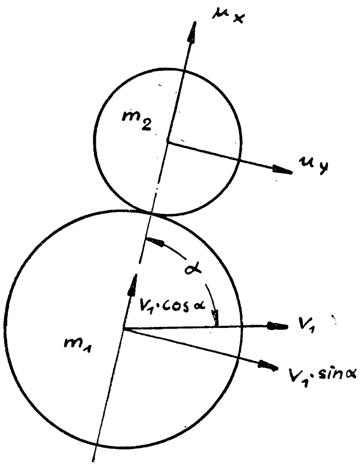
Die Verlustarbeit: $$\underline{\underline{A_2}} = \frac{(m_1 + m_2) c^2}{2} = \underline{\underline{700 \text{ mkg}}}$$

Die Schlagarbeit: $$A_1 = \frac{m_1 v_1^2}{2} - A_2 = \frac{m_1 v_1^2}{2} - \frac{m_1^2 v_1^2}{2(m_1 + m_2)}$$

$$\underline{\underline{A_1}} = \frac{m_1 m_2 v_1^2}{2(m_1 + m_2)} = \underline{\underline{14\,600 \text{ mkg}}}$$

Wirkungsgrad: $$\underline{\underline{\eta}} = \frac{14\,600}{14\,600 + 700} = \underline{\underline{0{,}95}}$$

Lösung 1132



Die Kugel 2 ist vor dem Stoß in Ruhe, es gilt also: $v_2 = 0$

1. Unelastischer Stoß:

$$m_1 v_1 \cos\alpha = (m_1 + m_2)\, u_x; \quad (u_x = u_{x_1} = u_{x_2})$$

$$m_1 v_1 \sin\alpha = m_1 u_{y_1}$$

$$u_x^2 + u_{y_1}^2 = u_1^2$$

$$\underline{\underline{u_1 = v_1 \sqrt{\sin^2\alpha + \left(\frac{m_1}{m_1 + m_2}\right)^2 \cos^2\alpha}}}$$

2. Elastischer Stoß, Stoßzahl k:

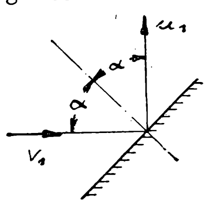
$$u_{x_1} = \frac{m_1 v_1 \cos\alpha - m_2 k v_1 \cos\alpha}{m_1 + m_2}; \quad u_{y_1} = v_1 \sin\alpha$$

$$\underline{\underline{u_1 = v_1 \sqrt{\sin^2\alpha + \left(\frac{m_1 - m_2 k}{m_1 + m_2}\right)^2 \cos^2\alpha}}}$$

$$u_{x_2} = \frac{m_1 v_1 \cos\alpha + m_1 v_1 k \cos\alpha}{m_1 + m_2}; \quad u_{y_2} = 0$$

$$\underline{\underline{u_2 = \frac{m_1 (1 + k)\, v_1 \cos\alpha}{m_1 + m_2}}}$$

Lösung 1133



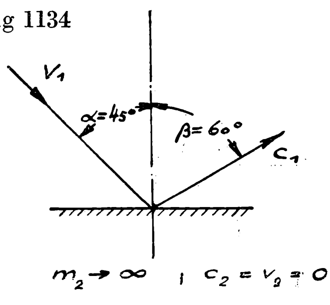
Da die Wand in Ruhe bleibt, kann ihre Masse $m_2 \to \infty$ gesetzt werden.

Nach Aufgabe 1132 ist somit:

$$u_1 = v_1$$

Nach dem Impulssatz ist der Einfallswinkel gleich dem Ausfallswinkel.

Lösung 1134



$$v_{1y} = v_1 \cos\alpha; \quad v_{1x} = v_1 \sin\alpha$$

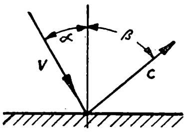
$$c_{1y} = c_1 \cos\beta; \quad c_{1x} = c_1 \sin\beta$$

$$c_{1y} = k v_{1y}; \quad c_{1x} = v_{1x}$$

$$k = \frac{c_1}{v_1} \cdot \frac{\cos\beta}{\cos\alpha} = \frac{\cos\beta \cdot \sin\alpha}{\cos\alpha \cdot \sin\beta}$$

$$k = \frac{\operatorname{tg}\alpha}{\operatorname{tg}\beta} = \underline{\underline{0{,}58}}$$

Lösung 1135



$$v \cdot \sin\alpha = c\sin\beta; \quad c\cos\beta = kv\cos\alpha$$

mit $c = v\frac{\sqrt{2}}{2}$ und $k = \frac{\sqrt{3}}{3}$ gilt:

$$\frac{\sqrt{2}}{2}\cos\beta = \frac{\sqrt{3}}{3}\cos\alpha; \quad \frac{1}{2}(1-\sin^2\beta) = \frac{1}{3}\cos^2\alpha$$

$$\sin\alpha = \frac{\sqrt{2}}{2}\sin\beta; \quad \sin^2\alpha = \frac{1}{2}\sin^2\beta$$

Daraus:

$$\underline{\underline{\alpha = \frac{\pi}{6}}}$$

$$\underline{\underline{\beta = \frac{\pi}{4}}}$$

Lösung 1136

Bei der Masse 2 ist nur die Geschwindigkeitskomponente $v_2\cos\alpha$ für den Stoß von Bedeutung. Es ist also:

$$\left\| \begin{array}{l} u_{1\tau} = 0 \\ u_{2\tau} = v\sin\alpha \end{array} \right.$$

Für u_n gelten die Gleichungen für vollkommen elastischen Stoß.

Allgemein gilt: $(\xrightarrow{v_1} \xrightarrow{v_2}; \xrightarrow{c_1} \xrightarrow{c_2})$

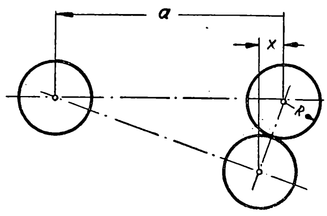
$$c_1 = \frac{(m_1 - m_2)\,v_1 + 2m_2v_2}{m_1 + m_2}; \quad c_2 = \frac{(m_2 - m_1)\,v_2 + 2m_1v_1}{(m_1 + m_2)}$$

Für $c_1 = u_{1n}$ und $c_2 = u_{2n}$ gilt mit $v_1 = v$; $v_2 = -v\cos\alpha$; $m_1 = m_2$:

$$\underline{\underline{u_{1n} = -v\cos\alpha}}; \quad \underline{\underline{u_{2n} = v}}$$

Lösung 1137

Lage der Kugeln im Augenblick des Stoßes:



$$\frac{x}{2R} = \frac{2R}{a}; \quad \underline{\underline{x = \frac{4R^2}{a}}}$$

Lösung 1138

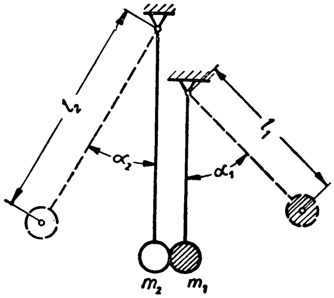
Kinetische Energie = Äußere Arbeit: $\frac{P_1 + P}{2g}c^2 = R\cdot\delta$

Geschwindigkeit nach dem Stoß: $c = \frac{\sqrt{2gh}\cdot P_1}{P_1 + P}$

$$R = \frac{(P_1+P)\cdot 2ghP_1^2}{2g\delta(P_1+P)^2} = \frac{P_1^2h}{\delta(P_1+P)}$$

$$\underline{\underline{R = 16{,}2\ \mathrm{t}}}$$

Lösung 1139



$$v_1 = \sqrt{2gl_1(1-\cos\alpha_1)}$$

$$c_2 = \sqrt{2gl_2(1-\cos\alpha_2)}$$

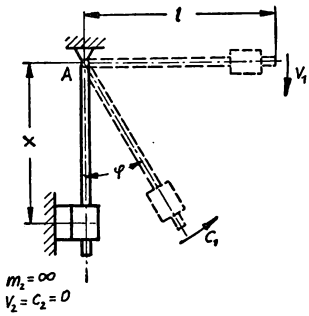
$$c_2 = \frac{m_1 v_1(1+k)}{m_1+m_2}$$

$$\sqrt{2gl_2(1-\cos\alpha_2)} = \frac{m_1(1+k)}{m_1+m_2}\sqrt{2gl_1(1-\cos\alpha_1)}$$

$$\sqrt{\frac{1-\cos\alpha}{2}} = \sin\frac{\alpha}{2}$$

$$\underline{\underline{\sin\frac{\alpha_2}{2} = \frac{m_1(1+k)}{m_1+m_2}\sqrt{\frac{l_1}{l_2}}\sin\frac{\alpha_1}{2}}}$$

Lösung 1140



$v_1 = \sqrt{2gx}$; $c_1 = \sqrt{2gx(1-\cos\varphi)}$

Stoßgesetz: $c_1 = kv_1$

$$\underline{\underline{k = \sqrt{1-\cos\varphi} = \sqrt{2}\sin\frac{\varphi}{2}}}$$

Das Prüfstück muß im Stoßmittelpunkt angebracht werden, damit das Gelenk keinen Stoß erhält.

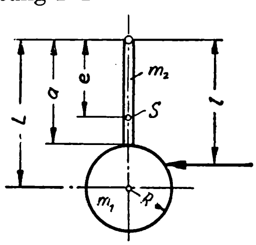
$x = \frac{\Theta_{\mathrm{St}A}}{m_{\mathrm{St}} \cdot e}$ $\quad\Theta_{\mathrm{St}A}$ = Trägheitsmoment des Stabes, bezogen auf A

m_{St} = Masse des Stabes

e = Schwerpunktsabstand des Stabes von A

$$\Theta_{\mathrm{St}A} = \frac{ml^2}{3}; \quad e = \frac{l}{2}; \quad \underline{\underline{x = \frac{2}{3}l}}$$

Lösung 1141



Gesucht ist der Stoßmittelpunkt

$$l = \frac{\Theta}{M \cdot e}; \quad m_1 + m_2 = M$$

$$\Theta = \frac{m_1 R^2}{2} + m_1 \cdot L^2 + \frac{m_2 a^2}{3}$$

$$e = \frac{m_1 \cdot L + m_2 \cdot \frac{a}{2}}{m_1 + m_2}$$

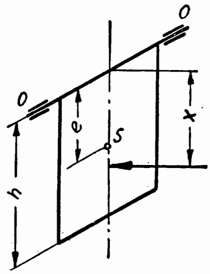
$$m_1 = \frac{\gamma}{g}\pi R^2 \cdot \delta; \quad m_2 = \frac{\gamma}{g}\pi r^2 \cdot a$$

$$\Theta = \frac{\gamma}{g}\pi\left[\frac{R^4\delta}{2} + R^2\delta L^2 + \frac{r^2 a^3}{3}\right]; \quad e = \frac{\frac{\gamma}{g}\pi\left[R^2\delta L + \frac{r^2 a^2}{2}\right]}{\frac{\gamma}{g}\pi[R^2\delta + r^2 a]}; \quad M = \frac{\gamma}{g}\pi[R^2\delta + r^2 a]$$

$$l=\frac{\Theta}{Me}=\frac{\frac{R^4\delta}{2}+R^2\delta L^2+\frac{r^2a^3}{3}}{R^2\delta L+\frac{r^2a^2}{2}};$$

$R=10$ cm
$\delta=5$ cm
$L=100$ cm $\qquad \underline{\underline{l=97{,}5\text{ cm}}}$
$r=1$ cm
$a=90$ cm

Lösung 1142



$$x=\frac{\Theta}{m\cdot e}$$

$$\underline{\underline{x=\frac{mh^2\cdot 2}{3m\cdot h}=\frac{2}{3}h}}$$

Lösung 1143 Bei gleichmäßiger Verteilung der Masse auf den Umfang gilt:

$$\Theta_1\omega_{10}+\Theta_2\omega_{20}=\Theta_1\omega_1+\Theta_2\omega_2;\qquad \omega_2=\omega_1\frac{R_1}{R_2}$$

$\Theta=mR^2=2\pi\varrho\cdot R^3\cdot F=B\cdot R^3;\quad F=$ Querschnitt des Ersatzringes

$$BR_1^3\omega_{10}+BR_2^3\omega_{20}=\omega_1\left[BR_1^3+BR_2^3\frac{R_1}{R_2}\right]$$

$$\underline{\underline{\omega_1=\frac{R_1^3\omega_{10}+R_2^3\omega_{20}}{R_1(R_1^2+R_2^2)}}};\qquad \underline{\underline{\omega_2=\frac{R_1^3\omega_{10}+R_2^3\omega_{20}}{R_2(R_1^2+R_2^2)}}}$$

Lösung 1144

$$\Theta_0\omega_{10}=\Theta_0\omega_1+\frac{Q}{g}r^2\omega_1;\qquad \omega_1=\frac{\Theta_0\omega_{10}}{\Theta_0+\frac{Q}{g}r^2}=\underline{\underline{6{,}15\ 1/\text{sek}}}$$

$$v_2=\omega_1\cdot r=\underline{\underline{1{,}23\text{ m/sek}}}$$

$$Q_{\text{mittel}}\cdot t=\frac{Q}{g}v_2=\frac{25\cdot 1{,}23}{9{,}81\cdot 0{,}05}=\underline{\underline{62{,}8\text{ kg}}}$$

Lösung 1145

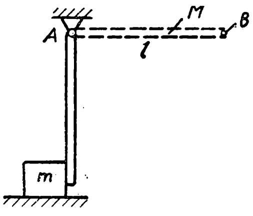
Drehimpulssatz: $mva=(M\varrho^2+ma^2)\,\omega$

Energiesatz: $(M\varrho^2+ma^2)\frac{\omega^2}{2}=g(Mh+ma)\,(1-\cos\alpha);\quad 1-\cos\alpha=2\sin^2\frac{\alpha}{2}$

$$v=\frac{(M\varrho^2+ma^2)\,\omega}{ma}=\frac{2\sin\frac{\alpha}{2}\sqrt{g(Mh+ma)\,(M\varrho^2+ma^2)}}{ma};\qquad \varrho^2=ah$$

$$\underline{\underline{v=2\left(1+\frac{Mh}{ma}\right)\sqrt{ga}\cdot\sin\frac{\alpha}{2}}}$$

Lösung 1146



$$\Theta_A = \frac{M l^2}{3}$$ Index v: vor dem Stoß

Energiesatz: $\frac{\Theta_A \omega_v^2}{2} = \frac{M g l}{2}$; $\omega_v^2 = \frac{3g}{l}$; Index N: nach dem Stoß

Drehimpulssatz: $\Theta_A \omega_v = \Theta_A \frac{v_{BN}}{l} + m l v_{BN}$

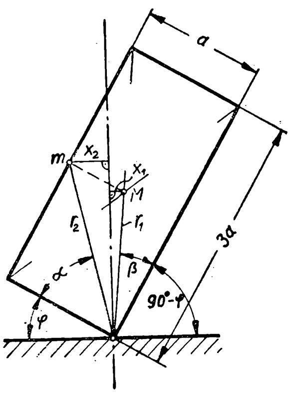
$$v_{BN} = \frac{M\sqrt{3gl}}{M+3m}$$

$$s = -\mu g \frac{t^2}{2} + v_{BN} \cdot t;$$

$v = -\mu g t + v_{BN}$; für $v = 0$; $s = s_{\max}$: $t = \frac{v_{BN}}{\mu g}$

$$\underline{\underline{s_{\max} = \frac{v_{BN}^2}{2\mu g} = \frac{3 M^2 l}{2\mu (M+3m)^2}}}$$

Lösung 1147



Im Moment des Umkippens herrscht labiles Gleichgewicht:

$$M \cdot x_1 = m x_2$$

$$x_1 = r_1 \sin(\varphi - \beta)$$

$$x_1 = r_1 [\sin\varphi \cos\beta - \cos\varphi \sin\beta]$$

$$\sin\beta = \frac{a}{2 r_1}; \quad \cos\beta = \frac{3}{2} \frac{a}{r_1}$$

$$x_1 = \frac{3}{2} a \sin\varphi - \frac{a}{2} \cos\varphi$$

$$x_2 = r_2 \cos(\varphi + \alpha)$$

$$x_2 = r_2 [\cos\alpha \cos\varphi - \sin\alpha \sin\varphi]$$

$$\cos\alpha = \frac{a}{r_2}; \quad \sin\alpha = \frac{3}{2} \frac{a}{r_2}$$

$$x_2 = a \cos\varphi - \frac{3}{2} a \sin\varphi$$

Somit: $\cos\varphi (2m + M) = \sin\varphi (3M + 3m)$

$$M = 3m$$

$$\operatorname{tg}\varphi = \frac{5}{12}$$

Energiesatz: $T + U = 0$

$$U = -ga\left\{m\left(\sin\varphi + \frac{3}{2}\cos\varphi - \frac{3}{2}\right) + M\left(\frac{3}{2}\cos\varphi + \frac{1}{2}\sin\varphi - \frac{3}{2}\right)\right\}; \quad M = 3m, \quad N = \sqrt{1 + \operatorname{tg}^2\varphi}$$

$$U = -gma\left[\frac{5}{12} \cdot \frac{1}{N} + \frac{3}{2} \cdot \frac{1}{N} - \frac{3}{2} + \frac{9}{2}\frac{1}{N} + \frac{3}{2} \cdot \frac{5}{12}\frac{1}{N} - \frac{9}{2}\right] \quad N = \frac{13}{12}$$

$$U = -gm \cdot \frac{a}{2}$$

$$T = \Theta_{\text{ges}} \cdot \frac{\dot{\varphi}^2}{2} = \frac{1}{2}\dot{\varphi}^2 \left[m\left(a^2 + \frac{9}{4}a^2\right) + 3m\left(\frac{a^2}{4} + \frac{9}{4}a^2\right) + \frac{30}{12}ma^2\right]$$

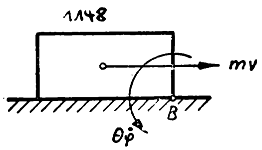
$$T = \frac{1}{2}\dot{\varphi}^2 \cdot \frac{53}{4}ma^2; \quad T + U = 0: \quad \dot{\varphi} = \sqrt{g \cdot \frac{1}{a}\,\frac{4}{53}}$$

Drehimpulssatz: $mv \cdot \frac{3}{2}a = \Theta\dot{\varphi}$

$$m \cdot v \cdot \frac{3}{2}a = \frac{53}{4}ma^2\sqrt{\frac{4g}{53a}}$$

$$\underline{\underline{v = \frac{1}{3}\sqrt{53ag}}}$$

Lösung 1148

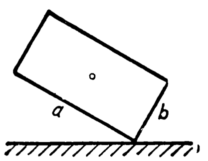


Drehimpulssatz: $mvh = \Theta_B \cdot \dot{\varphi}$

$$\Theta_B = m\varrho^2$$

$$\underline{\underline{\dot{\varphi} = \omega = \frac{v \cdot h}{\varrho^2}}}$$

Lösung 1149



Energiesatz: $T + U = 0$

$$T = \frac{1}{2}\Theta\dot{\varphi}^2 = \frac{1}{2}\dot{\varphi}^2\left\{\frac{m}{12}(a^2 + b^2) + m\left[\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2\right]\right\}$$

$$U = mg\frac{b}{2} - mg\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

Daraus: $\dot{\varphi}^2 = 3g\,\frac{\sqrt{a^2 + b^2} - b}{a^2 + b^2}$

Drallsatz: $\Theta \cdot \dot{\varphi} = mv \cdot \frac{b}{2}$; $\quad a = 4\,\text{m}$

$$b = 3\,\text{m}$$

$$\underline{\underline{v = \frac{10 \cdot 3{,}6}{9}\sqrt{6g} = 30{,}7\,\text{km/h}}}$$

45. Dynamik von Systemen mit veränderlicher Masse

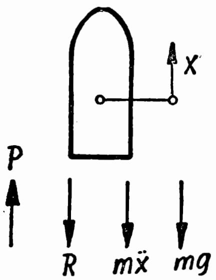
Lösung 1150

Da die herausgeschleuderten Teilchen die Geschwindigkeit der weiterfliegenden Masse haben, wird kein zusätzlicher Impuls auf die Masse ausgeübt. Es gilt deshalb:

$$m(t)\,l^2\ddot{\varphi} + m(t)\,l\sin\varphi + l^2\beta \cdot \dot{\varphi} = 0$$

$$\underline{\underline{\ddot{\varphi} + \frac{g}{l}\sin\varphi + \frac{\beta}{m(t)} \cdot \dot{\varphi} = 0}}$$

Lösung 1151



Antriebskraft: $P = -\frac{dm}{dt} \cdot v_r$

(v_r wirkt entgegen x)

$$m(\ddot{x}+g) + R - P = 0$$

$$m = m_0 f(t); \qquad R = \mathfrak{R}(x, \dot{x})$$

$$\ddot{x} = -g - \frac{m_0 \dot{f}(t)}{m_0 f(t)} \cdot v_r - \frac{R(x,\dot{x})}{m_0 f(t)}$$

$$\underline{\underline{\ddot{x} = -g - \frac{\dot{f}(t)}{f(t)} v_r - \frac{R(x,\dot{x})}{m_0 f(t)}}}$$

Lösung 1152

$$\ddot{x} = -g - \frac{\dot{f}(t)}{f(t)} v_r - \frac{R(x,\dot{x})}{m_0 f(t)}; \quad \text{mit } R = 0; \quad f(t) = 1 - \alpha t; \quad \dot{f}(t) = -\alpha \text{ gilt:}$$

$$\ddot{x} = -g + \frac{\alpha v_r}{1 - \alpha t}$$

$$\dot{x} = -gt - v_r \ln(1-\alpha t) + C_1; \quad t = 0; \quad \dot{x} = 0{:} \quad C_1 = 0$$

$$x = -\frac{gt^2}{2} + \frac{v_r}{\alpha}[(1-\alpha t)\ln(1-\alpha t) + \alpha t - 1] + C_2; \quad t = 0; \quad x = 0{:} \quad C_2 = \frac{v_r}{\alpha}$$

$$\underline{\underline{x_{(t)} = -\frac{gt^2}{2} + \frac{v_r}{\alpha}[(1-\alpha t)\ln(1-\alpha t) + \alpha t]}}$$

Nach Einsetzen der gegebenen Werte ergibt sich:

$$\underline{\underline{x(10) = 0{,}54\,\text{km}}}; \quad \underline{\underline{x(30) = 5{,}65\,\text{km}}}; \quad \underline{\underline{x(50) = 18{,}4\,\text{km}}}$$

Lösung 1153

Mit $R = 0$ gilt: $\ddot{x} = -g - \frac{\dot{f}(t)}{f(t)} \cdot v_r$; $\quad f(t) = e^{-\alpha t}$, $\dot{f}(t) = -\alpha e^{-\alpha t}$

$$\ddot{x} = -g + \alpha v_r$$

$$v(t_0) = (-g + \alpha v_r) \cdot t_0; \quad s(t_0) = (-g + \alpha v_r)\frac{t_0^2}{2}$$

$$H = s(t_0) + \frac{v(t_0)^2}{2g} = (-g + \alpha v_r)(2g + a v_r - g) \cdot \frac{t_0^2}{2g}$$

$$\underline{\underline{H = \frac{(\alpha^2 v_r^2 - g^2)\, t_0^2}{2g}}}$$

Lösung 1154

$$H = \frac{1}{2g}(\alpha^2 v_r^2 - g^2)\, t_0^2; \quad \mu = \alpha t_0$$

$$H = \frac{1}{2g}(\alpha^2 v_r^2 - g^2) \cdot \frac{\mu^2}{\alpha^2} = \frac{1}{2g}\left(v_r^2 \mu^2 - \frac{g^2 \mu^2}{\alpha^2}\right)$$

$H_{\max}$ wird erreicht für $\alpha = \infty$; $\underline{\underline{H_{\max} = \frac{v_r^2 \mu^2}{2g}}}$

Lösung 1155

$$P - \frac{d(m\dot{x})}{dt} - \beta\dot{x}^2 - mg = 0; \qquad \begin{aligned} m &= (Q+\gamma x)\cdot\frac{1}{g} \\ \dot{m} &= \frac{dm}{dt} = \gamma\dot{x}\cdot\frac{1}{g} \end{aligned}$$

$$P = m\ddot{x} + \dot{m}\dot{x} + \beta\dot{x}^2 + mg = \frac{1}{g}(Q+\gamma x)\ddot{x} + \frac{1}{g}\gamma\dot{x}^2 + \beta\dot{x}^2 + Q + \gamma x$$

$$\underline{\underline{\ddot{x} = -g + \frac{Pg}{Q+\gamma x} - \frac{\beta g+\gamma}{Q+\gamma x}\dot{x}^2}}$$

Lösung 1156

Aus Aufgabe 1155: $\ddot{x} = -g + \dfrac{Pg}{Q+\gamma x} - \dfrac{\beta g+\gamma}{Q+\gamma x}\cdot\dot{x}^2$

$$\ddot{x} = \frac{d\dot{x}}{dx}\cdot\frac{dx}{dt} = \frac{d\dot{x}}{dx}\cdot\dot{x} = \frac{1}{2}\,\frac{d(\dot{x}^2)}{dx}$$

$$\frac{1}{2}\,\frac{d(\dot{x}^2)}{dx} = -g + \frac{Pg}{Q+\gamma x} - \frac{\beta g + x}{Q+\gamma x}\cdot(\dot{x}^2); \qquad \dot{x}^2 = u\cdot v$$

$$\frac{1}{2}\left[\frac{du}{dx}\cdot v + \frac{dv}{dx}\cdot u\right] = -g + \frac{Pg}{Q+\gamma x} - \frac{\beta g+\gamma}{Q+\gamma x}\cdot u\cdot v$$

Diese Gleichung zerfällt in:

$$\frac{1}{2}\,\frac{dv}{dx}\cdot u = -\frac{\beta g+\gamma}{Q+\gamma x}\cdot u\cdot v \qquad (1)$$

und

$$\frac{1}{2}\,\frac{du}{dx}\cdot v = -g + \frac{Pg}{Q+\gamma x} \qquad (2)$$

Aus (1): $\dfrac{1}{v}\cdot\dfrac{dv}{dx} = -\dfrac{2(\beta g+\gamma)}{Q+\gamma x}; \qquad \ln v = -\dfrac{2(\beta g+\gamma)}{\gamma}\ln(Q+\gamma x)$

$$v = \left(\frac{1}{Q+\gamma x}\right)^{2\left(1+\frac{\beta g}{\gamma}\right)}$$

In (2) eingesetzt:

$$\frac{du}{dx} = 2g\left[P(Q+\gamma x)^{1+\frac{2\beta g}{\gamma}} - (Q+\gamma x)^{2\left(1+\frac{\beta g}{\gamma}\right)}\right]$$

$$u = 2g\left[\frac{P}{2\gamma+2\beta g}(Q+\gamma x)^{2\left(1+\frac{\beta g}{\gamma}\right)} - \frac{1}{3\gamma+2\beta g}(Q+\gamma x)^{\left(3+2\frac{\beta g}{\gamma}\right)} + C\right]$$

$$u = 0; \quad x = H_0: \quad C = \frac{Q+\gamma H_0}{3\gamma+2\beta g}^{\left(3+2\frac{\beta g}{\gamma}\right)} - \frac{P(Q+\gamma H_0)^{2\left(1+\frac{\beta g}{\gamma}\right)}}{2\gamma+2\beta g}$$

$$\underline{\underline{\dot{x}^2 = u\cdot v = \frac{Pg}{\beta g+\gamma}\left[1-\left(\frac{Q+\gamma H_0}{Q+\gamma x}\right)^{2\left(1+\frac{\beta g}{\gamma}\right)}\right] - \frac{2g(Q+\gamma x)}{2\beta g+3\gamma}\left[1-\left(\frac{Q+\gamma H_0}{Q+\gamma x}\right)^{3+2\beta\frac{g}{\gamma}}\right]}}$$

Lösung 1157

$$\frac{dV}{dt} = \alpha \cdot F; \quad 4\pi r^2 \cdot \frac{dr}{dt} = 4\pi r^2 \cdot \alpha; \quad \frac{dr}{dt} = \alpha; \quad \underline{\underline{r = r_0 + \alpha t}}$$

$$(m\dot{x})^{\cdot} = mg; \quad \text{mit} \quad \frac{d}{dt} = \alpha \frac{d}{dr}: \quad \alpha \frac{d}{dr}\left(m\alpha \frac{dx}{dr}\right) = mg$$

$$\alpha^2\left(\frac{1}{m}\frac{dm}{dr} \cdot \frac{dx}{dr} + \frac{d^2x}{dr^2}\right) = g$$

$$\frac{dm}{dr} = 4\pi \frac{\gamma}{g} r^2 = \frac{3m}{r} \qquad \left(\frac{3}{r}\frac{dx}{dr} + \frac{d^2x}{dr^2}\right) = \frac{g}{\alpha^2}$$ Eulersche Differentialgleichung

Ansatz der homogenen Lösung: $x = r^\lambda$: $3\lambda + \lambda(\lambda - 1) = 0$

$$\lambda_1 = 0; \quad \lambda_2 = -2$$

$$x = C_1 + \frac{C_2}{r^2} + \frac{g}{8\alpha^2} \cdot r^2$$

$$\dot{x} = v = \alpha \cdot \frac{dx}{dr} = -\frac{2\alpha C_2}{r^3} + \frac{gr}{4\alpha}$$

C_2 aus $v(r_0) = v_0$; $C_2 = \frac{r_0^3}{2\alpha}\left(\frac{gr_0}{4\alpha} - v_0\right)$

$$\underline{\underline{v = v_0 \frac{r_0^3}{r^3} + \frac{g}{4\alpha}\left(r - \frac{r_0^4}{r^3}\right)}}$$

C_1 aus $x(r_0) = h_0$: $C_1 = h_0 - \frac{r_0}{2\alpha}\left(\frac{gr_0}{4\alpha} - v_0\right) - \frac{g}{8\alpha^2} r_0^2 = h_0 + \frac{v_0 r}{2\alpha} - \frac{g r_0^2}{4\alpha^2}$

$$x = h_0 + \frac{v_0 r_0}{2\alpha} - \frac{g r_0^2}{4\alpha^2} + \frac{g r_0^4}{8\alpha^2 r^2} - \frac{v_0 r_0^3}{2\alpha r^2} + \frac{g r^2}{8\alpha^2}$$

$$\underline{\underline{x = h_0 + \frac{v_0 r_0}{2\alpha}\left(1 - \frac{r_0^2}{r^2}\right) + \frac{g}{8\alpha^2}\left(r^2 - 2r_0^2 + \frac{r_0^4}{r^2}\right)}}$$

Lösung 1158

Unter Berücksichtigung des Widerstandes gilt bei Verwendung von Aufgabe 1157:

$$m\alpha^2\left(\frac{3}{r}\frac{dx}{dr} + \frac{d^2x}{dr^2}\right) = mg - \frac{4\beta\pi r^2 v \cdot \gamma}{g}$$

Mit: $\frac{4\gamma}{g}\pi r^2 = \frac{3m}{r}$ und $v = \alpha \frac{dx}{dr}$ wird:

$$\frac{3}{r}\frac{dx}{dr} + \frac{d^2x}{dr^2} = \frac{g}{\alpha^2} - \frac{3\beta}{\alpha r} \cdot \frac{dx}{dr}$$

oder: $\frac{3(\alpha+\beta)}{\alpha r} \cdot \frac{dx}{dr} + \frac{d^2x}{dr^2} = \frac{g}{\alpha^2}$ Eulersche Differentialgleichung

Ansatz: $x = r^\lambda$; $\lambda\left(\frac{3(\alpha+\beta)}{\alpha} + \lambda - 1\right) = 0$; $\lambda_1 = 0$; $\lambda_2 = -\frac{(2\alpha + 3\beta)}{\alpha}$

$$x = C_1 + \frac{C_2}{r^{\frac{2\alpha+3\beta}{\alpha}}} + \frac{g}{2\alpha}\,\frac{r^2}{4\alpha + 3\beta}$$

$$v = \alpha \frac{dx}{dr} = -(2\alpha + 3\beta) \cdot \frac{C_2}{r^{\frac{3(\alpha+\beta)}{\alpha}}} + \frac{g r}{(4\alpha + 3\beta)}$$

$$C_2 \text{ aus } v(r_0) = v_0\text{:} \quad C_2 = \frac{r_0^{\frac{3(\alpha+\beta)}{\alpha}}}{2\alpha + 3\beta}\left(\frac{g r_0}{4\alpha + 3\beta} - v_0\right)$$

$$\underline{\underline{v = \frac{g r}{4\alpha + 3\beta} - \left[\frac{g r_0^{\frac{4\alpha+3\beta}{\alpha}}}{4\alpha + 3\beta} - v_0 r_0^{\frac{3(\alpha+\beta)}{\alpha}}\right] r^{-\frac{3(\alpha+\beta)}{\alpha}}}}$$

C_1 aus $x(r_0) = h_0$:

$$C_1 = h_0 - \frac{r_0}{2\alpha + 3\beta} \cdot \left(\frac{g r_0}{4\alpha + 3\beta} - v_0\right) = \frac{g r_0^2}{2\alpha(4\alpha + 3\beta)}$$

$$\underline{\underline{x = h_0 + \frac{1}{2\alpha + 3\beta}\left[\frac{g r_0^{\frac{4\alpha+3\beta}{\alpha}}}{4\alpha + 3\beta} - v_0 r_0^{\frac{3(\alpha+\beta)}{\alpha}}\right]\left[r^{-\frac{3\beta+2\alpha}{\alpha}} - r_0^{-\frac{3\beta+2\alpha}{\alpha}}\right] + \frac{g(r^2 - r_0^2)}{2\alpha(4\alpha + 3\beta)}}}$$

Lösung 1159

$$(m\dot{x})^{\cdot} = mg$$

$$\dot{m}\dot{x} + m\ddot{x} = mg; \quad m = k \cdot x; \quad \frac{dm}{dx} = k = \frac{m}{x}; \quad \dot{m} = \frac{dm}{dx} \cdot \dot{x} = m\frac{\dot{x}}{x}$$

$$\frac{\dot{x}^2}{x} + \ddot{x} = g; \quad \text{mit } \ddot{x} = \frac{1}{2}\frac{d(\dot{x}^2)}{dx} \text{ und } \dot{x}^2 = y \text{ ergibt sich:}$$

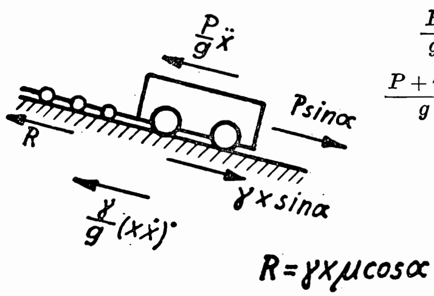
$$\frac{y}{x} + \frac{y'}{2} = g \quad \text{Eulersche Differentialgleichung}$$

Ansatz: $y = \frac{C_1}{x^2} + \frac{2}{3} g x$; für $x = 0$ ist $\dot{x} = 0$, also: $C_1 = 0$

$$\frac{\dot{x}}{\sqrt{x}} = \sqrt{\frac{2}{3} g}; \quad 2\sqrt{x} = \sqrt{\frac{2}{3} g}\, t + C_2; \quad \text{für } t = 0 \text{ ist } x = 0, \text{ also } C_2 = 0$$

$$\underline{\underline{x = \frac{g t^2}{6}}}$$

Lösung 1160



$$\frac{P}{g}\ddot{x} + \left(\frac{\gamma}{g} x\dot{x}\right)^{\cdot} = (P + \gamma x)\sin\alpha - \gamma x \mu \cos\alpha$$

$$\frac{P + \gamma x}{g} \cdot \ddot{x} + \frac{\gamma}{g}\dot{x}^2 = (P + \gamma x)\sin\alpha - \gamma x \mu \cos\alpha$$

Mit $\ddot{x} = \frac{1}{2}\frac{d(\dot{x}^2)}{dx}$ und $\dot{x}^2 = y$ gilt:

$$y' = 2g\sin\alpha - \frac{2\mu g \gamma x \cos\alpha}{P + \gamma x} - \frac{2\gamma y}{P + \gamma x}$$

$$y = u \cdot v; \quad y' = u'v + v'u$$

Die Differentialgleichung zerfällt somit in zwei Teile:

$$v' \cdot u = -\frac{2\gamma u v}{P+\gamma x}; \quad \ln v = -2\ln(P+\gamma x)$$

$$v = \frac{1}{(P+\gamma x)^2}$$

$$u' = (P+\gamma x)^2 \cdot 2g \sin\alpha - (P+\gamma x)\cdot 2\mu g x \cos\alpha$$

$$u = \frac{1}{3}(P+\gamma x)^3 \cdot \frac{2g}{\gamma}\sin\alpha - P\mu\gamma x^2\cos\alpha$$

$$-\frac{2}{3}\gamma\mu g x^3\cos\alpha + C$$

C aus $u(0)\cdot v(0) = v_0^2$, d. h. $u(0) = v_0^2 \cdot P^2$

$$C = P^2 v_0^2 - \frac{2}{3}\frac{P^3 g}{\gamma}\sin\alpha$$

$$\dot{x}^2 = u\cdot v = \frac{P^2 v_0^2}{(P+\gamma x)^2} + \frac{2Pg}{3\gamma}\sin\alpha\left[1-\frac{P^2}{(P+\gamma x)^2}\right] + \frac{2}{3}gx\sin\alpha + \frac{\mu P g\cos\alpha}{3\gamma}\left[1-\frac{P^2}{(P+x\gamma)^2}\right]$$

$$-\frac{2}{3}\mu g x\cos\alpha$$

Für $x\to\infty$ bleibt: $\dot{x}^2 = \frac{2}{3}gx(\sin\alpha - \mu\cos\alpha)$; dies wird Null für $\mu > \operatorname{tg}\alpha$

Lösung 1161

Die Gleichgewichtsbedingung in x-Richtung lautet:

$$m\frac{d^2x}{dt^2} + \frac{\Gamma\cdot M\cdot m}{r^2}\cdot\frac{x}{r} = 0; \quad \Gamma = \text{Gravitationskonstante}$$

$$\Gamma\cdot m = 1$$

$$\frac{d^2x}{dt^2} + \frac{M}{m}\cdot\frac{x}{r^3} = 0; \quad \xi = \frac{x}{1+\alpha t}; \quad \tau = \frac{1}{\alpha(1+\alpha t)}; \quad \xi = \alpha\cdot\tau\cdot x$$

$$\frac{d^2x}{dt^2} = \frac{d^2x}{d\tau^2}\left(\frac{d\tau}{dt}\right)^2 + \frac{dx}{d\tau}\cdot\frac{d^2\tau}{dt^2}; \qquad \frac{d\tau}{dt} = -\frac{1}{(1+\alpha t)^2} = -\alpha^2\tau^2$$

$$\frac{dx}{d\tau} = \frac{1}{\alpha}\cdot\frac{d\left(\frac{\xi}{\tau}\right)}{d\tau} = \frac{1}{\alpha}\left(\frac{1}{\tau}\frac{d\xi}{d\tau} - \frac{1}{\tau^2}\cdot\xi\right); \qquad \frac{d^2\tau}{dt^2} = 2\alpha^4\tau^3$$

$$\frac{d^2x}{d\tau^2} = \frac{1}{\alpha}\left(\frac{1}{\tau}\cdot\frac{d^2\xi}{d\tau^2} - \frac{2}{\tau^2}\cdot\frac{d\xi}{d\tau} + \frac{2}{\tau^3}\cdot\xi\right); \qquad \left(\frac{d\tau}{dt}\right)^2 = \alpha^4\tau^4$$

$$\frac{d^2x}{dt^2} = \alpha^3\left(\tau^3\frac{d^2\xi}{d\tau^2}\right); \quad \frac{x}{r^3} = \alpha^2\tau^2\frac{\xi}{\varrho^3}; \qquad x^2+y^2 = r^2;$$

$$\xi^2+\eta^2 = \varrho^2; \qquad \varrho = \alpha\tau r$$

$$M = \frac{M_0}{1+\alpha t}; \quad M = M_0\cdot\alpha\tau$$

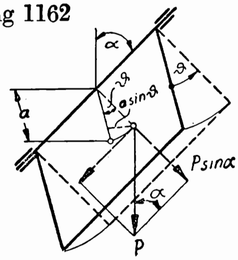
$$\frac{d^2\xi}{d\tau^2} + \frac{M_0}{m}\cdot\frac{\xi}{\varrho^3} = 0$$

Entsprechend ergibt sich:

$$\frac{d^2\eta}{d\tau^2} + \frac{M_0}{m}\cdot\frac{\eta}{\varrho^3} = 0$$

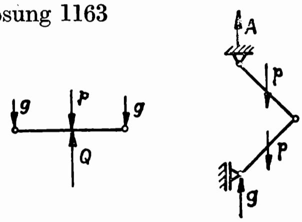
46. Analytische Statik

Lösung 1162



$$M = Pa \sin\alpha \sin\vartheta$$

Lösung 1163

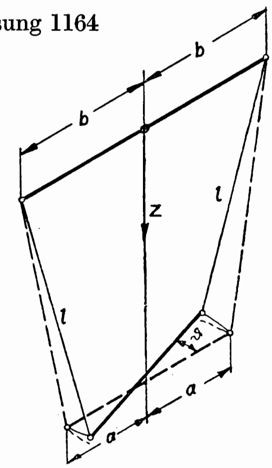


g = Gelenkdruck

$$Q = p + 2g; \qquad g = p = A$$

$$\underline{\underline{Q = 3p}}$$

Lösung 1164



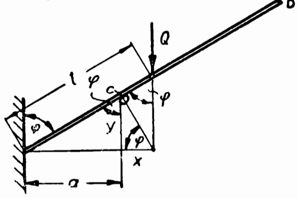
$$U = Q \cdot z = Q\sqrt{l^2 - (b - a\cos\vartheta)^2 - (a\sin\vartheta)^2}$$

$$U = Q\sqrt{l^2 - b^2 - a^2 + 2ab\cos\vartheta}$$

$$M = \frac{dU}{d\vartheta} = \frac{Q \cdot ab \cdot \sin\vartheta}{\sqrt{l^2 - b^2 - a^2 + 2ab\cos\vartheta}}$$

$$\underline{\underline{M\sqrt{l^2 - (a-b)^2 - 4ab\sin^2\frac{\vartheta}{2}} = Q \cdot ab\sin\vartheta}}$$

Lösung 1165



$$a + x = l\sin\varphi$$

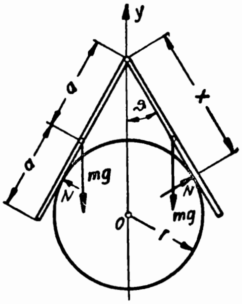
$$\operatorname{tg}\varphi = \frac{a}{y}; \quad y = \frac{a}{\operatorname{tg}\varphi}$$

$$\operatorname{tg}\varphi = \frac{y}{x}; \quad x = \frac{y^2}{a}$$

$$a + \frac{y^2}{a} = l\sin\varphi; \quad a\left(1 + \frac{1}{\operatorname{tg}^2\varphi}\right) = l\sin\varphi$$

$$\underline{\underline{\sin\varphi = \sqrt[3]{\frac{a}{l}}}}$$

Lösung 1166



$$\sum M_A = 0: \quad m g a \sin\vartheta - N \cdot x = 0$$

$$x = r \operatorname{ctg}\vartheta$$

$$\sum P_y = 0: \quad 2N\sin\vartheta = 2mg$$

$$m g a \sin\vartheta = \frac{mg}{\sin\vartheta} r \operatorname{ctg}\vartheta$$

$$\frac{a}{r}\sin^2\vartheta - \operatorname{ctg}\vartheta = 0$$

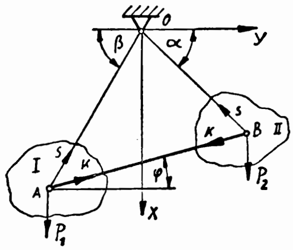
$$\frac{a}{r}\frac{\operatorname{tg}^2\vartheta}{1+\operatorname{tg}^2\vartheta} - \frac{1}{\operatorname{tg}\vartheta} = 0$$

$$\underline{\underline{a\operatorname{tg}^3\vartheta - r\operatorname{tg}^2\vartheta - r = 0}}$$

Lösung 1167

$$\begin{aligned} &\text{I} \quad K\cos\varphi = S\cos\beta \\ &\text{II} \quad K\cos\varphi = S\cos\alpha \end{aligned}; \quad \underline{\underline{\alpha = \beta}}$$

$$\operatorname{tg}\alpha = \operatorname{tg}\beta = \frac{x_1}{y_1} = \frac{x_2}{y_2}$$



$$P_1 : y_1 = P_2 y_2; \quad x_1^2 + y_1^2 = (\overline{AO})^2$$

$$y_1 = \frac{P_2}{P_1} y_2; \quad x_2^2 + y_2^2 = (\overline{BO})^2$$

$$x_1 = \frac{P_2}{P_1} x_2; \quad \left(\frac{P_2}{P_1}\right)^2 (x_2^2 + y_2^2) = (\overline{AO})^2$$

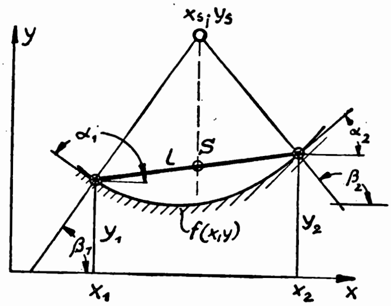
$$\underline{\underline{\frac{P_2}{P_1} = \frac{\overline{AO}}{\overline{BO}}}}$$

Ebenfalls herrscht Gleichgewicht für:

$$y_1 = y_2 = 0; \quad x_1 = \frac{1}{2}(L+l); \quad x_2 = \frac{1}{2}(L-l)$$

bzw.: $\quad x_1 = \frac{1}{2}(L-l); \quad x_2 = \frac{1}{2}(L+l)$

Lösung 1168



$$\underline{\underline{(x_2 - x_1)^2 + (y_2 - y_1)^2 = l^2;}} \quad \begin{aligned} f(x_1 y_1) = 0 \\ f(x_2 y_2) = 0 \end{aligned}$$

$$y' = \operatorname{tg}\alpha = -\frac{\dfrac{\partial f}{\partial x}}{\dfrac{\partial f}{\partial y}}$$

Gleichung der Normalen:

$$y_1'(y - y_1) = -(x - x_1)$$

$$\operatorname{tg}\beta_1 = -\frac{1}{y_1'} = 2\frac{y_S - y_1}{x_2 - x_1}$$

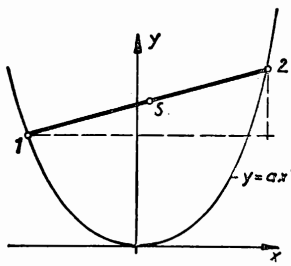
$$\operatorname{tg}(180 - \beta_2) = \frac{1}{y_2'} = 2\frac{y_S - y_2}{x_2 - x_1}$$

$$\frac{1}{y_2'} = 2\,\frac{y_1 - \dfrac{x_2 - x_1}{2y_1'} - y_2}{x_2 - x_1}$$

Daraus:

$$\underline{\underline{2(y_2 - y_1)\frac{\partial f}{\partial x_1}\cdot\frac{\partial f}{\partial x_2} = (x_2 - x_1)\left[\frac{\partial f}{\partial x_1}\cdot\frac{\partial f}{\partial y_2} + \frac{\partial f}{\partial y_1}\cdot\frac{\partial f}{\partial x_2}\right]}}$$

Lösung 1169



$$(y_2 - y_1)^2 + (x_2 - x_1)^2 - l^2 = 0; \quad y = x^2\cdot a$$

Daraus Zwangsbedingung:

$$F = a^2(x_2^2 - x_1^2)^2 + (x_2 - x_1)^2 - l^2 = 0 \qquad (1)$$

Schwerpunktshöhe: $y_S = \frac{a}{2}(x_1^2 + x_2^2)$

Um die Gleichgewichtsbedingung zu erfüllen, muß die potentielle Energie einen Extremwert haben. Also:

$$\frac{\partial y_S}{\partial x_1} + \lambda\frac{\partial F}{\partial x_1} = 0;$$

$$a x_1 + \lambda[-2a^2(x_2^2 - x_1^2)\cdot 2x_1 - 2(x_2 - x_1)] = 0$$

$$\frac{\partial y_S}{\partial x_2} + \lambda\frac{\partial F}{\partial x_2} = 0; \quad a x_2 + \lambda\{2a^2(x_2^2 - x_1^2)\cdot 2x_2 + 2(x_2 - x_1)\} = 0$$

$$a x_1 = \lambda[4a^2 x_1(x_2^2 - x_1^2) + 2(x_2 - x_1)]; \qquad -\frac{x_1}{x_2} = \frac{4a^2x_1(x_2^2 - x_1^2) + 2(x_2 - x_1)}{4a^2x_2(x_2^2 - x_1^2) + 2(x_2 - x_1)}$$

$$-a x_2 = \lambda[4a^2 x_2(x_2^2 - x_1^2) + 2(x_2 - x_1)];$$

$$(x_2^2 - x_1^2)(4a^2 x_1 x_2 + 1) = 0 \qquad (2)$$

Ansatz zur Lösung von (2) in Parameterform: $x_1 = -\frac{1}{2a}e^{-\xi}$; $x_2 = \frac{1}{2a}e^{\xi}$

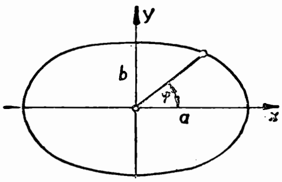
mit $y = ax^2$ ist: $y_1 = \frac{1}{4a}e^{-2\xi}$; $y_2 = \frac{1}{4a}e^{2\xi}$

Somit wird aus (1): $\underline{\underline{\mathfrak{Sin}^2 2\xi + 4\,\mathfrak{Cof}^2\xi = 4a^2l^2}}$

Die andere Gleichgewichtslage ist die Horizontale

$$x_2 = -x_1 = \frac{l}{2}; \quad y_1 = y_2 = \frac{al^2}{4}$$

Lösung 1170



Aus Aufgabe 1168 liegen folgende Gleichungen zugrunde:

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = l^2$$

$$2(y_2 - y_1)\frac{\partial f}{\partial x_1}\cdot\frac{\partial f}{\partial x_2} = (x_2 - x_1)\left[\frac{\partial f}{\partial x_1}\cdot\frac{\partial f}{\partial y_2} + \frac{\partial f}{\partial x_2}\cdot\frac{\partial f}{\partial y_1}\right]$$

Hier gilt: $x = a\cos\varphi$; $y = b\sin\varphi$; $\quad f = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$

Somit:

$$a^2(\cos\varphi_2 - \cos\varphi_1)^2 + b^2(\sin\varphi_2 - \sin\varphi_1)^2 - l^2 = 0$$

$$2b^2(\sin\varphi_2 - \sin\varphi_1) = a^2(\cos\varphi_2 - \cos\varphi_1)(\operatorname{tg}\varphi_1 + \operatorname{tg}\varphi_2)$$

$$4a^2\sin^2\frac{\varphi_2 + \varphi_1}{2}\sin^2\frac{\varphi_2 - \varphi_1}{2} + 4b^2\cos^2\frac{\varphi_2 + \varphi_1}{2}\sin^2\frac{\varphi_2 - \varphi_1}{2} = l^2 \qquad (1)$$

$$4b^2 \cos\frac{\varphi_2+\varphi_1}{2} \sin\frac{\varphi_2-\varphi_1}{2} = -2a^2 \sin\frac{\varphi_2+\varphi_1}{2} \sin\frac{\varphi_2-\varphi_1}{2} \cdot \frac{\sin(\varphi_1+\varphi_2)}{\cos\varphi_1 \cos\varphi_2}$$

$$b^2 \cos\frac{\varphi_2+\varphi_1}{2} [\cos(\varphi_1+\varphi_2) + \cos(\varphi_2-\varphi_1)] = -2a^2 \sin^2\frac{\varphi_2+\varphi_1}{2} \cos\frac{\varphi_2+\varphi_1}{2}$$

Daraus: 1. $\cos\frac{\varphi_2+\varphi_1}{2} = 0$; $\underline{\underline{\varphi_2+\varphi_1 = \pi}}$

in (1) eingesetzt: $\underline{\underline{\cos\varphi_2 = \frac{l}{2a}}}$

2. $b^2[\cos(\varphi_2+\varphi_1) + \cos(\varphi_2-\varphi_1)] = -2a^2 \sin^2\frac{\varphi_2+\varphi_1}{2}$

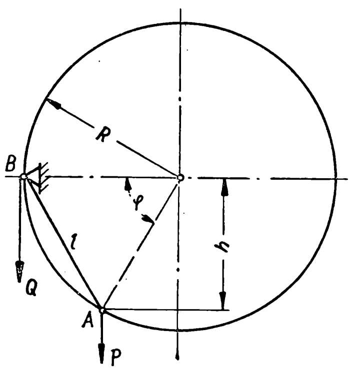
$$b^2\left[1 - 2\sin^2\frac{\varphi_1+\varphi_2}{2} + 2\cos^2\frac{\varphi_2-\varphi_1}{2} - 1\right] = -2a^2 \sin^2\frac{\varphi_1+\varphi_2}{2}$$

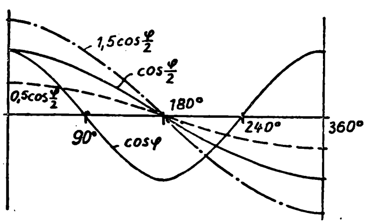
$$\cos^2\frac{\varphi_2-\varphi_1}{2} = \sin^2\frac{\varphi_1+\varphi_2}{2}\left(1-\frac{a^2}{b^2}\right)$$

$$\underline{\underline{\cos\frac{\varphi_2-\varphi_1}{2} = \sqrt{1-\frac{a^2}{b^2}} \sin\frac{\varphi_1+\varphi_2}{2}}}$$

in (1) eingesetzt: $\underline{\underline{\sin\frac{\varphi_2-\varphi_1}{2} = \sqrt{\frac{l}{2b}}}}$

Lösung 1171





Potentielle Energie:

$$U = Q \cdot l - P \cdot h; \qquad l = 2R\sin\frac{\varphi}{2}$$

$$h = R\sin\varphi$$

$$U = 2QR\sin\frac{\varphi}{2} - PR\sin\varphi$$

$$\frac{dU}{d\varphi} = 0: \quad \underline{\underline{\cos\varphi = \frac{Q}{P}\cos\frac{\varphi}{2}}}$$

Nach nebenstehender Abbildung wird diese Gleichung erfüllt bei folgenden Gleichgewichtslagen:

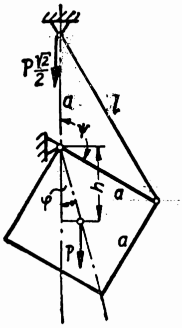
1. $0 \leq \frac{Q}{P} \leq 1$: $\quad 0 \leq \varphi \leq 90°$
   $240° \leq \varphi \leq 270°$
2. $\frac{Q}{P} \geq 1$: $\quad 180° \leq \varphi \leq 240°$

$$\frac{d^2U}{d\varphi^2} = R\left(-\frac{Q}{2}\sin\frac{\varphi}{2} + P\sin\varphi\right)$$

$\frac{d^2U}{d\varphi^2}$ für $180° \leq \varphi \leq 360°$ negativ: labil

$\frac{d^2U}{d\varphi^2}$ für $0° \leq \varphi \leq 90°$ positiv: stabil

Lösung 1172



Potentielle Energie:

$$U = \frac{P\sqrt{2}}{2} l - P \cdot h$$

$$l = 2a \sin\frac{\psi}{2}; \quad h = \frac{a}{2}\sqrt{2}\cos\varphi; \quad \varphi = 135 - \psi$$

$$h = \frac{a}{2}\sqrt{2}\cos(135^\circ - \psi)$$

$$U = P\frac{a\sqrt{2}}{2}\left[2\sin\frac{\psi}{2} - \cos(135^\circ - \psi)\right]$$

$$\frac{dU}{d\psi} = P\frac{a\sqrt{2}}{2}\left[\cos\frac{\psi}{2} - \sin(135^\circ - \psi)\right]$$

$$\frac{dU}{d\psi} = 0: \quad \cos\frac{\psi}{2} = \sin(135^\circ - \psi) = \cos(45^\circ - \psi)$$

Gleichgewichtslagen:

$$1.\ \frac{\psi}{2} = \left(\frac{\pi}{4} - \psi\right); \qquad \psi_1 = \frac{\pi}{6}$$

$$2.\ \frac{\psi}{2} = -\left(\frac{\pi}{4} - \psi\right); \qquad \psi_2 = \frac{\pi}{2}$$

$$3.\ \frac{\psi}{2} = 2\pi + \left(\frac{\pi}{4} - \psi\right); \qquad \psi_3 = \frac{3\pi}{2}$$

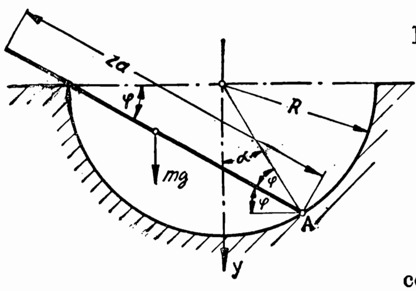
$$\frac{d^2U}{d\psi^2} = P\frac{a\sqrt{2}}{2}\left[-\frac{1}{2}\sin\frac{\psi}{2} + \cos(135^\circ - \psi)\right],$$

zu 1. $\frac{d^2U}{d\psi^2} = \text{neg.}$: labil

zu 2. $\frac{d^2U}{d\psi^2} = \text{pos.}$: stabil

zu 3. $\frac{d^2U}{d\psi^2} = \text{pos.}$: stabil

Lösung 1173



$$\alpha = 90 - 2\varphi$$

$$y_A = R(1 - \cos\alpha)$$

Potentielle Energie:

$$U = mg(y_A + a\sin\varphi)$$

$$U = mg[R(1 - \sin 2\varphi) + a\sin\varphi]$$

$$\frac{dU}{d\varphi} = mg[-2R\cos 2\varphi + a\cos\varphi]$$

$$\frac{dU}{d\varphi} = 0: \quad -2R(2\cos^2\varphi_0 - 1) + a\cos\varphi_0 = 0$$

$$\cos\varphi_0 = \frac{a}{8R} \pm \sqrt{\frac{a^2}{64R^2} + \frac{1}{2}}$$

$$\cos\varphi_0 = \frac{a \pm \sqrt{a^2 + 32R^2}}{8R}$$

$$\frac{d^2 U}{d\varphi^2} = mg\,(4R\sin 2\varphi_0 - a\sin\varphi_0) = \pm mg\sqrt{a^2 + 32R^2}\sin\varphi_0$$

Da $\sin\varphi_0 > 0$, gilt $(+)$ für stabiles Gleichgewicht und

$(-)$ für labiles Gleichgewicht.

Gleichgewichtsbedingungen: $\cos\varphi_0 \leqq 1;\quad (8R - a)^2 \geq a^2 + 32R^2$

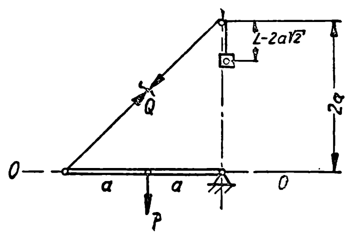
$$\underline{\underline{a \leq 2R}}$$

Damit der Stab am Rande aufliegt, gilt: $a \geq R\cos\varphi_0$

$$a \geq \frac{a + \sqrt{a^2 + 32R^2}}{8}$$

$$\underline{\underline{a \geq \sqrt{\frac{2}{3}}\,R}}$$

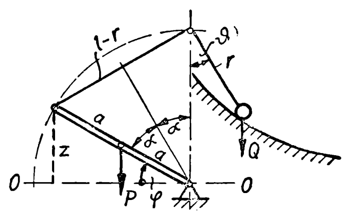
Lösung 1174



In der Anfangslage gilt:

$$Q \cdot a\sqrt{2} = Pa$$

$$\underline{\underline{Q = P\frac{\sqrt{2}}{2}}}$$



Bei indifferentem Gleichgewicht ist

$$U = U_0 = \text{const}$$

$$U_0 = Q[2a - (l - 2a\sqrt{2})]$$

$$U_0 = P\frac{\sqrt{2}}{2}[2a(1 + \sqrt{2}) - l]$$

$$U = P \cdot a\sin\varphi + Q\,(2a - r\cos\vartheta)$$

Aus den geometrischen Abmessungen folgt:

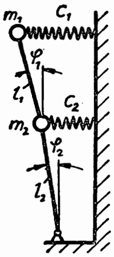
$$\sin\alpha = \frac{l - r}{4a};\qquad \sin\varphi = \cos 2\alpha = 1 - 2\sin^2\alpha = 1 - \frac{(l - r)^2}{8a^2}$$

Somit: $P\frac{\sqrt{2}}{2}[2a(1 + \sqrt{2}) - l] = P \cdot a\left[1 - \frac{(l - r)^2}{8a^2}\right] + P\frac{\sqrt{2}}{2}[2a - r\cos\vartheta]$

Daraus: $\underline{\underline{r^2 = 2r(l - 2\sqrt{2}\,a\cos\vartheta) - l^2 - 8a^2 + 4\sqrt{2}\,al}}$

Lösung 1175

$$U = m_1 g (l_1 \cos\varphi_1 + l_2 \cos\varphi_2) + m_2 g l_2 \cos\varphi_2$$
$$+ \frac{c_1}{2}(l_1 \sin\varphi_1 + l_2 \sin\varphi_2)^2 + \frac{c_2}{2} l_2^2 \sin^2\varphi_2$$



$$\frac{\partial U}{\partial \varphi_1} = -m_1 g l_1 \sin\varphi_1 + c_1 l_1 \cos\varphi_1 (l_1 \sin\varphi_1 + l_2 \sin\varphi_2)$$

$$\frac{\partial U}{\partial \varphi_2} = -(m_1 + m_2) g l_2 \sin\varphi_2 + c_1 l_2 \cos\varphi_2 (l_1 \sin\varphi_1 + l_2 \sin\varphi_2)$$
$$+ c_2 l_2^2 \sin\varphi_2 \cos\varphi_2$$

$$\left.\frac{\partial^2 U}{\partial \varphi_1^2}\right|_{\varphi_1=0} = -m_1 g l_1 + c_1 l_1^2 > 0; \quad \underline{\underline{c_1 l_1 > m_1 g}}$$

$$\left.\frac{\partial^2 U}{\partial \varphi_2^2}\right|_{\varphi_2=0} = -(m_1 + m_2) g l_2 + c_1 l_2^2 + c_2 l_2^2$$

$$\left.\frac{\partial^2 U}{\partial \varphi_1 \partial \varphi_2}\right|_{\varphi_1=\varphi_2=0} = c_1 l_1 l_2$$

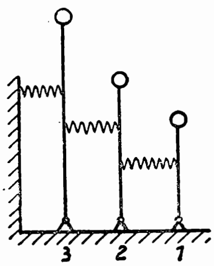
$$\frac{1}{l_1 l_2}\left[\frac{\partial^2 U}{\partial \varphi_1^2} \cdot \frac{\partial^2 U}{\partial \varphi_2^2} - \left(\frac{\partial^2 U}{\partial \varphi_1 \partial \varphi_2}\right)^2\right] = (c_1 l_1 - m_1 g)\,[(c_1 + c_2) l_2 - (m_1 + m_2) g] - c_1^2 l_1 l_2 > 0$$

$$\underline{\underline{[(c_1 + c_2) l_2 - (m_1 + m_2) g]\,(c_1 l_1 - m_1 g) > c_1^2 l_1 l_2}}$$

Lösung 1176

Potentielle Energie des Systems:

$$U = \frac{c h^2}{2}\left[(\varphi_1 - \varphi_2)^2 + 4(\varphi_2 - \varphi_3)^2 + 9\varphi_3^2\right]$$
$$+ m g h\,[2\cos\varphi_1 + 3\cos\varphi_2 + 4\cos\varphi_3]$$



Für Stabilität gilt:

$$\begin{vmatrix} U_{\varphi_1\varphi_1} & U_{\varphi_1\varphi_2} & U_{\varphi_1\varphi_3} \\ U_{\varphi_2\varphi_1} & U_{\varphi_2\varphi_2} & U_{\varphi_2\varphi_3} \\ U_{\varphi_3\varphi_1} & U_{\varphi_3\varphi_2} & U_{\varphi_3\varphi_3} \end{vmatrix} > 0$$

Also:

$$\begin{vmatrix} ch^2 - 2mgh & -ch^2 & 0 \\ -ch^2 & 5ch^2 - 3mgh & -4ch^2 \\ 0 & -4ch^2 & 13ch^2 - 4mgh \end{vmatrix} > 0$$

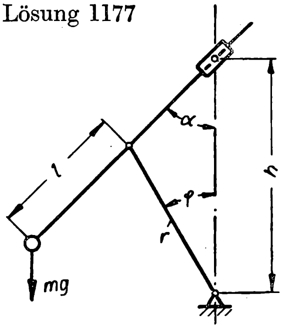
Daraus:

$$36c^3h^6 - 153mgc^2h^5 + 130m^2g^2ch^4 - 24m^3g^3h^3 > 0$$

Die Unterdeterminanten liefern:

$$13ch^2 - 4mgh > 0$$
$$49c^2h^4 - 59mgch^3 + 12m^2g^2h^2 > 0$$

Lösung 1177



Potentielle Energie:

$$U = mg(r\cos\varphi - l\cos\alpha)$$

$$\operatorname{tg}\alpha = \frac{r\sin\varphi}{h - r\cos\varphi}$$

$$U = mg\left[r\cos\varphi - l\sqrt{\frac{h^2 - 2rh\cos\varphi + r^2\cos^2\varphi}{h^2 - 2rh\cos\varphi + r^2}}\right]$$

$$U = mg[r\cos\varphi - l\varepsilon];$$

$$\frac{dU}{d\varphi} = mg\left[-r\sin\varphi - l\frac{d\varepsilon}{d\varphi}\right]$$

$$\frac{d\varepsilon}{d\varphi} = r\sin\varphi\frac{(h - r\cos\varphi)(h^2 - 2rh\cos\varphi + r^2) - h(h^2 - 2rh\cos\varphi + r^2\cos^2\varphi)}{\varepsilon(h^2 - 2rh\cos\varphi + r^2)^2}$$

$$\frac{d\varepsilon}{d\varphi} = r\sin\varphi\frac{\beta}{\lambda}; \quad \frac{d^2U}{d\varphi^2} = mg\left[-r\cos\varphi - l\frac{d^2\varepsilon}{d\varphi^2}\right]$$

$$= mg\left[-r\cos\varphi - l\left[r\cos\varphi\cdot\frac{\beta}{\varepsilon\lambda} + r\sin\varphi\cdot\frac{d\left(\frac{\beta}{\varepsilon\lambda}\right)}{d\varphi}\right]\right]$$

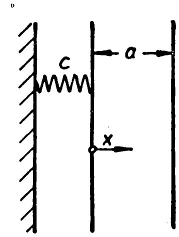
Für die vertikale Gleichgewichtslage ($\varphi = 0$) gilt: $\left.\frac{d^2U}{d\varphi^2}\right|_{\varphi=0} = mg\left(-r - lr\frac{\beta}{\varepsilon\lambda}\right)$

$$\lambda_{\varphi=0} = (h - r)^4; \quad \beta_{\varphi=0} = -r(h - r)^2; \quad \varepsilon_{\varphi=0} = 1$$

$\left.\frac{d^2U}{d\varphi^2}\right|_{\varphi=0} = mgr\left[lr\cdot\frac{1}{(h - r)^2} - 1\right]$; stab. Gleichgewichtsl.: $\frac{d^2U}{d\varphi^2} > 0$: $\underline{\underline{\sqrt{lr} > (h - r)}}$

lab. Gleichgewichtsl.: $\frac{d^2U}{d\varphi^2} < 0$: $\underline{\underline{\sqrt{lr} < (h - r)}}$

Lösung 1178



Potentielle Energie:

$$U = -\frac{c(x^2 - a^2)}{2} + \int\frac{2i_1i_2\cdot l}{(a - x)}dx + C$$

$$\frac{dU}{dx} = 0 = -cx + \frac{2i_1i_2\cdot l}{a - x}$$

$$x = \frac{a}{2} \pm \sqrt{\frac{a^2}{4} - \frac{2i_1i_2l}{c}}; \quad \frac{2i_1i_2l}{c} = \alpha$$

Stabilität herrscht bei: $\frac{d^2U}{dx^2} > 0$

$$\frac{d^2U}{dx^2} = -c + \frac{2i_1i_2\cdot l}{(a - x)^2} = c\left[\frac{\alpha}{(a - x)^2} - 1\right]$$

$$(a - x)^2 < \alpha$$

Somit gilt für $\alpha < \frac{a^2}{4}$:

$$x_1 = \frac{a}{2} - \sqrt{\frac{a^2}{4} - \alpha} \text{ stabil}$$

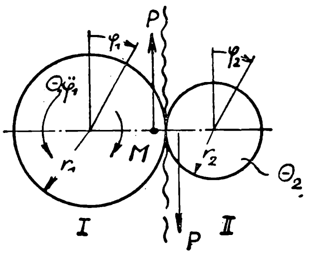
$$x_2 = \frac{a}{2} + \sqrt{\frac{a^2}{4} - \alpha} \text{ labil}$$

Für $\alpha > \frac{a^2}{4}$ ist $x = \frac{a}{2} \pm i \sqrt{\alpha - \frac{a^2}{4}}$; es gibt also keine Gleichgewichtslage.

Für $\alpha = \frac{a^2}{4}$ ist das Gleichgewicht indifferent.

47. Die Gleichungen von Lagrange

Lösung 1179



Kinetische Energie: $T = \Theta_1 \frac{\dot{\varphi}_1^2}{2} + \Theta_2 \frac{\dot{\varphi}_2^2}{2}$

Potentielle Energie: $U = 0$

Äußere Arbeit: $A = M_1 \varphi_1 + M_2 \varphi_2 - P(r_1 \varphi_1 + r_2 \varphi_2)$

$L = T - U$

$$\left(\frac{\partial L}{\partial \dot{\varphi}_1}\right)^{\cdot} - \frac{\partial L}{\partial \varphi_1} = \frac{\partial A}{\partial \varphi_1}: \quad \Theta_1 \ddot{\varphi}_1 = M_1 - P r_1$$

$$\left(\frac{\partial L}{\partial \dot{\varphi}_2}\right)^{\cdot} - \frac{\partial L}{\partial \varphi_2} = \frac{\partial A}{\partial \varphi_2}: \quad \Theta_2 \ddot{\varphi}_2 = M_2 - P r_2$$

$$-\varphi_1 = \frac{\varphi_2}{k}; \quad k = \frac{r_1}{r_2} = \frac{z_1}{z_2}$$

$$\frac{r_1}{r_2} = \frac{M_1 - \Theta_1 \ddot{\varphi}_1}{M_2 - \Theta_2 \ddot{\varphi}_2}$$

$$k = \frac{M_1 - \Theta_1 \ddot{\varphi}_1}{M_2 + \Theta_2 k \ddot{\varphi}_1}$$

$$\underline{\underline{\ddot{\varphi}_1 = \varepsilon_1 = \frac{M_1 - k M_2}{\Theta_1 + \Theta_2 k^2}}}$$

Lösung 1180

$$T = \frac{1}{2} [\Theta_1 \dot{\varphi}_1^2 + \Theta_2 \dot{\varphi}_2^2 + \Theta_3 \dot{\varphi}_3^2];$$

$$U = 0; \qquad \varphi_2 = \frac{r_1}{r_2} \varphi_1: \quad \varphi_3 = \frac{r_1}{r_3} \varphi_1$$

$$A = M_1 \varphi_1 - M_2 \varphi_2 - M_3 \varphi_3; \qquad L = T - U$$

$$\left(\frac{\partial L}{\partial \dot{\varphi}_1}\right)^{\cdot} - \frac{\partial L}{\partial \varphi_1} = \frac{\partial A}{\partial \varphi_1}: \quad \ddot{\varphi}_1 \left(\Theta_1 + \Theta_2 \cdot \frac{r_1^2}{r_2^2} + \Theta_3 \frac{r_1^2}{r_3^2}\right) = M_1 - \frac{r_1}{r_2} M_2 - \frac{r_1}{r_3} M_3$$

$$\Theta = \frac{m r^2}{2}$$

$$\underline{\underline{\ddot{\varphi}_1 = \varepsilon_1 = \frac{2\left(M_1 - \frac{r_1}{r_2} M_2 - \frac{r_1}{r_3} M_3\right)}{(m_1 + m_2 + m_3) r_1^2}}}$$

Lösung 1181

$$T = \Theta \cdot \frac{\dot\varphi^2}{2} + \frac{P}{g}\frac{\dot x^2}{2} + \frac{P_1 \dot x^2}{g \cdot 2}; \quad \Theta = \frac{P_2}{g} a^2; \quad a\dot\varphi = \dot x$$

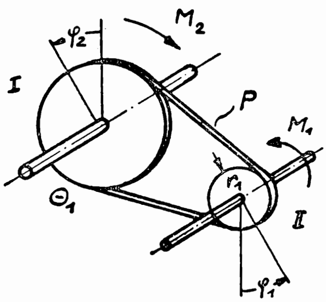
$$U = -P \cdot x - \frac{P_1 x_2}{2l}$$

$$L = T - U; \quad \left(\frac{\partial L}{\partial \dot x}\right)^{\cdot} - \frac{\partial L}{\partial x} = 0: \quad (P_1 + P_2 + P)\ddot x - \frac{P_1}{l} g x - P g = 0$$

Lösung der Differentialgleichung unter Berücksichtigung der Anfangsbedingungen $t = 0: x = l_0; \quad \dot x = 0:$

$$\underline{\underline{x = -\frac{P \cdot l}{P_1} + \left(l_0 + \frac{P \cdot l}{P_1}\right) \operatorname{Cof} \sqrt{\frac{P_1 g}{l(P + P_1 + P_2)}} \cdot t}}$$

Lösung 1182



$$T = \Theta_1 \frac{\dot\varphi_1^2}{2} + \Theta_2 \frac{\dot\varphi_2^2}{2} + \frac{P}{g}\frac{\dot x^2}{2}; \quad x = r_1 \varphi_1 = r_2 \varphi_2$$

$$U = 0; \qquad \frac{r_1}{r_2} = k$$

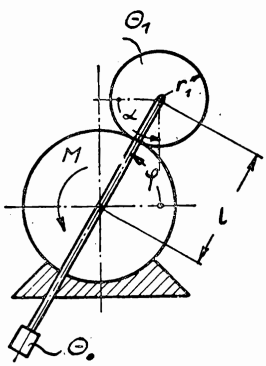
$$A = M_1 \varphi_1 - M_2 \varphi_2;$$

$$L = T - U; \quad \left(\frac{\partial L}{\partial \dot\varphi_1}\right)^{\cdot} - \frac{\partial L}{\partial \varphi_1} = \frac{\partial A}{\partial \varphi_1}:$$

$$\ddot\varphi_1 \left(\Theta_1 + \Theta_2 \frac{r_1^2}{r_2^2} + \frac{P}{g} r_1^2\right) = M_1 - M_2 \frac{r_1}{r_2}$$

$$\underline{\underline{\ddot\varphi_1 = \varepsilon_1 = g \frac{M_1 - k M_2}{(\Theta_1 + k^2 \Theta_2) g + P r_1^2}}}$$

Lösung 1183



$$T = \frac{\Theta_0 \cdot \dot\varphi^2}{2} + \frac{\Theta_1 \dot\alpha^2}{2} + m_1 \cdot l^2 \cdot \frac{\dot\varphi^2}{2}$$

$$U = 0; \quad \varphi \cdot l = \alpha r_1; \quad \dot\alpha = \frac{l}{r_1} \dot\varphi$$

$$A = M \cdot \varphi; \quad L = T - U$$

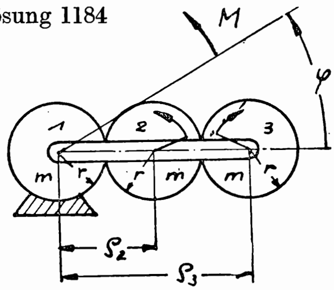
$$\left(\frac{\partial L}{\partial \dot\varphi}\right)^{\cdot} - \frac{\partial L}{\partial \varphi} = \frac{\partial A}{\partial \varphi}: \quad \ddot\varphi \left[\Theta_0 + \Theta_1 \left(\frac{l}{r_1}\right)^2 + m_1 l^2\right] = M$$

$$\underline{\underline{\ddot\varphi = \varepsilon = \frac{M}{\Theta_0 + \Theta_1 \left(\frac{l}{r_1}\right)^2 + m_1 l^2}}}$$

Umfangskraft: $S \cdot r_1 = \Theta_1 \cdot \ddot\alpha$

$$\underline{\underline{S = \frac{\Theta_1}{r_1^2} \cdot l \cdot \varepsilon}}$$

Lösung 1184



$$T = \Theta_2 \cdot \frac{\dot\varphi_2^2}{2} + m_2 \frac{\varrho_2^2 \cdot \dot\varphi^2}{2} + \Theta_3 \frac{\dot\varphi_3^2}{2} + m_3 \frac{\varrho_3^2 \varphi^2}{2}$$

$$m_2 = m_3 = m; \quad \Theta_2 = \Theta_3$$

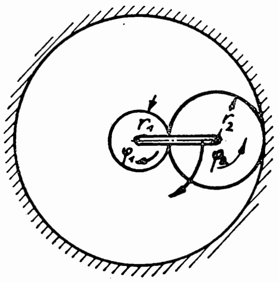
$$\varrho_2 = 2r; \quad \varphi_2 = 2\varphi$$

$$\varrho_3 = 4r; \quad \varphi_3 = 0$$

$$U = 0; \quad A = M\varphi; \quad L = T - U$$

$$\left(\frac{\partial L}{\partial \dot\varphi}\right)^{\bullet} - \frac{\partial L}{\partial \varphi} = \frac{\partial A}{\partial \varphi}: \quad \underline{\underline{\ddot\varphi = \varepsilon_1 = \frac{M}{22 m r^2}}}$$

Lösung 1185



$$T = \frac{\Theta_1}{2}\dot\varphi_1^2 + \frac{\Theta_2}{2}\dot\varphi_2^2 + \frac{m_2}{2}(r_1 + r_2)^2 \cdot \dot\varphi^2$$

$$U = 0; \quad A = M\varphi - M_1\varphi_1$$

$$\varphi_1 = 10\varphi;$$

$$\varphi_2 = \varphi\left(\frac{r_1 + 2r_2}{r_2} - 1\right) = \varphi\left(\frac{r_1}{r_2} + 1\right)$$

$$\varphi_1 = (\varphi_2 + \varphi)\frac{r_2}{r_1} + \varphi$$

$$10 = 2\left(\frac{r_2}{r_1} + 1\right); \quad \frac{r_2}{r_1} = 4$$

Somit: $\varphi_2 = \frac{5}{4}\varphi; \quad \varphi_1 = 10\varphi$

$$\Theta_2 = \Theta_1\left(\frac{r_2}{r_1}\right)^4 = 256\,\Theta_1; \quad m_2 = \frac{\Theta_2}{r_2^2}; \quad m_2(r_1 + r_2)^2 = 1600\,\Theta_1$$

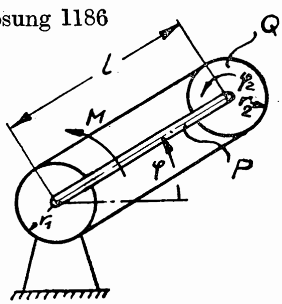
Somit: $T = \frac{\Theta_1}{2}100\dot\varphi^2 + \frac{256}{2}\Theta_1 \cdot \frac{25}{16}\dot\varphi^2 + \frac{1600}{2}\Theta_1\dot\varphi^2$

$$A = M\varphi - 10 M_1\varphi; \quad L = T - U$$

$$\left(\frac{\partial L}{\partial \dot\varphi}\right)^{\bullet} - \frac{\partial L}{\partial \varphi} = \frac{\partial A}{\partial \varphi}: \quad 100\,\Theta_1\ddot\varphi + 400\,\Theta_1\ddot\varphi + 800\,\Theta_1\ddot\varphi = M - 10 M_1$$

$$\underline{\underline{\ddot\varphi = \varepsilon = \frac{M - 10 M_1}{1300\,\Theta_1}}}$$

Lösung 1186



$$T = \Theta_K \cdot \frac{\dot\varphi^2}{2} + \frac{Q}{g} l^2 \cdot \dot\varphi^2; \quad \varphi_2 = 0$$

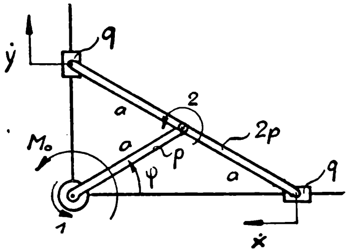
$$U = 0; \quad A = M \cdot \varphi; \quad \Theta_K = \frac{P}{g} \cdot \frac{l^2}{3}$$

$$L = T - U; \quad \left(\frac{\partial L}{\partial \dot\varphi}\right)^{\bullet} - \frac{\partial L}{\partial \varphi} = \frac{\partial A}{\partial \varphi}:$$

$$\ddot\varphi\left(\frac{P \cdot l^2}{g \cdot 3} + \frac{Q}{g} l^2\right) = M$$

$$\underline{\underline{\ddot\varphi = \varepsilon = \frac{3 g M}{l^2(P + 3Q)}}}$$

Lösung 1187



$$\Theta_1=\frac{p}{g}\,\frac{a^2}{3};\quad \Theta_2=\frac{2p}{g}\cdot\frac{a^2}{3};\quad \begin{aligned}x&=-2a\cos\varphi\\ y&=2a\sin\varphi\end{aligned}$$

$$T=\Theta_1\frac{\dot\varphi^2}{2}+\Theta_2\frac{\dot\varphi^2}{2}+\frac{2p}{g}a^2\cdot\frac{\dot\varphi^2}{2}+\frac{q}{2g}(\dot x^2+\dot y^2)$$

$$T=\left(\frac{3p}{g}a^2+\frac{4q}{g}a^2\right)\frac{\dot\varphi^2}{2}$$

$$A=M_0\cdot\varphi;\quad L=T-U;\quad U=0$$

$$\left(\frac{\partial l}{\partial\dot\varphi}\right)^{\cdot}-\frac{\partial l}{\partial\varphi}=A\varphi:\quad \underline{\underline{\ddot\varphi=\varepsilon=\frac{M_0\cdot g}{a^2(3p+4q)}}}$$

Lösung 1188

$$\varphi_2=\varphi_{\mathrm{I}}\left(1+\frac{r_1}{r_2}\right)=\varphi_3;\quad \varphi_3r_3=\varphi_{\mathrm{IV}}r_4;\quad \varphi_{\mathrm{IV}}=\varphi_{\mathrm{I}}\left(1-\frac{r_3r_1}{r_4r_2}\right)$$

$$\underline{\underline{\varepsilon_2=\varepsilon_1\left(1-\frac{r_3r_1}{r_4r_2}\right)}}$$

$$T=\frac{\Theta_1\dot\varphi_{\mathrm{I}}^2}{2}+\frac{2m_2(r_1+r_2)^2\dot\varphi_{\mathrm{I}}^2}{2}+\frac{2\Theta_2\dot\varphi_2^2}{2}+\frac{\Theta_4\cdot\dot\varphi_{\mathrm{IV}}^2}{2}$$

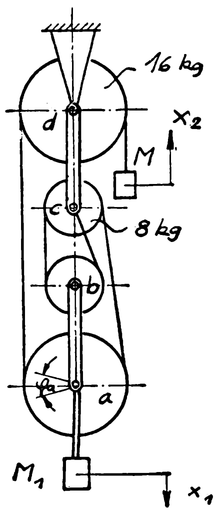
$$U=0;\quad A=M_1\varphi_{\mathrm{I}}-M_4\varphi_{\mathrm{IV}};\quad L=T-U$$

$$\left(\frac{\partial l}{\partial\dot\varphi_{\mathrm{I}}}\right)^{\cdot}-\frac{\partial l}{\partial\varphi_{\mathrm{I}}}=\frac{\partial A}{\partial\varphi_{\mathrm{I}}}:$$

$$\ddot\varphi_{\mathrm{I}}\left[\Theta_1+2m_2(r_1+r_2)^2+2\Theta_2\left(1+\frac{r_1}{r_2}\right)^2+\Theta_4\left(1-\frac{r_1r_3}{r_2r_4}\right)^2\right]=M_1-M_4\left(1-\frac{r_1r_3}{r_2r_4}\right)$$

$$\underline{\underline{\ddot\varphi_{\mathrm{I}}=\varepsilon_1=\frac{M_1-M_4\left(1-\frac{r_1r_3}{r_2r_4}\right)}{\Theta_1+2m_2(r_1+r_2)^2+2\Theta_2\left(1+\frac{r_1}{r_2}\right)^2+\Theta_4\left(1-\frac{r_1r_3}{r_2r_4}\right)^2}}}$$

Lösung 1189



$$T=M\frac{\dot x_2^2}{2}+M_1\frac{\dot x_1^2}{2}+(m_a+m_b)\frac{\dot x_1^2}{2}+\Theta_a\frac{\dot\varphi_a^2}{2}+\Theta_b\frac{\dot\varphi_b^2}{2}$$

$$+\Theta_c\frac{\dot\varphi_c^2}{2}+\Theta_d\frac{\dot\varphi_d^2}{2}$$

$$\varphi_a=\frac{3x_1}{r};\quad \varphi_b=\frac{x_1}{r_1};\quad \varphi_c=\frac{2x_1}{r_1};\quad \varphi_d=\frac{x_2}{r};\quad x_2=4x_1;$$

$$m_a=2m_b$$

$$T=\frac{\dot x_2^2}{2}\left[M+\frac{M_1}{16}+\frac{58}{32}m_a\right];\quad A=0$$

$$U=Mgx_2-\frac{M_1}{4}gx_2-\frac{3}{2}m_ag\frac{x_2}{4};\quad L=T-U$$

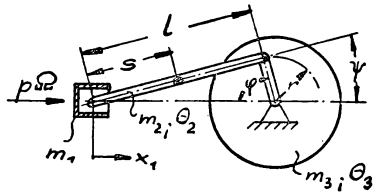
$$\left(\frac{\partial l}{\partial\dot x_2}\right)^{\cdot}-\frac{\partial l}{\partial x_2}=0:\quad \ddot x\left[M+\frac{M_1}{16}+\frac{58}{32}m_a\right]+Mg-\frac{M_1}{4}g$$

$$-\frac{3}{8}m_ag=0$$

$$\ddot x_2=-\frac{M-\frac{M_1}{4}-\frac{3}{8}m_a}{M+\frac{M_1}{16}+\frac{58}{32}m_a}\cdot g$$

$\ddot x_2=-0{,}1\,g$ Vorzeichen (—) besagt, daß die Last M sinkt.

Lösung 1190



$$T = \frac{m_1}{2}\dot{x}_1^2 + \frac{m_2}{2}(\dot{x}_2^2 + \dot{y}_2^2)$$

$$+ \frac{\Theta_2}{2}\dot{\psi}^2 + \frac{\Theta_3}{2}\dot{\varphi}^2$$

$$U = 0; \quad A = p\,\Omega x_1 - M\varphi$$

$\cos\psi = 1; \quad \sin\psi = \psi:$

$$x_1 = r(1 - \cos\varphi); \quad x_2 = s + r(1 - \cos\varphi); \quad y_2 = s\cdot\psi; \quad \psi = \frac{r}{l}\sin\varphi$$

$$\dot{x}_1 = r\sin\varphi\dot{\varphi}; \quad \dot{x}_2 = r\sin\varphi\dot{\varphi}; \quad \dot{y}_2 = s\cdot\dot{\psi}; \quad \dot{y}_2 = \frac{r}{l}s\cos\varphi\dot{\varphi}$$

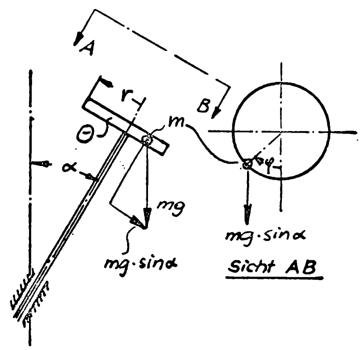
Somit:

$$L = T - U = \frac{m_1}{2}r^2\sin^2\varphi\dot{\varphi}^2 + \frac{m_2}{2}\left[r^2\sin^2\varphi\dot{\varphi}^2 + \left(\frac{r}{l}\right)^2 s^2\cos^2\varphi\dot{\varphi}^2\right] + \frac{\Theta_2}{2}\left(\frac{r}{l}\right)^2\cos^2\varphi\dot{\varphi}^2 + \frac{\Theta_3}{2}\dot{\varphi}^2$$

$$\left(\frac{\partial L}{\partial\dot{\varphi}}\right)^{\cdot} - \frac{\partial L}{\partial\varphi} = \frac{\partial A}{\partial\varphi}: \quad \ddot{\varphi}\left[(m_1 + m_2)r^2\sin^2\varphi + (\Theta_2 + ms^2)\left(\frac{r}{l}\right)^2\cos^2\varphi + \Theta_3\right]$$

$$+ \dot{\varphi}^2\cos\varphi\sin\varphi\left[(m_1 + m_2)r^2 - (\Theta_2 + ms^2)\left(\frac{r}{l}\right)^2\right] = -M + p\,\Omega r\sin\varphi$$

Lösung 1191



$$T = \frac{\Theta\dot{\varphi}^2}{2} + mr^2\cdot\frac{\dot{\varphi}^2}{2}$$

$$U = mg\sin\alpha\cdot r(1 - \cos\varphi)$$

$$L = T - U; \quad \left(\frac{\partial L}{\partial\dot{\varphi}}\right)^{\cdot} - \frac{\partial L}{\partial\varphi} = 0:$$

$$(\Theta + mr^2)\cdot\ddot{\varphi} + mgr\sin\alpha\sin\varphi = 0$$

Für kleine Ausschläge gilt: $\sin\varphi = \varphi$;

$$k = \sqrt{\frac{mgr\sin\alpha}{\Theta + mr^2}}$$

Lösung 1192

$$T = \frac{m}{2}\dot{s}^2$$

$$U = \int P\cdot ds; \quad P = mg\sin\varphi; \quad s = 4a\sin\varphi$$

$$P = mg\cdot\frac{s}{4a}$$

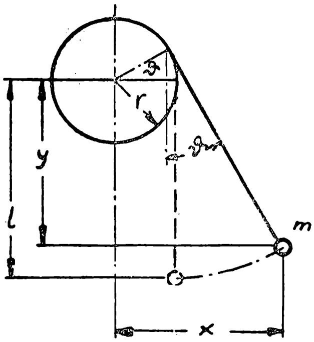
$$U = \int\frac{mg}{4a}\cdot s\,ds = \frac{mg}{8a}s^2$$

$$L = T - U; \quad \left(\frac{\partial L}{\partial\dot{s}}\right)^{\cdot} - \frac{\partial L}{\partial s} = 0:$$

$$\ddot{s} + \frac{g}{4a}s = 0$$

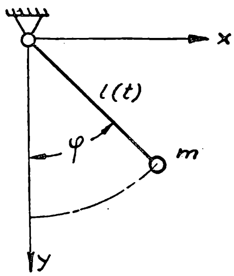
$$s = A\cdot\sin\left(\frac{1}{2}\sqrt{\frac{g}{a}}\,t + \varphi_0\right)$$

Lösung 1193



$$x = (l + r\vartheta)\sin\vartheta + r\cos\vartheta;$$
$$y = (l + r\vartheta)\cos\vartheta - r\sin\vartheta;$$
$$\left.\begin{aligned} \dot{x} &= (l + r\vartheta)\cos\vartheta\,\dot{\vartheta} \\ \dot{y} &= -(l + r\vartheta)\sin\vartheta\,\dot{\vartheta} \end{aligned}\right\} \begin{aligned} v^2 &= \dot{x}^2 + \dot{y}^2 \\ v^2 &= (l + r\vartheta)^2\dot{\vartheta}^2 \end{aligned}$$
$$L = \frac{m}{2}(l + r\vartheta)^2\dot{\vartheta}^2 + mg[(l + r\vartheta)\cos\vartheta - r\sin\vartheta]$$
$$\left(\frac{\partial L}{\partial\dot{\vartheta}}\right)^{\cdot} = m[(l + r\vartheta)^2\ddot{\vartheta} + 2(l + r\vartheta)r\dot{\vartheta}^2]$$
$$\left(\frac{\partial L}{\partial\vartheta}\right) = m[(l + r\vartheta)r\dot{\vartheta} - g(l + r\vartheta)\sin\vartheta]$$
$$\left(\frac{\partial L}{\partial\dot{\vartheta}}\right)^{\cdot} - \frac{\partial L}{\partial\vartheta} = 0: \quad \underline{\underline{(l + r\vartheta)\ddot{\vartheta} + r\dot{\vartheta}^2 + g\sin\vartheta = 0}}$$

Lösung 1194



$$l = l(t); \quad x = l\sin\varphi; \quad \dot{x} = \dot{l}\sin\varphi + l\cos\varphi\,\dot{\varphi}$$
$$y = l\cos\varphi; \quad \dot{y} = \dot{l}\cos\varphi + l\sin\varphi\,\dot{\varphi}$$
$$\dot{x}^2 + \dot{y}^2 = v^2 = \dot{l}^2 + l^2\dot{\varphi}^2$$
$$T = \frac{m}{2}(\dot{l}^2 + l^2\dot{\varphi}^2); \quad U = -mgl\cdot\cos\varphi$$
$$L = T - U; \quad \left(\frac{\partial L}{\partial\dot{\varphi}}\right)^{\cdot} - \frac{\partial L}{\partial\varphi} = 0:$$
$$ml^2\ddot{\varphi} + 2m\dot{\varphi}l\dot{l} + lmg\sin\varphi = 0$$
$$\underline{\underline{l\ddot{\varphi} + 2\dot{l}\dot{\varphi} + g\sin\varphi = 0}}$$

Lösung 1195

$$\ddot{\varphi} + 2\frac{\dot{l}}{l}\dot{\varphi} + \frac{g}{l}\varphi = 0; \qquad \frac{d}{dt} = c\frac{d}{dl}$$
$$\frac{d^2\varphi}{dl^2} + 2\frac{1}{l}\cdot\frac{d\varphi}{dl} + \frac{g}{c^2 l}\varphi = 0$$

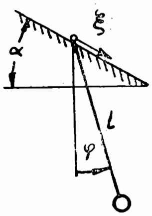
Nach Kamke, Differentialgleichungen Band 1, 4. Auflage, Seite 440, ist die Lösung:

$$\varphi = \frac{1}{\sqrt{l(t)}} Z_1\left(2\sqrt{\frac{g}{c^2}\cdot l(t)}\right)$$

bzw.: $$\varphi = \frac{1}{\sqrt{l(t)}}\left[C_1 J_1\left(2\sqrt{\frac{g}{c^2}l(t)}\right) + C_2 Y_1\left(2\sqrt{\frac{g}{c^2}l(t)}\right)\right],$$

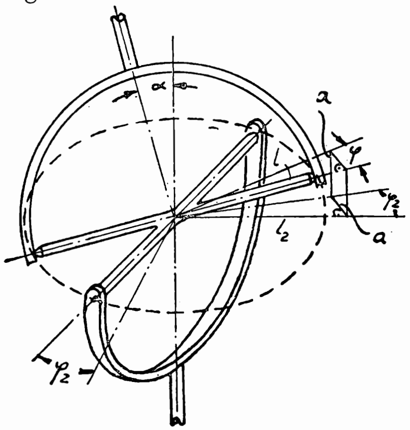
worin C_1 und C_2 willkürliche Konstante und J_1, Y_1 die Besselschen und Neumannschen Funktionen 1. Ordnung sind.

Lösung 1196



$$y = -l\cos\varphi + \xi\sin\alpha$$
$$x = l\sin\varphi + \xi\cos\alpha$$
$$T = \frac{m}{2}(\dot{x}^2 + \dot{y}^2)$$
$$U = mg(-l\cos\varphi + \xi\sin\alpha)$$
$$\dot{y} = l\sin\varphi\dot{\varphi} + \dot{\xi}\sin\alpha$$
$$\dot{x} = l\cos\varphi\dot{\varphi} + \dot{\xi}\cos\alpha$$
$$\dot{x}^2 + \dot{y}^2 = l^2\dot{\varphi}^2 + \dot{\xi}^2 + 2l\dot{\varphi}\dot{\xi}\cos(\varphi - \alpha)$$
$$L = T - U = \frac{m}{2}[l^2\dot{\varphi}^2 + \dot{\xi}^2 + 2l\dot{\varphi}\dot{\xi}\cos(\varphi-\alpha)] - mg\xi\sin\alpha + mgl\cos\alpha$$
$$\left(\frac{\partial L}{\partial\dot{\varphi}}\right)^{\cdot} - \frac{\partial L}{\partial\varphi} = 0: \quad m[l^2\ddot{\varphi} + l\,(\ddot{\xi}\cos(\varphi-\alpha) - \dot{\xi}\dot{\varphi}\sin(\varphi-\alpha)) + mgl\sin\varphi$$
$$+ ml\dot{\varphi}\dot{\xi}\sin(\varphi-\alpha)] = 0$$
$$\underline{\underline{\ddot{\varphi} + \frac{\ddot{\xi}}{l}\cos(\varphi-\alpha) + \frac{g}{l}\sin\varphi = 0}}$$

Lösung 1197



$$a = l\cdot\operatorname{tg}\varphi = l_2\operatorname{tg}\varphi_2$$
$$\operatorname{tg}\varphi_2 = \frac{l}{l_2}\cdot\operatorname{tg}\varphi$$
$$l_2 = l\cos\alpha$$
$$\operatorname{tg}\varphi_2 = \frac{\operatorname{tg}\varphi}{\cos\alpha}$$
$$\dot{\varphi}_2 = \frac{1}{\cos\alpha}\cdot\frac{\cos^2\varphi_2}{\cos^2\varphi}\dot{\varphi} = \frac{\cos\alpha\cdot\dot{\varphi}}{\cos^2\varphi(\cos^2\alpha + \operatorname{tg}^2\varphi)}$$
$$\dot{\varphi}_2 = \frac{\cos\alpha}{1 - \sin^2\alpha\cos^2\varphi}\cdot\dot{\varphi}$$
$$T = \Theta_1\frac{\dot{\varphi}^2}{2} + \Theta_2\frac{\dot{\varphi}_2^2}{2}$$
$$U = 0; \quad A = M_1\varphi - M_2\varphi_2$$
$$L = T - U$$
$$\frac{\partial L}{\partial\dot{\varphi}} = \Theta_1\dot{\varphi} + \Theta_2\dot{\varphi}\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\varphi}\right)^2;$$
$$\left(\frac{\partial L}{\partial\dot{\varphi}}\right)^{\cdot} = \left[\Theta_1 + \Theta_2\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\varphi}\right)^2\right]\ddot{\varphi} + \Theta_2\dot{\varphi}^2\frac{d}{d\varphi}\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\varphi}\right)^2$$
$$\frac{\partial L}{\partial\varphi} = \frac{\Theta_2}{2}\dot{\varphi}^2\frac{d}{d\varphi}\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\varphi}\right); \qquad \frac{\partial A}{\partial\varphi} = M_1 - M_2\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\varphi}$$
$$\left(\frac{\partial L}{\partial\dot{\varphi}}\right)^{\cdot} - \frac{\partial L}{\partial\varphi} = \frac{\partial A}{\partial\varphi}:$$
$$\underline{\underline{\left[\Theta_1 + \Theta_2\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\varphi}\right)^2\right]\ddot{\varphi} - \Theta_2\frac{\sin^2\alpha\cos^2\alpha\sin 2\varphi}{(1-\sin^2\alpha\cos^2\varphi)^3}\dot{\varphi}^2 = M_1 - M_2\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\varphi}}}$$

Lösung 1198

Für kleine Winkel kann gesetzt werden: $\cos\alpha = 1 - \frac{\alpha^2}{2}$; $\sin\alpha = \alpha$

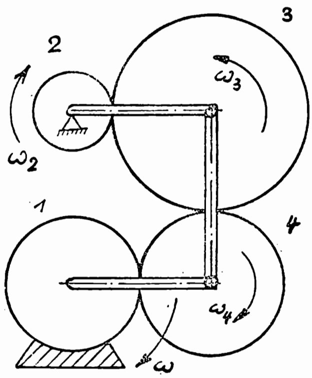
Somit wird nach Aufgabe 1197:

$$\left[\Theta_1 + \Theta_2\left(\frac{1-\frac{\alpha^2}{2}}{1-\alpha^2\cos^2\varphi}\right)^2\right]\ddot{\varphi} - \frac{\Theta_2\alpha^2\left(1-\frac{\alpha^2}{2}\right)^2\sin 2\varphi}{(1-\alpha^2\cos^2\varphi)^3}\dot{\varphi}^2 = M_1 - M_2\cdot\frac{1-\frac{\alpha^2}{2}}{1-\alpha^2\cos^2\varphi}$$

$(\Theta_1 + \Theta_2)\ddot{\varphi} = M_1 - M_2$ bei Vernachlässigung der Glieder mit α^2.

$$\ddot{\varphi} = \frac{M_1 - M_2}{\Theta_1 + \Theta_2}; \quad \underline{\underline{\varphi = \frac{M_1 - M_2}{\Theta_1 + \Theta_2}\cdot\frac{t^2}{2} + C_1 t + C_2}}$$

Lösung 1199



$$\omega_4 = \frac{r_1 + r_4}{r_4}\cdot\omega; \quad \omega_3 = \frac{r_1 + r_4}{r_3}\,\omega$$

$$\omega_2 = \frac{r_1 + r_2 + r_3 + r_4}{r_2}\,\omega; \quad r_1 + r_4 = r_2 + r_3 = l$$

$$\Theta_2 = \frac{m_2}{2}r_2^2; \quad \Theta_3 = \frac{m_3}{2}r_3^2; \quad \Theta_4 = \frac{m_4}{2}r_4^2$$

$$m_3 = m_2\left(\frac{r_3}{r_2}\right)^2; \quad m_4 = m_2\left(\frac{r_4}{r_2}\right)^2$$

$$T = \frac{1}{2}\left[\Theta_2\omega_2^2 + \Theta_3\omega_3^2 + \Theta_4\omega_4^2 + (m_4 + m_3)l^2\omega^2\right]$$

$$T = \frac{1}{2}\,\frac{m_2}{2}\,\omega^2\left[4l^2 + l^2\left(\frac{r_3}{r_2}\right)^2 + l^2\left(\frac{r_4}{r_2}\right)^2 + 2l^2\left(\left(\frac{r_3}{r_2}\right)^2 + \left(\frac{r_4}{r_2}\right)^2\right)\right]$$

$$T = \frac{m_2\omega^2 l^2}{4}\left[4 + 3\left(\frac{r_3}{r_2}\right)^2 + 3\left(\frac{r_4}{r_2}\right)^2\right] = \frac{\Theta^*}{2}\dot{\varphi}^2$$

$$\Theta^* = \frac{m_2}{2}l^2\left[4 + 3\left(\frac{r_3}{r_2}\right)^2 + 3\left(\frac{r_4}{r_2}\right)^2\right]$$

$$m_2 = \frac{M}{1 + \left(\frac{r_3}{r_2}\right)^2 + \left(\frac{r_4}{r_2}\right)^2}$$

$$\Theta^* = \frac{30\cdot 400}{981\,(1 + 2{,}25 + 3{,}52)\cdot 2}[4 + 3\cdot 2{,}25 + 3\cdot 3{,}52]$$

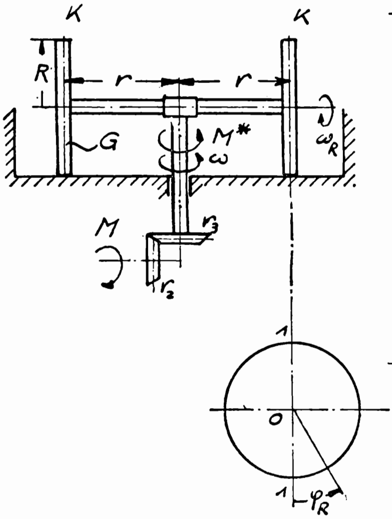
$$\Theta^* = 19{,}2 \text{ kgcmsek}^2$$

$$U = F\cdot l\sin\varphi$$

$$T - U = 0: \quad Fl\sin\varphi = \frac{\Theta^*}{2}\dot{\varphi}^2; \quad \int_0^{\frac{\pi}{6}}\frac{d\varphi}{\sqrt{\sin\varphi}} = \sqrt{\frac{F\cdot 2\cdot l}{\Theta^*}}\int_0^{\tau} dt$$

$$F = \frac{\Theta^*}{2\cdot l\cdot\tau^2}\left[\int_0^{\frac{\pi}{6}}\frac{d\varphi}{\sqrt{\sin\varphi}}\right]^2 = \underline{\underline{0{,}48\left[\int_0^{\frac{\pi}{6}}\frac{d\varphi}{\sqrt{\sin\varphi}}\right]^2 = 1{,}03 \text{ kg}}}$$

Lösung 1200



$$M^* = \frac{r_3}{r_2} \cdot M = \frac{3}{2} M$$

$$T = \frac{2G}{2g} r^2 \dot{\varphi}^2 + \frac{2\Theta_0}{2} \dot{\varphi}_R^2 + \frac{2\Theta_1}{2} \dot{\varphi}^2$$

$$\dot{\varphi}_R = \dot{\varphi} \cdot \frac{r}{R}; \quad \Theta_0 = \frac{G}{2g} R^2; \quad \Theta_1 = \frac{\Theta_0}{2}$$

$$T = \frac{G}{2g} \dot{\varphi}^2 \left[3r^2 + \frac{R^2}{2} \right]$$

$$A = M^* \cdot \varphi; \quad T = A$$

$$\dot{\varphi} = \sqrt{\frac{M^* \cdot \varphi \cdot 2g}{G\left(3r^2 + \frac{R^2}{2}\right)}} = \alpha \sqrt{\varphi}$$

$$\frac{d\varphi}{\sqrt{\varphi}} = \alpha\, dt; \quad 2\sqrt{\varphi} = \alpha(t + C)$$

$$\begin{matrix} t = 0 \\ \varphi = 0 \end{matrix} : \quad C = 0$$

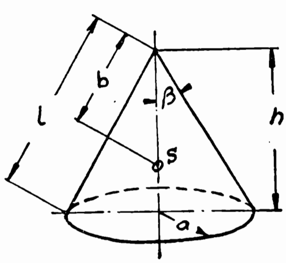
$$\sqrt{\varphi} = \frac{\alpha}{2} t$$

$$\dot{\varphi} = \frac{\alpha^2}{2} t$$

Daraus: $$M = \frac{2}{3} \frac{\dot{\varphi} G \left(3r^2 + \frac{R^2}{2}\right)}{g t}$$

$$\dot{\varphi} = \frac{\pi \cdot n}{30}; \quad \underline{\underline{M = 320 \text{ mkg}}}$$

Lösung 1201



Vorbetrachtungen:

1. Schwerpunkt des Kegels:

$$V \cdot s = \int_0^h r^2 \cdot \pi \cdot z\, dz = \int_0^h \left(\frac{a z}{h}\right)^2 \pi z\, dz = \frac{a^2 h^2}{4} \cdot \pi$$

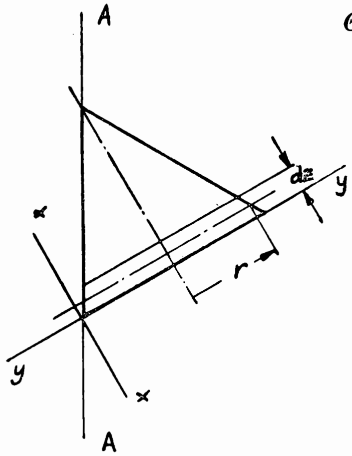
$$V = \frac{a^2 h \pi}{3}; \quad s = \frac{3h}{4}; \quad b = s \cos\beta = \frac{3}{4} l \cos^2\beta$$

2. Trägheitsmoment:

$$d\Theta_A = d(\Theta_x \cos^2\beta + \Theta_y \sin^2\beta)$$

$$d\Theta_A = \varrho \left[\frac{3 r^4}{2} \pi \cos^2\beta + \frac{r^4}{4} \pi \sin^2\beta \right] dz$$

$$\Theta_A = \frac{\varrho \pi}{4} (1 + 5\cos^2\beta) \int_0^h \left(\frac{a z}{h}\right)^4 dz$$



$$\Theta_A = \frac{\varrho\pi}{4}(1 + 5\cos^2\beta)\frac{a^4 h}{5} = \frac{3}{4} m a^2\left(\cos^2\beta + \frac{1}{5}\right)$$

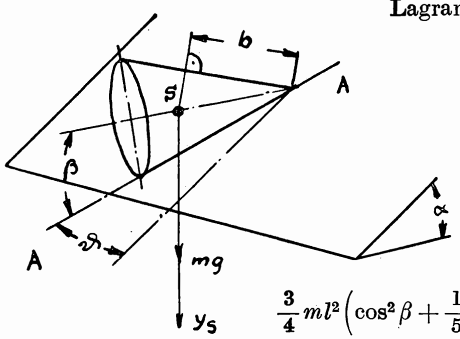
$$\Theta_A = \frac{3}{4} m l^2 \sin^2\beta\left(\cos^2\beta + \frac{1}{5}\right)$$

3. Schwerpunktsweg in Richtung der Schwerkraft:

$$y_S = b\cos\vartheta\sin\alpha = \frac{3}{4} l\cos^2\beta\sin\alpha\cos\vartheta$$

4. Winkelgeschwindigkeit der Kegeldrehung:

$$\omega_A \cdot l\,\mathrm{tg}\,\beta = l\dot\vartheta; \quad \omega_A = \dot\vartheta\cdot\frac{\cos\beta}{\sin\beta}$$



Lagrangesche Funktion:

$$L = T - U = \frac{\Theta_A}{2}\omega_A^2 + m g y_S$$

$$L = \frac{3}{8} m l^2\cos^2\beta\left(\cos^2\beta + \frac{1}{5}\right)\dot\vartheta^2 +$$

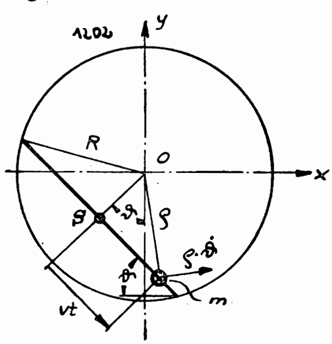
$$+ \frac{3}{4} m g l\cos^2\beta\sin\alpha\cos\beta$$

$$\left(\frac{\partial L}{\partial\dot\vartheta}\right)^{\cdot} - \frac{\partial L}{\partial\vartheta} = 0:$$

$$\frac{3}{4} m l^2\left(\cos^2\beta + \frac{1}{5}\right)\cos^2\beta\,\ddot\vartheta + \frac{3}{4} m g l\cos^2\beta\sin\alpha\sin\vartheta = 0$$

$$\underline{\underline{\ddot\vartheta + \frac{g}{l}\,\frac{\sin\alpha}{\left(\cos^2\beta + \frac{1}{5}\right)}\sin\vartheta = 0}}$$

Lösung 1202



Bewegung der Masse m:

$$x = x_s + vt\cos\vartheta; \quad x_s = -\sqrt{R^2 - a^2}\sin\vartheta$$

$$y = y_s - vt\sin\vartheta; \quad y_s = -\sqrt{R^2 - a^2}\cos\vartheta$$

$$\dot x^2 + \dot y^2 = (R^2 - a^2)\dot\vartheta^2 + v^2 t^2\dot\vartheta^2 + v^2 - 2\sqrt{R^2 - a^2}\,v\dot\vartheta$$

Trägheitsmoment des Stabes, bezogen auf 0:

$$\Theta_0 = \frac{M(2a)^2}{12} + M(R^2 - a^2) = M\left[R^2 - \frac{2a^2}{3}\right]$$

Somit:

$$T = \frac{m}{2}\left\{[R^2 - a^2 + v^2 t^2]\,\dot{\vartheta}^2 - 2\sqrt{R^2 - a^2}\,v\dot{\vartheta} + v^2\right\} + \frac{M}{2}\left(R^2 - \frac{2a^2}{3}\right)\dot{\vartheta}^2$$

$$\frac{\partial T}{\partial \dot{\vartheta}} = \text{const:} \qquad \frac{M}{m}\left(R^2 - \frac{2}{3}a^2\right)\dot{\vartheta} + (R^2 - a^2 + v^2 t^2)\,\dot{\vartheta} - \sqrt{R^2 - a^2}\,v = C_1$$

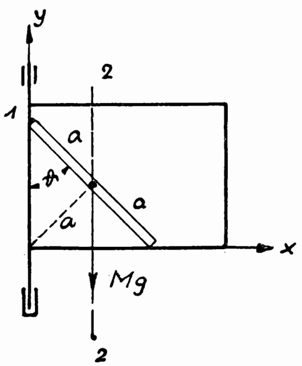
$$\dot{\vartheta}\left[(R^2 - a^2) + \frac{M}{m}\left(R^2 - \frac{2}{3}a^2\right) + v^2 t^2\right] = C_2$$

$$\vartheta - \vartheta_0 = C_2 \int_0^t \frac{d\tau}{\left(R^2 - a^2 + \frac{M}{m}\left(R^2 - \frac{2}{3}a^2\right) + v^2\tau^2\right.}$$

$$\vartheta - \vartheta_0 = C_2 \frac{1}{\sqrt{v^2\left[R^2 - a^2 + \frac{M}{m}\left(R^2 - \frac{2}{3}a^2\right)\right.}} \cdot \operatorname{arc\,tg} \frac{vt}{\sqrt{R^2 - a^2 + \frac{M}{m}\left(R^2 - \frac{2}{3}a^2\right)}}$$

$$\underline{\underline{\vartheta - \vartheta_0 = C \operatorname{arc\,tg} \frac{vt}{\sqrt{R^2 - a^2 + \frac{M}{m}\left(R^2 - \frac{2}{3}a^2\right)}}}}$$

Lösung 1203



$$T = \Theta_1 \frac{\dot{\vartheta}^2}{2} + \frac{M}{2}(\dot{x}^2 + \dot{y}^2) + \frac{M}{2}\omega^2 a^2 \sin^2\vartheta + \Theta_2 \frac{\omega^2}{2}$$

$$U = Mg \cdot y$$

$$x = a\sin\vartheta; \quad \dot{x} = a\cos\vartheta\,\dot{\vartheta}$$

$$y = a\cos\vartheta; \quad \dot{y} = -a\sin\vartheta\,\dot{\vartheta}$$

$$\dot{x}^2 + \dot{y}^2 = a^2\dot{\vartheta}^2$$

$$\Theta_1 = M\frac{a^2}{3}; \quad \Theta_2 = M\frac{a^2}{3}\sin^2\vartheta$$

$$L = T - U = \frac{M}{3}a^2\frac{\dot{\vartheta}^2}{2} + \frac{M}{2}a^2\dot{\vartheta}^2 + \frac{M}{2}\omega^2 a^2\sin^2\vartheta + \frac{M}{2}\omega^2\frac{a^2}{3}\sin^2\vartheta - Mga\cos\vartheta$$

$$\left(\frac{\partial L}{\partial \dot{\vartheta}}\right)^{\cdot} = \frac{M}{3}a^2\ddot{\vartheta} + Ma^2\ddot{\vartheta}$$

$$\frac{\partial L}{\partial \vartheta} = \frac{M}{2}\omega^2 a^2 \cdot 2\sin\vartheta\cos\vartheta + Mga\sin\vartheta + M\omega^2\frac{a^2}{3}\sin\vartheta\cos\vartheta$$

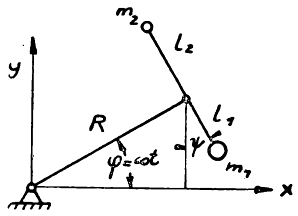
$$\left(\frac{\partial L}{\partial \dot{\vartheta}}\right)^{\cdot} - \frac{\partial L}{\partial \vartheta} = 0: \quad \underline{\underline{\frac{4}{3}Ma^2\ddot{\vartheta} - \frac{4}{3}M\omega^2 a^2\sin\vartheta\cos\vartheta - Mga\sin\vartheta = 0}}$$

Gleichgewichtslage: Für $\ddot{\vartheta} = 0$ gilt:

$$\sin\vartheta = 0$$

$$\underline{\underline{\vartheta = 0}}$$

Lösung 1204



$$x_1 = R\cos\omega t + l_1 \sin\psi$$
$$y_1 = R\sin\omega t - l_1 \cos\psi$$
$$\dot{x}_1 = -R\omega\sin\omega t + l_1 \psi\cos\psi$$
$$\dot{y}_1 = R\omega\cos\omega t + l_1 \dot{\psi}\sin\psi$$
$$v_1^2 = \dot{x}_1^2 + \dot{y}_1^2 = R^2 w^2 + l_1^2\dot{\psi}^2 + 2Rl_1\omega\dot{\psi}\sin(\psi - \omega t)$$

Entsprechend mit $(-l_2)$ für l_1:

$$v_2^2 = R^2\omega^2 + l_2^2\dot{\psi}^2 - 2Rl_2\omega\dot{\psi}\sin(\psi - \omega t)$$
$$T = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}; \quad U = 0; \quad L = T - U;$$
$$\left(\frac{\partial L}{\partial \dot{\psi}}\right)^{\cdot} - \frac{\partial L}{\partial \psi} = 0:$$
$$\underline{\underline{(m_1 l_1^2 + m_2 l_2^2)\ddot{\psi} - (m_1 l_1 - m_2 l_2) R\omega^2 \cos(\psi - \omega t) = 0}}$$

Gleichgewicht herrscht bei $\ddot{\psi} = 0$

a) $\underline{\underline{(m_1 l_1 - m_2 l_2) = 0}}$; Indifferentes Gleichgewicht

b) $\cos(\psi - \omega t) = 0$; $\underline{\underline{\psi = \omega t + \frac{\pi}{2}}}$; Relatives Gleichgewicht.

Lösung 1205

Unter Verwendung von Aufgabe 1204 folgt bei Hinzunahme der potentiellen Energie:

$$U = m_1 g y_1 + m_2 g y_2 = m_1 g (R\sin\varphi - l_1\cos\psi) + m_2 g (R\sin\varphi + l_2\cos\psi)$$
$$L = T - U; \quad \left(\frac{\partial L}{\partial \dot{\psi}}\right)^{\cdot} - \frac{\partial L}{\partial \psi} = 0:$$
$$\underline{\underline{(m_1 l_1^2 + m_2 l_2^2)\ddot{\psi} - (m_1 l_1 - m_2 l_2) R\omega^2\cos(\psi - \omega t) + (m_1 l_1 - m_2 l_2) g\sin\psi = 0}}$$

Gleichgewichtslage bei $\ddot{\psi} = 0$:

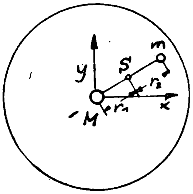
$$(m_1 l_1 - m_2 l_2)\,[g\sin\psi - R\omega^2\cos(\psi - \omega t)] = 0$$

a) $\underline{\underline{m_1 l_1 = m_2 l_2}}$; Indifferentes Gleichgewicht

b) $g\sin\psi - R\omega^2\cos(\psi - \omega t) = 0$ für: $\underline{\underline{\omega^2 = \frac{g}{R}}}$ und $\frac{\pi}{2} - \psi = \psi - \omega t$

$$\underline{\underline{\frac{\omega t}{2} + \frac{\pi}{4} = \psi}}$$

Lösung 1206

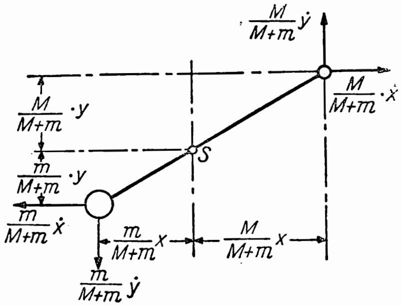


S = Gemeinsamer Schwerpunkt

$$\Theta_S = \Theta + M r_1^2 + m r_2^2; \qquad r_1^2 = \frac{m^2}{(m+M)^2}(x^2 + y^2)$$
$$\Theta_S = \Theta + \frac{mM}{M+m}(x^2 + y^2); \quad r_2^2 = \frac{M^2}{(m+M)^2}(x^2 + y^2)$$

Drehimpuls um den Schwerpunkt S

$$\Theta_S\dot{\varphi} + \frac{mM^2 + Mm^2}{(M+m)^2}(x\dot{y} - y\dot{x}) = \text{const}$$
$$\Theta_S\dot{\varphi} + \frac{Mm}{(m+M)}(x\dot{y} - y\dot{x}) = \text{const}$$



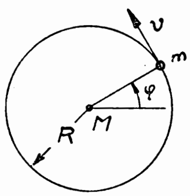
Da zu Anfang die Scheibe in Ruhe war, gilt:

$$\frac{mM}{m+M}(x_0\dot{y}_0-y_0\dot{x}_0)=\text{const}$$

$$\underline{\underline{\left(\Theta+\frac{mM}{m+M}(x^2+y^2)\right)\dot{\varphi}+\frac{mM}{m+M}(x\dot{y}-y\dot{x})}}$$

$$\underline{\underline{=\frac{mM}{m+M}(x_0\dot{y}_0-y_0\dot{x}_0)}}$$

Lösung 1207



$$R\cdot\varphi=s;\quad v=\alpha t;\quad s=\frac{\alpha t^2}{2}$$

$$\varphi=\frac{\alpha t^2}{2R}$$

$$x=R\cos\frac{\alpha t^2}{2R};\quad \dot{x}=-\alpha t\sin\frac{\alpha t^2}{2R}$$

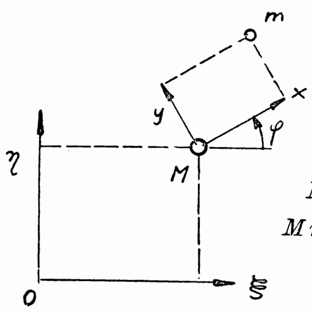
$$y=R\sin\frac{\alpha t^2}{2R};\quad \dot{y}=\alpha t\cos\frac{\alpha t^2}{2R}$$

$$x\dot{y}-y\dot{x}=R\alpha t;\quad x_0\dot{y}_0-y_0\dot{x}_0=0$$

Somit ergibt sich aus Aufgabe 1206:

$$\dot{\varphi}=-\frac{mM}{m+M}\cdot\frac{1}{\Theta+\frac{mMR^2}{m+M}}\cdot R\alpha t$$

$$\underline{\underline{\varphi=-\frac{mM}{2(m+M)}\cdot\frac{R\cdot\alpha}{\Theta+\frac{mM}{m+M}R^2}t^2=\frac{\beta}{2R}t^2}}$$



Impulssatz:

$$M\cdot\dot{\xi}+m(\dot{\xi}+\dot{x}\cos\varphi-\dot{y}\sin\varphi-x\dot{\varphi}\sin\varphi-y\dot{\varphi}\cos\varphi)=0$$

$$M\dot{\eta}+m(\dot{\eta}+\dot{x}\sin\varphi+\dot{y}\cos\varphi+x\dot{\varphi}\cos\varphi-y\dot{\varphi}\sin\varphi)=0$$

$$\dot{\xi}=\frac{m}{M+m}(-\dot{x}\cos\varphi+\dot{y}\sin\varphi+x\dot{\varphi}\sin\varphi+y\dot{\varphi}\cos\varphi)$$

$$\dot{\xi}=\frac{m(\alpha+\beta)}{M+m}t\sin\frac{(\alpha+\beta)}{2R}t^2$$

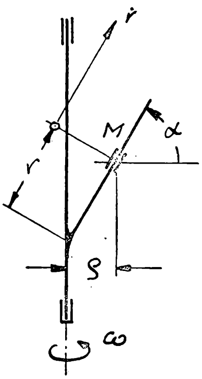
$$\underline{\underline{\xi=-\frac{mR}{M+m}\cos\frac{(\alpha+\beta)}{2R}t^2}}$$

$$\dot{\eta}=\frac{m}{M+m}(-\dot{x}\sin\varphi-\dot{y}\cos\varphi+y\dot{\varphi}\sin\varphi-x\dot{\varphi}\cos\varphi)$$

$$\dot{\eta}=-\frac{m(\alpha+\beta)t}{M+m}\cdot\cos\frac{(\alpha+\beta)}{2R}t^2$$

$$\underline{\underline{\eta=-\frac{mR}{M+m}\sin\frac{(\alpha+\beta)}{2R}t^2}}$$

Lösung 1208



$$T = \frac{m}{2}\dot r^2 + \frac{m\varrho^2}{2}\omega^2; \qquad \varrho = r\cos\alpha$$

$$T = \frac{m}{2}(\dot r^2 + r^2\cos^2\alpha\,\omega^2)$$

$$U = mgr\sin\alpha; \quad L = T - U$$

$$L = \frac{m}{2}(\dot r^2 + r^2\omega^2\cos^2\alpha) - mgr\sin\alpha$$

$$\left(\frac{\partial L}{\partial \dot r}\right)^{\cdot} - \frac{\partial L}{\partial r} = 0: \quad \ddot r - r\omega^2\cos^2\alpha = -g\sin\alpha$$

Lösungsansatz:

$$r = C_1 e^{\omega\cos\alpha\cdot t} + C_2 e^{-\omega\cos\alpha\cdot t} + D$$

$$\ddot r = C_1\omega^2\cos^2\alpha\, e^{\omega\cos\alpha\cdot t} + C_2\omega^2\cos^2\alpha\, e^{-\omega\cos\alpha\cdot t}$$

$$D = \frac{g\sin\alpha}{\omega^2\cos^2\alpha}$$

$$\underline{\underline{r = C_1 e^{\omega t\cos\alpha} + C_2 e^{-\omega t\cos\alpha} + \frac{g\sin\alpha}{\omega^2\cos^2\alpha}}}$$

Lösung 1209

$$T = ma^2\frac{\dot\vartheta^2}{2} + \frac{m}{2}\omega^2 a^2\sin^2\vartheta$$

$$U = -amg\cos\vartheta$$

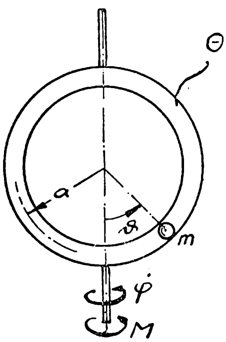
$$L = T - U = ma^2\frac{\dot\vartheta^2}{2} + \frac{m}{2}a^2\omega^2\sin^2\vartheta + amg\cos\vartheta$$

$$\left(\frac{\partial L}{\partial\dot\vartheta}\right)^{\cdot} - \frac{\partial L}{\partial\vartheta} = 0: \quad \underline{\underline{\ddot\vartheta + \left(\frac{g}{a} - \omega^2\cos\vartheta\right)\sin\vartheta = 0}}$$

$M = a\cdot\cos\vartheta\cdot m\cdot b_c$; Coriolisbeschleunigung: $b_c = 2\omega\dot\vartheta\cdot a\cdot\sin\vartheta$

$$\underline{\underline{M = 2ma^2\omega\dot\vartheta\sin\vartheta\cos\vartheta}}$$

Lösung 1210



$$T = \frac{\Theta}{2}\dot\varphi^2 + \frac{m}{2}a^2\dot\vartheta^2 + \frac{m}{2}a^2\sin^2\vartheta\,\dot\varphi^2$$

$$U = -mga\cos\vartheta; \quad A = M\varphi; \quad L = T - U$$

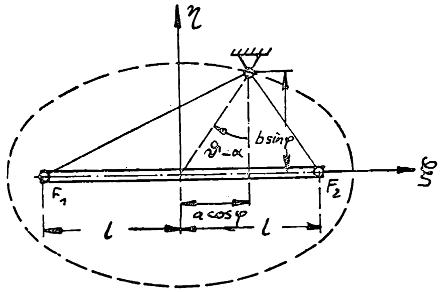
$$\left(\frac{\partial L}{\partial\dot\vartheta}\right)^{\cdot} - \frac{\partial L}{\partial\vartheta} = \frac{\partial A}{\partial\vartheta}:$$

$$\underline{\underline{ma^2\ddot\vartheta - ma^2\dot\varphi^2\sin\vartheta\cos\vartheta + mga\sin\vartheta = 0}}$$

$$\left(\frac{\partial L}{\partial\dot\varphi}\right)^{\cdot} - \frac{\partial L}{\partial\varphi} = \frac{\partial A}{\partial\varphi}:$$

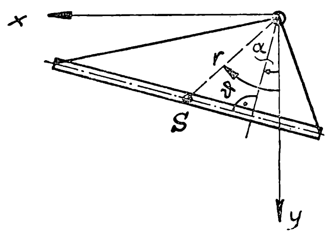
$$\underline{\underline{\ddot\varphi[\Theta + ma^2\sin^2\vartheta] + 2ma^2\dot\varphi\dot\vartheta\sin\vartheta\cos\vartheta = M}}$$

Lösung 1211



$$a^2 - b^2 = l^2$$

$$\operatorname{tg}(\vartheta-\alpha) = \frac{a\cos\varphi}{b\sin\varphi} \tag{1}$$



$$y_s = r\cos\vartheta; \quad x_s = r\sin\vartheta$$

$$\dot{x}_s^2 + \dot{y}_s^2 = \dot{r}^2 + r^2\dot{\vartheta}^2$$

$$r^2 = a^2\cos^2\varphi + b^2\sin^2\varphi$$

$$2r\dot{r} = -2l^2\sin\varphi\cos\varphi\cdot\dot{\varphi} \tag{2}$$

Aus (1): $-(1+\operatorname{tg}^2(\vartheta-\alpha))(\dot{\vartheta}-\dot{\alpha}) = \frac{a}{b}\cdot\frac{1}{\sin^2\varphi}\dot{\varphi}$

$$\dot{\vartheta} = \dot{\alpha} - \frac{a\dot{\varphi}}{b\sin^2\varphi(1+\operatorname{tg}^2(\vartheta-\alpha))}$$

$$\dot{\vartheta} = \dot{\alpha} - \frac{b\,a\dot{\varphi}}{a^2\cos^2\varphi + b^2\sin^2\varphi} \tag{3}$$

$$T = \frac{\Theta}{2}\dot{\alpha}^2 + \frac{m}{2}(\dot{x}_s^2 + \dot{y}_s^2); \quad U = -mgr\cos\vartheta; \quad \Theta = \frac{P}{g}\cdot\frac{l^2}{3}$$

$$T = \frac{P}{2g}\left[\frac{l^2}{3}\dot{\alpha}^2 + \dot{r}^2 + r^2\dot{\vartheta}^2\right]$$

$$T = \frac{P}{2g}\left[\left(\frac{l^2}{3} + a^2\cos^2\varphi + b^2\sin^2\varphi\right)\dot{\alpha}^2 - 2ab\dot{\alpha}\dot{\varphi} + \frac{a^2b^2 + l^4\sin^2\varphi\cos^2\varphi}{a^2\cos^2\varphi + b^2\sin^2\varphi}\cdot\dot{\varphi}^2\right]$$

$$U = -mgr\cos\vartheta = -P(b\sin\varphi\cos\alpha - a\cos\varphi\sin\alpha)$$

Lösung 1212

Für die $\begin{matrix}\text{Stabilität}\\ \text{Labilität}\end{matrix}$ gilt: $\Delta = \begin{vmatrix} U_{\varphi\varphi} & U_{\varphi\alpha} \\ U_{\alpha\varphi} & U_{\alpha\alpha}\end{vmatrix} \lessgtr 0$ $\begin{matrix}\text{Labilität}\\ \text{Stabilität}\end{matrix}$

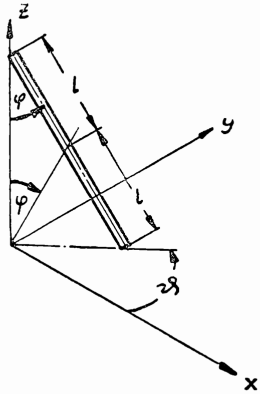
$$U_{\alpha\varphi} = \frac{\partial^2 U}{\partial\alpha\cdot\partial\varphi}; \quad U_{\varphi\varphi} = \frac{\partial^2 U}{\partial\varphi^2}; \quad U = -P[b\sin\varphi\cos\alpha - a\cos\varphi\sin\alpha]$$

$$\Delta = \begin{vmatrix} -b\sin\varphi\cos\alpha + a\cos\varphi\sin\alpha & -b\sin\alpha\cos\varphi + a\sin\varphi\cos\alpha \\ -b\sin\alpha\cos\varphi + a\sin\varphi\cos\alpha & -b\sin\varphi\cos\alpha + a\cos\varphi\sin\alpha \end{vmatrix}$$

Für $\alpha = 0$; $\varphi = \frac{\pi}{2}$ wird: $\Delta = \begin{vmatrix} -b & a \\ a & -b\end{vmatrix} = b^2 - a^2 < 0$

Da $a > b$, ist $\Delta < 0$, die Gleichgewichtslage ist labil ||

Lösung 1213



$$T = \Theta_S \frac{\dot\varphi^2}{2} + m l^2 \frac{\dot\varphi^2}{2} + \Theta_{ZZ} \frac{\dot\vartheta^2}{2}$$

$$U = m g l \cos\varphi; \quad \Theta_S = m \frac{l^2}{3}; \quad \Theta_{ZZ} = \frac{4}{3} m l^2 \sin^2\varphi$$

$$L = T - U; \quad L = \frac{2}{3} m l^2 \dot\varphi^2 + \frac{4}{3} m l^2 \frac{\dot\vartheta^2}{2} \sin^2\varphi - m g l \cos\varphi$$

$$\left(\frac{\partial L}{\partial \dot\varphi}\right)^{\cdot} - \frac{\partial L}{\partial \varphi} = 0: \quad \underline{\underline{\ddot\varphi - \dot\vartheta^2 \sin\varphi\cos\varphi = \frac{3}{4}\frac{g}{l}\sin\varphi}}$$

$$\left(\frac{\partial L}{\partial \dot\vartheta}\right)^{\cdot} - \frac{\partial L}{\partial \vartheta} = 0: \quad \left(\dot\vartheta \frac{4}{3} m l^2 \sin^2\varphi\right)^{\cdot} = 0 \qquad (*)$$

$$\underline{\underline{\ddot\vartheta \sin^2\varphi + 2\dot\vartheta\dot\varphi\sin\varphi\cos\varphi = 0}}$$

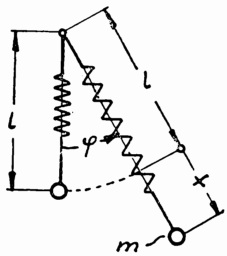
Integration: Aus (*) folgt:

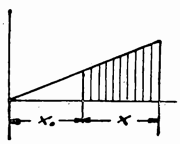
$$\underline{\underline{\dot\vartheta\sin^2\varphi = C_1}}$$

$$\int \dot\varphi\,(d\dot\varphi) - \int \dot\vartheta^2 \sin\varphi\cos\varphi\, d\varphi = \frac{3}{4}\frac{g}{l}\int \sin\varphi\, d\varphi + C_2$$

$$\underline{\underline{\dot\varphi^2 + \dot\vartheta^2 \sin^2\varphi + \frac{3}{2}\frac{g}{l}\cos\varphi = C_2}}$$

Lösung 1214





Feder-
diagramm

In der statischen Gleichgewichtslage ist die Feder um x_0 gedehnt.

$$x_0 = \frac{mg}{c}$$

$$T = m(l+x)^2 \frac{\dot\varphi^2}{2} + \frac{m\dot x^2}{2}$$

$$U = -g m (l+x)\cos\varphi + \frac{c(x+x_0)^2}{2} - \frac{c x_0^2}{2} - m g l$$

$$L = T - U; \quad \left(\frac{\partial l}{\partial \dot\varphi}\right)^{\cdot} = m\left(\ddot\varphi (l+x)^2 + 2\dot\varphi\dot x (l+x)\right)$$

$$\frac{\partial l}{\partial \varphi} = -m g (l+x)\sin\varphi$$

$$\left(\frac{\partial l}{\partial \dot x}\right)^{\cdot} = m\ddot x;$$

$$\frac{\partial l}{\partial x} = m\dot\varphi^2 (l+x) + m g\cos\varphi - m g - c x$$

Somit:
$$\ddot\varphi (l+x) + 2\dot\varphi\dot x + g\sin\varphi = 0$$

$$\ddot x - \dot\varphi^2 (l+x) + g(1-\cos\varphi) + \frac{c}{m} x = 0$$

mit $z - \frac{x}{l}$ als Dehnung des Fadens gilt:

$$\ddot\varphi (1+z) + 2\dot\varphi\dot z + \frac{g}{l}\sin\varphi = 0$$

$$\ddot z - \dot\varphi^2 (1+z) + \frac{g}{l}(1-\cos\varphi) + \frac{c}{m} z = 0$$

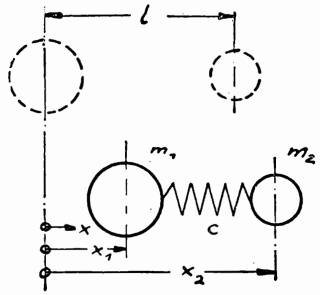
Lösung 1215

Aus den Differentialgleichungen der Aufgabe 1214 wird für kleine Ausschläge bei Vernachlässigung der quadratischen Glieder

$$\ddot{\varphi} + \frac{g}{l} \cdot \varphi = 0; \qquad \ddot{z} + \frac{c}{m} z = 0$$

$$\underline{\underline{\varphi = B \sin\left(\sqrt{\frac{g}{l}} \cdot t + \beta\right)}}; \quad \underline{\underline{z = A \sin\left(\sqrt{\frac{c}{m}} \cdot t + \alpha\right)}}$$

Lösung 1216



$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2}; \quad U = \frac{c}{2} (x_1 - x_2 - l)^2$$

$$L = T - U = \frac{m_1}{2} \dot{x}_1^2 + \frac{m_2}{2} \dot{x}_2^2 - \frac{c}{2} (x_1 - x_2 - l)^2$$

$$\left(\frac{\partial L}{\partial \dot{x}_1}\right)^{\cdot} - \frac{\partial L}{\partial x_1} = 0: \quad m_1 \ddot{x}_1 + c(x_1 - x_2 - l) = 0$$

$$\left(\frac{\partial L}{\partial \dot{x}_2}\right)^{\cdot} - \frac{\partial L}{\partial x_2} = 0: \quad m \ddot{x}_2 - c(x_1 - x_2 - l) = 0$$

Anfangsbedingungen: $t = 0$: $x_1 = 0$; $x_2 = l$
$\dot{x}_1 = u_0$; $\dot{x}_2 = 0$

$$m_1 \ddot{x}_1 + m_2 \ddot{x}_2 = 0$$

$$\ddot{x}_2 = -\frac{m_1}{m_2} \ddot{x}_1$$

$$\dot{x}_2 = -\frac{m_1}{m_2} (\dot{x}_1 - u_0); \quad x_2 = -\frac{m_1}{m_2} (x_1 - u_0 t) + l$$

$$m_1 \ddot{x}_1 + c\left(x_1 + \frac{m_1}{m_2} (x_1 - u_0 t)\right) = 0$$

$$m_1 \ddot{x}_1 + x_1 \left(1 + \frac{m_1}{m_2}\right) c = c \frac{m_1}{m_2} u_0 \cdot t; \quad k = \sqrt{c\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}$$

Lösungsansatz: $x_1 = A \sin kt + B \cos kt + Dt$; Das partikuläre Integral liefert:

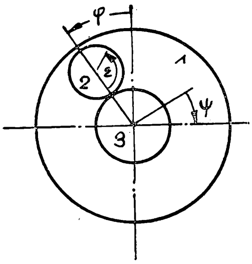
$$D = \frac{u_0 m_1}{m_1 m_2}$$

Aus den Anfangsbedingungen ergibt sich: $A = \frac{u_0 - D}{k}$; $B = 0$

Somit: $\underline{\underline{x_1 = \frac{1}{m_1 + m_2} \left\{ m_1 u_0 t + \frac{m_2 u_0}{k} \sin kt \right\}}}$

$$\underline{\underline{x_2 - l = \frac{1}{m_1 + m_2} \left\{ m_1 u_0 t - \frac{m_1 u_0}{k} \sin kt \right\}}}$$

Lösung 1217



$$T=\Theta_1\frac{\dot\varphi^2}{2}+\Theta_2\frac{\dot\varepsilon^2}{2}+m_2\cdot\frac{4a^2}{2}\dot\varphi^2+\Theta_3\frac{\dot\psi^2}{2}$$

$$U=\frac{c\psi^2}{2}-M\varphi;\qquad \varepsilon=2\varphi-\psi$$

$$L=T-U\qquad\qquad \dot\varepsilon=2\dot\varphi-\dot\psi$$

$$L=\Theta_1\frac{\dot\varphi^2}{2}+\frac{\Theta_2}{2}(4\dot\varphi^2+\dot\psi^2-4\dot\psi\dot\varphi)+\frac{4a^2m^2}{2}\dot\varphi^2$$

$$+\Theta_3\frac{\dot\psi^2}{2}+M\varphi-\frac{c\psi^2}{2}$$

$$\left(\frac{\partial L}{\partial\dot\psi}\right)^{\cdot}-\frac{\partial L}{\partial\psi}=0:\quad -2\Theta_2\ddot\varphi+\ddot\psi(\Theta_2+\Theta_3)+c\psi=0\quad (1)$$

$$\left(\frac{\partial L}{\partial\dot\varphi}\right)^{\cdot}-\frac{\partial L}{\partial\varphi}=0:\quad \ddot\varphi(\Theta_1+4\Theta_2+4m_2a^2)-2\Theta_2\ddot\psi-M=0\quad (2)$$

Aus (2):

$$-2\Theta_2\ddot\varphi=\frac{-(2\Theta_2\ddot\psi+M)\cdot 2\Theta_2}{\Theta_1+4\Theta_2+4m_2a^2}\quad (3)$$

Aus (1) und (3):

$$\left\{\Theta_2+\Theta_3-\frac{4\Theta_2^2}{\Theta_1+4\Theta_2+4m_2a^2}\right\}\ddot\psi+c\psi=\frac{2M\Theta_2}{\Theta_1+4\Theta_2+4m_2a^2}$$

$$\Theta_3=\Theta_2=\frac{ma^2}{2};\quad \Theta_1=20ma^2:\quad \frac{25}{26}ma^2\ddot\psi+c\psi=\frac{M}{26}$$

$$\underline{\underline{\psi=\frac{M}{26c}\left[1-\cos\left(1{,}02\sqrt{\frac{c}{ma^2}}\cdot t\right)\right]}}$$

Aus (3):

$$\ddot\varphi=\frac{2\Theta_2\ddot\psi+M}{\Theta_1+4\Theta_2+4ma^2};\quad \ddot\varphi=\frac{\ddot\psi}{26}+\frac{M}{26ma^2}$$

$$\dot\varphi=\frac{\dot\psi}{26}+\frac{M}{26ma^2}\cdot t+C_1$$

$$\varphi=\frac{\psi}{26}+\frac{M}{52ma^2}\cdot t^2+C_1t+C_2$$

Die Anfangsbedingungen ergeben: $C_1=0;\quad C_2=0$

$$\underline{\underline{\varphi=\frac{Mt^2}{52ma^2}+\frac{M}{676c}\left[1-\cos\left(1{,}02\sqrt{\frac{c}{ma^2}}\cdot t\right)\right]}}$$

Lösung 1218

$$T=\frac{m_1}{2}\dot y^2+\frac{m_2}{2}l^2\sin^2\varphi\dot\varphi^2+\frac{m_2}{2}(l\cos\varphi\dot\varphi+\dot y)^2$$

$$U=-m_2gl\cos\varphi$$

$$L=T-U=\frac{m_1}{2}\dot y^2+\frac{m_2}{2}l^2\dot\varphi^2+m_2l\cos\varphi\dot\varphi\dot y+\frac{m_2}{2}\dot y^2+m_2lg\cos\varphi$$

$$\left(\frac{\partial L}{\partial\dot\varphi}\right)^{\cdot}=(m_2l^2\dot\varphi+m_2l\dot y\cos\varphi)^{\cdot}=m_2l^2\ddot\varphi+m_2l\ddot y\cos\varphi-m_2l\dot y\sin\varphi\dot\varphi$$

$$\frac{\partial L}{\partial\varphi}=-m_2l\dot\varphi\dot y\sin\varphi-m_2gl\sin\varphi$$

$$\left(\frac{\partial L}{\partial\dot\varphi}\right)^{\cdot}-\frac{\partial L}{\partial\varphi}=0:\quad \underline{\underline{l\ddot\varphi+\ddot y\cos\varphi+g\sin\varphi=0}}$$

$$\left(\frac{\partial L}{\partial \dot y}\right)^{\cdot} = m_1\ddot y + (m_2 l\cos\varphi\,\dot\varphi)^{\cdot} + m_2\ddot y = \ddot y\,(m_1+m_2) + m_2 l(\ddot\varphi\cos\varphi - \dot\varphi^2\sin\varphi)$$

$$\frac{\partial L}{\partial y} = 0; \qquad \underline{\underline{\frac{d}{dt}\left[(m_1+m_2)\,\dot y + m_2 l\,\dot\varphi\cos\varphi\right] = 0}}$$

Lösung 1219

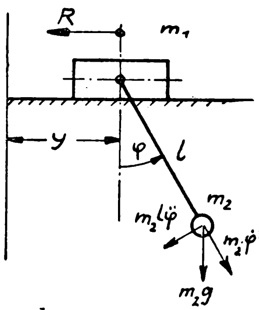
Für kleine Ausschläge wird aus den Differentialgleichungen **der Aufgabe 1218:**

$$l\ddot\varphi + \ddot y + g\varphi = 0$$

$$(m_1+m_2)\,\ddot y + m_2 l\,\ddot\varphi = 0$$

Daraus: $\ddot\varphi + \frac{g\,(m_1+m_2)}{l m_1}\cdot\varphi = 0;\quad \underline{\underline{T = 2\pi\sqrt{\frac{m_1}{m_1+m_2}\cdot\frac{l}{g}}}}$

Lösung 1220



$$T = \frac{m_1}{2}\dot y^2 + \frac{m_2}{2}\left[(y + l\sin\varphi)^{\cdot 2} + (l\cos\varphi)^{\cdot 2}\right]$$

$$U = -\,m_2 g\, l\cos\varphi$$

$$R = g\mu\left[m_1 + m_2 + \frac{m_2}{g}\,l\,(\dot\varphi^2\cos\varphi + \ddot\varphi\sin\varphi)\right]\operatorname{sign}\dot y$$

$$\operatorname{sign}\dot y = \begin{matrix} +1 \text{ bei } \dot y > 0 \\ -1 \text{ bei } \dot y < 0 \end{matrix}$$

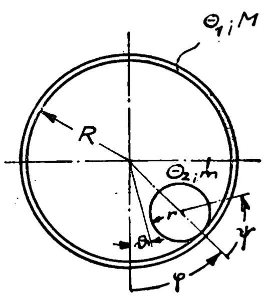
$$L = \frac{m_1}{2}\dot y^2 + \frac{m_2}{2}\left[\dot y^2 + l^2\dot\varphi^2 + 2\,\dot y\, l\,\dot\varphi\cos\varphi\right] + m_2 l g\cos\varphi$$

$$\left(\frac{\partial L}{\partial \dot y}\right)^{\cdot} - \frac{\partial L}{\partial y} = Q_y = R:$$

$$\underline{\underline{\frac{d}{dt}\left((m_1+m_2)\,\dot y + m_2 l\,\dot\varphi\cos\varphi\right) = -\,\mu\left[(m_1+m_2)\,g + m_2 l\,(\dot\varphi^2\cos\varphi + \ddot\varphi\sin\varphi)\right]\operatorname{sign}\dot y}}$$

$$\left(\frac{\partial L}{\partial \dot\varphi}\right)^{\cdot} - \frac{\partial L}{\partial \varphi} = Q_\varphi = 0: \quad \underline{\underline{l\ddot\varphi + \ddot y\cos\varphi + g\sin\varphi = 0}}$$

Lösung 1221



$$r\psi = \vartheta R - (R - r)\,\varphi$$

$$T = \frac{1}{2}\,\Theta_1\dot\vartheta^2 + \frac{1}{2}\,\Theta_2\dot\psi^2 + \frac{1}{2}\,m\,(R-r)^2\dot\varphi^2$$

$$U = -\,m_2 g\,(R-r)\cos\varphi;\quad \Theta_1 = MR^2;\quad \Theta_2 = \frac{mr^2}{2}$$

$$L = T - U = \frac{1}{2}MR^2\dot\vartheta^2 + \frac{1}{2}\cdot\frac{mr^2}{2}\left(\dot\vartheta\frac{R}{r} - \left(\frac{R}{r} - 1\right)\dot\varphi\right)^2 + \frac{1}{2}\,m(R-r)^2\dot\varphi^2 + mg(R-r)\cos\varphi$$

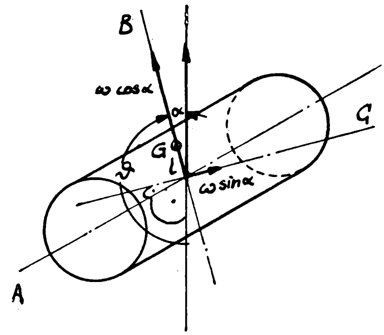
$\left(\frac{\partial L}{\partial \dot\vartheta}\right)^{\cdot} - \frac{\partial L}{\partial \vartheta} = 0$ ergibt den Drehimpulssatz:

$$MR^2\dot\vartheta - \frac{1}{2}\,mR\left[(R-r)\,\dot\varphi - R\dot\vartheta\right] = C_1$$

Energiesatz:

$$T + U = C_2;\quad \underline{\underline{\frac{1}{2}MR^2\dot\vartheta^2 + \frac{1}{4}\,m\left[(R-r)\,\dot\varphi - R\dot\vartheta\right]^2 + \frac{m}{2}(R-r)^2\dot\varphi^2 - mg(R-r)\cos\varphi = C_2}}$$

Lösung 1222



$$T = A\frac{\dot{\vartheta}^2}{2} + B\frac{\omega^2\cos^2\alpha}{2} + C\frac{\omega^2\sin^2\alpha}{2}$$

$$U = -P\cdot l\cdot\cos\vartheta;\quad \alpha = 180^\circ - \vartheta$$

$$L = T - U = A\frac{\dot{\vartheta}^2}{2} + \frac{\omega^2}{2}(B\cos^2\vartheta + C\sin^2\vartheta) + Pl\cos\vartheta$$

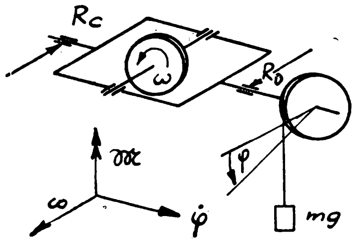
$$\left(\frac{\partial L}{\partial\dot{\vartheta}}\right)^{\bullet} = A\ddot{\vartheta}$$

$$\frac{\partial L}{\partial\vartheta} = \omega^2[-B\cos\vartheta\sin\vartheta + C\sin\vartheta\cos\vartheta] - Pl\sin\vartheta$$

$$\left(\frac{\partial L}{\partial\dot{\vartheta}}\right)^{\bullet} - \frac{\partial L}{\partial\vartheta} = 0;$$

$$\underline{\underline{A\ddot{\vartheta} - \omega^2(C-B)\sin\vartheta\cos\vartheta = -Pl\sin\vartheta}}$$

Lösung 1223



$$R_C = R_D = \frac{M}{b};\quad M = \Theta_{\text{Rotor}}\cdot\omega\cdot\dot{\varphi}$$

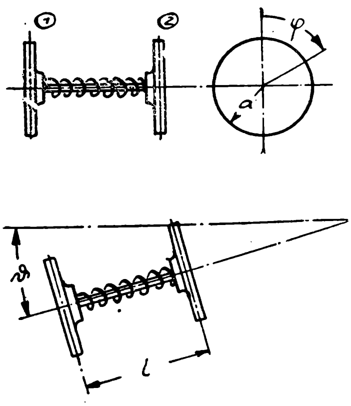
$$\Theta_x\cdot\frac{\dot{\varphi}^2}{2} = mg\cdot h;\quad \dot{\varphi} = \sqrt{\frac{2Ph}{\Theta_x}}$$

$$R_C = R_D = \frac{\Theta_{\text{Rotor}}\cdot\omega}{b}\sqrt{\frac{2Ph}{\Theta_x}}$$

$$\Theta_{\text{Rotor}} = C;\quad \Theta_x = A + A_1 + \frac{P}{g}r^2;\quad \omega = 2\pi n$$

$$\underline{\underline{R_C = R_D = \frac{2\pi n\cdot C}{b}\sqrt{\frac{2Ph}{A + A_1 + \frac{P}{g}r^2}}}}$$

Lösung 1224



$$T = C\frac{\dot{\varphi}_1^2}{2} + C\frac{\dot{\varphi}_2^2}{2} + \frac{M}{2}a^2\dot{\varphi}_1^2 + \frac{M}{2}a^2\dot{\varphi}_2^2 + 2A\frac{\dot{\vartheta}^2}{2}$$

$$U = \frac{c(\varphi_1 - \varphi_2)^2}{2};\quad \vartheta = \frac{a}{l}(\varphi_1 - \varphi_2)$$

$$\left(\frac{\partial L}{\partial\dot{\varphi}_1}\right)^{\bullet} - \frac{\partial L}{\partial\varphi_1} = 0:$$

$$\ddot{\varphi}_1(C + Ma^2) + 2A\left(\frac{a}{l}\right)^2(\ddot{\varphi}_1 - \ddot{\varphi}_2) + c(\varphi_1 - \varphi_2) = 0$$

$$\left(\frac{\partial L}{\partial\dot{\varphi}_2}\right)^{\bullet} - \frac{\partial L}{\partial\varphi_2} = 0:$$

$$\ddot{\varphi}_2(C + Ma^2) - 2A\left(\frac{a}{l}\right)^2(\ddot{\varphi}_1 - \ddot{\varphi}_2) - c(\varphi_1 - \varphi_2) = 0$$

Beide Gleichungen addiert, ergibt: $\ddot{\varphi}_1 + \ddot{\varphi}_2 = 0$. Nach Integration und unter Berücksichtigung der Anfangsbedingungen ergibt sich:

$$\varphi_1 = -\varphi_2 + \omega t$$

Somit: $\ddot{\varphi}_1\left(C + M a^2 + 4A\left(\frac{a}{l}\right)^2\right) + 2c\varphi_1 = c\omega t; \quad k = \sqrt{\frac{2c}{M a^2 + C + 4A\left(\frac{a}{l}\right)^2}}$

$$\underline{\underline{\varphi_1 = \frac{1}{2}\left(\omega t - \frac{\omega}{k}\sin kt\right)}}$$

$$\underline{\underline{\varphi_2 = \frac{1}{2}\left(\omega t + \frac{\omega}{k}\sin kt\right)}}$$

Lösung 1225

1. x = Bewegungskoordinate des Wagens

z = Bewegungskoordinate des Zylinders

$$T_1 = \frac{M\dot{x}^2}{2} + m\varrho^2\frac{(x-z)^{\cdot 2}}{2r^2} + m\frac{\dot{z}^2}{2}; \quad \frac{\partial T_1}{\partial \dot{z}} = m\dot{z} - m\varrho^2\frac{(x-z)^{\cdot}}{r^2} = \text{konst} = 0$$

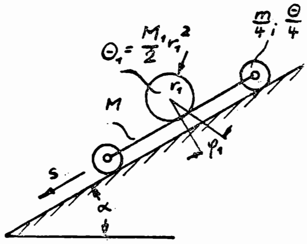
$$\dot{z} = \frac{\varrho^2}{r^2}(\dot{x} - \dot{z}); \quad \underline{\underline{\dot{z} = \dot{x}\frac{\varrho^2}{\varrho^2 + r^2}}}$$

$$\dot{x} = u: \quad A_1 = T_1 = \frac{M u^2}{2}\left(1 + \frac{m}{M}\left[\frac{r^2}{\varrho^2} + 1\right]\frac{\varrho^4}{(\varrho^2 + r^2)^2}\right)$$

$$\underline{\underline{A_1 = \frac{M u^2}{2}\left[1 + \frac{m}{M}\cdot\frac{\varrho^2}{\varrho^2 + r^2}\right]}}$$

2. $A_2 = T_2 = \frac{m + M}{2}u^2; \quad \underline{\underline{A_2 = \frac{M}{2}u^2\left(1 + \frac{m}{M}\right)}}$

Lösung 1226



$$T = \frac{M}{2}\dot{s}^2 + \frac{\Theta_1}{2}\dot{\varphi}_1^2 + \frac{M_1}{2}(s + r_1\varphi_1)^{\cdot 2} + \frac{m}{2}\dot{s}^2 + \frac{\Theta}{2}\dot{\varphi}^2$$

$$U = -(M + M_1 + m)gs\sin\alpha - M_1 g r_1\varphi_1\sin\alpha$$

$$L = \frac{M}{2}\dot{s}^2 + \frac{\Theta_1}{2}\dot{\varphi}_1^2 + \frac{M_1}{2}(\dot{s}^2 + r_1^2\dot{\varphi}_1^2 + 2r_1\dot{\varphi}_1\dot{s}) + \frac{m\dot{s}^2}{2}$$

$$+ \frac{1}{4}m\dot{s}^2 + (M + M_1 + m)gs\sin\alpha + M_1 g r_1\varphi_1\sin\alpha$$

$$\left(\frac{\partial L}{\partial \dot{s}}\right)^{\cdot} = M\ddot{s} + M_1\ddot{s} + \frac{3}{2}m\ddot{s} + M_1 r_1\ddot{\varphi}_1$$

$$\frac{\partial L}{\partial s} = (M + M_1 + m)g\sin\alpha$$

$$\left(\frac{\partial L}{\partial \dot{s}}\right)^{\cdot} - \frac{\partial L}{\partial s} = 0: \quad \ddot{s}\left[M + M_1 + \frac{3}{2}m\right] + M_1 r_1\ddot{\varphi}_1 = (M + M_1 + m)g\sin\alpha$$

$$\left(\frac{\partial L}{\partial \dot{\varphi}_1}\right)^{\cdot} = \Theta_1\ddot{\varphi}_1 + M_1 r_1^2\ddot{\varphi}_1 + M_1 r_1\ddot{s}; \quad \frac{\partial L}{\partial \varphi_1} = M_1 g r_1\sin\alpha$$

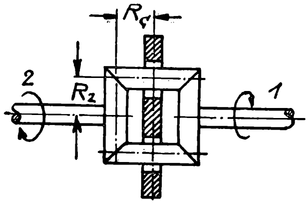
$$\left(\frac{\partial L}{\partial \dot{\varphi}_1}\right)^{\cdot} - \frac{\partial L}{\partial \varphi_1} = 0: \quad \frac{3}{2}r_1\ddot{\varphi}_1 + \ddot{s} = g\sin\alpha$$

$$\ddot{s}\left[M + M_1 + \frac{3}{2}m\right] + \ddot{\varphi}_1 M_1 r_1 = (M + M_1 + m)g\sin\alpha$$

$$\ddot{s} + \frac{3}{2}r_1\ddot{\varphi}_1 = g\sin\alpha$$

$$\underline{\underline{\ddot{s} = b = \frac{6M + 6m + 2M_1}{6M + 9m + 2M_1}g\sin\alpha}}$$

Lösung 1227



Aus der Anordnung ergibt sich:

$$\omega_C = \omega_2 \frac{R_2}{R_C} + \omega_1 \frac{R_2}{R_C}$$

$$v_C = \omega_2 \frac{R_2}{2} - \omega_1 \frac{R_2}{2}$$

$$\omega_D = \frac{\omega_2}{2} - \frac{\omega_1}{2}$$

$$T = \frac{\Theta_1}{2}(\omega_1^2 + \omega_2^2) + \frac{\Theta_C}{2}\frac{R_2^2}{R_C^2}(\omega_2 + \omega_1)^2 + \frac{\Theta_D + 4\Theta_C'}{8}(\omega_2 - \omega_1)^2$$

$$U = 0; \quad M_1 = n \cdot \omega_D = \frac{n}{2}(\omega_2 - \omega_1); \quad M_2 = -M_1$$

$$\left(\frac{\partial T}{\partial \omega_1}\right)^{\cdot} = M_1: \quad \Theta_1 \dot{\omega}_1 + \frac{\Theta_C R_2^2}{R_C^2}(\dot{\omega}_2 + \dot{\omega}_1) + \frac{\Theta_D + 4\Theta_C'}{4}(\dot{\omega}_1 - \dot{\omega}_2) = \frac{n}{2}(\omega_2 - \omega_1) \quad (1)$$

$$\left(\frac{\partial T}{\partial \omega_2}\right)^{\cdot} = M_2: \quad \Theta_1 \dot{\omega}_2 + \frac{\Theta_C R_2^2}{R_C^2}(\dot{\omega}_2 + \dot{\omega}_1) + \frac{\Theta_D + 4\Theta_C'}{4}(\dot{\omega}_2 - \dot{\omega}_1) = -\frac{n}{2}(\omega_2 - \omega_1) \quad (2)$$

Beide Gleichungen addiert: $\Theta_1(\dot{\omega}_1 + \dot{\omega}_2) + \frac{2\Theta_C R_2^2}{R_C^2}(\dot{\omega}_1 + \dot{\omega}_2) = 0$

$$\dot{\omega}_1 + \dot{\omega}_2 = 0$$

Gleichung (2) — Gleichung (1): $\frac{2\Theta_1 + \Theta_D + 4\Theta_C'}{2}(\dot{\omega}_1 - \dot{\omega}_2) + n(\omega_1 - \omega_2) = 0$

$$2\Theta_1 + \Theta_D + 4\Theta_C' = \Theta; \quad \frac{\Theta}{2n} = \lambda: \quad (\dot{\omega}_1 - \dot{\omega}_2) + \lambda(\omega_1 - \omega_2) = 0$$

Lösungsansatz: $\omega_1 - \omega_2 = C_1 e^{-\lambda t}$

$\omega_1 + \omega_2 = C_2$

Anfangsbedingungen: $t = 0$; $\omega_{10} - \omega_{20} = C_1$

$\omega_{10} + \omega_{20} = C_2$

Somit:

$$\underline{\underline{\omega_1 = \frac{1}{2}\omega_{10}(1 + e^{-\lambda t}) + \frac{1}{2}\omega_{20}(1 - e^{-\lambda t})}}$$

$$\underline{\underline{\omega_2 = \frac{1}{2}\omega_{10}(1 - e^{-\lambda t}) + \frac{1}{2}\omega_{20}(1 + e^{-\lambda t})}}$$

Lösung 1228

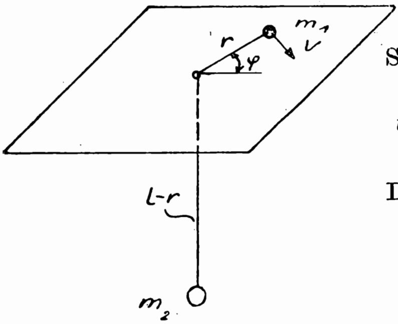
$$T = \frac{m_1}{2}(\dot{r}^2 + r^2\dot{\varphi}^2) + \frac{m_2}{2} \cdot \dot{r}^2$$

$$U = m_2 g (l - r)$$

$$T_0 = \frac{m_1}{2} v_0^2; \quad U_0 = m_2 g (l - r_0)$$

Drehimpulssatz: $\Theta_1 \frac{v_0}{r_0} = \Theta_2 \dot{\varphi}; \quad \Theta_1 = m_1 r_0^2; \quad \Theta_2 = m_1 \cdot r^2$

$$r\dot{\varphi} = v_0 \cdot \frac{r_0}{r}$$



Energiesatz: $T + U = T_0 + U_0$

$$(m_1 + m_2)\dot{r}^2 = m_1 v_0^2\left(1 - \frac{r_0^2}{r^2}\right) + 2m_2 g(r_0 - r)$$

Schwingungszeit:

$$t_s = \int_{r_0}^{r_1} 2\frac{\sqrt{m_1 + m_2}\, r\,dr}{\sqrt{m_1 v_0^2 (r^2 - r_0^2) + 2m_2 g r^2 (r_0 - r)}}$$

Die Geschwindigkeit $\dot{r}$ wird Null für:

$$m_1 v_0^2\left(1 - \frac{r_0^2}{r^2}\right) + 2m_2 g(r_0 - r) = 0$$

$$r = r_0$$

$$r_{1,2} = \frac{m_1 v_0^2}{4m_2 g} \pm \sqrt{\left(\frac{m_1 v_0^2}{4m_2 g}\right)^2 + \frac{m_1 v_0 r_0}{2m_2 g}}$$

Da das negative Vorzeichen nicht auftritt, gilt für die Amplitude:

$$\underline{\underline{a = r_0 - r_1}}$$

Somit:

$$\underline{\underline{t_s = \sqrt{\frac{2(m_1 + m_2)}{m_2 g}}\left|\int_{r_0}^{r_1} \frac{r\,dr}{\sqrt{(r_0 - r)(r - r_1)(r - r_2)}}\right|}}$$

Lösung 1229

$$T = M\frac{R^2}{2}\cdot\frac{\dot{\varphi}^2}{2} + \frac{m}{2}(\dot{x}^2 + \dot{y}^2);\quad U = -mg\cdot y$$

$$x = R\sin\varphi + l\sin\psi;\qquad \dot{x} = R\dot{\varphi}\cos\varphi + l\dot{\psi}\cos\psi$$

$$y = R\cos\varphi + l\cos\psi;\qquad -\dot{y} = R\dot{\varphi}\sin\varphi + l\dot{\psi}\sin\psi$$

$$\dot{x}^2 + \dot{y}^2 = R^2\dot{\varphi}^2 + l^2\dot{\psi}^2 + 2Rl\dot{\varphi}\dot{\psi}(\sin\varphi\sin\psi + \cos\varphi\cos\psi)$$

$$L = T - U = \frac{\dot{\varphi}^2}{2}\left[M\frac{R^2}{2} + mR^2\right] + \frac{m}{2}\left[l^2\dot{\psi}^2 + 2Rl\dot{\varphi}\dot{\psi}\cos(\varphi - \psi)\right]$$
$$+ mg(R\cos\varphi + l\cos\psi)$$

$$\left(\frac{\partial L}{\partial\dot{\varphi}}\right)^{\cdot} = \ddot{\varphi}R^2\left(\frac{M}{2} + m\right) + Rlm[\dot{\psi}\cos(\varphi - \psi)]^{\cdot}$$

$$\left(\frac{\partial L}{\partial\varphi}\right) = -mgR\sin\varphi - mRl\dot{\varphi}\dot{\psi}\sin(\varphi - \psi)$$

$$\left(\frac{\partial L}{\partial\dot{\varphi}}\right)^{\cdot} - \frac{\partial L}{\partial\varphi} = 0:\quad \underline{\underline{\left(m + \frac{M}{2}\right)R^2\ddot{\varphi} + mRl\cos(\varphi - \psi)\ddot{\psi} + mRl\sin(\varphi - \psi)\dot{\psi}^2}}$$
$$\underline{\underline{+ mgR\sin\varphi = 0}}$$

$$\left(\frac{\partial L}{\partial\dot{\psi}}\right)^{\cdot} = ml^2\ddot{\psi} + mRl[\ddot{\varphi}\cos(\varphi - \psi) - \dot{\varphi}(\dot{\varphi} - \dot{\psi})\sin(\varphi - \psi)]$$

$$\left(\frac{\partial L}{\partial\psi}\right) = -mRl\dot{\varphi}\dot{\psi}\sin(\varphi - \psi) - mgl\sin\psi$$

$$\left(\frac{\partial L}{\partial\dot{\psi}}\right)^{\cdot} - \frac{\partial L}{\partial\psi} = 0:\quad \underline{\underline{mRl\cos(\varphi - \psi)\ddot{\varphi} + ml^2\ddot{\psi} - mRl\sin(\varphi - \psi)\dot{\varphi}^2 + mgl\sin\psi = 0}}$$

Lösung 1230

Nach Aufgabe 1229 gilt für die Bewegung des Pendels:

$$R l \cos(\varphi - \psi)\,\ddot{\varphi} + l^2 \ddot{\psi} - R l \sin(\varphi - \psi)\,\dot{\varphi}^2 + g l \sin\psi = 0$$

mit $\varphi = \omega t$; $\dot{\varphi} = \omega$; $\ddot{\varphi} = 0$ wird hieraus:

$$\underline{\underline{\ddot{\psi} - \frac{R}{l}\,\omega^2 \sin(\omega t - \psi) + \frac{g}{l} \sin\psi = 0}}$$

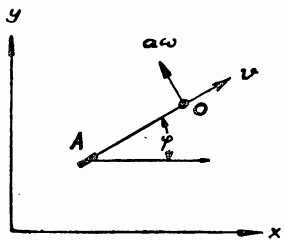
Mit $\omega t - \psi = -\gamma =$ Winkel zwischen dem Radius OA und dem Pendelstab:

$$\ddot{\gamma} + \frac{\omega^2 R}{l} \sin\gamma = -\frac{g}{l} \sin(\gamma + \omega t); \quad \text{für kleine } \gamma \text{ gilt:}$$

$$\ddot{\gamma} + \frac{\omega^2 R}{l}\,\gamma = -\frac{g}{l} \sin\omega t;$$

Danach: $\underline{\underline{l_{\text{red}} = l \cdot \frac{g}{R\,\omega^2}}}$

Lösung 1231



Energiesatz: $\frac{\Theta\,\omega^2}{2} + \frac{M v_S^2}{2} = C^*$

$$v_S^2 = v^2 + a^2 \omega^2$$

$$(\Theta + M a^2)\,\omega^2 + M v^2 = C$$

Auf den Schlitten wirkt keine äußere Kraft, also gilt:

$$M \dot{v}_S = 0; \quad \dot{v}_S = 0$$

$$\dot{v}_S = \dot{v} + a\,\omega^2 = 0$$

Somit: $M v^2 - (\Theta + M a^2)\,\frac{\dot{v}}{a} = C$

$$\frac{M a^2}{\Theta + M a^2} \cdot \frac{v^2}{a^2} - \frac{\dot{v}}{a} = \frac{C}{\Theta + M a^2}; \quad \frac{C}{\Theta + M a^2} = k_1^2$$

$$\frac{M a^2}{\Theta + M a^2} = k^2$$

$$k^2 \cdot \frac{v^2}{a^2} - \frac{\dot{v}}{a} = k_1^2; \quad \frac{\dot{v}}{a} = k^2 \frac{v^2}{a^2} - k_1^2$$

$$\frac{k_1 d\left(\frac{v}{a}\,\frac{k}{k_1}\right)}{k^2 \frac{v^2}{a^2} - k_1^2} = k\,dt; \quad -\frac{\frac{1}{k_1}\,d\left(\frac{k}{k_1}\,\frac{v}{a}\right)}{1 - \left(\frac{k}{k_1}\,\frac{v}{a}\right)^2} = k\,dt$$

$$-\frac{1}{k_1}\,\mathfrak{Ar}\,\mathfrak{Tg}\,\frac{k v}{k_1 a} = k \cdot t$$

$$\frac{k v}{k_1 a} = \mathfrak{Tg}\,(-k_1 k t); \quad \frac{k \dot{v}}{k_1 a} = \frac{-k_1 k}{\mathfrak{Cos}^2(-k_1 k t)}$$

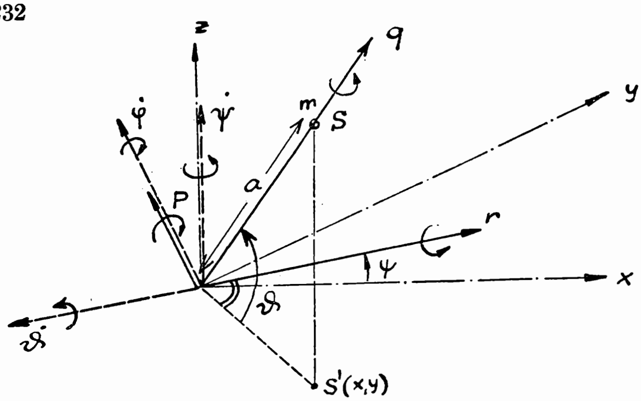
$$\omega^2 = \frac{k_1^2}{\mathfrak{Cos}^2(-k_1 k t)}; \quad \omega = \frac{k_1}{\mathfrak{Cos}\,(-k_1 k t)}$$

$$d\varphi = \frac{k_1\,dt}{\mathfrak{Cos}\,(-k_1 k t)};$$

$$\varphi + \varphi_0 = -\frac{1}{k}\,\mathrm{arc\,sin}\,\mathfrak{Tg}\,(-k_1 k t); \quad -\sin k(\varphi + \varphi_0) = \mathfrak{Tg}\,[-k_1 k t]$$

$$\underline{\underline{\sin[k(\varphi + \varphi_0)] = \mathfrak{Tg}\,ct}}$$

Lösung 1232



x; y; z = raumfeste Koordinaten (x; y geben Berührungspunkt mit der x-y-Ebene an)

p; q; r = körperfeste Koordinaten (Winkelgeschwindigkeiten um die Hauptachsen)

φ; ψ; ϑ = Eulersche Winkel

$$x_S = x + a\cos\vartheta\sin\psi; \quad \dot{x}_S = \dot{x} + a(-\dot{\vartheta}\sin\vartheta\sin\psi + \dot{\psi}\cos\vartheta\cos\psi)$$

$$y_S = y - a\cos\vartheta\cos\psi; \quad \dot{y}_S = \dot{y} + a(\dot{\vartheta}\sin\vartheta\cos\psi + \dot{\psi}\cos\vartheta\sin\psi)$$

$$z_S = a\sin\vartheta; \quad \dot{z}_S = a\cos\vartheta\,\dot{\vartheta}$$

$$v_S^2 = \dot{x}^2 + \dot{y}^2 + 2a(-\dot{x}\dot{\vartheta}\sin\vartheta\sin\psi + \dot{x}\dot{\psi}\cos\vartheta\cos\psi + \dot{y}\dot{\vartheta}\sin\vartheta\cos\psi + \dot{y}\dot{\psi}\cos\vartheta\sin\psi) + a^2(\dot{\vartheta}^2 + \dot{\psi}^2\cos^2\vartheta)$$

$$p = \dot{\varphi} + \dot{\psi}\cos\vartheta; \quad L = \frac{m}{2}v_S^2 + \frac{C}{2}p^2 + \frac{A}{2}(q^2 + r^2) - mgz_s$$

$$q = \dot{\psi}\sin\vartheta;$$

$$r = -\dot{\vartheta};$$

Nicht holonome Zwangsbedingungen:

$$F_1 = \dot{x} - a\dot{\varphi}\cos\psi = 0; \quad F_2 = \dot{y} - a\dot{\varphi}\sin\psi = 0$$

$$\left(\frac{\partial L}{\partial \dot{x}}\right)^{\cdot} = \lambda_1; \quad \left(\frac{\partial L}{\partial \dot{\varphi}}\right)^{\cdot} = \lambda_1(-a\cos\psi) + \lambda_2(-a\sin\psi)$$

$$\left(\frac{\partial L}{\partial \dot{y}}\right)^{\cdot} = \lambda_2; \quad \left(\frac{\partial L}{\partial \dot{\psi}}\right)^{\cdot} - \frac{\partial L}{\partial \psi} = 0; \quad \left(\frac{\partial L}{\partial \dot{\vartheta}}\right)^{\cdot} - \frac{\partial L}{\partial \vartheta} = 0$$

$$L = \frac{m}{2}[\dot{x}^2 + \dot{y}^2 + 2a\{\dot{\vartheta}\sin\vartheta(\dot{y}\cos\psi - \dot{x}\sin\psi) + \dot{\psi}\cos\vartheta(\dot{x}\cos\psi + \dot{y}\sin\psi)\}$$

$$+ a^2(\dot{\vartheta}^2 + \dot{\psi}^2\cos^2\vartheta)] + \frac{C}{2}(\dot{\varphi} + \dot{\psi}\cos\vartheta)^2 + \frac{A}{2}(\dot{\psi}^2\sin^2\vartheta + \dot{\vartheta}^2) - mga\sin\vartheta$$

$$\left(\frac{\partial L}{\partial \dot{x}}\right)^{\cdot} = \lambda_1: \quad m[\dot{x} + a(\dot{\psi}\cos\vartheta\cos\psi - \dot{\vartheta}\sin\vartheta\sin\psi)]^{\cdot} = \lambda_1 \tag{1}$$

$$\left(\frac{\partial L}{\partial \dot{y}}\right)^{\cdot} = \lambda_2: \quad m[\dot{y} + a(\dot{\psi}\cos\vartheta\sin\psi + \dot{\vartheta}\sin\vartheta\cos\psi)]^{\cdot} = \lambda_2 \tag{2}$$

$$\left(\frac{\partial L}{\partial \dot{\varphi}}\right)^{\cdot} = \lambda_1(-a\cos\psi) + \lambda_2(-a\sin\psi): \quad C(\dot{\varphi} + \dot{\psi}\cos\vartheta)^{\cdot} = -a(\lambda_1\cos\psi + \lambda_2\sin\psi) \tag{3}$$

$$\begin{aligned}\left(\frac{\partial L}{\partial \dot{\psi}}\right)^{\cdot} - \left(\frac{\partial L}{\partial \psi}\right) = 0: \quad & ma[\cos\vartheta(\dot{x}\cos\psi + \dot{y}\sin\psi) + a\dot{\psi}\cos^2\vartheta]^{\cdot} \\ & + C[\cos\vartheta(\dot{\varphi} + \dot{\psi}\cos\vartheta)]^{\cdot} + A(\dot{\psi}\sin^2\vartheta)^{\cdot} \\ & - ma[\dot{\psi}\cos\vartheta(\dot{y}\cos\psi - \dot{x}\sin\psi) \\ & - \dot{\vartheta}\sin\vartheta(\dot{y}\sin\psi + \dot{x}\cos\psi)] = 0\end{aligned} \tag{4}$$

$$\begin{aligned}\left(\frac{\partial L}{\partial \dot{\vartheta}}\right)^{\cdot} - \left(\frac{\partial L}{\partial \vartheta}\right) = 0: \quad & ma[\sin\vartheta(\dot{y}\cos\psi - \dot{x}\sin\psi)]^{\cdot} + (A + ma^2)\ddot{\vartheta} \\ & - ma[\dot{\vartheta}\cos\vartheta(\dot{y}\cos\psi - \dot{x}\sin\psi) \\ & - \dot{\psi}\sin\vartheta(\dot{x}\cos\psi + \dot{y}\sin\psi)] \\ & + ma^2\dot{\psi}^2\sin\vartheta\cos\vartheta + C\dot{\psi}\sin\vartheta(\dot{\varphi} + \dot{\psi}\cos\vartheta) \\ & - A\dot{\psi}\sin\vartheta\cos\vartheta + mga\cos\vartheta = 0\end{aligned} \tag{5}$$

Gl. (1) und Gl. (2) in Gl. (3) eingesetzt, ergibt bei Berücksichtigung von

$\dot{y}\cos\psi - \dot{x}\sin\psi = 0$ und $\dot{y}\sin\psi + \dot{x}\cos\psi = a\dot{\varphi}$ (aus den Zwangsbedingungen) (6)

$$\underline{\underline{(ma^2 + C)(\dot{\varphi} + \dot{\psi}\cos\vartheta)^{\cdot} - ma^2\dot{\vartheta}\dot{\psi}\sin\vartheta = 0}} \tag{7}$$

Aus (4) und (6): $$\begin{aligned}&(ma^2 + C)[\cos\vartheta(\dot{\varphi} + \dot{\psi}\cos\vartheta)]^{\cdot} + A(\dot{\psi}\sin^2\vartheta)^{\cdot} \\ &\qquad + ma^2\dot{\varphi}\dot{\vartheta}\sin\vartheta = 0\end{aligned} \tag{8}$$

Aus (5) und (6): $$\begin{aligned}&(A + ma^2)\ddot{\vartheta} + (ma^2 + C)\dot{\psi}\sin\vartheta(\dot{\varphi} + \dot{\psi}\cos\vartheta) \\ &\qquad \underline{\underline{- A\dot{\psi}^2\sin\vartheta\cos\vartheta + mga\cos\vartheta = 0}}\end{aligned} \tag{9}$$

Aus (8) und (7): $$\underline{\underline{A(\dot{\psi}\sin^2\vartheta)^{\cdot} - C\dot{\vartheta}\sin\vartheta(\dot{\varphi} + \dot{\psi}\cos\vartheta) = 0}} \tag{10}$$

Lösung 1233

$$T = \frac{m}{2}\dot{x}^2 + \frac{L}{2}\dot{q}^2$$

$$U = \frac{c}{2}(x + x_0)^2 + \frac{(q + q_0)^2(a - x)}{2C_0 a} - Eq$$

$$L = \frac{m}{2}\dot{x}^2 + \frac{L}{2}\dot{q}^2 - \frac{c}{2}(x + x_0)^2 - \frac{(q + q_0)^2(a - x)}{2C_0 a} + Eq$$

$$\left(\frac{\partial L}{\partial \dot{x}}\right)^{\cdot} - \frac{\partial L}{\partial x} = p(t): \quad m\ddot{x} + c(x + x_0) - \frac{(q + q_0)^2}{2C_0 a} = p(t)$$

$$\left(\frac{\partial L}{\partial \dot{q}}\right)^{\cdot} - \frac{\partial L}{\partial q} = -R\dot{q}: \quad L\ddot{q} + \frac{(q + q_0)(a - x)}{C_0 \cdot a} - E = -R\dot{q}$$

mit $cx_0 = \frac{Eq_0}{2a}$; $q_0 = EC_0$ wird hieraus:

$$\underline{\underline{m\ddot{x} + cx - \frac{q^2}{2C_0 a} - \frac{E}{a}q = p(t)}}$$

$$\underline{\underline{L\ddot{q} + R\dot{q} - \frac{E}{a}x + \frac{q}{C_0} - \frac{qx}{aC_0} = 0}}$$

Lösung 1234

Für kleine freie Schwingungen wird aus den Differentialgleichungen der Aufgabe 1233:

$$m\ddot{x} + cx - \frac{E}{a}q = 0$$

$$L\ddot{q} + \frac{q}{C_0} - \frac{E}{a}x = 0$$

Ansatz: $x = A\sin kt$; $q = B\sin kt$

$$A(c - mk^2) \qquad -\frac{E}{a}B = 0$$

$$-\frac{E}{a}A \qquad + B\left(\frac{1}{C_0} - Lk^2\right) = 0$$

mit $E = \frac{q_0}{C_0}$ wird hieraus:

$$k^4 - k^2\left(\frac{c}{m} + \frac{1}{C_0 L}\right) = \frac{q_0^2}{a^2 C_0^2 mL} - \frac{c}{m}\cdot\frac{1}{C_0\cdot L}$$

$$\underline{\underline{k_{1,2} = \sqrt{\frac{1}{2}\left[\left(\frac{c}{m} + \frac{1}{C_0 L}\right) \pm \sqrt{\left(\frac{c}{m} - \frac{1}{LC_0}\right)^2 + 4\frac{q_0^2}{a^2 C_0^2 mL}}\right]}}}$$

Lösung 1235

Unter den gegebenen Bedingungen wird aus den Differentialgleichungen der Aufgabe 1233:

$$cx - \frac{E}{a}q = p_0$$

$$L\ddot{q} - \frac{E}{a}x + \frac{q}{C_0} = 0$$

$$\ddot{q} + \frac{1}{L}\left[\frac{1}{C_0} - \frac{E^2}{ca^2}\right]\cdot q = \frac{Ep_0}{Lac}; \quad \omega^2 = \frac{1}{LC_0}\left[1 - \frac{q_0^2}{C_0 a^2 c}\right]$$

Ansatz: $q = A + B\cos\omega t + D\sin\omega t$; $A = \dfrac{q_0 p_0}{ac\left(1 - \dfrac{q_0^2}{C_0 a^2 c}\right)}$

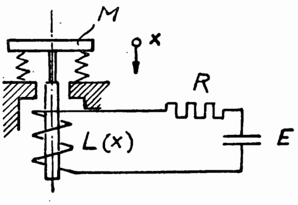
Anfangsbedingungen: $t = 0$; $q = 0$: $0 = A + B$

$\dot{q} = 0$: $D = 0$

Somit: $q = A(1 - \cos\omega t)$

$$\underline{\underline{q = \frac{q_0 p_0}{ac\left(1 - \frac{q_0^2}{C_0 a^2 c}\right)}\left[1 - \cos\sqrt{\left(1 - \frac{q_0^2}{C_0 a^2 c}\right)\cdot\frac{1}{LC_0}}\cdot t\right]}}$$

Lösung 1236



$$T = \frac{M}{2}\dot{x}^2 + \frac{L(x)}{2}\dot{q}^2;$$

$$U = \frac{c\,x^2}{2} - Mg\cdot x - Eq; \quad L = T - U$$

$$\left(\frac{\partial L}{\partial \dot{q}}\right)^{\cdot} - \frac{\partial L}{\partial q} = -R\dot{q}: \quad \underline{\underline{L\ddot{q} + \dot{q}\dot{x}\frac{dL}{dx} = E - R\dot{q}}}$$

$$\left(\frac{\partial L}{\partial \dot{x}}\right)^{\cdot} - \frac{\partial L}{\partial x} = 0: \quad \underline{\underline{M\ddot{x} + cx - \frac{1}{2}\dot{q}^2\frac{dL}{dx} = Mg}}$$

Gleichgewichtslage: $\ddot{x} = 0; \quad x = x_0$

$$i = \dot{q} = i_0 = \frac{E}{R}$$

$$\underline{\underline{c\,x_0 = Mg + \frac{1}{2}\left[\frac{dL}{dx}\right]_{x=x_0}\cdot i_0^2}}$$

Lösung 1237

Aus Aufgabe 1236 wird mit: $L = L_0 + L_1\cdot\xi$

$$\dot{q} = i_0 + \dot{e}; \quad x = x_0 + \xi$$

$$L_0\ddot{e} + Ri_0 + R\dot{e} + L_1 i_0\cdot\dot{\xi} = E$$

$$M\ddot{\xi} - \frac{1}{2}L_1 i_0^2 - L_1 i_0\dot{e} + c\,x_0 + c\,\xi = Mg$$

Nach Aufgabe 1236 gilt für die Gleichgewichtslage: $E = R\cdot i_0$

$$c\,x_0 = Mg + \frac{1}{2}L_1 i_0^2$$

Somit:
$$\left.\begin{aligned} L_0\ddot{e} + R\dot{e} + L_1 i_0\cdot\dot{\xi} &= 0 \\ M\ddot{\xi} - L_1 i_0\dot{e} + c\,\xi &= 0 \end{aligned}\right\|$$

Lösung 1238

Aus Aufgabe 1237 folgt mit i für $\dot{e}$ und x für ξ:

$$L_0\dot{i} + Ri + L_1 i_0\dot{x} = 0 \tag{1}$$

$$M\ddot{x} + cx - L_1 i_0 i = M\ddot{\xi} = -M\xi_0\omega^2\sin\omega t \tag{2}$$

Ansatz: $i = b\cos\omega t + a\sin\omega t$

$x = f\cos\omega t + g\sin\omega t$

Durch Koeffizientenvergleich ergeben sich folgende Gleichungen:

$$L_0\omega a + Rb + L_1 i_0\omega g = 0 \quad (3) \qquad (c - M\omega^2)f - L_1 i_0 b = 0 \quad (5)$$

$$-L_0\omega b + Ra - L_1 i_0\omega f = 0 \quad (4) \qquad (c - M\omega^2)g - L_1 i_0 a = -M\xi_0\omega^2 \quad (6)$$

Aus (4) und (5): $\omega b[(c - M\omega^2)L_0 + L_1^2 i_0^2] - R(c - M\omega^2)a = 0$

Aus (3) und (6): $Rb(c - M\omega^2) + \omega[(c - M\omega^2)L_0 + L_1^2 i_0^2]a = M\xi_0\omega^2 L_1 i_0 \omega$

Daraus: $$a = M\xi_0\omega^2 L_1 i_0 \omega \cdot \frac{\omega[L_1^2 i_0^2 + L_0(c - M\omega^2)]}{\Delta}$$

$$b = M\xi_0\omega^2 L_1 i_0 \omega \cdot \frac{R(c - M\omega^2)}{\Delta}$$

mit $$\Delta = R^2(c - M\omega^2)^2 + \omega^2[L_1^2 i_0^2 + L_0(c - M\omega^2)]^2$$

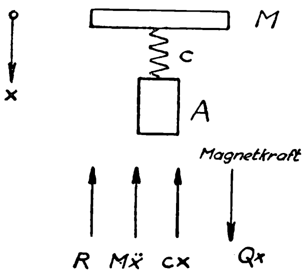
Aus (5): $$f = M\xi_0\omega^2 R L_1^2 i_0^2 \omega \cdot \frac{1}{\Delta}$$

Aus (6): $$g = -M\xi_0\omega^2 \cdot \frac{L_1^2 i_0^2 L_0\omega^2 + (R^2 + L_0^2\omega^2)(c - M\omega^2)}{\Delta}$$

Somit: $$\underline{\underline{i = \frac{M\xi_0\omega^3}{\Delta} \cdot L_1 i_0[R(c - M\omega^2)\cdot\cos\omega t + \omega[L_1^2 i_0^2 + L_0(c - M\omega^2)]\sin\omega t]}}$$

$$\underline{\underline{x = \frac{M\xi_0\omega^2}{\Delta}[RL_1^2 i_0^2\omega\cos\omega t - [L_1^2 i_0^2 L_0\omega^2 + (R^2 + L_0^2\omega^2)(c - M\omega^2)]\sin\omega t]}}$$

Lösung 1239



Gleichgewicht der mechanischen Kräfte:

$$\underline{\underline{M\ddot{x} + cx + \beta\dot{x} - 2\pi rnB\cdot\dot{q} = 0}}$$

Gleichgewicht der Spannungen

$$\underline{\underline{L\ddot{q} + R\dot{q} + 2\pi rnB\dot{x} = v(t)}}$$

Lösung 1240

Die durch die Erregung hervorgerufene Kraft ist: $M\ddot{\xi}$

$$\xi = \xi_0\sin\omega t; \quad \ddot{\xi} = -\xi_0\omega^2\sin\omega t$$

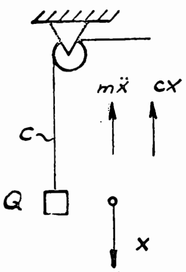
Somit entsprechend Aufgabe 1239: $$\underline{\underline{M\ddot{x} + \beta\dot{x} + cx - 2\pi rnB\dot{q} = M\xi_0\omega^2\sin\omega t}}$$

$$\underline{\underline{L\ddot{q} + R\dot{q} + 2\pi rnB\dot{x} = 0}}$$

X. Theorie der Schwingungen

48. Schwingungen von Systemen mit einem Freiheitsgrad

Lösung 1241



Schwingungsgleichung:

$$m\ddot{x} + cx = 0$$

$$\frac{c}{m} = \alpha^2; \quad \alpha = 30\,\frac{1}{\text{sek}}$$

$$\ddot{x} + \alpha^2 x = 0$$

$$x = A\sin\alpha t + B\cos\alpha t$$

Anfangsbedingungen:

$$t = 0: \quad \left.\begin{matrix} x = 0 \\ \dot{x} = u \end{matrix}\right\} \text{ daraus folgt:}$$

$$B = 0$$

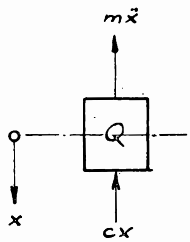
$$\dot{x} = \alpha A\cos\alpha t$$

$$u = \alpha A$$

$$A = \frac{u}{\alpha} = 0{,}1\text{ m}$$

$$\underline{\underline{x = 0{,}1\sin 30\ t\text{ m}}}$$

Lösung 1242



Schwingungsgleichung:

$$m\ddot{x} + cx = 0$$

$$\ddot{x} + \frac{c\cdot g}{Q}x = 0$$

$$T = 2\pi\sqrt{\frac{Q}{c\cdot g}}$$

$$c = \lambda\cdot S = 3\cdot 50\cdot 10000$$

$$= 15\cdot 10^5\text{ kg/cm}^2$$

$$\underline{\underline{T}} = 2\pi\sqrt{\frac{147\,000}{981\cdot 15\cdot 10^5}} = \underline{\underline{6{,}28\cdot 10^{-2}\text{ sek}}}$$

Lösung 1243

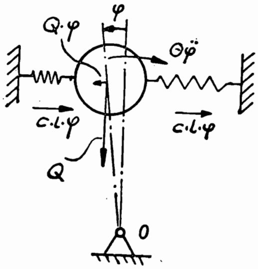
Schwingungsgleichung:

$$\frac{h}{a}l\cdot\ddot{\varphi} + g\varphi = 0$$

$$\ddot{\varphi} + \frac{g\cdot a}{h\cdot l}\varphi = 0$$

$$\underline{\underline{T = 2\pi\sqrt{\frac{h\cdot l}{g\cdot a}}}}$$

Lösung 1244



$$\Sigma M_0 = 0:$$

$$\Theta\ddot{\varphi} + 2cl^2\varphi - mgl\varphi = 0$$

$$ml^2\ddot{\varphi} + (2cl^2 - mgl)\varphi = 0$$

$$\ddot{\varphi} + \frac{2cl^2 - mgl}{ml^2}\,\varphi = 0$$

$$\underline{\underline{T = \frac{2\pi}{\sqrt{\frac{2c}{m} - \frac{g}{l}}}}}$$

Lösung 1245

$$\Sigma M_0 = 0: \quad ml^2\ddot{\varphi} + (mgl + 2ca^2)\,\varphi = 0$$

$$\ddot{\varphi} + \frac{mgl + 2ca^2}{ml^2}\,\varphi = 0$$

$$\underline{\underline{T = \frac{2\pi}{\sqrt{\frac{2ca^2}{ml^2} + \frac{g}{l}}}}}$$

Lösung 1246

$$\Sigma M_0 = 0: \quad ml^2\ddot{\varphi} + (2ca^2 - mgl)\,\varphi = 0$$

$$\ddot{\varphi} + \left(\frac{2ca^2}{ml^2} - \frac{g}{l}\right)\varphi = 0$$

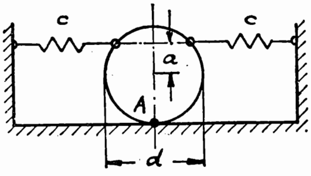
$$\underline{\underline{T = \frac{2\pi}{\sqrt{\frac{2ca^2}{ml^2} - \frac{g}{l}}}}}$$

Die vertikale Gleichgewichtslage des Pendels ist stabil, wenn die Wurzel reell ist.

Also:

$$\underline{\underline{a^2 > \frac{mlg}{2c}}}$$

Lösung 1247



$$\Sigma M_A = 0:$$

$$\Theta_A\ddot{\varphi} + 2c\left(a + \frac{d}{2}\right)^2\varphi = 0$$

$$\Theta_A + \Theta_0 + m\frac{d^2}{4} = \frac{3}{8}\,md^2$$

$$\ddot{\varphi} + \frac{2c\left(a + \frac{d}{2}\right)^2}{\frac{3}{8}\,md^2}\,\varphi = 0$$

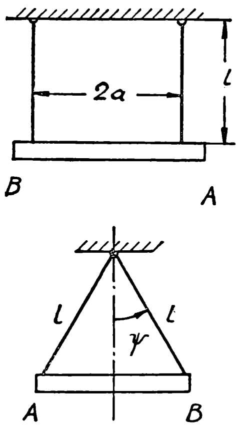
$$\underline{\underline{T = \frac{\pi\sqrt{3}}{1 + \frac{2a}{d}}\sqrt{\frac{m}{c}}}}$$

Lösung 1248 $\Sigma M_0 = 0$: $(\Theta_0 + m s^2)\ddot{\varphi} + (M s_0 - m s) g \varphi = 0$

$$\ddot{\varphi} + \frac{M s_0 - m s}{\Theta_0 + m s^2}\, g \varphi = 0$$

$$\underline{\underline{T = 2\pi \sqrt{\frac{\Theta_0 + m s^2}{(M s_0 - m s) g}}}}$$

Lösung 1249



Momentengleichgewicht:

$$\Theta \ddot{\varphi} + m g a \psi = 0$$

$$\psi \cdot l = a \varphi$$

$$\ddot{\varphi} + \frac{m g a^2}{\Theta \cdot l}\, \varphi = 0$$

mit $\varrho = \sqrt{\frac{\Theta}{m}}$

$$\underline{\underline{T = 2\pi \sqrt{\frac{l \cdot \Theta}{m g a^2}} = 2\pi \frac{\varrho}{a} \sqrt{\frac{l}{g}}}}$$

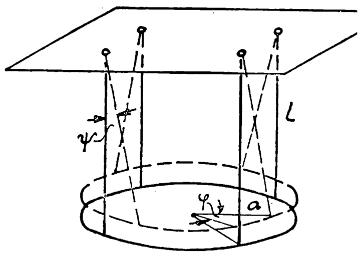
Lösung 1250 Die Schwingungszeit ist von der Anzahl der Fäden unabhängig.

$$\Theta \ddot{\varphi} + m g a \psi = 0$$

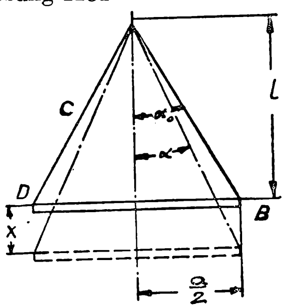
$$\Theta_{\text{Reifen}} = m a^2; \quad \varphi \cdot a = \psi \cdot l$$

$$m a^2 \ddot{\varphi} + m g \frac{a^2}{l} \varphi = 0$$

$$\ddot{\varphi} + \frac{g}{l} \varphi = 0; \quad \underline{\underline{T = 2\pi \sqrt{\frac{l}{g}}}}$$



Lösung 1251



In der Gleichgewichtslage ist die Federkraft:

$$F_0 = \frac{M g}{4 \cos \alpha_0}$$

Allgemein:

$$F = F_0 + \frac{c a}{2} \left(\frac{1}{\sin \alpha} - \frac{1}{\sin \alpha_0} \right)$$

Nach der Taylorschen Formel gilt:

$$\frac{1}{\sin \alpha} = \frac{1}{\sin \alpha_0} + \left(\frac{1}{\sin \alpha} \right)'_0 d\alpha \cdots = \frac{1}{\sin \alpha_0} - \frac{\cos \alpha_0}{\sin^2 \alpha_0} d\alpha$$

Somit: $F = F_0 - \frac{ca}{2} \cdot \frac{\cos\alpha_0}{\sin^2\alpha_0} \cdot d\alpha; \quad l = \frac{a}{2}\operatorname{ctg}\alpha_0$

$x = \frac{a}{2}\operatorname{ctg}\alpha - l = -\frac{a}{2}\frac{1}{\sin^2\alpha_0} d\alpha$

Resultierende der Federkräfte (vertikal):

$R = 4F\cos\alpha = 4F\cos\alpha_0 - 4F\sin\alpha_0 d\alpha$

Dabei wurde das quadratische Glied von $d\alpha$ vernachlässigt.

$M\ddot{x} - Mg + R = 0$

$$M\frac{a}{2} \cdot \frac{1}{\sin^2\alpha_0}(d\alpha)^{\cdot\cdot} + 2ca\left(\frac{\cos^2\alpha_0}{\sin^2\alpha_0}\right) d\alpha + \frac{Mg \cdot \sin\alpha_0}{\cos\alpha_0} d\alpha = 0$$

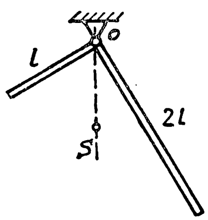
$$(d\alpha)^{\cdot\cdot} + \left(\frac{4c}{M}\cos^2\alpha_0 + \frac{2g\sin^3\alpha_0}{a\cos\alpha_0}\right) d\alpha = 0$$

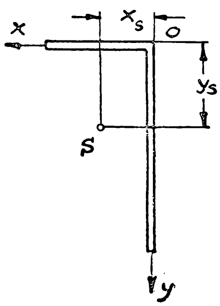
Mit $\operatorname{tg}\alpha_0 = \frac{a}{2l}$ wird:

$$(d\alpha)^{\cdot\cdot} + \frac{4c}{M}\frac{4l^2}{4l^2 + a^2}\left(1 + \frac{Mga^2}{16cl^3}\right) d\alpha = 0$$

$$\underline{\underline{T = 2\pi\sqrt{\frac{M}{4c} \cdot \frac{(4l^2 + a^2)}{4l^2} \cdot \frac{1}{1 + \frac{Mga^2}{16cl^3}}}}}$$

Lösung 1252





$\sum M_0 = 0$:

$$\Theta_0\ddot{\varphi} + Mg \cdot \overline{OS} \cdot \varphi = 0$$

$$y_S = \frac{2l \cdot l}{3l} = \frac{2}{3}l$$

$$x_S = \frac{l^2}{2 \cdot 3l} = \frac{1}{6}l$$

$$\overline{OS} = \sqrt{x_S^2 + y_S^2} = \frac{\sqrt{17}}{6}l$$

$$\Theta_0 = \frac{m_1}{3}l^2 + \frac{m_2}{3}4l^2$$

$$= \frac{l^2}{3}\left(\frac{M}{3} + \frac{4 \cdot 2}{3}M\right)$$

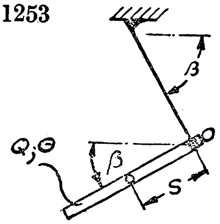
$$\Theta_0 = Ml^2$$

$$Ml^2\ddot{\varphi} + \frac{\sqrt{17}}{6}lMg\varphi = 0$$

$$\ddot{\varphi} + \frac{\sqrt{17}g}{6l}\varphi = 0$$

$$\underline{\underline{T = 2\pi\frac{\sqrt{6}}{\sqrt[4]{17}}\sqrt{\frac{l}{g}} = 7{,}53\sqrt{\frac{l}{g}}}}$$

Lösung 1253



$$\Sigma M_0 = 0:$$

$$\Theta \ddot{\varphi} + Q s \cos\beta\, \varphi = 0$$

$$\ddot{\varphi} + \frac{Q}{\Theta} s \cos\beta\, \varphi = 0$$

$$\underline{\underline{T = 2\pi \sqrt{\frac{\Theta}{Q \cdot s \cdot \cos\beta}}}}$$

Lösung 1254

$$\Sigma M_0 = 0: \quad \left(\Theta + \frac{Q}{g} a^2\right) \ddot{\varphi} + (c_2 b^2 + c_1 a^2)\, \varphi = 0$$

$$\ddot{\varphi} + \frac{c_2 b^2 + c_1 a^2}{\Theta g + Q a^2} g \varphi = 0$$

$$\underline{\underline{T = 2\pi \sqrt{\frac{\Theta g + Q a^2}{(c_2 b^2 + c_1 a^2)\, g}}}}$$

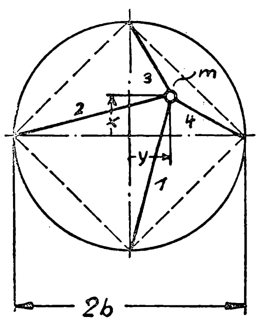
Lösung 1255

$$\Sigma M_0 = 0: \quad \left(\Theta + \frac{Q}{g} a^2\right) \ddot{\varphi} + (c_1 + c_2 b^2) \varphi = 0$$

$$\ddot{\varphi} + \frac{c_1 + c_2 b^2}{\Theta g + \Theta a^2} g \varphi = 0$$

$$\underline{\underline{T = 2\pi \sqrt{\frac{\Theta g + Q a^2}{(c_1 + c_2 b^2) g}}}}$$

Lösung 1256



Ableitung am Viereck:

Lagrangesche Funktion:

$$L = T - U = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{c_1}{2}\left[\sqrt{(b+x)^2 + y^2} - a\right]^2$$

$$- \frac{c_3}{2}\left[\sqrt{(b-x)^2 + y^2} - a\right]^2 - \frac{c_2}{2}\left[\sqrt{(b+y)^2 + x^2} - a\right]^2$$

$$- \frac{c_4}{2}\left[\sqrt{(b-y)^2 + x^2} - a\right]^2$$

Entwicklung der Ausdrücke in den eckigen Klammern in eine binomische Reihe gibt mit

$$c_1 = c_2 = c_3 = c_4 = c:$$

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{c}{2}\left\{\left[b + x + \frac{y^2}{2(b+x)} - a\right]^2\right.$$

$$+ \left[b - x + \frac{y^2}{2(b-x)} - a\right]^2 + \left[b + y + \frac{x^2}{2(b+y)} - a\right]^2$$

$$\left. + \left[b - y + \frac{x^2}{2(b-y)} - a\right]^2\right\}$$

Lagrangesche Gleichung:

$$\left(\frac{\partial L}{\partial \dot x}\right)^{\cdot} - \frac{\partial L}{\partial x} = 0$$

$$\begin{aligned} m\ddot x + c\Bigg\{&\left[b+x+\frac{y^2}{2(b+x)}-a\right]\left(1-\frac{y^2}{2(b+x)^2}\right)\\ &+\left[b-x+\frac{y^2}{2(b-x)}-a\right]\cdot\left(-1+\frac{y^2}{2(b-x)^2}\right)+\left[b+y+\frac{x^2}{2(b+y)}-a\right]\frac{x}{b+y}\\ &+\left[b-y+\frac{x^2}{2(b-y)}-a\right]\frac{x}{b-y}\Bigg\}=0\end{aligned}$$

Vernachlässigt man Glieder höherer Ordnung, so ist

$$m\ddot x + c\left(2x+2x-2x\frac{a}{b}\right)=0=m\ddot x+2c\left(2-\frac{a}{b}\right)x$$

Für n Federn:

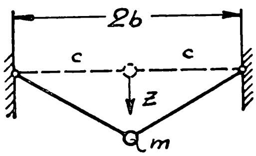
$$m\ddot x + \frac{n}{2}c\left(\frac{2b-a}{b}\right)x=0$$

$$\underline{\underline{k=\sqrt{\frac{nc}{2m}\cdot\frac{2b-a}{b}}}}$$

Lösung 1257

$$L=T-U=m\frac{\dot z^2}{2}-\frac{nc}{2}\cdot\Delta l^2$$

$$=m\frac{\dot z^2}{2}-\frac{nc}{2}\left[\sqrt{b^2+z^2}-a\right]^2$$



Entwicklung der Klammer in eine binomische Reihe:

$$L=m\frac{\dot z^2}{2}-\frac{nc}{2}\left[b+\frac{1}{2b}z^2-a\right]^2$$

$$\left(\frac{\partial L}{\partial \dot z}\right)^{\cdot}-\frac{\partial L}{\partial z}=0$$

$$m\ddot z+nc\left(b+\frac{z^2}{2b}-a\right)\frac{z}{b}=0$$

Vernachlässigung der Glieder höherer Ordnung:

$$m\ddot z+nc\frac{b-a}{b}z=0$$

$$\underline{\underline{k=\sqrt{\frac{nc(b-a)}{mb}}}}$$

Lösung 1258

$$k=\sqrt{\frac{c^*}{m}}=\sqrt{\frac{g}{\eta_E}}$$

$$\eta_E=\frac{\eta_D+\eta_F}{2};\quad \eta_F=\frac{mg}{2c_4};\quad \eta_D=\eta_B+\frac{mg}{2c_3}$$

$$\eta_B=\frac{\eta_A+\eta_C}{2};\quad \eta_A=\frac{mg}{4c_1};\quad \eta_C=\frac{mg}{4c_2}$$

$$\eta_D = \frac{mg}{2}\left(\frac{1}{4c_1} + \frac{1}{4c_2} + \frac{1}{c_3}\right)$$

$$\eta_E = \frac{mg}{4}\left(\frac{1}{4c_1} + \frac{1}{4c_2} + \frac{1}{c_3} + \frac{1}{c_4}\right)$$

$$\underline{\underline{k = \sqrt{\frac{4}{m\left(\frac{1}{4c_1} + \frac{1}{4c_2} + \frac{1}{c_3} + \frac{1}{c_4}\right)}}}}$$

Lösung 1259

Bezeichnungen:

$x;\ y =$ Koordinaten der Massen m

$X;\ Y =$ Koordinaten der Masse M

$\alpha;\ \beta =$ Winkel in Ruhelage

$\alpha^*;\ \beta^* =$ Momentanwinkel bei der Bewegung

$d\alpha;\ d\beta =$ Winkeländerung bei der Bewegung; $\alpha^* = \alpha + d\alpha;\ \beta^* = \beta + d\beta$

$\dot{x}^2 + \dot{y}^2 = a^2\dot{\alpha}^{*2} = a^2 \cdot (\dot{d\alpha})^2;\quad y = a\cos\alpha^* = a\cos\alpha - a\sin\alpha\, d\alpha - a\cos\alpha\frac{(d\alpha)^2}{2}$

(Entwicklung in eine Taylor-Reihe)

$$X = a(\sin\alpha^* + \sin\beta^*) = a(\sin\alpha + \sin\beta) + a(\cos\alpha\, d\alpha + \cos\beta\, d\beta)$$
$$- \frac{a}{2}\left(\sin\alpha(d\alpha)^2 + \sin\beta(d\beta)^2\right) = \text{konst.}$$

Somit: $a(\cos\alpha\, d\alpha + \cos\beta\, d\beta) - \frac{a}{2}\left(\sin\alpha(d\alpha)^2 + \sin\beta(d\beta)^2\right) = 0$

$d\beta = -\frac{\cos\alpha}{\cos\beta}d\alpha + \frac{1}{2}\left(\frac{\sin\alpha\cos^2\beta + \sin\beta\cos^2\alpha}{\cos^3\beta}\right)(d\alpha)^2$ unter Vernachlässigung der Glieder dritter Ordnung

$$Y = a(\cos\alpha^* + \cos\beta^*) = a(\cos\alpha + \cos\beta) - a\left(\sin\alpha\, d\alpha - \sin\beta \cdot \frac{\cos\alpha}{\cos\beta}d\alpha\right)$$
$$- \frac{a}{2}\left(\cos\alpha(d\alpha)^2 + \cos\beta\frac{\cos^2\alpha}{\cos^2\beta}(d\alpha)^2 + \sin\alpha\frac{\sin\beta}{\cos\beta}(d\alpha)^2\right.$$
$$\left. + \frac{\sin^2\beta\cos^2\alpha}{\cos^3\beta}(d\alpha)^2\right)$$

$$T = \frac{2}{2}ma^2(\dot{d\alpha})^2 + \frac{M}{2}a^2\frac{\sin^2(\beta-\alpha)}{\cos^2\beta}(\dot{d\alpha})^2$$

$$U = 2mg \cdot a\sin\alpha\, d\alpha - Mga\frac{\sin(\beta-\alpha)}{\cos\beta}d\alpha + \frac{2}{2}mga\cos\alpha(d\alpha)^2$$
$$+ \frac{Mga}{2}\left[\frac{\cos\alpha}{\cos\beta}(\cos\beta + \cos\alpha) + \frac{\sin\beta}{\cos^3\beta}(\sin\alpha\cos^2\beta + \sin\beta\cos^2\alpha)\right](d\alpha)^2$$

Abgekürzt: $T = \frac{A}{2}(\dot{d\alpha})^2;\quad U = B\,d\alpha + \frac{C}{2}(d\alpha)^2;\quad L = T - U$

Damit Gleichgewicht herrscht, muß sein: $\frac{\partial U}{\partial(d\alpha)} = 0$: $\underline{\underline{\frac{2m}{M} = \frac{\sin(\beta-\alpha)}{\sin\alpha\cos\beta}}}$

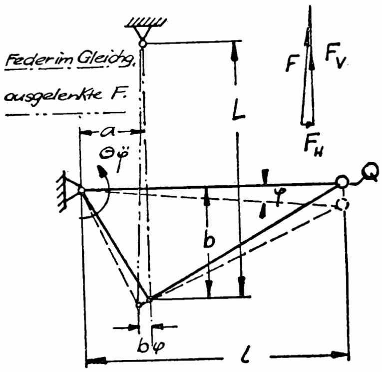
Somit aus: $\left(\frac{\partial L}{\partial(d\alpha)^{\cdot}}\right)^2 - \frac{\partial L}{\partial(d\alpha)} = 0$:

$$k = \sqrt{\frac{g}{a}\,\frac{\frac{2m}{M}\cos\alpha + \frac{\cos\alpha}{\cos\beta}(\cos\beta+\cos\alpha) + \frac{\sin\beta}{\cos^3\beta}(\sin\alpha\cos^2\beta + \sin\beta\cos^2\alpha)}{\frac{2m}{M} + \frac{\sin^2(\beta-\alpha)}{\cos^2\beta}}}$$

Daraus:

$$\underline{\underline{k = \sqrt{\frac{g}{a}\,\frac{\cos^2\beta\sin\beta + \cos^2\alpha\sin\alpha}{\cos\beta\sin(\beta-\alpha)\cos\alpha\cos(\beta-\alpha)}}}}$$

Lösung 1260



$$F_V = F_0 + ca\varphi$$

$$F_H = F_0 \cdot \frac{b\varphi}{L}$$

$\Sigma M_0 = 0$:

$$\Theta\ddot{\varphi} - Ql + F_V(a - b\varphi) + F_H \cdot b = 0$$

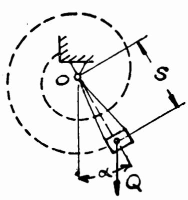
Unter Vernachlässigung der quadratischen Glieder von φ wird:

$$\Theta\ddot{\varphi} - Ql + F_0 a + \left(-F_0 b + ca^2 + \frac{F_0 \cdot b^2}{L}\right)\varphi = 0$$

$F_0 = \frac{Ql}{a}$: $\Theta\ddot{\varphi} + \left[ca - F_0 b\left(1 - \frac{b}{L}\right)\right]\varphi = 0$

$$\underline{\underline{k = \sqrt{\frac{ca^2 - F_0 b\left(1-\frac{b}{L}\right)}{\Theta}}}}$$

Lösung 1261



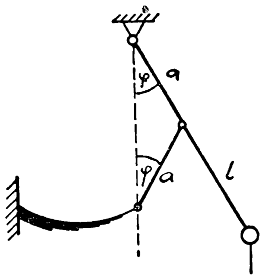
$\Sigma M_0 = 0$: $\Theta\ddot{\varphi} + c\varphi + mgs\cos\alpha\varphi = 0$; $mg = Q$

$$\underline{\underline{T = \frac{2\pi}{k} = 2\pi\sqrt{\frac{\Theta}{Q\cdot s\cdot\cos\alpha + c}}}}$$

Lösung 1262

$\Sigma M_0 = 0$: $\Theta\ddot{\varphi} + c\varphi + Qa\varphi = 0$; $\underline{\underline{T}} = 2\pi\sqrt{\frac{\Theta}{c + Q\cdot a}} = 2\pi\sqrt{\frac{0{,}03}{2\cdot 4{,}5}} = \underline{\underline{0{,}364 \text{ sek}}}$

Lösung 1263



$$U = Q \cdot l\,(1-\cos\varphi) + \frac{c}{2}\left[\frac{F_0}{c} - 2a\,(1-\cos\varphi)\right]^2$$

$$T = \frac{Q l^2}{2g}\cdot\dot\varphi^2; \quad L = T - U$$

$$U \cong Ql\left(\frac{\varphi^2}{2} - \frac{\varphi^4}{24} + \frac{\varphi^6}{720}\right) + \frac{c}{2}\left[\frac{F_0}{c} - a\left(\varphi^2 - \frac{\varphi^4}{12} + \frac{\varphi^6}{360}\right)\right]^2$$

$$U \cong Ql\left(\frac{\varphi^2}{2} - \frac{\varphi^4}{24} + \frac{\varphi^6}{720}\right) + \frac{F_0^2}{2c}$$

$$-F_0 a\varphi^2 + \left(F_0\frac{a}{12} + \frac{c}{2}a^2\right)\varphi^4 - \left(F_0\frac{a}{360} + \frac{ca^2}{12}\right)\varphi^6$$

$$\left(\frac{\partial L}{\partial\dot\varphi}\right)^{\cdot} - \frac{\partial L}{\partial\varphi} = 0:$$

$$\frac{Ql^2}{g}\ddot\varphi + (Ql - 2F_0 a)\,\varphi + \left(-\frac{Ql}{6} + \frac{F_0 a}{3} + 2ca^2\right)\varphi^3 + \left(\frac{Ql}{120} - \frac{F_0 a}{60} - \frac{ca^2}{2}\right)\varphi^5 = 0$$

Der Faktor von φ^3 wird Null gesetzt: $\underline{\underline{Ql - 2aF_0 - 12a^2 c}}$

$$\underline{\underline{T = 2\pi\sqrt{\frac{l}{g\left(1-\frac{2F_0 a}{Q\cdot l}\right)}}}}$$ Das Glied mit φ^5 wurde vernachlässigt.

Die Ruhelage des Pendels ist senkrecht.

Lösung 1264

Aus Aufgabe 1263: $\frac{l}{g}\ddot\varphi + \left(1 - \frac{2F_0 a}{Ql}\right)\varphi + \left(\frac{1}{120} - \frac{F_0 a}{60Ql} - \frac{ca^2}{2Ql}\right)\varphi^5 = 0$

mit $\frac{ca^2}{Ql} = \frac{1}{12}\left(1 - \frac{2F_0 a}{Ql}\right)$ wird:

$$\frac{l}{g}\ddot\varphi + \left(1 - \frac{2F_0 a}{Ql}\right)\left(\varphi - \frac{\varphi^5}{30}\right) = 0 \quad \text{oder:} \quad \ddot\varphi + \omega^2\left(\varphi - \frac{\varphi^5}{30}\right) = 0$$

integriert: $\frac{\dot\varphi^2}{2} + \omega^2\left(\frac{\varphi^2}{2} - \frac{\varphi^6}{180}\right) = \omega^2\left(\frac{\varphi_0^2}{2} - \frac{\varphi_0^6}{180}\right)$

$$\dot\varphi = \omega\sqrt{(\varphi_0^2 - \varphi^2) - \frac{1}{90}(\varphi_0^6 - \varphi^6)}$$

$$\frac{T}{4} = \int_0^{\varphi_0}\frac{d\varphi}{\omega\sqrt{(\varphi_0^2-\varphi^2) - \frac{1}{90}(\varphi_0^6-\varphi^6)}} \approx \frac{1}{\omega}\int_0^{\varphi_0}\frac{1}{\sqrt{\varphi_0^2-\varphi^2}}\left(1 + \frac{\varphi_0^4 + \varphi_0^2\varphi^2 + \varphi^4}{180}\right)d\varphi$$

mit $\frac{\varphi}{\varphi_0} = \sin z$ ergibt sich:

$$\int_0^{\varphi_0}\frac{d\varphi}{\sqrt{\varphi_0^2-\varphi^2}} = \frac{\pi}{2}; \quad \int_0^{\varphi_0}\frac{\varphi^2\,d\varphi}{\sqrt{\varphi_0^2-\varphi^2}} = \frac{1}{2}\,\frac{\pi}{2}\,\varphi_0^2; \quad \int_0^{\varphi_0}\frac{\varphi^4\,d\varphi}{\sqrt{\varphi_0^2-\varphi^2}} = \frac{\pi}{2}\,\varphi_0^4$$

$$T=\frac{2\pi}{\omega}\left[1+\frac{\varphi_0^4}{180}\left(1+\frac{1}{3}+\frac{3}{8}\right)\right]=T_0\left[1+\frac{\varphi_0^4}{96}\right];\quad T_0=2\pi\sqrt{\frac{l}{g\left(1-\frac{2F_0a}{Q\cdot l}\right)}}$$ vergleiche Aufgabe 1263

$$\underline{\underline{T=2\pi\sqrt{\frac{l}{g}}\,\frac{1}{\sqrt{1-\frac{2F_0a}{Ql}}}\left(1+\frac{\varphi^4}{96}\right)}}$$

$$\frac{\varphi_0^4}{96}=\frac{\left(\frac{\pi}{4}\right)^4}{96}=0{,}004;$$ Der Fehler beträgt also 0,4 %

Lösung 1265

Mit $Q\cdot l=2aF_0$ wird nach Aufgabe 1263:

$$U=\frac{F_0^2}{2c}+2ca^2(1-\cos\varphi)^2;\quad T=\frac{Ql^2}{2g}\dot\varphi^2$$

$$T+U=\text{konst.}:\quad \frac{Ql^2}{2g}\dot\varphi^2+2ca^2(1-\cos\varphi)^2+\frac{F_0^2}{2c}=\frac{F_0^2}{2c}+2ca^2(1-\cos\varphi_0)^2$$

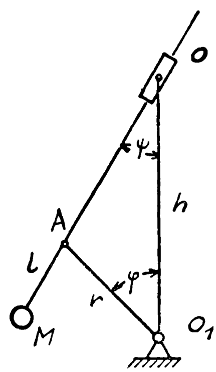
bzw.: $$\frac{Ql^2}{2g}\dot\varphi^2+2ca^2[(\cos\varphi_0-\cos\varphi)(2-\cos\varphi_0-\cos\varphi)]=0$$

Schwingungszeit: $$T=2\sqrt{\frac{Ql^2}{cga^2}}\int\limits_0^{\varphi_0}\frac{d\varphi}{\sqrt{(-\cos\varphi_0+\cos\varphi)(2-\cos\varphi_0-\cos\varphi)}}$$

$$T=\frac{4}{\varphi_0}\sqrt{\frac{Ql^2}{cga^2}}\int\limits_0^1\frac{d\left(\frac{\varphi}{\varphi_0}\right)}{\sqrt{1-\left(\frac{\varphi}{\varphi_0}\right)^4}};\quad \text{mit}\quad \int\limits_0^1\frac{dx}{\sqrt{1-x^4}}=1{,}31\quad \text{ergibt sich:}$$

$$\underline{\underline{T=5{,}24\,\frac{l}{a\varphi_0}\sqrt{\frac{Q}{cg}}}}$$

Lösung 1266



Für kleine Ausschläge gilt:

$$\frac{\varphi}{r}=\frac{\varphi+\psi}{h};\quad \varphi=\psi\left(\frac{h}{r}-1\right)$$

$$\cos\alpha=1-\frac{\alpha^2}{2}+\cdots$$

$$U=Mg(-l\cos\psi+r\cos\varphi);$$

$$U=\frac{Mg}{2}\psi^2\left[l-\frac{(h-r)^2}{r}\right]+Mg(r-l)$$

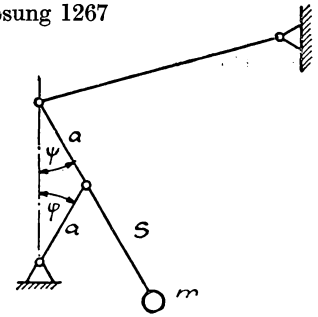
$$T=\frac{M}{2}(r\dot\varphi+l\dot\psi)^2=\frac{M}{2}\dot\psi^2(l+h-r)^2$$

$$L=T-U;\quad \left(\frac{\partial L}{\partial\dot\psi}\right)^{\cdot}-\frac{\partial L}{\partial\psi}=0:$$

$$M\left[(l+h-r)^2\ddot\psi+g\left(l-\frac{(h-r)^2}{r}\right)\psi\right]=0$$

$$\underline{\underline{T=2\pi\sqrt{\frac{(l+h-r)^2\cdot r}{g[lr-(h-r)^2]}}}};\quad (h-r)<\sqrt{rl}$$

Lösung 1267



Für kleine Ausschläge gilt: $\varphi = \psi$

Entsprechend Aufgabe 1266:

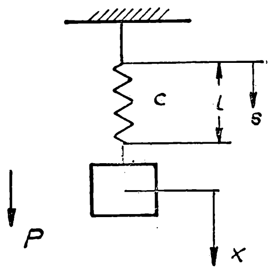
$$U = \frac{mg}{2}\varphi^2(s-a)$$

$$T = \frac{m}{2}\dot{\varphi}^2(a+s)^2; \quad L = T - U$$

$$\left(\frac{\partial L}{\partial \dot{\varphi}}\right)^{\cdot} - \frac{\partial L}{\partial \varphi} = 0: \quad m[(a+s)^2\ddot{\varphi} + g(s-a)\varphi] = 0$$

$$\underline{\underline{T = 2\pi\sqrt{\frac{(a+s)^2}{g(s-a)}}; \quad s > a}}$$

Lösung 1268



Weg des Federteilchens im Abstand s:

$$x(s) = x \cdot \frac{s}{l}$$

$$\dot{x}(s) = \dot{x} \cdot \frac{s}{l}$$

$$\dot{x}^2(s) = \dot{x}^2 \cdot \frac{s^2}{l^2}$$

$$T = \frac{1}{2}\left[\frac{P}{g}\dot{x}^2 + \frac{P_0\dot{x}^2}{gl}\int_0^l \frac{s^2}{l^2}\cdot ds\right] = \frac{P + \frac{1}{3}P_0}{2g}\cdot\dot{x}^2$$

$$U = \frac{c}{2}x^2; \quad L = T - U$$

$$\left(\frac{\partial L}{\partial \dot{x}}\right)^{\cdot} - \frac{\partial L}{\partial x} = 0: \quad \frac{\left(P + \frac{1}{3}P_0\right)\ddot{x}}{g} + cx = 0$$

$$\underline{\underline{T = 2\pi\sqrt{\frac{P + \frac{1}{3}P_0}{cg}}}}$$

Lösung 1269 Entsprechend Aufgabe 1268 gilt:

$$\varphi(s) = \varphi \cdot \frac{s}{l}; \quad \dot{\varphi}^2(s) = \dot{\varphi}^2 \cdot \frac{s^2}{l^2}$$

$$T = \frac{\Theta\dot{\varphi}^2}{2} + \frac{\Theta_0\dot{\varphi}}{2l}\int_0^l \frac{s^2}{l^2}\cdot ds = \frac{\Theta + \frac{\Theta_0}{3}}{2}\dot{\varphi}^2$$

$$U = \frac{c}{2}\varphi^2; \quad L = T - U$$

$$\left(\frac{\partial L}{\partial \dot{\varphi}}\right)^{\cdot} - \frac{\partial L}{\partial \varphi} = 0:$$

$$\left(\Theta + \frac{1}{3}\Theta_0\right)\ddot{\varphi} + c\varphi = 0$$

$$\underline{\underline{T = 2\pi\sqrt{\frac{\Theta + \frac{1}{3}\Theta_0}{c}}}}$$

Lösung 1270

Die Federkonstante des frei aufliegenden Balkens mit Mittellast ist:

$$c = \frac{48\,EJ}{l^3}$$

Kreisfrequenz $\omega = \sqrt{\frac{c}{m}}$; $n = \frac{\omega \cdot 30}{\pi} = \frac{30}{\pi}\sqrt{\frac{48\,EJ}{Q\,l^3}\cdot g} = \underline{\underline{2080\sqrt{\frac{EJ}{Q\,l^3}}}}$

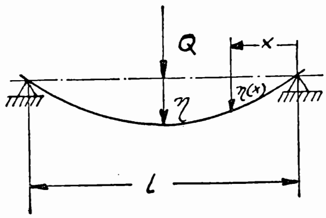
(Längen in cm)

Lösung 1271

$$\eta = \frac{P\,l^3}{48\,EJ}; \quad \eta(x) = 3\frac{x}{l}\left(1 - \frac{4x^2}{3l^2}\right)\eta$$

$$\dot\eta(x) = 3\frac{x}{l}\left(1 - \frac{4x^2}{3l^2}\right)\dot\eta$$

$$\dot\eta^2(x) = 9\frac{x^2}{l^2}\left(1 - \frac{8x^2}{3l^2} + \frac{16x^4}{9l^4}\right)\cdot\dot\eta^2$$



$$\int_0^{\frac{l}{2}} \dot\eta(x)\,dx = 9\dot\eta^2 l\left(\frac{1}{24} - \frac{1}{60} + \frac{1}{978}\right) = \dot\eta^2 l\cdot\frac{17}{70}$$

Mittlere Geschwindigkeit:

$$v^2 = \frac{2}{l}\int_0^{\frac{l}{2}} \dot\eta^2(x) = \dot\eta^2\,\frac{17}{35}$$

$$T = \frac{\left(Q + \frac{17}{35}Q_1\right)\dot\eta^2}{2}; \quad L = T - U$$

$$U = \frac{c}{2}\eta^2; \quad c = \frac{48\,EJ}{l^3}$$

$$\left(\frac{\partial L}{\partial\dot\eta}\right)^{\bullet} - \frac{\partial L}{\partial\eta} = 0: \quad \left(Q + \frac{17}{35}Q_1\right)\ddot\eta + c\eta = 0$$

Somit entsprechend Aufgabe 1270:

$$\underline{\underline{n = 2080\sqrt{\frac{EJ}{\left(Q + \frac{17}{35}Q_1\right)l^3}}}} \quad \text{(Längen in cm)}$$

Lösung 1272

Unter Verwendung der Aufgaben 1271 und 1270 ergibt sich mit $k = \frac{n}{60}$:

$\underline{\underline{k_1 = 10{,}1\ 1/\text{sek}; \quad k_2 = 10{,}2\ 1/\text{sek}}}$

Lösung 1273

Unter Verwendung der Aufgaben 1271 und 1270 ergibt sich mit $k = \frac{n}{60}$:

$\underline{\underline{k_1 = 4{,}56\ 1/\text{sek}; \quad k_2 = 5{,}34\ 1/\text{sek}}}$

Lösung 1274

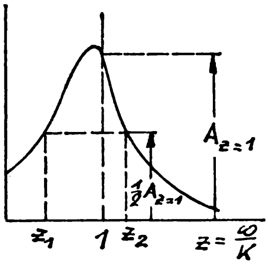
$$T = 2\pi\sqrt{\frac{\eta}{g}};\quad \eta = \eta_1 + \eta_2 = \frac{Q l^3}{48EJ} + \frac{Q}{2c}$$

$$T = 2\pi\sqrt{\frac{Q}{g}\left(\frac{l^3}{48EJ} + \frac{1}{2c}\right)} = 6{,}28\sqrt{\frac{200}{9{,}81}\left(\frac{64\cdot 10^6}{48\cdot 2\cdot 10^6\cdot 180} + \frac{1}{2\cdot 150}\right)} = \underline{\underline{0{,}238 \text{ sek}}}$$

Lösung 1275

$$T = 2\pi\sqrt{\frac{\eta}{g}};\quad \eta = \frac{Q l^3}{3EJ};\quad \underline{\underline{E = \frac{4\pi^2 Q l^3}{T^2 3 J g}}}$$

Lösung 1276



Allgemein lautet die Differentialgleichung einer erzwungenen gedämpften Schwingung:

$$\ddot{x} + 2n\dot{x} + k^2 x = C\sin(\omega t + \varphi)$$

Das partikuläre Integral lautet: $x = A\sin\omega t$

$$A\left((k^2 - \omega^2)\sin\omega t + 2n\omega\cos\omega t\right) = C\sin(\omega t + \varphi)$$

$$A = \frac{C}{\sqrt{(k^2-\omega^2)^2 + 4n^2\omega^2}}$$

mit $z = \frac{\omega}{k}$ und $\delta = \frac{n}{k}$ wird

$$A = \frac{\frac{C}{k^2}}{\sqrt{(1-z^2)^2 + 4\delta^2 z^2}}$$

$$A_{(z=1)} = \frac{\frac{C}{k^2}}{2\delta};\quad A_{(z)} = \frac{1}{2}A_{(z=1)} = \frac{\frac{C}{k^2}}{4\delta}$$

oder: $16\delta^2 = (1-z^2)^2 + 4\delta^2 z^2$

$$z^4 - 2z^2(1-2\delta^2) = 16\delta^2 - 1$$

$$z_{1,2}^2 = (1-2\delta^2) \mp 2\delta\sqrt{3+\delta^2}$$

$$\underline{\underline{\Delta = z_2 - z_1 = \sqrt{(1-2\delta^2) + 2\delta\sqrt{3+\delta^2}} - \sqrt{(1-2\delta^2) - 2\delta\sqrt{3+\delta^2}}}}$$

Für $\delta \ll 1$ gilt: $\Delta = \sqrt{1 + 2\delta\sqrt{3}} - \sqrt{1 - 2\delta\sqrt{3}}$

Beide Ausdrücke in Potenzreihen entwickelt ergibt:

$$\underline{\underline{\Delta \approx 2\delta\sqrt{3}}}$$

Lösung 1277

$$\Theta\ddot{\varphi} + c\varphi = \frac{Qa}{g}\ddot{z};\quad z = 2\sin 25t = h\sin pt$$

Ansatz: $\varphi = A\sin pt;\quad A(\Theta p^2 - c) = Qap^2 h$

$$A = \frac{Qah}{g\left(\Theta - \frac{c}{p^2}\right)} = \frac{10\cdot 0{,}2}{981\left(0{,}4 - \frac{0{,}1}{625}\right)} = 0{,}0051;\quad \underline{\underline{\varphi = 0{,}0051\sin 25t \text{ cm}}}$$

Lösung 1278

$$\Theta\ddot\varphi + k\dot\varphi + c\varphi = \frac{Q\cdot a}{g}\ddot z;\quad z = h\sin pt$$

Ansatz: $\varphi = A\sin(pt-\varepsilon)$: $A[(\Theta p^2 - c)\sin(pt-\varepsilon) - kp\cos(pt-\varepsilon)]$

$$= \frac{Qap^2h}{g}\sin pt$$

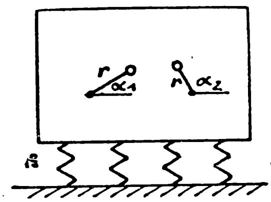
Daraus durch Koeffizientenvergleich: $A[(\Theta p^2 - c)\cos\varepsilon - kp\sin\varepsilon] = \frac{Qap^2h}{g}$

$$(\Theta p^2 - c)\sin\varepsilon + kp\cos\varepsilon = 0$$

$$\underline{\underline{\operatorname{tg}\varepsilon = \frac{\frac{kp}{c}}{1-\frac{\Theta p^2}{c}}}}$$

$$A = \frac{Qah\cos\varepsilon}{\Theta g\left[1-\frac{c}{\Theta p^2}\right]};\quad \underline{\underline{\varphi = \frac{Qah\cdot\cos\varepsilon}{\Theta g\left[1-\frac{c}{\Theta p^2}\right]}\cdot\sin(pt-\varepsilon)}}$$

Lösung 1279



$$\left(\frac{P+2Q}{g}\right)\ddot x + cx = \frac{Qr\omega^2}{g}\left[\sin(\alpha_1+\omega t) + \sin(\alpha_2-\omega t)\right]$$

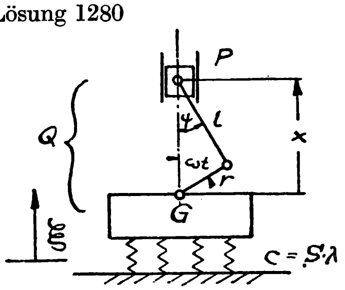
$$\left(\frac{P+2Q}{g}\right)\ddot x + cx = \frac{2Qr\omega^2}{g}\left[\sin\left(\frac{\alpha_1+\alpha_2}{2}\right)\cos\left(\frac{\alpha_1-\alpha_2}{2}+\omega t\right)\right]$$

Ansatz: $x = A\cos\left(\frac{\alpha_1-\alpha_2}{2}+\omega t\right)$

$$A\left[c-\frac{P+2Q}{g}\omega^2\right] = \frac{2Qr\omega^2}{g}\sin\frac{\alpha_1+\alpha_2}{2}$$

$$\underline{\underline{A = \frac{2Qr}{\frac{cg}{\omega^2}-(P+2Q)}\sin\frac{\alpha_1+\alpha_2}{2}}}$$

Lösung 1280



$l \gg r$: $\sin\psi = \frac{r}{l}\sin\omega t$

$$\cos\psi = 1 - \frac{r^2}{2l^2}\sin^2\omega t$$

$$= 1 - \frac{r^2}{4l^2}(1-\cos 2\omega t)$$

$$x = r\cos\omega t + l\cos\psi$$

$$x = \left(1-\frac{r^2}{4l^2}\right) + r\cos\omega t + \frac{r^2}{4l}\cos 2\omega t$$

$$\left(\frac{Q+G}{g}\right)\ddot\xi + S\cdot\lambda\xi + \frac{P}{g}\ddot x = 0$$

Ansatz: $\xi = A\cos\omega t + B\cos 2\omega t$. Der Koeffizientenvergleich liefert:

$$A\left(-\frac{Q+G}{g}\,\omega^2 + S\lambda\right) = \frac{Pr}{g}\,\omega^2$$

$$B\left(-\frac{Q+G}{g}\,4\omega^2 + S\lambda\right) = \frac{Pr^2}{gl}\,\omega^2$$

mit $k = \sqrt{\frac{\lambda S g}{Q+G}}$ wird:

$$\underline{\underline{\xi = \frac{Pr\omega^2}{(Q+G)(k^2-\omega^2)}\cos\omega t + \frac{r}{l}\,\frac{Pr\omega^2}{(Q+G)(k^2-4\omega^2)}\cos 2\omega t}}$$

Lösung 1281

Aus 1280 folgt mit $\frac{r}{l} \to 0$

$$\xi_{\max} = A = -\frac{Pr\omega^2}{(Q+G)\,(k^2-\omega^2)} \quad \text{(überkritisch)} \quad k^2 = \frac{S\lambda g}{Q+G}$$

$$A = -\frac{Pr\omega^2}{S\lambda g - (Q+G)\,\omega^2}\,; \qquad G = \frac{S\lambda g}{\omega^2} + \frac{Pr}{A} - Q$$

$Q = 10$ t; $r = 30$ cm

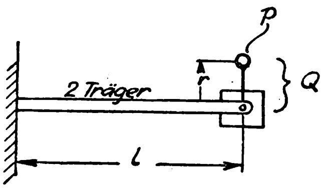
$\omega = 8\pi$ 1/sek; $A = 0{,}025$ cm

$S\lambda = 50$ t/cm; $P = 0{,}25$ t

$$G = \left(\frac{50\cdot 981}{64\pi^2} + \frac{7{,}5}{0{,}025} - 10\right)\text{t}$$

$$\underline{\underline{G = 366{,}6\,\text{t}}}$$

Lösung 1282



$$\eta = \frac{1}{2}\,\frac{Pl^3}{3EJ} = \frac{P}{c}\,; \quad c = \frac{6EJ}{l^3}$$

$$\frac{Q}{g}\,\ddot{x} + cx = \frac{pr\omega^2}{g}\sin\omega t$$

Ansatz: $x = \pm A\sin\omega t$

$$A = \frac{\pm\frac{Pr}{g}\,\omega^2}{c - \frac{Q}{g}\,\omega^2}$$

$$c = \frac{Q}{g}\,\omega^2 \pm \frac{pr\omega^2}{Ag}\,; \quad \frac{1}{J} = \frac{6E}{l^3}\,\frac{g}{Q\omega^2 \pm \frac{pr\omega^2}{A}}$$

$E = 2\cdot 10^6$ kg/cm²

$l = 150$ cm

$Q = 1200$ kg $\qquad J = 7{,}17\,(1200 \pm 20)$

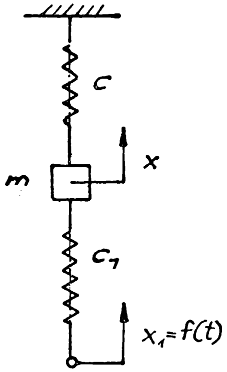
$p = 200$ kg

$r = 0{,}005$ cm $\qquad \underline{\underline{J_1 = 8740\ \text{cm}^4}}$

$\omega = 50\,\pi$ 1/sek

$A = 0{,}05$ cm $\qquad \underline{\underline{J_2 = 8480\ \text{cm}^4}}$

Lösung 1283



$$x_1 = a\,(1-\cos\omega t) \quad \text{für} \quad 0 \leqq t \leqq \frac{2\pi}{\omega}$$

$$x_1 = 0 \quad \text{für} \quad t > \frac{2\pi}{\omega}$$

a) $0 \leqq t \leqq \frac{2\pi}{\omega}$

$$m\ddot{x} + (c+c_1)\,x - c_1 a\,(1-\cos\omega t) = 0$$

Allgemeines Integral: $x_a = A\cos kt + B\sin kt$

mit: $k = \sqrt{\frac{c+c_1}{m}}$

Partikuläres Integral: $x_p = \frac{c_1 a}{c+c_1} - \frac{c_1 a\cos\omega t}{(c+c_1) - m\omega^2}$

$$x_p = \frac{c_1 a}{mk^2} - \frac{c_1 a\cos\omega t}{m\,(k^2-\omega^2)}$$

$$x = x_a + x_p$$

Randbedingungen: $t=0$; $x=0$; $\dot{x}=0$

$$0 = A + \frac{c_1 a}{m}\left(\frac{1}{k^2} - \frac{1}{k^2-\omega^2}\right)$$

$$0 = Bk$$

$$\underline{\underline{x = \frac{c_1 a}{m}\left[\frac{1}{k^2}\,(1-\cos kt) + \frac{1}{k^2-\omega^2}\,(\cos kt - \cos\omega t)\right]}}$$

b) $t > \frac{2\pi}{\omega}$: $\quad m\ddot{x} + (c+c_1)\,x = 0$

$$x = C\cos k\,t + D\cos k\left(t - \frac{2\pi}{\omega}\right)$$

Die Konstanten werden bestimmt aus der Bedingung, daß die Bewegungsgleichungen von a) und b) an ihrer Schranke übereinstimmen müssen.

aus a) $x_{\left(t=\frac{2\pi}{\omega}\right)} = \frac{c_1 a}{m}\left(\frac{1}{k^2} - \frac{1}{k^2-\omega^2}\right)\left(1 - \cos\frac{2\pi k}{\omega}\right)$

$$\dot{x}_{\left(t=\frac{2\pi}{\omega}\right)} = \frac{c_1 a}{m}\left(\frac{1}{k^2} - \frac{1}{k^2-\omega^2}\right)\sin\frac{2\pi k}{\omega}$$

aus b) $x_{\left(t=\frac{2\pi}{\omega}\right)} = C\cos\frac{2\pi k}{\omega} + D$; $\quad \dot{x}_{\left(t=\frac{2\pi}{\omega}\right)} = -k\left(C\sin\frac{2\pi k}{\omega}\right)$

Vergleich: $C = -\frac{c_1 a}{m}\left(\frac{1}{k^2} - \frac{1}{k^2-\omega^2}\right)$; $\quad D = -C$

Somit gilt für b): $\underline{\underline{x = \frac{c_1 a}{m}\left[\frac{1}{k^2-\omega^2} - \frac{1}{k^2}\right]\left[\cos kt - \cos k\left(t - \frac{2\pi}{\omega}\right)\right]}}$

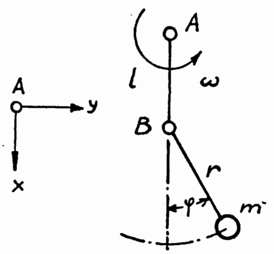
Lösung 1284

$$\Theta\ddot{\psi} + c(\psi - \varphi^*) = 0; \quad \varphi = \omega t + \varphi_0 \sin\omega t = \omega t + \varphi^*$$

Ansatz: $\psi = A\sin\omega t; \quad (-\Theta\omega^2 + c)A = c\varphi_0; \quad A = \dfrac{c\varphi_0}{c - \Theta\omega^2}$

$$\underline{\underline{\psi = \frac{\frac{c}{\Theta}\varphi_0}{\frac{c}{\Theta} - \omega^2}\sin\omega t}}$$

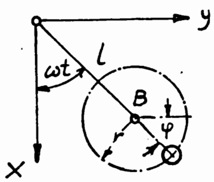
Lösung 1285



$$x = l\cos\omega t + r\cos(\omega t + \varphi)$$
$$y = l\sin\omega t + r\sin(\omega t + \varphi)$$
$$\dot{x} = -l\omega\sin\omega t - r(\omega + \dot{\varphi})\sin(\omega t + \varphi)$$
$$\dot{y} = l\omega\cos\omega t + r(\omega + \dot{\varphi})\cos(\omega t + \varphi)$$
$$v^2 = \dot{x}^2 + \dot{y}^2 = l^2\omega^2 + r^2(\omega + \dot{\varphi})^2 + 2rl\omega(\omega + \dot{\varphi})\cos\varphi$$

$T = \dfrac{m}{2}v^2; \quad U = 0$ (Schwerkraft wird vernachlässigt)

$$L = T - U$$

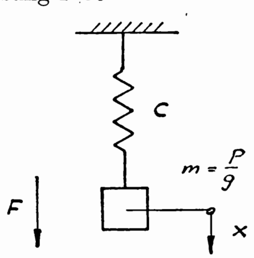


$$\left(\frac{\partial L}{\partial\dot{\varphi}}\right)^{\cdot} - \frac{\partial L}{\partial\varphi} = 0:$$
$$m[r^2\ddot{\varphi} - r(l\omega\sin\varphi\dot{\varphi}) + rl\omega\sin\varphi(\omega + \dot{\varphi})] = 0$$

Für kleine Winkel φ gilt: $r\ddot{\varphi} + l\omega^2\varphi = 0$

Ansatz: $\varphi = A\sin kt \quad \underline{\underline{k = \omega\sqrt{\dfrac{l}{r}}}}$

Lösung 1286



vergl. Aufgabe 1283

a) $\dfrac{P}{g}\ddot{x} + cx = F \quad$ für $0 \leqq t \leqq \tau$

$$x = \frac{F}{c} + A\cos\sqrt{\frac{cg}{P}}\,t + B\sin\sqrt{\frac{cg}{P}}\,t$$

Anfangsbed.: $t = 0$: $x = 0$; $\dot{x} = 0$

$$0 = \frac{F}{c} + A; \qquad 0 = B\sqrt{\frac{cg}{P}}$$

$$\underline{\underline{x = \frac{F}{c}\left(1 - \cos\sqrt{\frac{cg}{P}}\,t\right)}} \quad \text{für } 0 \leqq t \leqq \tau$$

b) $\dfrac{P}{g}\ddot{x} + cx = 0 \quad$ für $t > \tau$: $\quad x = C\cos\sqrt{\dfrac{cg}{P}}\,t + D\cos\sqrt{\dfrac{cg}{P}}\,(t - \tau)$

aus a) $\quad x(\tau) = \dfrac{F}{c}\left(1 - \cos\sqrt{\dfrac{cg}{P}}\,\tau\right); \quad \dot{x}(\tau) = \dfrac{F}{c}\sqrt{\dfrac{cg}{P}}\sin\sqrt{\dfrac{cg}{P}}\,\tau$

aus b) $\quad x(\tau) = C\cos\sqrt{\frac{cg}{P}}\,\tau + D; \quad \dot{x}(\tau) = -C\sin\sqrt{\frac{cg}{P}}\,\tau$

Vergleich von a) und b): $\quad C = -\frac{F}{c}; \quad D = -C$

$$\underline{\underline{x = \frac{F}{c}\left[\cos\sqrt{\frac{cg}{P}}(t-\tau) - \cos\sqrt{\frac{cg}{P}}\,t\right] \quad \text{für } t > \tau}}$$

Lösung 1287

Aus 1286 folgt, daß für $\tau < \frac{T}{2}$ $\dot{x}(\tau)$ positiv ist. Der Maximalwert wird also für $t > \tau$ erreicht; $\quad T = 2\pi\sqrt{\frac{P}{cg}}$

$$\dot{x} = \frac{F}{c}\sqrt{\frac{cg}{P}}\left[\sin\sqrt{\frac{cg}{P}}\,t - \sin\sqrt{\frac{cg}{P}}(t-\tau)\right]$$

$$\dot{x} = 0\colon \quad \sqrt{\frac{cg}{P}}(2t-\tau) = \pi; \quad \sqrt{\frac{2g}{P}}\,t = \frac{\pi}{2} + \sqrt{\frac{cg}{P}}\cdot\frac{\tau}{2}$$

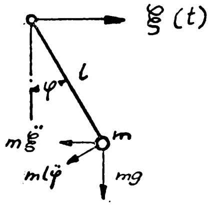
$$x = \frac{F}{c}\left[\cos\left(\frac{\pi}{2} - \sqrt{\frac{cg}{P}}\,\frac{\tau}{2}\right) - \cos\left(\frac{\pi}{2} + \sqrt{\frac{cg}{P}}\cdot\frac{\tau}{2}\right)\right] = 2\,\frac{F}{c}\sin\sqrt{\frac{cg}{P}}\cdot\frac{\tau}{2}$$

a) $\quad \lim\limits_{\tau \to 0} F\cdot\tau = S; \qquad \underline{\underline{x = \sqrt{\frac{g}{Pc}}\cdot S}}$

b) $\quad \tau = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{P}{cg}}; \qquad \underline{\underline{x = \sqrt{2}\cdot\frac{F}{c}}}$

c) $\quad \tau = \frac{T}{2} = \frac{\pi}{\sqrt{\frac{cg}{P}}}; \qquad \underline{\underline{x = 2\,\frac{F}{c}}}$

Lösung 1288



$$ml\ddot{\varphi} + mg\sin\varphi + m\ddot{\xi}\cos\varphi = 0$$

$$\ddot{\varphi} + k^2\sin\varphi + \frac{\ddot{\xi}}{l}\cos\varphi = 0; \quad k^2 = \frac{g}{l}$$

Für kleine φ gilt: $\quad \ddot{\varphi} + k^2\varphi = -\frac{\ddot{\xi}}{l}$

Lösung der Differentialgleichung durch Variation der Konstanten:

$$\varphi = c_1(t)\sin kt + c_2(t)\cos kt$$

$$\dot{\varphi} = \dot{c}_1(t)\sin kt + \dot{c}_2(t)\cos kt + c_1(t)\,k\cos kt - c_2(t)\,k\sin kt;$$

$$\dot{c}_1(t)\sin kt + \dot{c}_2(t)\cos kt = 0$$

$$\ddot{\varphi} = \dot{c}_1(t)\,k\cos kt - \dot{c}_2(t)\,k\sin kt - c_1(t)\,k^2\sin kt - c_2(t)\,k^2\cos kt$$

Somit: $\quad \dot{c}_1(t)\cos kt - \dot{c}_2(t)\sin kt = -\frac{\ddot{\xi}}{kl}; \quad \dot{c}_1 = -\frac{\ddot{\xi}}{kl}\cos kt$

$\qquad\qquad \dot{c}_1(t)\sin kt + \dot{c}_2(t)\cos kt = 0; \qquad \dot{c}_2 = +\frac{\ddot{\xi}}{kl}\sin kt$

$$c_1(t) = -\frac{1}{kl}\int \ddot{\xi}\cos k\tau\, d\tau + c_1 = -\frac{1}{kl}\left[\dot{\xi}\cos kt + k\int \dot{\xi}\sin k\tau\right] + c_1$$

(partielle Integration)

$$c_2(t) = \frac{1}{kl}\int \ddot{\xi}\sin k\tau\, d\tau + c_2 = \frac{1}{kl}\left[\dot{\xi}\sin kt - k\int \dot{\xi}\cos k\tau\, d\tau\right] + c_2$$

$$c_1(t) = \frac{1}{kl}\left[-\dot{\xi}\cos kt - k\,\xi\sin kt + k^2\int \xi\cos k\tau\, d\tau\right] + c_1$$

$$c_2(t) = \frac{1}{kl}\left[\dot{\xi}\sin kt - k\,\xi\cos kt - k^2\int \xi\sin k\tau\, d\tau\right] + c_2$$

Somit: $$\underline{\underline{\varphi = c_1\sin kt + c_2\cos kt - \frac{\xi(t)}{l} + \frac{k}{l}\int_0^t \xi(\tau)\sin k(t-\tau)\, d\tau}}$$

Lösung 1289

Für $0 < t < \tau$ gilt: $\frac{P}{g}\ddot{x} + c\,x = \frac{t}{\tau}F_0$

$$x_1 = C_1\cos kt + C_2\sin kt + \frac{F_0\cdot t}{c\cdot\tau}; \qquad k = \sqrt{\frac{cg}{P}}$$

Anfangsbedingungen: $t = 0$: $x = 0$; $\dot{x} = 0$

$$C_1 = 0$$

$$0 = C_2 k + \frac{F_0}{c\tau}$$

$$x_1 = \frac{F_0}{c\tau}\left[t - \frac{1}{k}\sin kt\right]$$

Für $t > \tau$ gilt: $x_2 = C_3\cos kt + C_4\sin kt + \frac{F_0}{c}$

Für $t = \tau$ müssen beide Gleichungen erfüllt werden, also $x_1 = x_2$; $\dot{x}_1 = \dot{x}_2$:

$$-C_3\sin k\tau + C_4\cos k\tau = \frac{F_0}{c\tau k}(1 - \cos kt)$$

$$C_3\cos k\tau + C_4\sin k\tau = -\frac{F_0}{c}\,\frac{\sin k\tau}{k\tau}$$

Daraus: $C_3 = \frac{F_0}{ck\tau}\cdot(-\sin k\tau)$; $C_4 = \frac{F_0}{ck\tau}(\cos k\tau - 1)$

$$x_2 = \frac{F_0}{ck\tau}\left[-\sin k\tau\cos kt + (\cos k\tau - 1)\sin kt + k\tau\right]$$

$$\underline{\underline{x_2 = x = \frac{F_0}{c}\left[1 - \frac{2}{k\tau}\sin\frac{k\tau}{2}\cos k\left(t - \frac{\tau}{2}\right)\right]}}$$

Die Amplitude beträgt: $\underline{\underline{A = \frac{2F_0}{ck\tau}\cdot\sin\frac{k\tau}{2}}}$

Lösung 1290

$m\ddot{x} + cx = F\,|\sin\omega t|$; für $0 < t < \frac{\pi}{\omega}$ können die Betragstriche wegfallen, da hierfür kein Vorzeichenwechsel bei $\sin\omega t$ eintritt.

$$x_1 = C_1 \cos kt + C_2 \sin kt + \frac{F}{m(k^2 - \omega^2)} \sin\omega t; \quad k = \sqrt{\frac{cg}{P}}$$

Damit eine periodische Bewegung stattfindet, muß gelten:

$$\dot{x}(0) = 0; \qquad x(0) = x\left(\frac{\pi}{\omega}\right)$$

$$C_2 k + \frac{F\omega}{m(k^2 - \omega^2)} = 0; \qquad C_2 = \frac{F\omega}{mk(\omega^2 - k^2)}$$

$$C_1 = C_1 \cos\frac{k\pi}{\omega} + C_2 \sin\frac{k\pi}{\omega}; \quad C_1 = C_2 \cdot \operatorname{ctg}\frac{k\pi}{2\omega}$$

$$\underline{\underline{x = \frac{F\omega}{mk(\omega^2 - k^2)}\left[\sin kt + \operatorname{ctg}\frac{k\pi}{2\omega}\cos kt\right] - \frac{F}{m(\omega^2 - k^2)}\sin\omega t}}$$

Lösung 1291

$$m\ddot{x}_2 + c_2 x_2 = 0; \qquad x_2 = x_0 \cos k_2 t; \qquad k_2 = \sqrt{\frac{c_2}{m}}$$

$$\text{gilt für } t < \frac{\pi}{2k_2}$$

$$m\ddot{x}_1 + c_1 x_1 = 0; \qquad x_1 = a\cos k_1 t + b\sin k_1 t; \qquad k_1 = \sqrt{\frac{c_1}{m}}$$

$$\text{für } t = \frac{\pi}{2k_2} \text{ gilt:} \quad x_2 = 0; \quad \dot{x}_2 = -x_0 \cdot k_2$$

$$x_1 = a\cos\frac{\pi k_1}{2k_2} + b\sin\frac{\pi k_1}{2k_2}$$

$$\dot{x}_1 = -ak_1 \sin\frac{\pi k_1}{2k_2} + bk_1 \cos\frac{\pi k_1}{2k_2}$$

$$x_2 = x_1: \qquad a\cos\frac{\pi}{2}\,\frac{k_1}{k_2} + b\sin\frac{\pi k_1}{2k_2} = 0$$

$$\dot{x}_1 = \dot{x}_2: \quad -a\sin\frac{\pi}{2}\,\frac{k_1}{k_2} + b\cos\frac{\pi k_1}{2k_2} = -x_0\frac{k_2}{k_1}$$

$$a = x_0\frac{k_2}{k_1}\sin\frac{\pi k_1}{2k_2}; \quad b = -x_0\frac{k_2}{k_1}\cos\frac{\pi}{2}\,\frac{k_1}{k_2}$$

$$\underline{\underline{x = -x_0 \cdot \frac{k_2}{k_1}\sin\left(k_1 t - \frac{\pi}{2}\,\frac{k_1}{k_2}\right)}}; \quad \text{für } \frac{\pi}{2k_2} < t < \frac{\pi}{2k_2} + \frac{\pi}{2k_1}$$

$$\underline{\underline{T = \pi\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}}$$

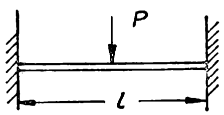
Lösung 1292

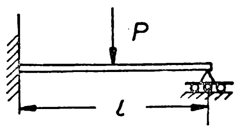
Für $t=\frac{\pi}{2k_2}+\frac{\pi}{2k_1}$ gilt nach Aufgabe 1291: $x=-x_0\cdot\frac{k_2}{k_1}$

Die Amplituden vermindern sich also nach einer geometrischen Reihe mit dem Faktor $\frac{k_2}{k_1}$;

$$\frac{k_2}{k_1}=\sqrt{\frac{c_2}{c_1}}=\frac{7{,}05}{13{,}0}; \qquad \underline{\underline{\frac{c_1}{c_2}}}=\left(\frac{13{,}0}{7{,}05}\right)^2=\underline{\underline{3{,}4}}$$

Lösung 1293





Allgemein: $\omega_{\text{krit}}=\sqrt{\frac{g}{\eta}}$

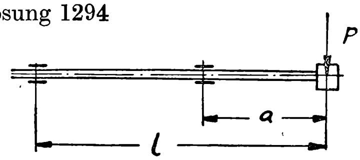
Für die Belastungsfälle gilt:

$$\eta_1=\frac{Pl^3}{192\,EJ}; \qquad \eta_2=\frac{7\,Pl^3}{768\,EJ}$$

Somit:

$$\underline{\underline{\omega_{1\,\text{krit}}=\sqrt{\frac{192\,EJ\cdot g}{Pl^3}}}}; \qquad \underline{\underline{\omega_{2\,\text{krit}}=\sqrt{\frac{768\,EJg}{7\,Pl^3}}}}$$

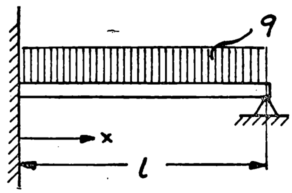
Lösung 1294



Entsprechend Aufgabe 1293 gilt:

$$\underline{\underline{\omega_{\text{krit}}}}=\sqrt{\frac{g}{\eta}}=\underline{\underline{\sqrt{\frac{3\,EJg}{Pla^2}}}}$$

Lösung 1295



Die Differentialgleichung für die Biegeschwingung von Stäben lautet:

$$\eta^{\mathrm{IV}}+\frac{q\ddot{\eta}}{EJg}=0; \qquad \text{Ansatz: } \eta=\bar{\eta}\sin\omega t$$

$$\bar{\eta}^{\mathrm{IV}}-\frac{q\,\omega^2}{EJg}\bar{\eta}=0; \qquad \frac{q\,\omega^2}{EJg}=\alpha^4$$

$$\bar{\eta}^{\mathrm{IV}}-\alpha^4\bar{\eta}=0$$

Daraus: $\bar{\eta}=c_1\cos\alpha x+c_2\sin\alpha x+c_3\operatorname{Cos}\alpha x+c_4\operatorname{Sin}\alpha x$

Randbedingungen: $x=0$: $\bar{\eta}=0$; $\bar{\eta}'=0$

$x=l$: $\bar{\eta}=0$; $\bar{\eta}''=0$

$$c_1+c_3=0; \qquad c_2+c_4=0$$

$$c_1(\cos\alpha l-\operatorname{Cos}\alpha l)+c_2(\sin\alpha l-\operatorname{Sin}\alpha l)=0$$

$$c_1(\cos\alpha l+\operatorname{Cos}\alpha l)+c_2(\sin\alpha l+\operatorname{Sin}\alpha l)=0$$

Dieses homogene Gleichungssystem hat nur dann eine von Null verschiedene Lösung, wenn seine Koeffizientendeterminante Null ist.

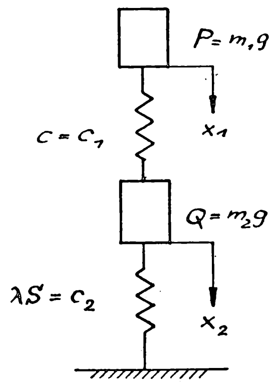
$$2(\mathfrak{Sin}\,\alpha l \cos\alpha l - \mathfrak{Cof}\,\alpha l \sin\alpha l) = 0$$

$$\mathfrak{Tg}\,\alpha l = \operatorname{tg}\alpha l; \qquad \alpha l = \frac{5}{4}\pi$$

$$\omega^2 = \frac{EJ\cdot\alpha^4}{q} = \frac{EJg}{ql^4}\cdot\left(\frac{5}{4}\pi\right)^4; \qquad \underline{\underline{\omega = 15{,}4\sqrt{\frac{EJg}{ql^4}}}}$$

49. Schwingungen mit kleinen Ausschlägen von Systemen mit mehreren Freiheitsgraden

Lösung 1296



Gleichgewichtsbedingungen:

$$m_1\ddot{x}_1 + c_1(x_1 - x_2) = 0$$
$$m_2\ddot{x}_2 - c_1(x_1 - x_2) + c_2 x_2 = 0$$

Der Ansatz $x_1 = A\sin kt$

$x_2 = B\sin kt$ führt zu der Koeffizientendeterminante des homogenen Gleichungssystems:

$$\begin{vmatrix} -k^2 m_1 + c_1 & -c_2 \\ -c_1 & -k^2 m_2 + c_1 + c_2 \end{vmatrix} = 0$$

$$k^4 - k^2\left(\frac{c_1+c_2}{m_2} + \frac{c_1}{m_1}\right) + \frac{c_1 c_2}{m_1 m_2} = 0$$

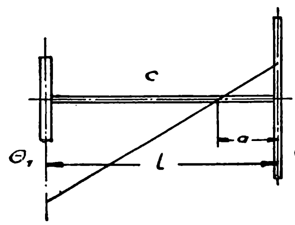
$$k_{1,2} = \sqrt{\frac{1}{2}\left(\frac{c_1+c_2}{m_2} + \frac{c_1}{m_1}\right) \mp \sqrt{\frac{1}{4}\left(\frac{c_1+c_2}{m_2} + \frac{c_1}{m_1}\right)^2 - \frac{c_1 c_2}{m_1 m_2}}}$$

$$m_1 = \frac{P}{g} = 0{,}5\,\mathrm{t}\,\frac{\mathrm{sek}^2}{\mathrm{m}}; \qquad c_1 = c = 5000\,\mathrm{t/m}$$

$$m_2 = \frac{Q}{g} = 10{,}2\,\mathrm{t}\,\frac{\mathrm{sek}^2}{\mathrm{m}}; \qquad c_2 = \lambda\cdot S = 102\,000\,\mathrm{t/m}$$

$$\underline{\underline{k_2 = 111{,}7\ 1/\mathrm{sek}}}; \qquad \underline{\underline{k_1 = 89{,}5\ 1/\mathrm{sek}}}$$

Lösung 1297



$$\Theta_1\ddot{\varphi}_1 + c(\varphi_1 - \varphi_2) = 0$$
$$\Theta_2\ddot{\varphi}_2 - c(\varphi_1 - \varphi_2) = 0$$

Ansatz: $\varphi_1 = A\sin kt$; $\varphi_2 = B\sin kt$

Koeffizientendeterminante:

$$\begin{vmatrix} -\Theta k^2 + c & -c \\ -c & -\Theta_2 k^2 + c \end{vmatrix} = 0$$

$$\Theta_1\Theta_2 k^4 + c^2 - \Theta_1 k^2 c - \Theta_2 k^2 c - c^2 = 0$$

$$k^2 = c\,\frac{\Theta_1+\Theta_2}{\Theta_1\Theta_2}; \qquad T = 2\pi\sqrt{\frac{1}{k^2}}$$

$$c = \frac{G\cdot\pi d^4}{32\cdot l} = 2{,}33\cdot 10^4\,\mathrm{kg/cm}; \qquad \frac{\Theta_1\cdot\Theta_2}{\Theta_1+\Theta_2} = 4{,}83\,\mathrm{kgcm/sek^2}; \qquad \underline{\underline{T}} = \frac{2\pi}{100}\sqrt{\frac{4{,}83}{2{,}33}} = \underline{\underline{0{,}09\,\mathrm{sek}}}$$

$$\underline{\underline{a}} = \frac{\Theta_1}{\Theta_1+\Theta_2}\cdot l = \underline{\underline{50\,\mathrm{mm}}}$$

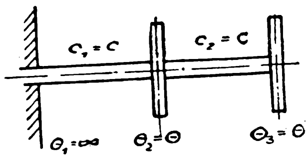
Lösung 1298

Entsprechend Aufgabe 1297 ergibt sich:

$$k=\sqrt{c\,\frac{\Theta_1+\Theta_2}{\Theta_1\Theta_2}};\quad c=\frac{GJ_p}{l}=\frac{G\pi d^4}{32l}=\frac{8{,}8\cdot 10^5\cdot\pi 35^4}{32\cdot 5000}=260\cdot 10^5\ \text{kg/cm}$$

$$\underline{k}=\sqrt{260\cdot 10^5\cdot\frac{459}{390\cdot 69\cdot 10^3}}=\underline{\underline{21{,}4\ 1/\text{sek}}}$$

Lösung 1299



$$\Theta_1\ddot\varphi_1+c_1(\varphi_1-\varphi_2)=0$$
$$\Theta_2\ddot\varphi_2-c_1(\varphi_1-\varphi_2)+c_2(\varphi_2-\varphi_3)=0$$
$$\Theta_3\ddot\varphi_3-c_2(\varphi_2-\varphi_3)=0$$

Ansatz: $\varphi_1=A_1\sin kt$
$\varphi_2=A_2\sin kt$
$\varphi_3=A_3\sin kt$

$$\begin{aligned}
&A_1(-\Theta_1k^2+c_1)-c_2A_2 &&=0\\
&-A_1c_1+A_2(-\Theta_2k^2+c_1+c_2)-A_3c_3 &&=0\\
&-A_2c_2+A_3(-\Theta_3k^2+c_3) &&=0
\end{aligned}$$

Daraus die Koeffizientendeterminante:

$$\begin{vmatrix}\frac{c_1}{\Theta_1}-k^2 & -\frac{c_2}{\Theta_1} & 0\\ -\frac{c_1}{\Theta_2} & \frac{c_1+c_2}{\Theta_2}-k^2 & -\frac{c_3}{\Theta_2}\\ 0 & -\frac{c_2}{\Theta_3} & \frac{c_3}{\Theta_3}-k^2\end{vmatrix}=0$$

Mit $\Theta_1=\infty$; $\Theta_2=\Theta_3=\Theta$; $c_1=c_2=c$ wird:

$$\begin{vmatrix}-k^2 & 0 & 0\\ -\frac{c}{\Theta} & \frac{2c}{\Theta}-k^2 & -\frac{c}{\Theta}\\ 0 & -\frac{c}{\Theta} & \frac{c}{\Theta}-k^2\end{vmatrix}=0$$

Daraus: $-k^2\left(\frac{2c}{\Theta}-k^2\right)\left(\frac{c}{\Theta}-k^2\right)+\frac{c^2}{\Theta^2}k^2=0;\quad k^4-\frac{3c}{\Theta}k^2+\frac{c^2}{\Theta^2}=0$

$$k_{2,1}^2=\frac{c}{\Theta}\left[\frac{3\pm\sqrt5}{2}\right];\quad \underline{\underline{k_1=0{,}62\sqrt{\frac{c}{\Theta}}}};\quad \underline{\underline{k_2=1{,}62\sqrt{\frac{c}{\Theta}}}}$$

Lösung 1300

Entsprechend Aufgabe 1299 gilt mit $\Theta_1=\Theta_2=\Theta_3=\Theta$; $c_1=c_2=c$:

$$\begin{vmatrix}\frac{c}{\Theta}-k^2 & -\frac{c}{\Theta} & 0\\ -\frac{c}{\Theta} & \frac{2c}{\Theta}-k^2 & -\frac{c}{\Theta}\\ 0 & -\frac{c}{\Theta} & \frac{c}{\Theta}-k^2\end{vmatrix}=0$$

$$-k^6+k^4\frac{4c}{\Theta}-k^2\cdot 3\left(\frac{c}{\Theta}\right)^2=0$$

$$k_1^2=\frac{c}{\Theta};\quad k_2^2=\frac{3c}{\Theta}$$

$$\underline{\underline{k_1=\sqrt{\frac{c}{\Theta}}}};\quad \underline{\underline{k_2=\sqrt{\frac{3c}{\Theta}}}}$$

Lösung 1301

Die Lagrangesche Funktion lautet: $L = T - U = \frac{ml^2\dot{\varphi}_1^2}{2} + \frac{ml^2\dot{\varphi}_2^2}{2} - mgl(1-\cos\varphi_1)$

$$- mgl(1-\cos\varphi_2) - \frac{ch^2}{2}(\varphi_1 - \varphi_2)^2$$

$$\left(\frac{\partial L}{\partial \dot{\varphi}_1}\right)^{\cdot} - \frac{\partial L}{\partial \varphi_1} = 0: \quad ml^2\ddot{\varphi}_1 + mgl\varphi_1 + ch^2(\varphi_1 - \varphi_2) = 0 \tag{1}$$

$$\left(\frac{\partial L}{\partial \dot{\varphi}_2}\right)^{\cdot} - \frac{\partial L}{\partial \varphi_2} = 0: \quad ml^2\ddot{\varphi}_2 + mgl\varphi_2 - ch^2(\varphi_1 - \varphi_2) = 0 \tag{2}$$

Gl. (1) + Gl. (2): $ml^2(\varphi_1 + \varphi_2)^{\cdot\cdot} + mgl(\varphi_1 + \varphi_2) = 0$

Gl. (1) — Gl. (2): $ml^2(\varphi_1 - \varphi_2)^{\cdot\cdot} + (mgl + 2ch^2)(\varphi_1 - \varphi_2) = 0$

Ansatz: $\varphi_1 + \varphi_2 = A\sin k_1 t + B\cos k_1 t; \quad k_1 = \sqrt{\frac{g}{l}}$

$\varphi_1 - \varphi_2 = C\sin k_2 t + D\cos k_2 t; \quad k_2 = \sqrt{\frac{g}{l} + \frac{2ch^2}{ml^2}}$

Anfangsbedingungen: $t = 0$: $\dot{\varphi}_1 = \dot{\varphi}_2 = 0; \quad A = 0; \quad C = 0$

$\varphi_1 = \alpha; \quad \varphi_2 = 0; \quad B = \alpha; \quad D = \alpha$

$\varphi_1 + \varphi_2 = \alpha\cos k_1 t;$

$\varphi_1 - \varphi_2 = \alpha\cos k_2 t;$

$$\underline{\underline{\varphi_1}} = \frac{\alpha}{2}(\cos k_1 t + \cos k_2 t) = \underline{\underline{\alpha\cos\frac{k_1+k_2}{2}t\cdot\cos\frac{k_2-k_1}{2}t}}$$

$$\underline{\underline{\varphi_2}} = \frac{\alpha}{2}(\cos k_1 t - \cos k_2 t) = \underline{\underline{\alpha\sin\frac{k_1+k_2}{2}t\cdot\sin\frac{k_2-k_1}{2}t}}$$

Lösung 1302

$$L = T - U = \frac{M\dot{x}^2}{2} + m\frac{(x+l\varphi)^{\cdot 2}}{2} - mgl(1-\cos\varphi) - \frac{cx^2}{2}$$

$\left(\frac{\partial L}{\partial \dot{x}}\right)^{\cdot} - \frac{\partial L}{\partial x} = 0: \quad M\ddot{x} + m(x + l\varphi)^{\cdot\cdot} + cx = 0;$ Ansatz: $x = A\sin kt$, $\varphi = B\sin kt$

$$\left(\frac{\partial L}{\partial \dot{\varphi}}\right)^{\cdot} - \frac{\partial L}{\partial \varphi} = 0: \quad ml(x + l\varphi)^{\cdot\cdot} + mlg\varphi = 0$$

Koeffizientendeterminante: $\begin{vmatrix} -k^2(M+m)+c & -mlk^2 \\ -mlk^2 & -ml^2k^2 + mgl \end{vmatrix} = 0$

$$Mmk^4l^2 - k^2[Mmgl + m^2gl + cml^2] + mglc = 0$$

$$\underline{\underline{k^4 - k^2\left[\frac{g}{l}\,\frac{M+m}{M} + \frac{c}{M}\right] + \frac{gc}{Ml} = 0}}$$

Lösung 1303

$$L = T - U = \frac{\Theta \dot\varphi_1^2}{2} + \frac{\Theta \dot\varphi_2^2}{2} - mga(1-\cos\varphi_1) - mga(1-\cos\varphi_2) - \frac{cb^2}{2}(\varphi_1 - \varphi_2)^2$$

$$\left(\frac{\partial L}{\partial \dot\varphi_1}\right)^{\bullet} - \frac{\partial L}{\partial \varphi_1} = 0: \quad \Theta\ddot\varphi_1 + mga\varphi_1 + cb^2(\varphi_1 - \varphi_2) = 0 \quad (1); \qquad mg = P$$

$$\left(\frac{\partial L}{\partial \dot\varphi_2}\right)^{\bullet} - \frac{\partial L}{\partial \varphi_2} = 0: \quad \Theta\ddot\varphi_2 + mga\varphi_2 - cb^2(\varphi_1 - \varphi_2) = 0 \quad (2); \qquad \Theta = m(a^2 + \varrho^2)$$

Gl. (1) + Gl. (2): $\quad m(a^2+\varrho^2)(\varphi_1+\varphi_2)^{\bullet\bullet} + mga(\varphi_1+\varphi_2) = 0; \quad \underline{\underline{k_1^2 = \frac{ga}{a^2+\varrho^2}}}$

Gl. (1) − Gl. (2): $\quad m(a^2+\varrho^2)(\varphi_1-\varphi_2)^{\bullet\bullet} + mga(\varphi_1-\varphi_2) + 2cb^2(\varphi_1-\varphi_2) = 0$

$$\underline{\underline{k_2^2 = \frac{Pa + 2cb^2}{P(a^2+\varrho^2)} \cdot g}}$$

Aus Gl. (1) ergibt sich: $\quad (-m(a^2+\varrho^2)k^2 + mga + cb^2)\varphi_1 - cb^2\varphi_2 = 0 \quad (3)$

In diese Gleichung k_1^2 eingesetzt ergibt: $\quad \varphi_1 = \varphi_2; \quad$ also: $\quad \frac{A_1^{(1)}}{A_2^{(1)}} = 1$

In Gl. (3) k_2^2 eingesetzt ergibt: $\quad \varphi_1 = -\varphi_2; \quad$ also: $\quad \frac{A_1^{(2)}}{A_2^{(2)}} = -1$

Lösung 1304

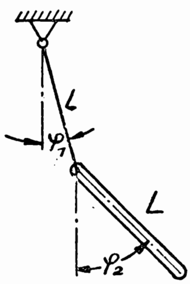
Nach Aufgabe 1303 gilt mit den gegebenen Zahlenwerten:

$$\underline{\underline{k_1}} = \sqrt{\frac{ga}{a^2+\varrho^2}} = \sqrt{\frac{P \cdot a}{\Theta}} = \underline{\underline{4{,}8 \ 1/\mathrm{sek}}}$$

$$\underline{\underline{k_2}} = \sqrt{\frac{Pa + 2cb^2}{P(a^2+\varrho^2)} \cdot g} = \sqrt{\frac{Pa + 2ch^2}{\Theta}} = \underline{\underline{6{,}1 \ 1/\mathrm{sek}}}$$

$$\underline{\underline{\frac{A_1^{(1)}}{A_2^{(1)}} = 1; \quad \frac{A_1^{(2)}}{A_2^{(2)}} = -1}}$$

Lösung 1305



$$T = \frac{m}{2}\left(l\dot\varphi_1 + \frac{L\dot\varphi_2}{2}\right)^2 + \frac{mL^2}{2 \cdot 12} \cdot \dot\varphi_2^2; \qquad L = T - U$$

$$U = -mg\left(l\cos\varphi_1 + \frac{L}{2}\cos\varphi_2\right);$$

$$\left(\frac{\partial L}{\partial \dot\varphi_1}\right)^{\bullet} - \frac{\partial L}{\partial \varphi_1} = 0: \quad ml\left(l\ddot\varphi_1 + \frac{L}{2}\ddot\varphi_2\right) + mgl\sin\varphi_1 = 0$$

$$\left(\frac{\partial L}{\partial \dot\varphi_2}\right)^{\bullet} - \frac{\partial L}{\partial \varphi_2} = 0: \quad m\left[\frac{L}{2}\left(l\ddot\varphi_1 + \frac{L}{2}\ddot\varphi_2\right) + \frac{L^2}{12}\ddot\varphi_2\right] + mgL\frac{\sin\varphi_2}{2} = 0$$

mit $\ddot\varphi_n = -k^2\varphi_n$ und $\sin\varphi_n = \varphi_n$ ergibt sich:

$$(g - lk^2)\varphi_1 - \frac{L}{2}k^2\varphi_2 = 0$$

$$-lk^2\varphi_1 + \left(g - \frac{2}{3}Lk^2\right)\varphi_2 = 0$$

Die Koeffizientendeterminante ergibt: $g^2 - g k^2\left(l + \frac{2}{3}L\right) + k^2\frac{Ll}{6} = 0$

$$\text{oder:}\quad k^4 - 2k^2 g\left(\frac{3}{L} + \frac{2}{l}\right) = -\frac{6g^2}{Ll}$$

$$k^2_{\mathrm{I,\,II}} = g\left[\left(\frac{3}{L} + \frac{2}{l}\right) \mp \sqrt{\frac{9}{L^2} + \frac{6}{Ll} + \frac{4}{l^2}}\right]$$

mit $L = 2l$ wird: $k^2_{\mathrm{I,\,II}} = \frac{g}{l}\left[\frac{7}{2} \mp \frac{\sqrt{37}}{2}\right]$ $\quad \underline{\underline{k_{\mathrm{I}} = 0{,}677\sqrt{\frac{g}{l}}}}$

$$\underline{\underline{k_{\mathrm{II}} = 2{,}558\sqrt{\frac{g}{l}}}}$$

Aus $-lk^2\varphi_1 + \left(g - \frac{2}{3}Lk^2\right)\varphi_2 = 0$ folgt mit k_{I}: $\underline{\underline{\varphi_1 = 0{,}847\,\varphi_2}}$

mit k_{II}: $\underline{\underline{\varphi_1 = -1{,}180\,\varphi_2}}$

Lösung 1306

Aus Aufgabe 1305: $k_{\mathrm{I}}^2 = \frac{g}{L}\left[\left(3 + 2\frac{L}{l}\right) - 3\sqrt{1 + \frac{2}{3}\frac{L}{l} + \frac{4}{9}\frac{L^2}{l^2}}\right]$

$$k_{\mathrm{I}}^2 \approx \frac{g}{L}\left[3 + 2\frac{L}{l} - 3\left(1 + \frac{1}{3}\frac{L}{l} + \frac{1}{6}\frac{L^2}{l^2}\right)\right]$$

$$k_{\mathrm{I}}^2 \approx \frac{g}{l}\left[1 - \frac{1}{2}\frac{L}{l}\right]$$

$$k_{\mathrm{I}} \approx \sqrt{\frac{g}{l}}\left(1 - \frac{1}{4}\frac{L}{l}\right)$$

$$k_{\mathrm{math}} = \sqrt{\frac{g}{l}}; \qquad \underline{\underline{k_{\mathrm{I}} = k_{\mathrm{math}}\left(1 - \frac{1}{4}\frac{L}{l}\right)}}$$

Lösung 1307

Aus Aufgabe 1305: $k_{\mathrm{I}}^2 = \frac{g}{l}\left[\left(2 + \frac{3l}{L}\right) - 2\sqrt{1 + \frac{3}{2}\frac{l}{L} + \frac{9}{4}\frac{l^2}{L^2}}\right]$

$$k_{\mathrm{I}}^2 \approx \frac{g}{l}\left[2 + \frac{3l}{L} - 2\left(1 + \frac{3}{4}\frac{l}{L} + \frac{27}{32}\frac{l^2}{L^2}\right)\right]$$

$$k_{\mathrm{I}}^2 \approx \frac{3g}{2L}\left[1 - \frac{9}{8}\frac{l}{L}\right] \approx \sqrt{\frac{3g}{2L}}\left(1 - \frac{9}{16}\frac{l}{L}\right)$$

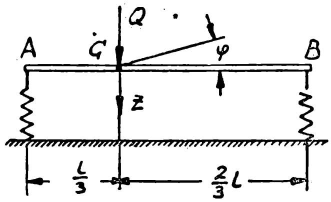
$$k_{\mathrm{Ph}} = \sqrt{\frac{mg\frac{L}{2}}{m\frac{L^2}{3}}} = \sqrt{\frac{3g}{2L}}; \qquad \underline{\underline{k_{\mathrm{I}} = k_{\mathrm{Ph}}\left(1 - \frac{9}{16}\frac{l}{L}\right)}}$$

Lösung 1308

$$i = \frac{l}{5}; \quad \Theta = \frac{Q}{g} \cdot \frac{l^2}{25}$$

$$\frac{Q}{g}\ddot{z} + c\left(z - \frac{2l}{3}\varphi\right) + c\left(z + \frac{l}{3}\varphi\right) = 0$$

$$\frac{Q}{g} \cdot \frac{l^2}{25}\ddot{\varphi} + c\frac{2l}{3}\left(-z + \frac{2l}{3}\varphi\right) + c\left(\frac{l}{3}\varphi + z\right) \cdot \frac{l}{3} = 0$$



Ansatz: $z = A \sin kt; \quad l\varphi = B \sin kt$

$$\left[-\frac{Q}{cg}k^2 + 2\right]A - \frac{1}{3}B = 0 \qquad (1)$$

$$-\frac{1}{3}A + \left[\frac{Q}{cg}\frac{k^2}{25} + \frac{5}{9}\right]B = 0 \qquad (2)$$

Die Koeffizientendeterminante ergibt

$$\frac{1}{25}\left(\frac{Q}{cg}k^2\right)^2 - \frac{143}{225}\frac{Q}{cg}k^2 + 1 = 0$$

$$k_{1,2}^2 = \frac{cg}{Q}\left[\frac{143}{18} \pm \sqrt{\left(\frac{143}{18}\right)^2 - 25}\right]$$

$$k_1 = 1{,}330\sqrt{\frac{cg}{Q}}; \quad k_2 = 3{,}758\sqrt{\frac{cg}{Q}}$$

Aus Gl. (1): $\frac{B}{A} = -\frac{3Q}{cg}k^2 + 6; \quad \frac{B_1}{A_1} = 0{,}69; \quad \frac{B_2}{A_2} = -36{,}15$

Anfangsbedingungen: $t = 0$: $\frac{Q}{g}\dot{z} = S; \quad \Theta\dot{\varphi} = 0$

$$S = \frac{Q}{g}[A_1k_1 + A_2k_2] = \sqrt{\frac{cQ}{g}}[1{,}330A_1 + 3{,}758A_2]$$

$$0 = B_1k_1 + B_2k_2 = \sqrt{\frac{cg}{Q}}[0{,}69 \cdot 1{,}330A_1 - 3{,}758 \cdot 36{,}15A_2]$$

Daraus: $A_1 = 0{,}738\sqrt{\frac{g}{cQ}} \cdot S; \quad A_2 = 0{,}00496\sqrt{\frac{g}{cQ}} \cdot S$

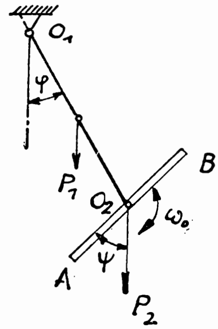
$$B_1 = 0{,}69A_1 = 0{,}509\sqrt{\frac{g}{cQ}} \cdot S; \quad B_2 = -36{,}15A_2 = -0{,}180\sqrt{\frac{g}{cQ}} \cdot S$$

Eingesetzt ergibt:

$$\underline{\underline{z = \sqrt{\frac{g}{cQ}} \cdot S\left[0{,}738\sin 1{,}330\sqrt{\frac{cg}{Q}}t + 0{,}00496\sin 3{,}758\sqrt{\frac{cg}{Q}}t\right]}}$$

$$\underline{\underline{l\varphi = \sqrt{\frac{g}{cQ}} \cdot S\left[0{,}509\sin 1{,}330\sqrt{\frac{cg}{Q}}t - 0{,}180\sin 3{,}758\sqrt{\frac{cg}{Q}}t\right]}}$$

Lösung 1309



$$\underline{\underline{\psi = \omega_0 t}}$$

Der Stab AB wirkt nur mit seiner Masse

$$\Theta_{0_1} = \frac{P_1}{g}\frac{(2a_1)^2}{3} + \frac{P_2}{g}(2a_1)^2$$

$$mgs = P_1 \cdot a_1 + P_2 \cdot 2a_1$$

$$\underline{\underline{\varphi = \varphi_0 \cdot \cos kt}}; \quad k = \sqrt{\frac{mgs}{\Theta_{0_1}}}$$

$$\underline{\underline{k = \sqrt{\frac{3}{4}\frac{g}{a_1}\frac{1+\frac{2P_2}{P_1}}{1+\frac{3P_2}{P_1}}}}}$$

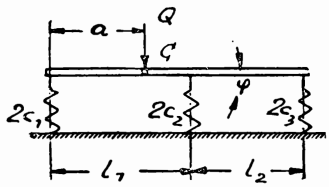
Lösung 1310

$$l_1 = l_2 = l; \quad \Theta\ddot{\varphi} + 2l^2(c_1 + c_2 + c_3)\varphi = 0$$

Ansatz: $\underline{\underline{\varphi = A\sin(kt + \alpha)}}$; $\quad k = \sqrt{\frac{2l^2(c_1 + c_2 + c_3)}{\Theta}}$

$$\underline{\underline{k = \sqrt{\frac{2{,}5^2 \cdot 418}{6}} = 20{,}88\ 1/\text{sek}}}$$

Lösung 1311



$\Sigma P_z = 0$:

$$\frac{Q}{2g}\ddot{z} + (c_1 + c_2 + c_3)z$$
$$- [c_1 a + c_2(a - l_1) + c_3(a - l_1 - l_2)]\varphi = 0$$

$\Sigma M_c = 0$:

$$\frac{\Theta\ddot{\varphi}}{2} + [c_1 a^2 + c_2(a - l_1)^2 + c_3(a - l_1 - l_2)^2]\varphi$$
$$- [c_1 a + c_2(a - l_1) + c_3(a - l_1 - l_2)]z = 0$$

Unter Einsetzen der gegebenen Zahlenwerte ergibt sich:

$$\frac{13}{9{,}81}\ddot{z} + 418 \cdot z + 107\varphi = 0 \qquad (1)$$

$$110\ddot{\varphi} + 953\varphi + 107z = 0 \qquad (2)$$

Ansatz: $\varphi = \varphi_0 \sin kt$; $\quad z = A\sin kt$

$$\ddot{\varphi} = -k^2\varphi; \quad \ddot{z} = -k^2 z$$

Koeffizientendeterminante:

$$\begin{vmatrix} 418 - \frac{13}{9{,}81}k^2 & 107 \\ 107 & 943 - 110k^2 \end{vmatrix} = 0$$

Daraus: $382725 - 47230\,k^2 + 145{,}77\,k^4 = 0;\quad k^2 = 162 \mp \sqrt{23618{,}5}$

$$\underline{\underline{k_1 = 2{,}88\ 1/\text{sek}}};\quad \underline{\underline{k_2 = 17{,}76\ 1/\text{sek}}}$$

Aus (2): $-110\,k^2\varphi + 943\,\varphi + 107\,z = 0;\quad \beta = \frac{z}{\varphi} = \frac{-943 + 110\,k^2}{107}$

$$\underline{\underline{\beta_1 = -0{,}263\,\frac{\text{m}}{\text{Bg}}}};\quad \underline{\underline{\beta_2 = 318\,\frac{\text{m}}{\text{Bg}}}}$$

Lösung 1312

$$\frac{Q}{g}\ddot{x} + 2cx = 0;\qquad x = A\sin\left(\sqrt{\frac{2cg}{Q}}\,t + \alpha\right)$$

$$\frac{Q}{g}\varrho^2\ddot{\psi} + 2cl^2\psi = 0;\qquad \psi = B\sin\left(\sqrt{\frac{2cl^2g}{Q\varrho^2}}\,t + \beta\right)$$

Lösung 1313

$$\frac{Q}{g}\ddot{x}_1 + p\left[\frac{x_1}{a} + \frac{x_1 - x_2}{2b}\right] = 0;$$

$$\frac{Q}{g}\ddot{x}_2 + p\left[\frac{x_2}{a} + \frac{x_2 - x_1}{2b}\right] = 0;$$

Daraus:

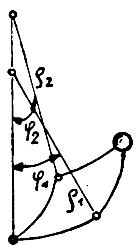
$$\frac{Q}{g}\left(\frac{x_1 + x_2}{2}\right)^{\cdot\cdot} + \frac{p}{a}\,\frac{x_1 + x_2}{2} = 0$$

$$\frac{Q}{g}\left(\frac{x_2 - x_1}{2}\right)^{\cdot\cdot} + p\left(\frac{1}{a} + \frac{1}{b}\right)\left(\frac{x_2 - x_1}{2}\right) = 0$$

Die Hauptkoordinaten sind demnach: $\Theta_1 = \frac{x_1 + x_2}{2};\quad \Theta_2 = \frac{x_2 - x_1}{2}$

Die Frequenzen: $\underline{\underline{k_1 = \sqrt{\frac{pg}{Q\cdot a}}}};\quad \underline{\underline{k_2 = \sqrt{\frac{pg}{Q}\left(\frac{1}{a} + \frac{1}{b}\right)}}}$

Lösung 1314



$$T = \frac{m}{2}\left[(\varrho_1\dot{\varphi}_1)^2 + (\varrho_2\dot{\varphi}_2)^2\right];\quad \cos\varphi = 1 - \frac{\varphi^2}{2}$$

$$U = \frac{mg}{2}\left[\varrho_1\varphi_1^2 + \varrho_2\varphi_2^2\right];\quad L = T - U$$

$$\left(\frac{\partial L}{\partial\dot{\varphi}_1}\right)^{\cdot} - \frac{\partial L}{\partial\varphi_1} = 0:\quad m\varrho_1^2\ddot{\varphi}_1 + mg\varrho_1\varphi_1 = 0$$

$$\underline{\underline{k_1 = \sqrt{\frac{g}{\varrho_1}}}}$$

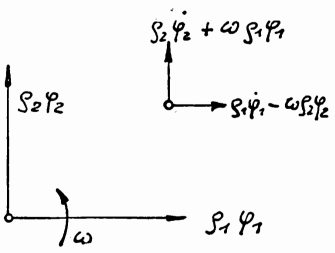
$$\left(\frac{\partial L}{\partial\varphi_2}\right)^{\cdot} - \frac{\partial L}{\partial\varphi_2} = 0:\quad m\varrho_2^2\ddot{\varphi}_2 + mg\varrho_2\varphi_2 = 0$$

$$\underline{\underline{k_2 = \sqrt{\frac{g}{\varrho_2}}}}$$

Lösung 1315

$$T=\frac{m}{2}\left[(\varrho_1\dot\varphi_1-\omega\varrho_2\varphi_2)^2+(\varrho_2\dot\varphi_2+\omega\varrho_1\varphi_1)^2\right]$$

von oben auf die Fläche gesehen:



$$U=\frac{mg}{2}(\varrho_1\varphi_1^2+\varrho_2\varphi_2^2);\quad L=T-U$$

$$\left(\frac{\partial L}{\partial\dot\varphi_1}\right)^{\cdot}-\frac{\partial L}{\partial\varphi_1}=0:\quad m\left[\varrho_1(\varrho_1\ddot\varphi_1-\omega\varrho_2\dot\varphi_2)\right.$$
$$\left.-\omega\varrho_1(\varrho_2\dot\varphi_2+\omega\varrho_1\varphi_1)+g\varrho_1\varphi_1\right]=0$$

$$\left(\frac{\partial L}{\partial\dot\varphi_2}\right)^{\cdot}-\frac{\partial L}{\partial\varphi_2}=0:\quad m\left[\varrho_2(\varrho_2\ddot\varphi_2+\omega\varrho_1\dot\varphi_1)\right.$$
$$\left.+\omega\varrho_2(\varrho_1\dot\varphi_1-\omega\varrho_2\varphi_2)+g\varrho_2\varphi_2\right]=0$$

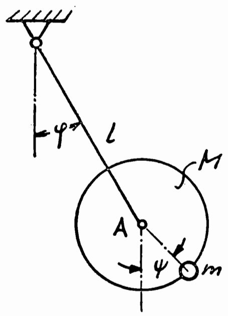
Die Koeffizientendeterminante des hieraus folgenden homogenen Gleichungssystems liefert:

$$[g-\varrho_1(\omega^2+k^2)][g-\varrho_2(\omega^2+k^2)]-4\omega^2k^2\varrho_1\varrho_2=0$$

oder:

$$k^4-k^2\left[2\omega^2+\frac{g}{\varrho_1}+\frac{g}{\varrho_2}\right]+\left(\omega^2-\frac{g}{\varrho_1}\right)\left(\omega^2-\frac{g}{\varrho_2}\right)=0$$

Lösung 1316



$$\sum M_0=0:$$

$$Ml^2\ddot\varphi+ml(l\ddot\varphi+r\ddot\psi)+g(Ml\varphi+ml\varphi)=0$$

$$\sum M_A=0:$$

$$\frac{Mr^2}{2}\ddot\psi+mr(l\ddot\varphi+r\ddot\psi)+gmr\psi=0$$

Ansatz: $\varphi=\varphi_0\cdot\sin kt;\quad \psi=\psi_0\sin kt$

$\ddot\varphi=-k^2\varphi;\qquad \ddot\psi=-k^2\psi$

Somit die Koeffizientendeterminante des homogenen Gleichungssystems:

$$\begin{vmatrix} -k^2[Ml^2+ml^2]+g(Ml+ml) & -k^2mrt \\ -k^2mrl & -k^2\left(\frac{Mr^2}{2}+mr^2\right)+gmr \end{vmatrix}=0$$

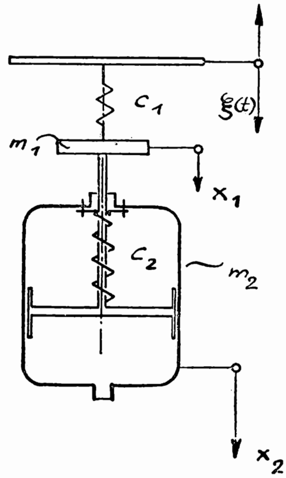
Daraus:

$$k^4\frac{(M^2+3M+m)rl}{2}-k^2g\frac{(M+m)[(M+2m)r+2ml]}{2}+g^2(M+m)m=0$$

oder:

$$\underline{\underline{k^4-\frac{M+m}{M+3m}\left[1+2\frac{m(r+l)}{Mr}\right]\frac{g}{l}k^2+\frac{2m(M+m)}{M(M+3m)}\cdot\frac{g^2}{lr}=0}}$$

Lösung 1317



Kräfte, die an der Masse m_1 angreifen:

$$c_1(x_1 - \xi) + c_2(x_1 - x_2) + \beta(\dot{x}_1 - \dot{x}_2) + m_1\ddot{x}_1 = 0$$

oder:

$$\underline{\underline{m_1\ddot{x}_1 + \beta(\dot{x}_1 - \dot{x}_2) + (c_1 + c_2)x_1 - c_2x_2 = c_1\xi(t)}}$$

Kräfte, die an der Masse m_2 angreifen:

$$c_2(x_1 - x_2) + \beta(\dot{x}_1 - \dot{x}_2) - m_2\ddot{x}_2 = 0$$

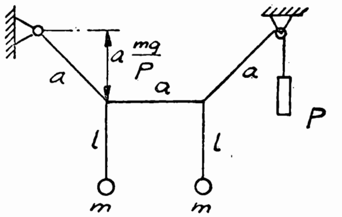
oder:

$$\underline{\underline{m_2\ddot{x}_2 - \beta(\dot{x}_1 - \dot{x}_2) - c_2(x_1 - x_2) = 0}}$$

Lösung 1318

Die Masse der Rolle B ist so groß, daß eine Verschiebung von P während der Schwingung nicht eintritt.

1. Beide Pendel liegen in einer schwingenden Ebene



$$k_1 = \sqrt{\frac{g}{l_1}}$$

$$l_1 = l\left(1 + \frac{a}{l}\frac{mg}{P}\right)$$

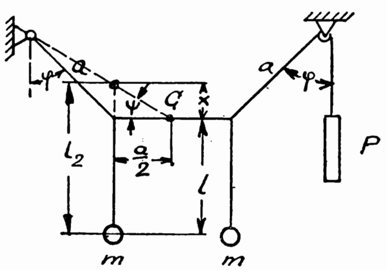
Für kleine Werte von $\frac{a}{l}\frac{mg}{P}$ ergibt sich durch Reihenentwicklung:

$$\frac{1}{1 + \frac{a}{l}\frac{mg}{P}} \approx 1 - \frac{a}{l}\frac{mg}{P}$$

$$\underline{\underline{k_1 = \sqrt{\frac{g}{l}\left(1 - \frac{amg}{lP}\right)}}}$$

2. Beide Pendel schwingen entgegengesetzt:

Aus Symmetriegründen bildet sich bei C ein Schwingungsknoten aus.



$$\operatorname{tg}\psi = \frac{a\cos\varphi}{\frac{1}{2}a + a\sin\varphi}$$

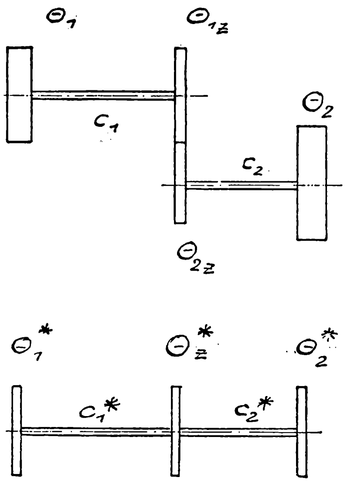
$$x = \frac{a}{2}\operatorname{tg}\psi; \quad \cos\varphi = \frac{mg}{P}$$

$$l_2 = l + x = l + \frac{a\cos\varphi}{1 + 2\sin\varphi}$$

$$l_2 = l + \frac{\frac{amg}{P}}{1 + 2\sqrt{1 - \left(\frac{mg}{P}\right)^2}}$$

$$\frac{mg}{P} \ll 1: \qquad \underline{\underline{k_2 = \sqrt{\frac{g}{l}\left(1 - \frac{amg}{3lP}\right)}}}$$

Lösung 1319



Bildwelle

Für die Bildwelle gilt:

$$\Theta_1^* = \Theta_1; \quad \Theta_z^* = \Theta_{1z} + i^2\Theta_{2z}$$

$$\Theta_2^* = i^2\Theta_2; \quad c_1^* = c_1; \quad c_2^* = i^2 c_2$$

Entsprechend Aufgabe 1299 gilt für die Koeffizientendeterminante:

$$\begin{vmatrix} -\Theta_1^* k^2 + c_1^* & -c_1^* & 0 \\ -c_1^* & -\Theta_z^* k^2 + c_1^* + c_2^* & -c_2^* \\ 0 & -c_2^* & -\Theta_2^* k^2 + c_2^* \end{vmatrix} = 0$$

Daraus:

$$k^4 - k^2\left[\frac{c_1^*}{\Theta_1^*} + \frac{c_2^*}{\Theta_2^*} + \frac{c_1^* + c_2^*}{\Theta_z^*}\right] + \frac{(\Theta_1^* + \Theta_2^* + \Theta_z^*)\, c_1^* c_2^*}{\Theta_1^* \cdot \Theta_2^* \cdot \Theta_z^*} = 0$$

Mit den gegebenen Werten ergibt sich:

$$k^4 - 5657{,}457 \cdot 10^3 k^2 + 3014{,}8 \cdot 10^6 = 0$$

Daraus: $\underline{\underline{k_1 = 23{,}1 \ 1/\text{sek}}}$

$\underline{\underline{k_2 = 2474 \ 1/\text{sek}}}$

Lösung 1320

Aus Aufgabe 1319 folgt mit $\Theta_z^* = 0$:

$$k^2(c_1^* + c_2^*) - \frac{\Theta_1^* + \Theta_2^*}{\Theta_1^* \cdot \Theta_2^*} \cdot c_1^* \cdot c_2^* = 0$$

$$\underline{\underline{k = 23{,}0 \ 1/\text{sek}}}$$

Lösung 1321

Für die vertikale Schwingung ergibt sich:

$$\frac{Q}{g}\ddot{x} + c_z \cdot F \cdot x = 0$$

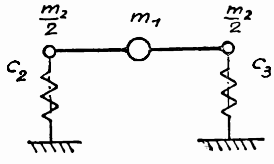
$$\underline{\underline{k_1 = \sqrt{\frac{c_z \cdot F \cdot g}{Q}}}}$$

Für die Winkelschwingung gilt:

$$\Theta_D \ddot{\varphi} + c_z \cdot J_C \varphi = 0$$

$$\underline{\underline{k_2 = \sqrt{\frac{c_z \cdot J_C}{\Theta_D}}}}$$

Lösung 1322



$c_2 = c_3$

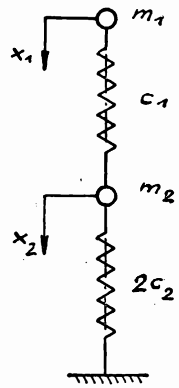
$$m_1 \ddot{x}_1 + c_1 (x_1 - x_2) = 0$$
$$m_2 \ddot{x}_2 + 2 c_2 x_2 - c_1 (x_1 - x_2) = 0$$

Ansatz: $x_1 = A \sin kt; \quad x_2 = B \sin kt$

$$\ddot{x}_1 = -k^2 x_1; \quad \ddot{x}_2 = -k^2 x_2$$

$$\begin{aligned} (-m_1 k^2 + c_1) x_1 - c_2 x_2 &= 0 \\ -c_1 x_1 \qquad (-m_2 k^2 + 2 c_2 + c_1) x_2 &= 0 \end{aligned}$$

Ersatzsystem:



Koeffizientendeterminante:

$$\begin{vmatrix} -m_1 k^2 + c_1 & -c_2 \\ -c_1 & -m k^2 + 2 c_2 + c_1 \end{vmatrix} = 0$$

Daraus:

$$k^4 - \left[\frac{c_1 + 2 c_2}{m_2} + \frac{c_1}{m_1} \right] k^2 + \frac{2 c_1 c_2}{m_1 m_2} = 0$$

$$c_1 = 1{,}15 \cdot 10^6 \text{ kg/cm}; \quad m_1 = \frac{5 \cdot 10^3}{981} \frac{\text{kgsek}^2}{\text{cm}}$$

$$2 c_2 = 9{,}52 \cdot 10^6 \text{ kg/cm}; \quad m_2 = \frac{15{,}3 \cdot 10^3}{981} \frac{\text{kgsek}^2}{\text{cm}}$$

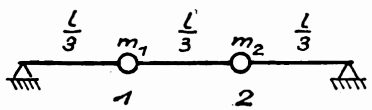
$$k^4 - 0{,}909 \cdot 10^6 k^2 + 0{,}1377 \cdot 10^{12} = 0$$

$$k_2^2 = 0{,}717 \cdot 10^6 \frac{1}{\text{sek}^2}; \quad k_1^2 = 0{,}192 \cdot 10^6 \frac{1}{\text{sek}^2}$$

$$n = \frac{k \cdot 60}{2\pi} = 9{,}55\, k$$

$$\underline{\underline{n_2 = 8080 \text{ U/min}}}; \quad \underline{\underline{n_1 = 4180 \text{ U/min}}}$$

Lösung 1323



$\alpha_{11}; \quad \alpha_{22}; \quad \alpha_{21}; \quad \alpha_{12} =$ Einflußzahlen

Erklärung: Die Kraft eins (1 kg) ruft an der Stelle 1 die Durchbiegung α_{11}, an der Stelle 2 die Durchbiegung α_{12} hervor.

Kräftegleichgewicht an der Stelle 1:

$$\frac{\eta_1}{\alpha_{11}} + m_1 \ddot{\eta}_1 + m_2 \ddot{\eta}_2 \frac{\alpha_{12}}{\alpha_{11}} = 0$$

$$\eta_1 + m_1 \alpha_{11} \ddot{\eta}_1 + m_2 \alpha_{12} \ddot{\eta}_2 = 0$$

Kräftegleichgewicht an der Stelle 2:

$$\frac{\eta_2}{\alpha_{22}} + m_2 \ddot{\eta}_2 + m_1 \ddot{\eta}_1 \frac{\alpha_{21}}{\alpha_{22}} = 0$$

$$\eta_2 + m_2 \alpha_{22} \ddot{\eta}_2 + m_1 \alpha_{21} \ddot{\eta}_1 = 0$$

Für den gegebenen Fall gilt: $\alpha_{11}=\alpha_{22}=\frac{8}{486}\cdot\frac{l^3}{EJ}$; $m_1=\frac{Q}{g}$

$\alpha_{12}=\alpha_{21}=\frac{7}{486}\cdot\frac{l^3}{EJ}$; $m_2=\frac{Q}{g}$

Ansatz: $\eta_1=A_1\sin kt$; $\eta_2=A_2\sin kt$

$$A_1(1-m_1\alpha_{11}k^2)-A_2m_2\alpha_{12}k^2=0;$$

$$-A_1m_1\alpha_{21}k^2+A_2(1-m_2\alpha_{22}k^2)=0;$$

Mit $x=\frac{Q}{g}\,\frac{l^3}{486\,EJ}k^2$ wird hieraus: $A_1(1-8x)-A_2 7x=0$ (1)

$-A_1 7x+A_2(1-8x)=0$ (2)

Aus der Koeffizientendeterminante: $(1-8x)^2-(7x)^2=0$

$$x_1=\frac{1}{15};\quad x_2=1$$

$$k=\sqrt{\frac{EJg}{Ql^3}}\sqrt{486x};\quad \underline{\underline{k_1=5{,}69\sqrt{\frac{EJg}{Ql^3}}}};\quad \underline{\underline{k_2=22{,}04\sqrt{\frac{EJg}{Ql^3}}}}$$

Aus Gl. (2): $\underline{\underline{\frac{A_1^{(1)}}{A_2^{(1)}}=1}}$; $\underline{\underline{\frac{A_1^{(2)}}{A_2^{(2)}}=-1}}$

Lösung 1324

Entsprechend Aufgabe 1323 gilt mit: $m_1=\frac{Q}{g}$; $m_2=\frac{Q}{2g}$:

$A_1(1-8x)-A_2\cdot\frac{7}{2}x=0$ (1)

$-A_1\cdot 7x+A_2(1-4x)=0$ (2)

Die Koeffizientendeterminante liefert: $1-12x+32x^2-\frac{49}{2}x^2=0$

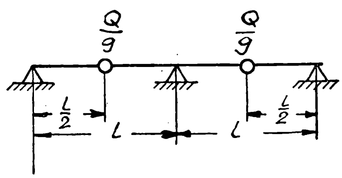
$$x_{1,2}=\frac{4}{5}\mp\sqrt{\frac{38}{75}}$$

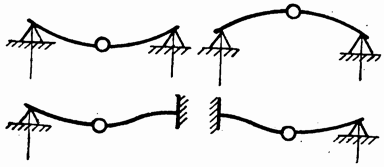
$$x_1=0{,}088;\quad x_2=1{,}512$$

Somit: $\underline{\underline{k_1=6{,}55\sqrt{\frac{EJg}{Ql^3}}}}$; $\underline{\underline{k_2=27{,}2\sqrt{\frac{EJg}{Ql^3}}}}$

Aus Gl. (2): $\underline{\underline{\frac{A_2^{(1)}}{A_1^{(1)}}=0{,}95}}$; $\underline{\underline{\frac{A_2^{(2)}}{A_1^{(2)}}=-2{,}10}}$

Lösung 1325



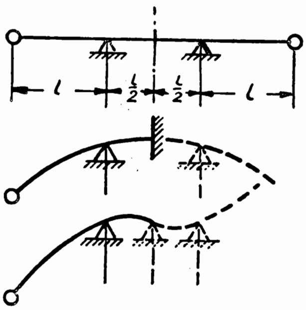


$$(\alpha)_1 = \frac{l^3}{48\,EJ}$$

$$(\alpha)_2 = \frac{7\,l^3}{768\,EJ}$$

$$k = \sqrt{\frac{g}{Q\cdot\alpha}}; \qquad k_1 = 6{,}93\sqrt{\frac{EJ\,g}{Q\,l^3}}; \qquad k_2 = 10{,}46\sqrt{\frac{EJ\,g}{Q\,l^3}}$$

Lösung 1326

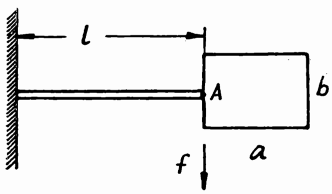


$$(\alpha)_1 = \frac{5}{6}\;\frac{l_3}{EJ}$$

$$(\alpha)_2 = \frac{l^3}{2\,EJ}$$

$$k = \sqrt{\frac{1}{m\alpha}}; \qquad k_1 = \sqrt{\frac{6}{5}\;\frac{EJ}{m\,l^3}}; \qquad k_2 = \sqrt{\frac{2\,EJ}{m\,l^3}}$$

Lösung 1327



$$a = 0{,}2\,l; \qquad b = 0{,}1\,l$$

$$\Theta_A = \frac{m}{12}(a^2 + b^2) + \frac{m\,a^2}{4} = m\,l^2\cdot\frac{17}{1200}$$

$$Q = -m(\ddot{f} + 0{,}1\,l\ddot{\varphi}) \qquad (1)$$

$$M = -(\Theta_A\ddot{\varphi} + m\,0{,}1\,l\ddot{f}) \qquad (2)$$

$$f = p\,Q + s\,M; \qquad \varphi = s\,Q + q\,M \qquad (3)$$

$$\text{Ansatz:}\quad f = A\sin kt; \qquad \varphi = \frac{B}{l}\sin kt$$

Setzt man Gl. (1) und Gl. (2) in die Gln. (3) ein, so erhält man unter Verwendung des Ansatzes:

$$A\,[1 - k^2(mp + m0{,}1\,ls)] - \frac{B}{l}k^2(m\cdot 0{,}1\cdot l\cdot p + \Theta_A\cdot s) = 0$$

$$-A\,k^2(ms + m\cdot 0{,}1\cdot l\cdot q) + \frac{B}{l}k^2[1 - k^2(m\cdot 0{,}1\cdot l\cdot s + \Theta_A q)] = 0$$

Mit $p = \frac{l^3}{3\,EJ}$; $q = \frac{l}{EJ}$; $s = \frac{l^2}{2\,EJ}$; $x = k^2\frac{m\,l^3}{3\,EJ}$

Somit:

$$A\left(1-\frac{23}{20}x\right)-B\,\frac{97}{800}\,x=0 \tag{4}$$

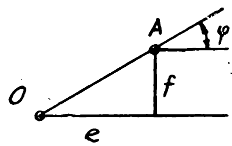
$$-A\,\frac{9}{5}\,x+B\left(1-\frac{77}{400}\,x\right)=0 \tag{5}$$

Die Koeffizientendeterminante ergibt: $1-\frac{537}{400}\,x+\frac{25}{8000}\,x^2=0$

$$x_1=0{,}746;\quad x_2=418{,}854$$

$$\underline{\underline{k_1=0{,}864\sqrt{\frac{3\,EJ}{m\,l^3}}}};\quad \underline{\underline{k_2=20{,}47\sqrt{\frac{3\,EJ}{m\,l^3}}}}$$

$$OA=\frac{f}{\varphi}=\frac{A}{B}\cdot l$$



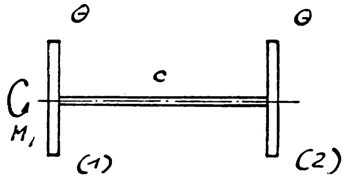
Aus Gl. (5) $OA=l\cdot\dfrac{1-\frac{77}{400}\,x}{\frac{9}{5}\,x}$

$$O_1A=l\cdot\frac{0{,}8564}{1{,}343}=\underline{\underline{0{,}638\,l}}$$

$$O_2A=-l\,\frac{79{,}629}{753{,}94}=\underline{\underline{-0{,}1056\,l}}$$

Lösung 1328

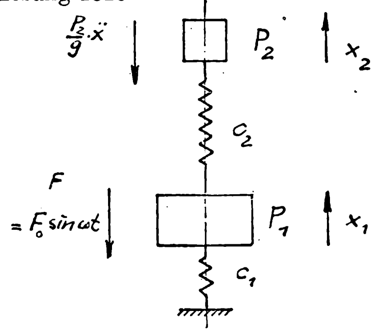
$$\Theta\ddot{\varphi}_1+c(\varphi_1-\varphi_2)=M$$
$$\Theta\ddot{\varphi}_2+c(\varphi_2-\varphi_1)=0$$



Durch Addition bzw. Subtraktion ergibt sich:

$$\Theta(\ddot{\varphi}_1+\ddot{\varphi}_2)=M$$
$$\Theta(\ddot{\varphi}_1-\ddot{\varphi}_2)+2c(\varphi_1-\varphi_2)=M$$
$$\varphi_1+\varphi_2=\frac{M}{2\Theta}\,t^2$$
$$\varphi_1-\varphi_2=\frac{M}{2c}\left(1-\cos\sqrt{\frac{2c}{\Theta}}\,t\right)$$
$$\varphi_1=\frac{M}{4\Theta}\,t^2+\frac{M}{4c}\left(1-\cos\sqrt{\frac{2c}{\Theta}}\,t\right)$$
$$\varphi_2=\frac{M}{4\Theta}\,t^2-\frac{M}{4c}\left(1-\cos\sqrt{\frac{2c}{\Theta}}\,t\right)$$

Lösung 1329



Da $x_1=0$ gefordert wird, ist:

$$F=-P_2\,\frac{\ddot{x}_2}{g}=P_2\,\frac{\omega^2 A\sin\omega t}{g}$$

$$\omega^2=\frac{c_2 g}{P_2};\quad F=F_0\sin\omega t$$

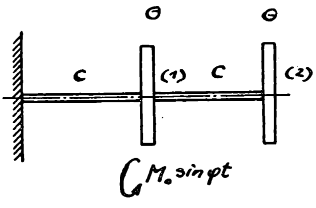
$$P_2=\frac{F_0 g}{A\,\omega^2};\quad c_2=\frac{P_2\omega^2}{g}=\frac{F_0}{A}$$

$$\underline{\underline{P_2}}=\frac{10\cdot 981}{0{,}2\cdot 10^4}=\underline{\underline{4{,}9\text{ t}}}$$

$$c_2=\frac{10\text{ t}}{0{,}2\text{ cm}}=50\text{ t/cm}$$

$$\underline{\underline{c_2=5000\text{ t/m}}}$$

Lösung 1330



$$\Theta\ddot{\varphi}_1 + c(2\varphi_1 - \varphi_2) = M_0 \sin pt$$
$$\Theta\ddot{\varphi}_2 + c(\varphi_2 - \varphi_1) = 0$$

Ansatz: $\varphi_1 = A \sin pt;\quad \varphi_2 = B \sin pt$

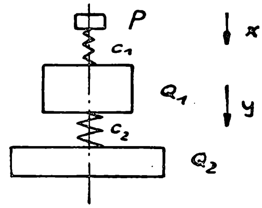
$$(-\Theta p^2 + 2c)A - cB = M_0$$
$$-cA + (-\Theta p^2 + c)B = 0$$

Daraus:

$$[(c - \Theta p^2)(2c - \Theta p^2) - c^2]A = M_0 (c - \Theta p^2)$$
$$A = \frac{M_0 (c - \Theta p^2)}{\Theta^2 \left[p^4 - \frac{3c}{\Theta} p^2 + \frac{c^2}{\Theta^2}\right]} = \frac{M_0 (c - \Theta p^2)}{\Theta^2 (p^2 - k_1^2)(p^2 - k_2^2)}$$
$$B = \frac{cA}{c - \Theta p^2} = \frac{M_0 \cdot c}{\Theta^2 (p^2 - k_1^2)(p^2 - k_2^2)}$$

$$\underline{\underline{\varphi_1 = \frac{M_0 (c - \Theta p^2) \cdot \sin pt}{\Theta^2 (p^2 - k_1^2)(p^2 - k_2^2)}}};\quad \underline{\underline{\varphi_2 = \frac{M_0 \cdot c \cdot \sin pt}{\Theta^2 (p^2 - k_1^2)(p^2 - k_2^2)}}}$$

Lösung 1331



$$\frac{P}{g}\ddot{x} + c_1 (x - y) = \frac{P}{g} r\omega^2 \sin \omega t$$
$$\left(\frac{Q_1 + \frac{1}{3} Q_2}{g}\right)\ddot{y} + c_1 (y - x) + c_2 y = 0$$

Ansatz: $x = A \sin \omega t$

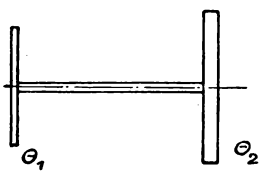
$y = B \sin \omega t$

$$A (c_1 g - P\omega^2) - B c_1 g = P r \omega^2 \qquad \Big| \cdot c_1 g$$
$$-A c_1 g + B \left[c_2 g + c_1 g - \left(Q_1 + \frac{1}{3} Q_2\right)\omega^2\right] = 0 \qquad \Big| \cdot (c_1 g - P\omega^2)$$

$$B\left[c_1 c_2 g^2 - \left\{(c_1 + c_2) P + c_1 \left(Q_1 + \frac{1}{3} Q_2\right)\right\} g\omega^2 + P\left(Q_1 + \frac{1}{3} Q_2\right)\omega^4\right] = P r \omega^2 c_1 g$$

$$\underline{\underline{y = \frac{P r \omega^2 c_1 g \cdot \sin \omega t}{c_1 c_2 g^2 - \left[(c_1 + c_2) P + c_1 \left(Q_1 + \frac{1}{3} Q_2\right)\right] g\omega^2 + P\left(Q_1 + \frac{1}{3} Q_2\right)\omega^4}}}$$

Lösung 1332



$$\Theta_1 = \frac{P_1 R^2}{g};\quad \Theta_2 = \frac{P_2 R^2}{g};\quad c = \frac{GJ}{L}$$
$$k_{\mathrm{kr}} = \sqrt{\frac{c(\Theta_1 + \Theta_2)}{\Theta_1 \cdot \Theta_2}} = \sqrt{\frac{GJ}{L}\,\frac{(P_1 + P_2) g}{P_1 \cdot P_2 \cdot R^2}}$$
$$n_{\mathrm{kr}} = \frac{30}{\pi} \cdot k_{\mathrm{kr}} \ [\mathrm{U/min}]$$
$$n_{\mathrm{kr}} = 476\ \mathrm{U/min}$$

Der Dreizylinder-Zweitakt-Motor überträgt 3 Impulse pro Umdrehung.
Da $3n_1 < n_{\mathrm{kr}} < 3n_2$ wurden die Torsionsschwingungen bis zum Bruch erregt.

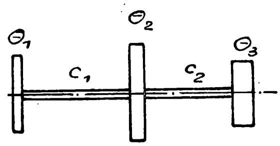
Lösung 1333

$$c=\frac{G\cdot J_p}{l};\quad c_1=\frac{10^{10}}{373}=2{,}68\cdot 10^6\ \mathrm{kgcm}$$

$$c_2=\frac{10^{10}}{239}=4{,}18\cdot 10^6\ \mathrm{kgcm}$$

$$\Theta_1=1{,}78\cdot 10^3\ \mathrm{kgcm\,sek^2}$$

$$\Theta_2=5\,\Theta_1;\quad \Theta_3=50\,\Theta_1$$



Entsprechend Aufgabe 1337 gilt:

$$p^4-\left[\frac{c_1}{\Theta_1}+\frac{c_1+c_2}{\Theta_2}+\frac{c_2}{\Theta_3}\right]p^2+(\Theta_1+\Theta_2+\Theta_3)\frac{c_1c_2}{\Theta_1\Theta_2\Theta_3}=0$$

$$p^4-\frac{4{,}13\cdot 10^3}{1{,}78}p^2+\frac{2{,}51\cdot 10^6}{1{,}78^2}=0$$

$$\underline{\underline{p_1=64{,}3\,\frac{1}{\mathrm{sek}}}};\quad \underline{\underline{p_2=138\,\frac{1}{\mathrm{sek}}}}$$

$$\omega_{\mathrm{kr}}=\frac{2}{3}\,p_{\mathrm{kr}};\quad \omega_{\mathrm{kr}\,1}=42{,}6\,\frac{1}{\mathrm{sek}}$$

$$\omega_{\mathrm{kr}\,2}=92{,}0\,\frac{1}{\mathrm{sek}}$$

Aus der ersten und dritten Schwingungsgleichung ergibt sich:

$$A_1(c_1-\Theta_1p^2)=A_2c_1$$

$$A_3(c_2-\Theta_3p^2)=A_2c_2$$

$$\frac{A_2}{A}=1-\frac{\Theta_1}{c_1}\cdot p^2;\quad \frac{A_2}{A_3}=1-\frac{\Theta_3}{c_2}p^2$$

Somit: $\underline{\underline{\dfrac{A_2^{(1)}}{A_1^{(1)}}=\ 0{,}724;\quad \dfrac{A_2^{(2)}}{A_1^{(2)}}=-0{,}265}}$

$$\underline{\underline{\frac{A_3^{(1)}}{A_1^{(1)}}=-0{,}092;\quad \frac{A_3^{(2)}}{A_1^{(2)}}=\ 0{,}0096}}$$

Lösung 1334

$$T=\frac{\Theta_1}{2}\dot\varphi_1^2+\frac{\Theta_2}{2}\dot\varphi_2^2+\frac{\Theta_3}{2}\dot\varphi_3^2+\frac{m}{2}\left(l^2\dot\varphi_2^2+a^2\dot\varphi_3^2+2al\dot\varphi_2\dot\varphi_3\cos(\varphi_2-\varphi_3)\right)$$

$$U=\frac{c_1}{2}(\varphi_1-\varphi_2)^2;\quad L=T-U$$

Allgemein gilt: $\left(\dfrac{\partial L}{\partial\dot\varphi_k}\right)^{\cdot}-\dfrac{\partial L}{\partial\varphi_k}=Q_k;\quad Q_1=Q_3=0;\quad Q_2=M_0\sin\omega t$

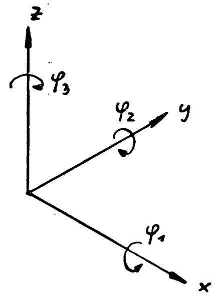
Somit: $\underline{\underline{\Theta_1\ddot\varphi_1+c_1(\varphi_1-\varphi_2)=0}}$

$$\underline{\underline{(\Theta_2+ml^2)\ddot\varphi_2+mal\ddot\varphi_3\cos(\varphi_2-\varphi_3)+mal\dot\varphi_3^2\sin(\varphi_2-\varphi_3)+c_1(\varphi_2-\varphi_1)}}$$

$$\underline{\underline{=M_0\sin\omega t}}$$

$$\underline{\underline{(\Theta_3+ma^2)\ddot\varphi_3+mal\ddot\varphi_2\cos(\varphi_2-\varphi_3)-mal\dot\varphi_2^2\sin(\varphi_2-\varphi_3)=0}}$$

Lösung 1335



$$T=\frac{m}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2)+\frac{\Theta}{2}(\dot{\varphi}_1^2+\dot{\varphi}_2^2+\dot{\varphi}_3^2)$$

$$U=\frac{1}{4}\Big[c_x\{(x-a\varphi_2-a\varphi_3)^2+(x-a\varphi_2+a\varphi_3)^2\} + c_y\{(y+a\varphi_1-a\varphi_3)^2+(y+a\varphi_1+a\varphi_3)^2\} + \frac{c_z}{2}\{(z+a\varphi_1+a\varphi_2)^2+(z+a\varphi_1-a\varphi_2)^2 + (z-a\varphi_1+a\varphi_2)^2+(z-a\varphi_1-a\varphi_2)^2\}\Big]$$

$$U=\frac{1}{2}\left[c_x\{(x-a\varphi_2)^2+(a\varphi_3)^2\}+c_y\{(y+a\varphi_1)^2 + (a\varphi_3)^2\}+c_z\{z^2+(a\varphi_1)^2+(a\varphi_2)^2\}\right]$$

Allgemein gilt: $\left(\frac{\partial L}{\partial \dot{q}}\right)^{\cdot}-\frac{\partial L}{\partial q}=0$

$$\begin{array}{ll} q=x: & m\ddot{x}+c_x(x-a\varphi_2)=0 \\ q=y: & m\ddot{y}+c_y(y+a\varphi_1)=0 \\ q=z: & m\ddot{z}+c_z\cdot z=0 \\ q=\varphi_1: & \Theta\ddot{\varphi}_1+c_ya(y+a\varphi_1)+c_za^2\varphi_1=0 \\ q=\varphi_2: & \Theta\ddot{\varphi}_2+c_xa(a\varphi_2-x)+c_za^2\varphi_2=0 \\ q=\varphi_3: & \Theta\ddot{\varphi}_3+c_xa^2\varphi_3+c_ya^2\varphi_3=0 \end{array}$$

Für $c_y=c_x$ gilt: $m(x+y)^{\cdot\cdot}+c_x\{(x+y)+a(\varphi_1-\varphi_2)\}=0$; $m\ddot{z}+c_zz=0$

$\Theta(\varphi_1-\varphi_2)^{\cdot\cdot}+c_xa\{(x+a)+a(\varphi_1-\varphi_2)\}+c_za^2(\varphi_1-\varphi_2)=0$; $\Theta\ddot{\varphi}_3+2c_xa^2\varphi_3=0$

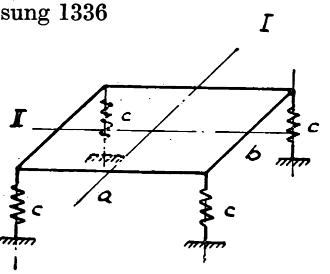
Daraus: $\underline{\underline{k_z=\sqrt{\frac{c_z\cdot g}{P}}}}$; $\underline{\underline{k_{\varphi_3}=\sqrt{\frac{2c_xa^2}{\Theta}}}}$

Die restlichen Frequenzen folgen aus:

$$\begin{vmatrix} c_x-mk^2 & c_xa \\ c_xa & c_za^2+c_xa^2-\Theta k^2 \end{vmatrix}=0$$

$$\underline{\underline{\Theta mk^4-\{m(c_x+c_z)a^2+c_x\Theta\}k^2+c_xc_za^2=0}}$$

Lösung 1336



Vertikale Schwingung:

$$M\ddot{x}+4cx=0;\quad \underline{\underline{k_1=\sqrt{\frac{4c}{M}}}}$$

Schwingung um Achse I:

$$\Theta_{\mathrm{I}}\ddot{\varphi}_{\mathrm{I}}+4c\left(\frac{a}{2}\varphi_{\mathrm{I}}\right)\frac{a}{2}=0$$

$$\Theta_{\mathrm{I}}=M\cdot\frac{a^2}{12}$$

$$\underline{\underline{k_2=\sqrt{\frac{12c}{M}}}}$$

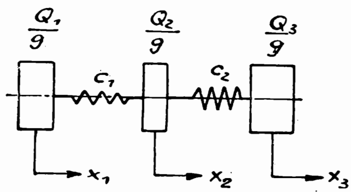
Schwingung um Achse II:

$$\Theta_{II}\,\ddot{\varphi}_{II} + 4c\left(\frac{b}{2}\,\varphi_{II}\right)\frac{b}{2} = 0$$

$$\Theta_{II} = M\,\frac{b^2}{12}$$

$$\underline{\underline{k_3 = \sqrt{\frac{12\,c}{M}}}}$$

Lösung 1337



$$\frac{Q_1}{g}\,\ddot{x}_1 + c_1(x_1 - x_2) = 0$$

$$\frac{Q_2}{g}\,\ddot{x}_2 + c_1(x_2 - x_1) + c_2(x_2 - x_3) = 0$$

$$\frac{Q_3}{g}\,\ddot{x}_3 + c_2(x_3 - x_2) = 0$$

Ansatz: $x_1 = A\cos kt$

$x_2 = B\cos kt$

$x_3 = C\cos kt$

Somit die Koeffizientendeterminante:

$$\begin{vmatrix} c_1 - Q_1 k^2 & -c_1 & 0 \\ -c_1 & c_1 + c_2 - Q_2 k^2 & -c_2 \\ 0 & -c_2 & c_2 - Q_3 k^2 \end{vmatrix} = 0$$

Daraus: $Q_1 Q_2 Q_3 k^6 + \{c_1 Q_2 Q_3 + (c_1 + c_2) Q_1 Q_3 + c_2 Q_1 Q_2\} k^4$

$$+ c_1 c_2 (Q_1 + Q_2 + Q_3) k^2 = 0$$

$$\underline{\underline{k^4 - k^2 g\left(\frac{c_1}{Q_1} + \frac{c_1 + c_2}{Q_2} + \frac{c_2}{Q_3}\right) + \left(\frac{c_1 c_2}{Q_1 Q_2} + \frac{c_1 c_2}{Q_1 Q_3} + \frac{c_1 c_2}{Q_2 Q_3}\right) g^2 = 0}}$$

Lösung 1338

Aus Aufgabe 1337 wird mit $Q_1 = Q_2 = Q_3 = Q$

und $c_1 = c_2 = c$:

$$k^6 - \frac{4cg}{Q}\,k^4 + \frac{3c^2g^2}{Q^2}\,k^2 = 0; \quad \underline{\underline{k_1^2 = 0}}; \quad \underline{\underline{k_2^2 = \frac{cg}{Q}}}; \quad \underline{\underline{k_3^2 = \frac{3cg}{Q}}}$$

Aus dem homogenen Gleichungssystem der Aufgabe 1337

$$A(c_1 - Q_1 k^2) - c_1 B = 0$$

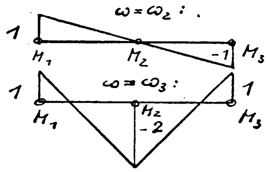
$$-A c_1 + B(c_1 + c_2 - Q_2 k^2) - c_2 C = 0$$

$$-B c_2 + C(c_2 - Q_3 k^2) = 0$$

folgt mit $k = k_1$: $B = C = A$

$k = k_2$: $B = 0$; $C = -A$

$k = k_3$: $B = -2A$; $C = A$



Somit: $x_1 = A_1 + A_2 \cos k_2 t + A_3 \cos k_3 t$

$x_2 = A_1 \qquad - 2A_3 \cos k_3 t$

$x_3 = A_1 - A_2 \cos k_2 t + A_3 \cos k_3 t$

A_1; A_2; A_3 ergeben sich aus den Randbedingungen: $x_1(0) = 0$

$x_2(0) = 0$

$x_3(0) = x_0$

$A_1 + A_2 + A_3 = 0$; $A_2 = -2A_3$

$A_1 - 2A_3 = 0$; $A_1 = 2A_3$

$A_1 - A_2 + A_3 = x_0$; $A_3 = \frac{x_0}{6}$; $A_1 = \frac{x_0}{3}$; $A_2 = -\frac{x_0}{2}$

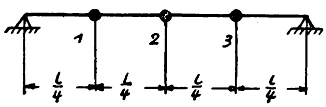
Dies eingesetzt ergibt die Lösung:

$$x_1 = \frac{x_0}{3} - \frac{x_0}{2} \cos k_2 t + \frac{x_0}{6} \cos k_3 t$$

$$x_2 = \frac{x_0}{3} - \frac{x_0}{3} \cos k_3 t$$

$$x_3 = \frac{x_0}{3} + \frac{x_0}{2} \cos k_2 t + \frac{x_0}{6} \cos k_3 t$$

Lösung 1339 (vergleiche Aufgabe 1323)



$$\alpha_{11} = \alpha_{33} = \frac{3}{256} \frac{l^3}{EJ}$$

$$\alpha_{22} = \frac{1}{48} \frac{l^3}{EJ}$$

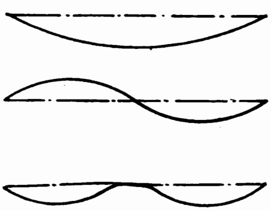
$$\alpha_{12} = \alpha_{21} = \alpha_{23} = \alpha_{32} = \frac{11}{768} \cdot \frac{l^3}{EJ}$$

$$\alpha_{13} = \alpha_{31} = \frac{7}{768} \cdot \frac{l^3}{EJ}$$

$$\eta_k = \sum \alpha_{kl} p_l = \sum \alpha_{kl} m \ddot{\eta}_l = -\sum \alpha_{kl} m \omega^2 \eta_l$$

Koeffizientendeterminante der η_k mit $\frac{m\omega^2 l^3}{768 EJ} = x$

$$\begin{vmatrix} 1 - 9x & -11x & -7x \\ -11x & 1 - 16x & -11x \\ -7x & -11x & 1 - 9x \end{vmatrix} = 0; \quad 1 - 34x + 78x^2 - 28x^3 = 0$$



$$x_1 = 0{,}0316; \quad k_1 = 4{,}93\sqrt{\frac{EJ}{ml^3}}$$

$$1 - 32x + 14x^2 = 0$$

$$x_2 = \frac{1}{2}; \quad k_2 = 19{,}6\sqrt{\frac{EJ}{ml^3}}$$

$$x_3 = 2{,}26; \quad k_3 = 41{,}8\sqrt{\frac{EJ}{ml^3}}$$

Lösung 1340

$$m\ddot{x}_1 + c\,(2x_1 - x_0 \quad - x_2) \quad = 0$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$m\ddot{x}_n + c\,(2x_n - x_{n-1} - x_{n+1}) = 0$$

Ansatz: $x_n = A e^{n\lambda} \sin\omega t$

$$-m\omega^2 + c\,(2 - e^{-\lambda} - e^{+\lambda}) = 0; \quad -m\omega^2 + c\left(-4\,\mathfrak{Sin}^2\frac{\lambda}{2}\right) = 0$$

$$\mathfrak{Sin}^2\frac{\lambda}{2} = -\frac{\omega^2}{4\,\frac{c}{m}}; \quad \pm\,\mathfrak{Sin}\frac{\lambda}{2} = i\,\frac{\omega}{2\sqrt{\frac{c}{m}}}; \quad \pm\frac{\lambda}{2} = x + i\varrho$$

$$\mathfrak{Sin}\,x\cos\varrho + i\,\mathfrak{Cof}\,x\sin\varrho = i\,\frac{\omega}{2\sqrt{\frac{c}{m}}}$$

a) $\omega < 2\sqrt{\frac{c}{m}}$: $\quad x = 0; \quad \sin\varrho = \dfrac{\omega}{2\sqrt{\frac{c}{m}}}$

b) $\omega > 2\sqrt{\frac{c}{m}}$: $\quad \varrho = \dfrac{\pi}{2}; \quad \mathfrak{Cof}\,x = \dfrac{\omega}{2\sqrt{\frac{c}{m}}}$

Im Falle b) enthält λ einen Realteil, daher Abklingen der Schwingungen, also

$$\underline{\underline{0 < \omega < 2\sqrt{\frac{c}{m}}}}$$

Lösung 1341

Entsprechend Aufgabe 1340 gilt mit: $m \to \Theta$

$$x \to \vartheta$$

$$2\varrho \to \mu$$

$$\vartheta_0 = \vartheta\sin\omega t$$

$$\sin\frac{\mu}{2} = \frac{\omega}{2}\sqrt{\frac{\Theta}{c}}; \quad 0 < \omega < 2\sqrt{\frac{c}{\Theta}};$$

$$\vartheta_k = B e^{k\lambda}\sin\omega t; \quad \pm\frac{\lambda}{2} = i\,\frac{\mu}{2}$$

also: $\vartheta_k = (b\cos\mu k + c_1\sin\mu k)\sin\omega t; \quad b = \vartheta$

$$\underline{\underline{\vartheta_k = (\vartheta\cos\mu k + c_1\sin\mu k)\sin\omega t}}$$

Lösung 1342

$$m\ddot{x}_n + c(2x_n - x_{n-1} - x_{n+1}) + c_1 x_n = 0$$

Ansatz: $x_n = A e^{\lambda n} \sin \omega t$

$$-m\omega^2 + c_1 - c\cdot 4\,\mathfrak{Sin}^2\frac{\lambda}{2} = 0; \quad \text{(vgl. Aufgabe 1340)}$$

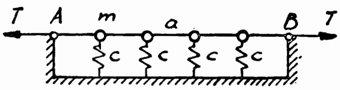
$$\mathfrak{Sin}^2\frac{\lambda}{2} = \underbrace{-\frac{m\omega^2 + c_1}{4c}}_{k};$$ λ ist nur rein imaginär für: $-1 < k < 0$

Also: $-1 < -\dfrac{m\omega^2 + c_1}{4c} < 0$

$$\frac{c_1}{m} < \omega^2 < \frac{c_1 + 4c}{m}$$

$$\sqrt{\frac{c_1}{m}} < \omega < \sqrt{\frac{c_1 + 4c}{m}}$$

Lösung 1343



$$m\ddot{x}_n + c x_n + \frac{T}{a}(2x_n - x_{n-1} - x_{n+1}) = 0$$

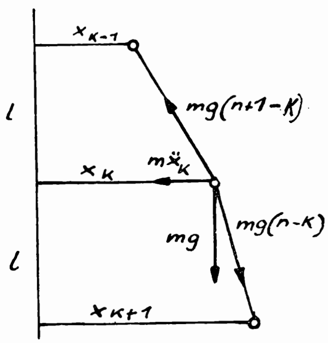
Entsprechend Aufgabe 1342 ergibt sich mit

$$c \to \frac{T}{a}$$

$$c_1 \to c$$

$$\sqrt{\frac{c}{m}} < \omega < \sqrt{\frac{c + 4\dfrac{T}{a}}{m}}$$

Lösung 1344



Oberste Masse: $k = 1$

$$m\ddot{x}_k + mg(n+1-k)\frac{x_k - x_{k-1}}{l} + mg(n-k)\frac{x_k - x_{k+1}}{l} = 0$$

$$m\ddot{x}_k = -\frac{mg}{l}[(2n - 2k + 1)x_k - (n-k+1)x_{k-1} - (n-k)x_{k+1}]$$

$n = 3; \quad x_0 = 0:$

$k = 1: \quad \ddot{x}_1 = -\frac{g}{l}[5x_1 - 2x_2]$

$k = 2: \quad \ddot{x}_2 = -\frac{g}{l}[3x_2 - 2x_1 - x_3]$

$k = 3: \quad \ddot{x}_3 = -\frac{g}{l}[x_3 - x_2]$

Ansatz: $x_k = a_k \sin \omega t$

Koeffizientendeterminante:

$$
\begin{array}{c}
 \\
k=1 \\
k=2 \\
k=3
\end{array}
\begin{array}{ccc}
a_1 & a_2 & a_3 \\
\end{array}
$$

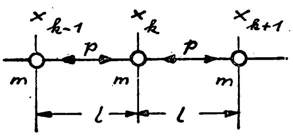
$$
\begin{array}{c|ccc|}
 & a_1 & a_2 & a_3 \\
\hline
k=1 & 5\frac{g}{l}-\omega^2 & -\frac{2g}{l} & 0 \\
k=2 & -\frac{2g}{l} & 3\frac{g}{l}-\omega^2 & -\frac{g}{l} \\
k=3 & 0 & -\frac{g}{l} & \frac{g}{l}-\omega^2
\end{array} = 0
$$

Daraus: $\quad -\omega^6+9\frac{g}{l}\,\omega^4-18\frac{g^2}{l^2}\,\omega^2+6\frac{g^3}{l^3}=0$

$$\underline{\underline{\omega_1=0{,}646\sqrt{\frac{g}{l}}}}\,; \quad \underline{\underline{\omega_2=1{,}515\sqrt{\frac{g}{l}}}}\,; \quad \underline{\underline{\omega_3=2{,}505\sqrt{\frac{g}{l}}}}$$

Lösung 1345

$$m\ddot{x}_k+\frac{P}{l}\,(2x_k-x_{k-1}-x_{k+1})=0$$



Ansatz: $\quad x_k=A\,e^{\lambda k}\sin\omega t$

$$-m\omega^2+\frac{P}{l}\left(-4\,\mathfrak{Sin}^2\frac{\lambda}{2}\right)=0$$

(vgl. Aufgabe 1340)

$$\mathfrak{Sin}^2\frac{\lambda}{2}=-\frac{ml\omega^2}{4P}\,; \qquad \pm\frac{\lambda}{2}=i\varrho$$

$$\sin\varrho=\sqrt{\frac{ml\omega^2}{4P}}$$

$$x_k=(A_1e^{2i\varrho k}+A_2e^{-2i\varrho k})\sin\omega t$$

Randbedingungen: $\quad x_0=x_1;\quad x_n=x_{n+1}$ (freie Enden)

Somit: $\quad A_1(1-e^{2i\varrho})+A_2(1-e^{-2i\varrho})=0$

$$A_1\,(e^{2i\varrho n}-e^{2i\varrho(n+1)})+A_2\,(e^{-2i\varrho n}-e^{-2i\varrho(n+1)})=0$$

Die Koeffizientendeterminante ergibt:

$$2e^{-2i\varrho n}-e^{-2i\varrho(n+1)}-e^{-2i\varrho(n-1)}-2e^{2i\varrho n}+e^{-2i\varrho(n+1)}+e^{-2i\varrho(n-1)}=0$$

oder: $\quad 2\sin 2\varrho n-\sin 2\varrho\,(n+1)-\sin 2\varrho\,(n-1)=0$

$$2\sin 2\varrho n-2\sin 2\varrho n\cos 2\varrho=0;\quad \text{da } \varrho\neq 0, \text{ gilt:}$$

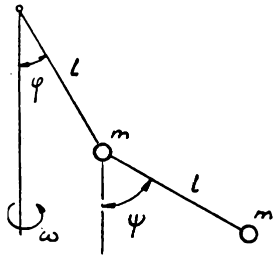
$$\sin 2\varrho n=0$$

$$2\varrho n=s\cdot\pi$$

$$\underline{\underline{\omega=2\sqrt{\frac{P}{ml}}\sin\varrho=2\sqrt{\frac{P}{ml}}\sin\frac{s\pi}{2n}}}\,; \quad \underline{\underline{s=1,\,2,\,3\cdots n-1}}$$

50. Dynamische Stabilität

Lösung 1346



$$T = \frac{ml^2\omega^2}{2}[\varphi^2 + (\varphi+\psi)^2]$$

$$U = -mgl[2\cos\varphi + \cos\psi]$$

$$L = T - U$$

Eine Gleichgewichtslage ist stabil, wenn gilt:

a) $\frac{\partial^2 L}{\partial\varphi^2} < 0;\quad \frac{\partial^2 L}{\partial\psi^2} < 0$

b) $\frac{\partial^2 L}{\partial\varphi^2}\cdot\frac{\partial^2 L}{\partial\psi^2} - \left(\frac{\partial^2 L}{\partial\varphi\,\partial\psi}\right)^2 > 0$

Gleichgewicht herrscht für: $\quad\varphi = 0;\quad \psi = 0$

$$\left(\frac{\partial^2 L}{\partial\varphi^2}\right)_{\varphi=0} = ml^2\omega^2\cdot 2 - 2mgl;\qquad \left(\frac{\partial^2 L}{\partial\psi^2}\right)_{\psi=0} = ml^2\omega^2 - mgl$$

$$\left(\frac{\partial^2 L}{\partial\varphi\,\partial\psi}\right)_{\substack{\varphi=0\\ \psi=0}} = ml^2\omega^2$$

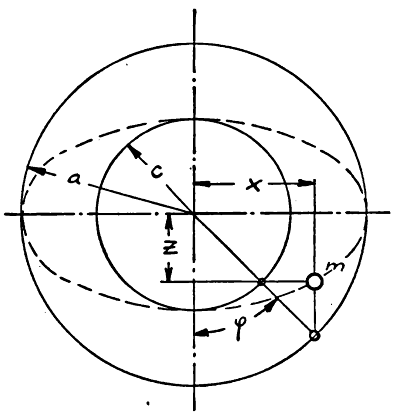
Aus a) folgt: $\quad \omega^2 < \frac{g}{l}$

Aus b) folgt: $\quad 2(l\omega^2 - g)^2 > (l\omega^2)^2;\quad -\sqrt{2}(l\omega^2 - g) > l\omega^2;\quad l\omega^2 - g < 0$

$$\sqrt{2}g > l\omega^2(1+\sqrt{2});$$

$$\underline{\underline{\frac{g}{l\omega^2} > 1 + \frac{1}{\sqrt{2}}}}$$

Lösung 1347



$$x = a\sin\varphi;\quad \dot{x}^2 = a^2\cos^2\varphi\,\dot\varphi^2$$

$$z = c\cos\varphi;\quad \dot{z}^2 = c^2\sin^2\varphi\,\dot\varphi^2$$

$$T = \frac{m}{2}[a^2\omega^2\sin^2\varphi + \dot\varphi^2(a^2\cos^2\varphi + c^2\sin^2\varphi)]$$

$$U = -mgc\cos\varphi;\quad L = T - U$$

Gleichgewicht herrscht bei $\dot\varphi = 0$

$$\left(\frac{\partial L}{\partial\dot\varphi}\right)^{\cdot} - \frac{\partial L}{\partial\varphi}\Bigg|_{\substack{\dot\varphi=0\\ \ddot\varphi=0}}$$

$$= ma^2\omega^2\sin\varphi\cos\varphi - mgc\sin\varphi = 0$$

a) $\omega^2 \leqq \frac{gc}{a^2};\quad \sin\varphi = 0$

b) $\omega^2 > \frac{gc}{a^2};\quad \sin\varphi = 0\quad$ oder

$$\cos\varphi = \frac{gc}{a^2\omega^2}$$

$$\frac{\partial^2 L}{\partial\varphi^2}\Bigg|_{\dot\varphi=0} = ma^2\omega^2(\cos^2\varphi - \sin^2\varphi) - mgc\cos\varphi$$

a) $\varphi = 0$, d. h. $x = 0$; $z = c$

$$\frac{\partial^2 L}{\partial \varphi^2} < 0: \text{ stabil}$$

$\varphi = \pi$, d. h. $x = 0$; $z = -c$

$$\frac{\partial^2 L}{\partial \varphi^2} > 0: \text{ labil}$$

b) $\begin{array}{l} \varphi = 0 \\ \varphi = \pi \end{array}$ d. h. $\begin{array}{ll} x = 0; & z = c \\ x = 0; & z = -c \end{array}$: labil

$$\cos\varphi = \frac{g c}{a^2 \omega^2}: \quad \left.\frac{\partial^2 L}{\partial \varphi^2}\right|_{\dot\varphi = 0} = m(2gc - a^2\omega^2) - m\frac{g^2 c^2}{a^2\omega^2} < 0: \text{ stabil}$$

$$z = c\cos\varphi = \frac{g c^2}{a^2 \omega^2}$$

Lösung 1348

$$L_{(\dot x = 0)} = \frac{m}{2} x^2 \omega^2 - m g \frac{x^2}{2p}$$

$$\left(\frac{\partial L}{\partial \dot x}\right)^{\bullet} - \left.\frac{\partial L}{\partial x}\right|_{\substack{\dot x = 0 \\ \ddot x = 0}} = m\left(\omega^2 - \frac{g}{p}\right) x; \quad \text{also Gleichgewicht für } x = 0$$

bzw. $z = 0$

$$\left.\frac{\partial^2 L}{\partial x^2}\right|_{\dot x = 0} = m\left(\omega^2 - \frac{g}{p}\right); \qquad \frac{\partial^2 L}{\partial x^2} < 0 \text{ für } \omega^2 < \frac{g}{p}: \text{ stabil}$$

$$\frac{\partial^2 L}{\partial x^2} > 0 \text{ für } \omega^2 > \frac{g}{p}: \text{ labil}$$

$$\frac{\partial^2 L}{\partial x^2} = 0 \text{ für } \omega^2 = \frac{g}{p}: \text{ indifferent}$$

Lösung 1349

$$L = \frac{m}{2}(r^2(s)\omega^2 + \dot s^2) - V(s)$$

$$\left(\frac{\partial L}{\partial \dot s}\right)^{\bullet} - \frac{\partial L}{\partial s} = m\ddot s + \left[\frac{dV}{ds} - m\omega^2\left(r\frac{dr}{ds}\right)\right] = 0$$

Entwicklung des Klammerausdruckes in eine Taylor-Reihe:

$$\left[\frac{dV}{ds} - m\omega^2\left(r\frac{dr}{ds}\right)\right] = \left[\frac{dV}{ds} - m\omega^2 r\frac{dr}{ds}\right]_{s = s_0} + s\left[\frac{d^2 V}{ds^2} - m\omega^2\frac{d}{ds}\left(r\frac{dr}{ds}\right)\right]_{s = s_0}$$

Gleichgewicht herrscht für $\ddot s = 0$, also:

$$\underline{\underline{\left(\frac{dV}{ds}\right)_{s = s_0} = \omega^2\left(m r\frac{dr}{ds}\right)_{s = s_0}}}$$

Mit $\ddot s = -k^2 s$ ergibt sich die Eigenfrequenz aus:

$$-mk^2 s + \left[\frac{d^2 V}{ds^2} - m\omega^2\frac{d}{ds}\left(r\frac{dr}{ds}\right)\right]_{s = s_0} \cdot s = 0$$

$$\underline{\underline{k^2 = \frac{1}{m}\left[\frac{d^2 V}{ds^2} - \frac{d}{ds}\left(r\frac{dr}{ds}\right) m\omega^2\right]_{s = s_0}}}$$

Lösung 1350

$$T=\frac{m}{2}(\dot r^2+r^2\dot\varphi^2);\qquad U=\int P\cdot r=\frac{a}{n+1}r^{n+1};\qquad L=T-U$$

$$\frac{\partial L}{\partial\dot\varphi}=\text{const}=mr^2\dot\varphi=mh\qquad\text{(Flächensatz)}$$

$$\frac{\partial L}{\partial r}=+mr\dot\varphi^2-ar^n=m\frac{h^2}{r^3}-ar^n$$

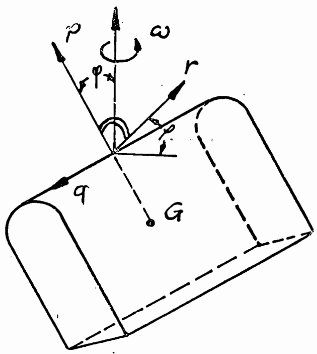
Gleichgewichtslage $\left(\frac{\partial L}{\partial r}\right)_{r=r_0}=0:\quad {r_0}^{n+3}=\frac{mh^2}{a}$

$$\left(\frac{\partial^2 L}{\partial r^2}\right)_{r=r_0}=-\frac{3mh^2}{r_0^4}-an{r_0}^{n-1}=a(-3{r_0}^{n-1}-n{r_0}^{n-1})$$

$$\frac{\partial^2 L}{\partial r^2}<0\quad\text{für}\quad n>-3:\quad\text{stabil}$$

$$\frac{\partial^2 L}{\partial r^2}>0\quad\text{für}\quad n<-3:\quad\text{labil}$$

Lösung 1351



$$p=\omega\cos\varphi$$
$$q=\dot\varphi$$
$$r=\omega\sin\varphi$$

$$L(\dot\varphi=0)=\frac{1}{2}[\omega^2(C\cos^2\varphi+B\sin^2\varphi)]+Mgh\cos\varphi$$

$$\left(\frac{\partial L}{\partial\varphi}\right)_{\dot\varphi=0}=\omega^2(B-C)\sin\varphi\cos\varphi-Mgh\sin\varphi$$

Gleichgewicht für $\frac{\partial L}{\partial\varphi}=0$:

a) $\sin\varphi=0$

b) $\cos\varphi=\frac{Mgh}{(B-C)\,\omega^2}$

$$\left(\frac{\partial^2 L}{\partial\varphi^2}\right)_{\dot\varphi=0}=\omega^2(B-C)(\cos^2\varphi-\sin^2\varphi)-Mgh\cos\varphi$$

a) $\varphi=0$: $\frac{\partial^2 L}{\partial\varphi^2}=\omega^2(B-C)-Mgh$

für $\omega^2>\frac{Mgh}{|B-C|}$ labil

für $\omega^2<\frac{Mgh}{|B-C|}$ stabil mit $B>C$

$\varphi=\pi$: $\frac{\partial^2 L}{\partial\varphi^2}=\omega^2(B-C)+Mgh$

für $C>B$ und $\omega^2>\frac{Mgh}{C-B}$ stabil

für $\omega^2<\frac{Mgh}{|C-B|}$ labil

b) Nur für $\omega^2 > \frac{Mgh}{|B-C|}$; $\varphi = \varphi_0 = \arccos \frac{Mgh}{(B-C)\,\omega^2}$

$$\frac{\partial^2 L}{\partial \varphi^2} = 2\,Mgh - \omega^2 (B-C) - \frac{(Mgh)^2}{\omega^2 (B-C)^2}$$

$$B > C \quad \text{stabil}$$

$$B < C \quad \text{labil}$$

Lösung 1352

Entsprechend Aufgabe 1351 folgt mit $B = A + Mh^2$

$$L = \frac{1}{2}\,[\omega^2 (C\cos^2\varphi + B\sin^2\varphi) + B\dot{\varphi}^2] + Mgh\cos\varphi$$

$$B\ddot{\varphi} + Mgh\sin\varphi - \omega^2 (B-C)\sin\varphi\cos\varphi = 0$$

Die Gleichgewichtslage folgt aus Aufgabe 1351: $\varphi_0 = \arccos \frac{Mgh}{(B-C)\,\omega^2}$

mit $\varphi = \varphi_0 + \vartheta$ wird:

$$B\ddot{\vartheta} + [Mgh\cos\varphi_0 - \omega^2 (B-C)\,(2\cos^2\varphi_0 - 1)]\,\vartheta = 0$$

$$k = \sqrt{\frac{-M^2 g^2 h^2 + (B-C)^2\,\omega^4}{B\,(B-C)\,\omega^2}}$$

$$\underline{\underline{T = \frac{2\pi}{k} = 2\pi\omega\sqrt{\frac{(A+Mh^2)\,(A+Mh^2-C)}{(A+Mh^2-C)^2\omega^4 - M^2 g^2 h^2}}}}$$

Lösung 1353

Momentengleichgewicht um eine Achse durch A parallel zu der Feder mit c_2:

$$\Theta_I\,\omega\dot{\psi} + \Theta_{II}\ddot{\varphi} - Q\cdot l\cdot\varphi + L^2 c_1 \varphi = 0$$

Momentengleichgewicht um eine Achse durch A parallel zu der Feder mit c_1:

$$-\Theta_I\,\omega\dot{\varphi} + \Theta_{II}\ddot{\psi} - Q\cdot l\cdot\psi + L^2 c_2 \psi = 0$$

Ansatz: $\varphi = a_1\cos\alpha t$; $\psi = a_2 \sin\alpha t$

Koeffizientendeterminante:

$$\begin{vmatrix} -\Theta_{II}\alpha^2 + L^2 c_1 - Ql & \Theta_I\,\omega\alpha \\ \Theta_I\,\omega\alpha & -\Theta_{II}\alpha^2 + L^2 c_2 - Ql \end{vmatrix} = 0$$

Daraus: $\Theta_{II}^2\alpha^4 - (\Theta_{II} L^2 c_1 + \Theta_{II} L^2 c_2 - 2\,\Theta_{II} Ql + \Theta_I^2\omega^2)\,\alpha^2$
$+ (L^2 c_1 - Ql)\,(L^2 c_2 - Ql) = 0$

Abkürzung: $A\alpha^4 - B\alpha^2 + C = 0$

Bei stabilem Gleichgewicht müssen α_1^2 und α_2^2 positiv und reell sein, also:

$$(1)\quad \frac{C}{A} > 0; \qquad (2)\quad B^2 - 4AC > 0$$

Aus (1): Bei $L^2c_1 < Ql < L^2c_2$ ist das System bei jeder Winkelgeschwindigkeit labil.

Bei $Ql < c_1L^2$ ist das System bei jeder Winkelgeschwindigkeit stabil.

Aus (2): Bei $Ql > c_2L^2$ ist das System stabil, wenn $\omega > \omega^*$ ist.

$B > 2\sqrt{AC}$

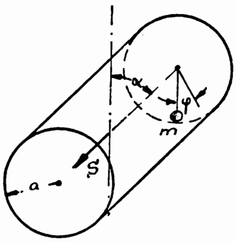
$$\Theta_I^2\omega^2 > 2\Theta_{II}Ql - \Theta_{II}(c_1L^2 + c_2L^2) + 2\Theta_{II}\sqrt{(L^2c_1 - Ql)(L^2c_2 - Ql)}$$

$$\omega^{*2} = \frac{\Theta_{II}}{\Theta_I^2}\left[(Ql - c_1L^2) + 2\sqrt{(Ql - c_1L^2)(Ql - c_2L^2)} + (Ql - c_2L^2)\right]$$

Mit $\Theta_{II} = \frac{Q}{g}\left(\frac{r^2}{4} + l^2\right)$; $\quad\Theta_I = \frac{Q}{g}\,\frac{r^2}{2}$ wird:

$$\omega^* = \sqrt{\frac{gl(4l^2 + r^2)}{r^4}}\left[\sqrt{1 - \frac{c_1L^2}{Ql}} + \sqrt{1 - \frac{c_2L^2}{Ql}}\right]$$

Lösung 1354



$$T = \frac{m}{2}(\dot{s}^2 + a^2\dot{\varphi}^2)$$

$$U = -mg[s\cos\alpha + a\cos\varphi\sin\alpha]$$

$$\left(\frac{\partial L}{\partial\dot{\varphi}}\right)^{\cdot} - \frac{\partial L}{\partial\varphi} = ma^2\ddot{\varphi} + mga\sin\alpha\sin\varphi = 0$$

Gleichgewicht: $\frac{\partial L}{\partial\varphi} = 0$

a) $\varphi = 0$; stabil: $T = 2\pi\sqrt{\frac{a}{g\sin\alpha}}$

b) $\varphi = \pi$ labil

Lösung 1355

$$x = (a + b\cos\vartheta)\cos\psi$$
$$y = (a + b\cos\vartheta)\sin\psi$$
$$z = b\sin\vartheta$$

$$\dot{x} = -b\sin\vartheta\cos\psi\dot{\vartheta} - (a + b\cos\vartheta)\sin\psi\dot{\psi}$$
$$\dot{y} = -b\sin\vartheta\sin\psi\dot{\vartheta} + (a + b\cos\vartheta)\cos\psi\dot{\psi}$$
$$\dot{z} = b\cos\vartheta\dot{\vartheta}$$

$$v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2 = b^2\dot{\vartheta}^2 + (a + b\cos\vartheta)\dot{\psi}^2$$

$$L = \frac{m}{2}v^2 - mgz$$

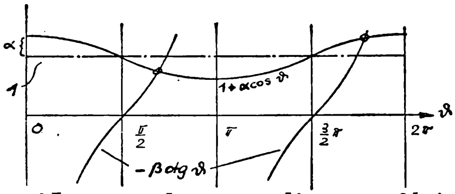
$$\frac{\partial L}{\partial\dot{\psi}} = \text{konst.} = m(a + b\cos\vartheta)^2\dot{\psi} = mh$$

$$\left(\frac{\partial L}{\partial\dot{\vartheta}}\right)^{\cdot} - \frac{\partial L}{\partial\vartheta} = m[b^2\ddot{\vartheta} + b\sin\vartheta(a + b\cos\vartheta)\dot{\psi}^2 + gb\cos\vartheta] = 0$$

$$b\ddot{\vartheta} + \sin\vartheta\cdot\frac{h^2}{(a + b\cos\vartheta)^3} + g\cos\vartheta = 0$$

Gleichgewicht für: $\sin\vartheta_i \frac{\dot\psi^2 (a+b\cos\vartheta_i)^2}{(a+b\cos\vartheta_i)^3} + g\cos\vartheta_i = 0$

oder: $1 + \frac{b}{a}\cos\vartheta_i = -\frac{g \operatorname{ctg}\vartheta_i}{a\dot\psi^2}$



$(1+\alpha\cos\vartheta_i) = -\beta \operatorname{ctg}\vartheta_i$

$\frac{3\pi}{2} < \vartheta_1 < 2\pi$

bzw.: $-\frac{\pi}{2} < \vartheta_1 < 0$ (1)

$\frac{\pi}{2} < \vartheta_2 < \pi$ (2)

$$\frac{\partial^2 L}{\partial\vartheta^2} = -mb\left[\cos\vartheta \frac{h^2}{(a+b\cos\vartheta)^3} + \frac{3b\sin^2\vartheta\, h^2}{(a+b\cos\vartheta)^4} - g\sin\vartheta\right] = 0$$

$\frac{\partial^2 L}{\partial\vartheta^2} < 0$ für $-\frac{\pi}{2} < \vartheta_1 < 0$: stabil; $\frac{\partial^2 L}{\partial\vartheta^2} > 0$ für $\frac{\pi}{2} < \vartheta_2 < \pi$: labil

Lösung 1356

Nach Aufgabe 1232 gilt mit: $\vartheta = 90 + \varepsilon$; $\varphi = \omega t + \gamma$; $\varepsilon, \gamma, \psi$: klein:

Aus Gl. (8): $A\ddot\psi - C\dot\varepsilon\omega = 0$

Aus Gl. (9): $(A + ma^2)\ddot\varepsilon + (ma^2 + C)\dot\psi\omega - mga\varepsilon = 0$

$$(A + ma^2)\ddot\varepsilon + \left(\frac{(ma^2 + C)\,C\omega^2}{A} - mga\right)\cdot\varepsilon = 0$$

Für Stabilität muß gelten:

$$\frac{(ma^2 + C)\,C\omega^2}{A} - mga > 0; \quad \omega^2 > \frac{A\cdot mga}{C\,(ma^2 + C)}$$

mit $C = ma^2$ und $A = \frac{ma^2}{2}$ wird: $\underline{\underline{\omega^2 > \frac{g}{4a}}}$

Lösung 1357

Aus Aufgabe 1356 folgt mit:

$$C = 2A - \frac{2\pi\cdot a^2 + \frac{4}{3}a^2}{2\pi + 4}\, m:$$

$$\underline{\underline{v^2 = a^2\omega^2 > ga\frac{\pi + 2}{4\pi + \frac{16}{3}}}}$$

Lösung 1358

Nach Aufgabe 1232 gilt mit: $\vartheta = 90 + \varepsilon$; $\psi = \omega t + \gamma$; $\varepsilon, \varphi, \gamma$: klein:

Aus Gl. (7): $(ma^2 + C)(\ddot\varphi - \omega\dot\varepsilon) - ma^2\dot\varepsilon\omega = 0$

Aus Gl. (9): $(A + ma^2)\ddot\varepsilon + (ma^2 + C)\,\omega\,(\dot\varphi - \omega\varepsilon) + A\omega^2\varepsilon - mga\varepsilon = 0$

$$(A + ma^2)\ddot\varepsilon + \varepsilon\,[ma^2\omega^2 + A\omega^2 - mga] = 0$$

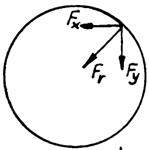
Für Stabilität muß gelten: $ma^2\omega^2 + A\omega^2 - mga > 0$

$$\omega^2 > \frac{mga}{A + ma^2}$$

mit $A = \frac{ma^2}{2}$ wird: $\underline{\underline{\omega^2 > \frac{2}{3}\frac{g}{a}}}$

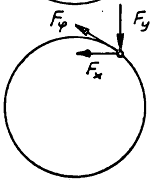
Lösung 1359

Von oben auf die Bewegungsebene gesehen:



$$F_x = \frac{x}{r} F_r$$

$$F_y = \frac{y}{r} F_r$$



$$F_x = \frac{y}{r} F_\varphi$$

$$F_y = -\frac{x}{r} F_\varphi$$

$$m\ddot{x} + c_{11} x + c_{12} y = 0$$

$$m\ddot{y} + c_{11} y - c_{12} x = 0$$

Ansatz: $x = A \sin \omega t$

$y = B \sin \omega t$

Koeffizientendeterminante:

$$\begin{vmatrix} -m\omega^2 + c_{11} & +c_{12} \\ -c_{12} & -m\omega^2 + c_{11} \end{vmatrix} = 0$$

Daraus: $(c_{11} - m\omega^2)^2 = -c_{12}^2$

Es ist somit kein reelles ω möglich und das Gleichgewicht also labil.

Lösung 1360

Entsprechend Aufgabe 1359 gilt:

$$m\ddot{x} + c_{11} x + c_{12} y + \beta \dot{x} = 0$$

$$m\ddot{y} + c_{11} y - c_{12} x + \beta \dot{y} = 0$$

Ansatz: $x = A e^{\lambda t}$; $y = B e^{\lambda t}$

Koeffizientendeterminante: $\begin{vmatrix} m\lambda^2 + \beta\lambda + c_{11} & +c_{12} \\ -c_{12} & m\lambda^2 + \beta\lambda + c_{11} \end{vmatrix} = 0$

Daraus: $m\lambda^2 + \beta\lambda + c_{11} = -i c_{12}$

$$\lambda = -\frac{\beta}{2m} \pm \underbrace{\sqrt{\frac{\beta^2}{4m^2} - \frac{(c_{11} \pm i c_{12})}{m}}}_{\mu \mp i\varrho} \qquad \mu;\quad \varrho > 0$$

Realteil: $\mu^2 - \varrho^2 = \frac{\beta^2}{4m^2} - \frac{c_{11}}{m}$

Imaginärteil: $2\mu\varrho = \frac{c_{12}}{m}$

Daraus: $\mu^4 - \frac{c_{12}^2}{4m^2} = \left(\frac{\beta^2}{4m^2} - \frac{c_{11}}{m}\right)\mu^2$;

μ muß kleiner als $\frac{\beta}{2m}$ sein, damit der Realteil von λ negativ wird.

$$\frac{\beta^2}{4m^2} > \mu^2 = \left(\frac{\beta^2}{8m} - \frac{c_{11}}{2m}\right) + \sqrt{\left(\frac{\beta^2}{8m} - \frac{c_{11}}{2m}\right)^2 + \frac{c_{12}^2}{4m^2}}$$

$$\left(\frac{\beta^2}{8m^2} + \frac{c_{11}}{2m}\right)^2 > \left(\frac{\beta^2}{8m^2} - \frac{c_{11}}{2m}\right)^2 + \frac{c_{12}^2}{4m^2}; \qquad \underline{\underline{\frac{\beta^2 c_{11}}{m} > c_{12}^2}}$$

Lösung 1361

$$m\ddot{x} + c_{11}x - c_{12}y = 0$$
$$m\ddot{y} + c_{22}y - c_{12}x = 0$$

Ansatz: $x = A \sin \omega t$

$y = B \sin \omega t$

Koeffizientendeterminante: $\begin{vmatrix} -m\omega^2 + c_{11} & -c_{12} \\ -c_{21} & -m\omega^2 + c_{22} \end{vmatrix} = 0$

Daraus: $m^2\omega^4 - m\omega^2(c_{11} + c_{22}) + c_{11}c_{22} - c_{12}c_{21} = 0$

$$\omega^2 = \frac{c_{11} + c_{22}}{2m} \pm \sqrt{\frac{(c_{11} + c_{22})^2}{4m^2} - \frac{(c_{11}c_{22} - c_{12}c_{21})}{m^2}}$$

Für Stabilität muß gelten: $\frac{(c_{11} + c_{22})^2}{4m^2} - \frac{(c_{11}c_{22} - c_{12}c_{21})}{m^2} > 0$

Somit: $\underline{\underline{(c_{11} - c_{22})^2 + 4c_{12}c_{21} > 0}}$

Lösung 1362

$$m\ddot{x} + \beta\dot{x} + cx - A(\omega - \omega_0) = 0$$
$$\Theta\dot{\omega} + Bx = 0$$

Ansatz: $x = ae^{\lambda t}$; $\omega - \omega_0 = be^{\lambda t}$

Koeffizientendeterminante: $\begin{vmatrix} m\lambda^2 + \beta\lambda + c & -A \\ B & \Theta\lambda \end{vmatrix} = 0$

$$\Theta m\lambda^3 + \Theta\beta\lambda^2 + c\Theta\lambda + AB = 0$$
$$\lambda^3 + \frac{\beta}{m}\lambda^2 + \frac{c}{m}\lambda + \frac{AB}{\Theta m} = 0$$

Der Realteil von λ muß negativ sein, man kann also schreiben:

$\lambda_1 = -\mu$; $\quad\mu$; $\nu > 0$

$\lambda_2 = -\nu + i\varrho$;

$\lambda_3 = -\nu - i\varrho$; $\quad$ Grenzfälle μ bzw. ν gleich Null;

μ wird positiv für $AB > 0$

ν wird positiv für $\dfrac{\frac{AB}{\Theta m}}{\frac{\beta}{m}} < \dfrac{\frac{c}{m}}{1}$

also: $\underline{\underline{0 < AB < \frac{c\beta\Theta}{m}}}$

Lösung 1363

Die mit ()′ versehenen Größen beziehen sich auf den unteren Kreisel.

$$x_S = h \sin\vartheta' \cos\psi' + c \sin\vartheta \cos\psi$$
$$y_S = h \sin\vartheta' \sin\psi' + c \sin\vartheta \sin\psi$$
$$z_S = h \cos\vartheta' + c \cos\vartheta$$

$$\dot{x}_S = -h \sin\vartheta' \sin\psi' \dot{\psi}' + h \cos\vartheta' \cos\psi' \dot{\vartheta}' - c \sin\vartheta \sin\psi \dot{\psi} + c \cos\vartheta \cos\psi \dot{\vartheta}$$
$$\dot{y}_S = h \sin\vartheta' \cos\psi' \dot{\psi}' + h \cos\vartheta' \sin\psi' \dot{\vartheta}' + c \sin\vartheta \cos\psi \dot{\psi} + c \cos\vartheta \sin\psi \dot{\vartheta}$$
$$\dot{z}_S = -h \sin\vartheta' \dot{\vartheta}' - c \sin\vartheta \dot{\vartheta}$$

$$v_S^2 = h^2(\dot{\vartheta}'^2 + \dot{\psi}'^2 \sin^2\vartheta') + c^2(\dot{\vartheta}^2 + \dot{\psi}^2 \sin^2\vartheta) + 2hc \cdot V$$

$$V = \{\cos\vartheta \cos\vartheta' \cos(\psi' - \psi) + \sin\vartheta \sin\vartheta'\} \dot{\vartheta}' \dot{\vartheta} + \cos\vartheta' \sin\vartheta \sin(\psi' - \psi) \dot{\vartheta}' \dot{\psi}$$
$$- \sin\vartheta' \cos\vartheta \sin(\psi' - \psi) \dot{\psi}' \dot{\vartheta} + \sin\vartheta' \sin\vartheta \cos(\psi' - \psi) \dot{\psi}' \dot{\psi}$$

$$L = \frac{A' + Mh^2}{2}(\dot{\vartheta}'^2 + \dot{\psi}'^2 \sin^2\vartheta') + \frac{A}{2}(\dot{\vartheta}^2 + \dot{\psi}^2 \sin^2\vartheta) + \frac{c'}{2}(\dot{\varphi}' + \dot{\psi}' \cos\vartheta')^2$$
$$+ \frac{c}{2}(\dot{\varphi} + \dot{\psi} \cos\vartheta)^2 + Mhc \cdot V - (M'c' + Mh) g \cos\vartheta' - Mcg \cos\vartheta$$

Aus $\dfrac{\partial L}{\partial \dot{\varphi}'} = \text{konst}$: $\quad \dot{\varphi}' + \dot{\psi}' \cos\vartheta' = \omega'$

Aus $\dfrac{\partial L}{\partial \dot{\varphi}} = \text{konst}$: $\quad \dot{\varphi} + \dot{\psi} \cos\vartheta = \omega$

In der Gleichgewichtslage ist: $\vartheta = \vartheta' = \dot{\vartheta} = \dot{\vartheta}' = 0$

Dafür: $\dfrac{\partial^2 L}{\partial \vartheta'^2} = (A' + Mh^2)\dot{\psi}'^2 - C'\omega'\dot{\psi}' + (M'c' + Mh) g$

$$\frac{\partial^2 L}{\partial \vartheta^2} = A\dot{\psi}^2 - C\omega\dot{\psi} + Mcg$$

$$\frac{\partial^2 L}{\partial \vartheta' \partial \vartheta} = Mhc \cos(\psi' - \psi)\dot{\psi}'\dot{\psi}$$

Setzt man $\dot{\psi} = \dot{\psi}' = -\lambda$ und bildet: $\begin{vmatrix} \dfrac{\partial^2 L}{\partial \vartheta'^2} & \dfrac{\partial^2 L}{\partial \vartheta' \partial \vartheta} \\ \dfrac{\partial^2 L}{\partial \vartheta' \partial \vartheta} & \dfrac{\partial^2 L}{\partial \vartheta^2} \end{vmatrix} = 0$

So erhält man:

$$[AA' + Mh^2(A - Mc^2)]\lambda^4 + [AC'\omega' + C\omega(A' + h^2 M)]\lambda^3$$
$$+ [A(M'c' + Mh) g + (A' + Mh^2) Mcg + CC'\omega\omega']\lambda^2$$
$$+ [C\omega(M'c' + Mh) g + C'\omega' Mcg]\lambda + Mc(M'c' + Mh) g = 0$$